ОБЪЕДИНЕННЫЙ
ИНСТИТУТ
ЯДЕРНЫХ
ИССЛЕДОВАНИЙ
ДУБНА

E2 - 9170

D.I.Kazakov, V.N.Pervushin, M.K.Volkov

ON RENORMALIZATIONS IN NONLINEAR CHIRAL FIELD THEORY

1975

E2 - 9170

D.I.Kazakov, V.N.Pervushin, M.K.Volkov

ON RENORMALIZATIONS IN NONLINEAR CHIRAL FIELD THEORY

Submitted to ТМФ

1. Introduction

Recent investigations [1-4] have shown that the one-loop approximation in quantum chiral field theory is in good agreement with the known experimental data in the description of strong, weak, and electromagnetic interactions of mesons at low energies ($\sqrt{s} \ll$ 1 Gev). hen the one-loop approximation is considered in such theories there arise a number of problems. In particular, these concern the removal of ultraviolet divergences in the non-polynomial theory and correctness of perturbation expansion in the strong coupling constant.

In the present paper these problems are investigated in the nonlinear model of πN -interactions invariant under the SU(2) x SU(2) group with the Lagrangian [5]

$$\mathcal{L} = \bar{\Psi} i \hat{\partial} \Psi - M \bar{\Psi} U\left(\frac{\pi}{F_\pi}\right) \Psi + \frac{F_\pi^2}{4} Sp\left(\partial_\mu U\left(\frac{\pi}{F_\pi}\right) \partial_\mu U^+\left(\frac{\pi}{F_\pi}\right)\right), \quad (1)$$

where $\mathcal{U}(\frac{\pi}{F_\pi})$ is the unitary matrix depending on the pion field π, $F_\pi \cong 92$ Mev is the weak pion decay coupling constant and M is the nucleon mass. In the chiral theory under consideration the role of the strong coupling of $\pi\mathcal{N}$-interaction plays the quantity M/F_π. The phenomenological constant of $\pi\mathcal{N}$-interaction g is connected with M/F_π by the Goldberger-Treiman relation [6]

$$g = g_A \frac{M}{F_\pi} ,$$

where g_A is the axial current constant.

As to the origin of the constant g_A there exist two points of view. First, one can regard the origin of g_A to be the effect of higher orders of perturbation theory in the constant $1/F_\pi$. Second, g_A arises when for the realization of Goldberger-Treiman relation in the tree-approximation, one inserts, into the Lagrangian, the nonminimal in the number of derivatives terms of πN-interaction[7]. We keep here the Lehmann point of view[1] that the origin of $g_A \neq 1$ is connected with the higher orders of perturbation theory.

In the present paper the renormalization procedure for a nonpolynomial Lagrangian is formulated and a number of relations between the renormalization constants (like the Ward identities in chiral-invariant theory) is obtained. For the removal of ultraviolet divergences arising while investigating nonrenormalizable part of the Lagrangian we use the superpropagator method[8,9].

We have considered the strong vertex of $\pi\mathcal{N}$-interaction and the weak vertex of β-decay in the one-loop approximation. When taking into account the renormalizations they lead to the same value of g_A. This follows from the axial current conservation and confirms the chiral invariance of the accepted renormalization procedure.

Finally, the last question we discuss in this paper concerns the estimation of the contributions of higher orders of perturbation theory in the constant $\left(\frac{M}{F_\pi}\right)$. In this connection we consider the contributions of the two-loop approximation to the pion polarization operator. The performed calculations show that in spite of the fact that the expansion parameter is rather large ($M_N^2/(4\pi F_\pi)^2 \cong 0.66$) the contributions of the two-loop approximation are relatively small and are not beyond the accuracy of chiral theory (~ 20-30%). It should be noted, that our calculations have a model character. We do not take into account, for instance, the interactions with all other hyperons which contribution may be very considerable[2,3]. Therefore, the quantitative results obtained, for example, for the value of g_A, should be treated rather carefully.

2. The renormalization scheme. The Ward identities

Consider the nonlinear Lagrangian (1) invariant under the SU(2) x SU(2) group taken in the exponential parametrization.

To calculate divergent diagrams in perturbation theory we shall use the superpropagator method, according to which for the removal of the divergences one should calculate the whole set of diagrams with the fixed number of vertices and arbitrary number of internal lines. In this case our theory will be finite without inserting the counterterms . However, irrespective of problem of ultraviolet divergences the calculations by perturbation theory require the renormalization of the physical quantities. According to this we shall insert, into the Lagrangian, only such counterterms which renormalize the quantities of the initial Lagrangian (1) (fields, masses, charges) and shall not insert the counterterms containing any new structures.

One should, however, take care that the renormalizations do not break the initial chiral symmetry. This requirement puts definite bounds between the renormalization constants that results in the existence of definite chiral Ward identities.

Thus, the insertion of the counterterms into the Lagrangian is equivalent to the renormalization of the quantities in the theory:

$$\mathcal{L}(M,F,\psi,\pi)+\Delta\mathcal{L}(M,F,\psi,\pi)=\mathcal{L}(MZ_M,\, FZ,\, \psi\sqrt{Z_\psi},\, \pi\sqrt{Z_\pi}) \quad (2)$$

The counterterms arising under the renormalization of the Green functions are: $\bar\psi i\hat\partial\psi(Z_\psi-1)$, $M\bar\psi\psi(Z_2-1)$,

$\frac{M}{F}\bar\psi\pi\psi(Z_3-1)$, ..., $\frac{M}{F^n}\bar\psi\pi^n\psi(Z_{n+2}-1)$,

+ counterterms which do not contain the spinor fields.

From eq. (2) we have:

$$\begin{aligned} Z_2 &= Z_M Z_\psi, \\ Z_3 &= Z_M Z_\psi Z_\pi^{1/2} Z^{-1}, \\ &\ldots\ldots \\ Z_{n+2} &= Z_M Z_\psi Z_\pi^{n/2} Z^{-n}. \end{aligned} \quad (3)$$

Now, if we want the counterterms in the Lagrangian to be chiral-invariant and to reproduce the structure of the Lagrangian, we shall demand the equality

$$Z_2 = Z_3 = \ldots = Z_{n+2} = \ldots \quad \text{to be held.} \quad (4)$$

This, in turn, leads to the relation

$$Z = Z_\pi^{1/2}. \quad (5)$$

The obtained Ward identities (3), (4) and (5) follow from eq. (2) and from the chiral symmetry of the Lagrangian. The consideration of pion-pion interactions also leads to a number of identities between the renormalization constants of pion-pion vertices.

Hence, we have 3 independent renormalization constants: Z_ψ, Z_π, Z_M . They are fixed by the normalization of the proper renormalized Green functions (see fig. 1):

$$\Sigma_R(M)=0, \quad \Sigma'_R(M)=0, \quad \Pi'_R(0)=0.^{*)}$$

Fig.1.

*) In the case of $m_\pi=0$, $\Pi_R(0)=0$.

Taking into account the identities (3), (4) and (5) we obtain the unique finite expressions for all the Green functions:

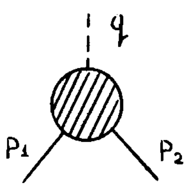
$$G_R = Z_\psi^{-1} G\,,\quad D_R = Z_\pi^{-1} D\,,\quad \Gamma_R = Z_\Gamma \Gamma\,,\quad \Box_R = Z_\Box \Box\,, \tag{6}$$

where Γ and $\Box$ are pion-nucleon and pion-pion vertices, resp. The renormalization constants are determined by the following equations:

$$Z_\psi = 1 + \Sigma'(M)\,,\quad Z_M = 1 - \frac{\Sigma(M)}{M}\,,\quad Z_\pi = 1 + \Pi'(0)\,,$$
$$Z_\Gamma = Z_M Z_\psi = 1 + \Sigma'(M) - \frac{\Sigma(M)}{M}\,,\quad Z_\Box = Z_\pi\,. \tag{7}$$

3. <u>The calculation of the strong vertrex</u>

As an example consider the three-point vertex function (see fig. 2)



$$\Gamma(p_1, p_2, q, p_1^2, p_2^2, q^2) = 1 + \ldots$$

$$\Gamma_R = Z_\Gamma \Gamma$$

Fig. 2.

Let us calculate at first the renormalization constant $Z_\Gamma = Z_M Z_\psi$. In the one-loop approximation the contribution to the Z_Γ comes only from one diagram (Fig. 3)

Fig. 3.

$$\Sigma(p) = -i\frac{3M^2}{(2\pi)^4 F^2}\int \frac{dk\,(M-\hat{k})}{(k^2-M^2+i\epsilon)[(p-k)^2+i\epsilon]}\,. \tag{8}$$

This integral as well as the following ones can be expressed by the same divergent integral J :

$$\Sigma(M) = -\frac{3}{2}M\beta(1+J)\,,\quad \Sigma'(M) = \frac{3}{2}\beta(1+J)\,, \tag{9}$$

where $\beta = \left(\frac{M}{4\pi F}\right)^2 \simeq 0.66$ and

$$J(M) = \frac{i}{\pi^2}\int\frac{dk}{(k^2-2kp+i\epsilon)(k^2+i\epsilon)}\,,\quad p^2 = M^2. \tag{10}$$

The integral $J(M)$ can be calculated by the superpropagator technique (see. Appendix). As a result, we have

$$J^{s.p.}(M) = \left[\ln\left(\frac{M}{4\pi F}\right)^2 + 3C - \frac{5}{3}\right] \cong -0.35\,, \tag{11}$$

where $C = 0.577\ldots$ is the Euler constant. Then, according to eq. (7), we find

$$Z_M = 1 - \frac{\Sigma(M)}{M} = 1 + \frac{3}{2}\beta(1+J)\,,$$
$$Z_\psi = 1 + \Sigma'(M) = 1 + \frac{3}{2}\beta(1+J)\,, \tag{12}$$
$$Z_\Gamma = Z_M Z_\psi = 1 + 3\beta(1+J)\,.$$

Consider now the vertex function Γ. In the given approximation there are 3 vertex diagrams (Fig.4)

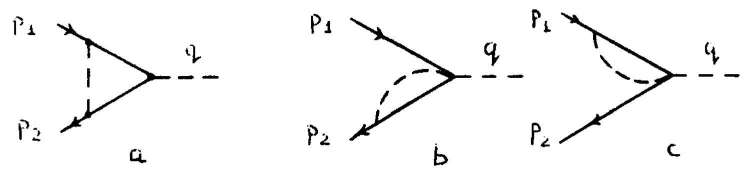


Fig. 4.

Separating the Born term, we come to the following expression for the vertex function

$$\Gamma(p_1, p_2, q, p_1^2, p_2^2, q^2) = 1 + \Gamma^{(1)} + 2\Gamma^{(2)}, \qquad (13)$$

where $\Gamma^{(1)}$ corresponds to the diagram 4a and $\Gamma^{(2)}$ corresponds to 4b and 4c. On the mass shell we have

$$\Gamma^{(1)} = -\frac{iM^2}{(2\pi)^4 F^2}\int\frac{dk}{(k^2-2kp_1+i\epsilon)^2} = -\beta(2+J),$$
$$\Gamma^{(2)} = \frac{2}{3}\,\frac{\Sigma(M)}{M} = -\beta(1+J). \qquad (14)$$

Hence for the renormalized vertex function on the mass shell we find

$$\Gamma_R(M, M, 0, M^2, M^2, 0) = [1+3\beta(1+J)]\times[1-\beta(3+2J)] = 1+\beta J = g_A. \qquad (15)$$

Substituting the value β =0.66 and J =-0.35, obtained by the s.p.-method, for g_A we finally have

$$g_A = 1 - 0.23\ . \qquad (16)$$

4. <u>The calculation of the axial constant of β -decay</u>

To show the consistency of the accepted renormalization

scheme and perturbation theory with the basic requirements of chiral-symmetrical theory, in particular, with the Goldberger-Treiman relation, consider the renormalization of the axial constant in the weak β -decay in our model. For this purpose we shall add, to the initial Lagrangian (1), the term describing the interaction of hadrons with the weak lepton current

$$\mathcal{L}' = \vec{J}_{5\mu}\vec{L}_{5\mu}\ , \qquad (17)$$

where $\vec{J}_{5\mu} = D_\mu\vec{\pi}F_\pi + \frac{1}{2}\bar{\Psi}\vec{\tau}\gamma_\mu\gamma_5\Psi$

and D_μ is a covariant derivative of the form:

$$D_\mu\pi^i = \partial_\mu\pi^i + \left(\delta_{ij} - \frac{\pi^i\pi^j}{\pi^2}\right)\left(\frac{\sin Z}{Z} - 1\right)\partial_\mu\pi^j, \quad Z = \sqrt{\frac{\pi^2}{F^2}} \qquad (18)$$

The relations between the renormalization constants, obtained below, require the insertion of the quite defined counterterms for this interaction, namely:

$$\Delta\mathcal{L}' = \left[D_\mu\vec{\pi}F_\pi(Z_\pi - 1) + \frac{1}{2}\bar{\Psi}\vec{\tau}\gamma_\mu\gamma_5\Psi(Z_\psi - 1)\right]\vec{L}_{5\mu} \qquad (19)$$

Hence, the renormalized axial vertex of β -decay is

$$\Gamma_R^\beta = Z_\psi\Gamma_\beta\ , \qquad (20)$$

where the contribution to Γ_β comes from the following diagrams:

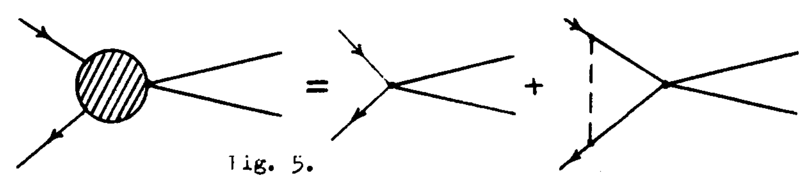


Fig. 5.

Calculating Γ_β in the one-loop approximation, we obtain

$$\gamma_\mu \gamma_5 \Gamma_\beta = \gamma_\mu \gamma_5 + \frac{1}{i\pi^2} \frac{M^2}{(4\pi F)^2} \int \frac{(M + \hat{q} - \hat{k})\gamma_\mu (M - \hat{k})\gamma_5 \, dk}{[k^2 - M^2 + i\epsilon][(k-q)^2 - M^2 + i\epsilon][(p-k)^2 + i\epsilon]} . \tag{21}$$

On the nucleon mass shell for the transfer momentum $q \to 0$ we get

$$\Gamma_\beta = 1 - \frac{1}{2}\beta\,(3 + J) . \tag{22}$$

Performing the renormalization, according to eq. (20), we obtain the following expression for Γ_R^β

$$\Gamma_R^\beta = 1 + \beta J = g_A , \tag{23}$$

coinciding with eq. (16). This is the consequence of the axial current conservation. Thus, the πN -interaction vertex and the axial vertex of β -decay are equally renormalized*).

5. The estimation of the two-loop contribution

While computing such quantities as electromagnetic pion form-factor and pion polarizability in the one-loop approximation we considered the diagram with two vertices of the strong type (see/3/). Here naturally the question arises about the contribution of the higher orders of perturbation theory in the strong coupling. To answer this question we shall consider the one-loop diagram with two strong vertices and estimate the contribution of the next two-loop approximation (see fig. 6a-e).

*) The vector current analogously with the electrodynamics, is not renormalized in the given model.

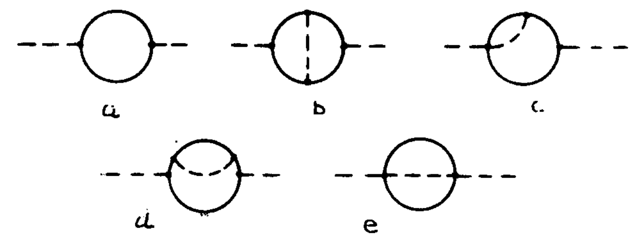


Fig. 6.

To diagram 6a there corresponds the following expression

$$\delta_{ij}\Pi^{(1)}(p^2) = \delta_{ij}\left(p^2 R(p^2) + \mathrm{const}\right) = i\frac{M^2}{(2\pi)^4 F^2}\int dk\, Sp\Big\{\gamma_5 \tau_i \times (M - \hat{k})^{-1}\gamma_5 \tau_j (M - \hat{k} - \hat{p})^{-1}\Big\} , \tag{24}$$

where

$$R(p^2) = -\frac{i}{\pi^2}\left(\frac{M}{4\pi F}\right)^2 \int \frac{dk}{(k^2 - M^2 + i\epsilon)((k+p)^2 - M^2 + i\epsilon)} .$$

It is easy to convince oneself that, for instance, the electromagnetic pion radius, calculated in the one-loop approximation, coincides with the value of the second derivative of the function $\Pi^{(1)}(p^2)$ taken at $p^2 = m_\pi^2$ *)

$$\frac{1}{2}\left(\frac{\partial}{\partial p^2}\right)^2 \Pi^{(1)}(p^2)\Big|_{p^2 = m_\pi^2} = \frac{2}{3}\frac{1}{(4\pi F_\pi)^2} = a . \tag{25}$$

Let us show now what kind of corrections will give the next two-loop approximation (fig. 6b-e).

Note, first of all, that four diagrams, represented in figs. 6b-e with the vertices corresponding to the interaction Lagrangian

*) Further for simplicity we shall put $m_\pi = 0$.

(1) are equivalent to three diagrams represented in figs. 7a-c, here one of the vertices with the pseudoscalar coupling is replaced by the vertex with the pseudovector coupling.

Fig. 7.

To these diagrams there correspond the following expression

$$\Pi^{(2)} = \Pi^{(2)}_a + \Pi^{(2)}_b + \Pi^{(2)}_c \,, \tag{26}$$

where

$$\delta_{ij}\Pi^{(2)}_a = \frac{i^2}{(2\pi)^8}\frac{M^4}{F^4}\int \frac{dk\,dq}{[-(k-q)^2]} Sp\left\{\gamma_5 T_i G(k) T_k \gamma_5 G(q) \gamma_5 T_j \times G(p+k)\right\} = -\frac{4}{(4\pi F)^2}\beta(2A+B_1)\delta_{ij} \,,$$

$$\delta_{ij}\Pi^{(2)}_b = \frac{i^2}{(2\pi)^8}\frac{M^4}{F^4}2\int \frac{dk\,dq}{[-(k-q)^2]} Sp\left\{G(q)\gamma_5 T_i G(k)\hat{p}\gamma_5 T_j \times G(p+k)\right\} = -\frac{2\,4}{(4\pi F)^2}\beta(-2A+B_2)\delta_{ij} \,,$$

$$\delta_{ij}\Pi^{(2)}_c = \frac{i^2}{(2\pi)^8}\frac{M^4}{F^4}2\int \frac{dk\,dq}{[-(k-q)^2]} Sp\left\{\gamma_5 T_i G(k)\gamma_5 T_k G(q) \times \gamma_5 T_k G(k)\hat{p}\gamma_5 T_j G(p+k)\right\} = -\frac{6\cdot 4}{(4\pi F)^2}\beta(A+B_3)\delta_{ij} \,.$$

Here A and B are expressed in terms of divergent integrals

$$A = \frac{M^2}{\pi^4}\int dk\,dq \frac{p(k-q)}{[(p+k)^2-M^2][k^2-M^2][q^2-M^2](k-q)^2} \,, \quad B_1 = \frac{M^2p^2}{\pi^4}\left[\int \frac{dk}{[(p+k)^2-M^2][k^2-M^2]}\right]^2$$

$$B_2 = \frac{1}{\pi^4}\int dk\,dq \frac{(pq - p^2/2)}{[(p+k)^2-M^2][(q+k)^2-M^2]q^2} \,, \quad B_3 = \frac{M^4p^2}{\pi^4}\int \frac{dk}{[(p+k)^2-M^2][k^2-M^2]^2}\int \frac{dq}{(q^2-M^2)q^2}$$

The second derivatives of these quantities, unlike (25), cannot be expressed in terms of finite integrals and contain logarithmical divergences. It appears that all these quantities can again be expresed in terms of the integral J (see eq. (10)), which is calculated by the superpropagator method. It appears that the form of the superpropagator is the same. Then we obtain

$$\frac{1}{2}\left(\frac{\partial}{\partial p^2}\right)^2 A = -\frac{1}{24}(2+J) \;;\; \frac{1}{2}\left(\frac{\partial}{\partial p^2}\right)^2 B_1 = \frac{1}{3}(2+J) \,,$$
$$\frac{1}{2}\left(\frac{\partial}{\partial p^2}\right)^2 B_2 = \frac{1}{24}\left(\frac{7}{6}+J\right) \;;\; \frac{1}{2}\left(\frac{\partial}{\partial p^2}\right)^2 B_3 = -\frac{1}{12}J \tag{27}$$

Thus, the total contribution from the nonrenormalized diagrams 7a-c (or 6b-e) turns out to be

$$a x_1 = a\beta\left(\frac{3}{2}J + \frac{5}{12} - 3\right). \tag{28}$$

The account of the vertex renormalization in diagram 6a gives the additional contribution (see eq. (12))

$$a x_2 = 6 a\beta(1 + J) \,. \tag{29}$$

At last we have to take into account the contribution from the renormalization of the nucleon Green function and nucleon mass (see eqs.(7) and (12)). These renormalizations lead, resp. to the quantities:

$$a x_3 = -3a\beta(1+J), \tag{30}$$

$$a x_4 = -3a\beta. \tag{31}$$

Summarizing expressions (28)-(31), we finally get for the two-loop approximation

$$x = \beta\left(\frac{3}{2}J + \frac{5}{12}\right) \cong 0.1\beta = 0.066 \tag{32}$$

Thus, the second derivative with respect to p^2 of the expression corresponding to diagrams 6(a-e) can be written down in the following way

$$\frac{1}{2}\left(\frac{\partial}{\partial p^2}\right)^2 \left[\Pi^{(1)}(p^2) + \Pi^{(2)}(p^2)\right]\Big|_{p^2=0} = (1+x)\frac{1}{2}\left(\frac{\partial}{\partial p^2}\right)^2 \Pi^{(1)}(p^2), \tag{33}$$

where x is the quantity essentially smaller than unity. Hence, we can see that the basic contribution to the pion Green function with the legs on the mass shell comes from the one-loop approximation.

6. Conclusion

Summarizing all the results we have obtained it should be noted the following: On the basis of the chiral-invariant model of interactions we have shown how the consistency renormalization theory is constructed using the superpropagator method for the regularization of divergent expressions. A number of relations between the renormalization constants is found and the fulfilment of the Goldberger-Treiman relation in the accepted perturbation theory is tested. For this purpose in the one-loop approximation there are calculated the corrections to the $\pi\gamma$-vertex in the limit of small transfer momenta and those to the weak coupling constant with axial current (g_A). The two-loop corrections to the selfenergy operator are computed, as well.

It should be noted also that all the calculated corrections due to higher orders of perturbation theory turn out to be small in spite of the absence of small parameter in the perturbation theory employed.

The authors would like to express their deep gratitude to D.I.Blokhintsev and D.V.Shirkov for regular interest in the work and also to V.V.Serebryakov, A.A.Slavnov and L.D.Faddeev for fruitful discussions.