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1. A great part of the presently known experimental facts on high energy particle scattering finds its natural explanation in the framework of various assumptions on the composite structure of hadrons (see, e.g., [1-10]). On the other hand composite models together with a principle of automodelity [11,12] lead to a number of predictions, experimental verification of which seems to be very actual.

Interest to the study of relativistic composite objects is supported also by the future [13] and presently performed [14] experiments with beams of relativistic nuclei.

In refs. [15-22] problems of relativistic description of composite systems have been considered on the basis of the Logunot-Tavkhelidze quasipotential approach [23]. The "light front" [24] form of the quasipotential approach [19,20] seems to be very effective tool in the investigations of such a kind (see, e.g., [21,22]).

In the study of interactions of composite particles a necessity arises to consider the scattering of composite particle on the elementary one (hadron-quark and/or deuteron-nucleon scattering). In the theoretical analysis of these problems we will exploit the results of refs.[19-22]. Special form of the three-dimensional formalism will be developed for the description of relativistic three-body systems (see in this connection [25-31]). In the framework of some simple assumptions on the particle inteactions expressions for the scattering amplitude of the composite particle on the elementary one will be obtained.

2. Consider the three-particle Green function

$$'G_+(x^{(1)}_\mu, x^{(2)}_\mu, x^{(3)}_\mu; y^{(1)}_\mu, y^{(2)}_\mu, y^{(3)}_\mu) =$$

$$= \langle 0| T_+(\phi_1(x^{(1)}_\mu)\phi_2(x^{(2)}_\mu)\phi_3(x^{(3)}_\mu)\phi^+_1(y^{(1)}_\mu)\phi^+_2(y^{(2)}_\mu)\phi^+_3(y^{(3)}_\mu))|0\rangle =$$

$$= [(2\pi)^{-4}]^3 \int \prod_{i=1}^{3} d^4p\, d^4q\, \exp[-i\sum_{i=1}^{3} (p^{(i)}x^{(i)} - q^{(i)}y^{(i)})] \times$$

$$\times\, 'G_+(P^{(123)*}; p^{(1)}, p^{(2)}, p^{(3)}; q^{(1)}, q^{(2)}, q^{(3)}) . \qquad (1)$$

The sign "+" denotes /1/ that the ordering in (1) is carried out with respect to the variables $x^{(i)}_+ = x^{(i)}_0 + x^{(i)}_3)/2$. In order to distinguish between the space-time variables $x^{(i)}_\mu$ and the variables $x^{(i)}$, which will occur later, the former are accompanied by the Lorentz index $\mu = 0,1,2,3$.

Define the Fourier transform of the "two-time" Green function of three particles by the relation:

$$\tilde{G}(P^{(123)}; [p^{(i)}_+, \vec p^{(i)}_\perp]; [q^{(i)}_+, \vec q^{(i)}_\perp]) = \qquad (2)$$

$$= \int_{-\infty}^{\infty} \prod_{i=1} dp^{(i)}_-\, dq^{(i)}_-\, \delta(P^{(123)}_- - \sum_{i=1}^{3} p^{(i)}_-)\, 'G(P^{(123)}; [p^{(i)}_\mu]; [q^{(i)}_\mu])$$

Here $p^{(i)}_\pm = p^{(i)}_0 \pm p^{(i)}_3$; $\vec p^{(i)}_\perp = (p^{(i)}_1, p^{(i)}_2)$

$$[p^{(i)}_+, \vec p^{(i)}_\perp] = p^{(1)}_+, \vec p^{(1)}_\perp, \; p^{(2)}_+, \vec p^{(2)}_\perp, p^{(3)}_+, \vec p^{(3)}_\perp$$

$$[p^{(i)}_\mu] = p^{(1)}_\mu, p^{(2)}_\mu, p^{(3)}_\mu ; \quad P^{(123)}_\mu = p^{(1)}_\mu + p^{(2)}_\mu + p^{(3)}_\mu$$

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*In what follows we omit the sign "+" everywhere and this will not cause any misunderstanding.

Iterating the equation

$$'G = g_0 + g_0 K\, 'G$$

for the three-particle Green function and performing the integration according to the definition (2), we obtain:

$$\tilde G = \tilde g_0 + \widetilde{g_0 K g_0} + \widetilde{g_0 K g_0 K g_0} \qquad (3)$$

g_0 is the Green function for three non-interacting particles, $\tilde g_0$ is defined by the following relation

$$\tilde g_0(P^{(123)}; [p^{(i)}_+, \vec p^{(i)}_\perp]; [q^{(i)}_+, \vec q^{(i)}_\perp]) =$$

$$= \tilde g_0(P^{(123)}[p^{(i)}_+, \vec p^{(i)}_\perp]) \prod_{i=1}^{3} \delta(p^{(i)}_+ - q^{(i)}_+)\, \delta^{(2)}(\vec p^{(i)}_\perp - \vec q^{(i)}_\perp) ,$$

where

$$\tilde g_0(P^{(123)}[p^{(i)}_+, \vec p^{(i)}_\perp]) = (2i)^3 (2\pi i)^2 (P^{(123)}_+)^2 \prod_{i=1}^{3} (x^{(i)})^{-1} \times$$

$$\times \left[P^{(123)2} - \sum_{i=1}^{3} \frac{(\vec p^{(i)} - x^{(i)} \vec P^{(123)})^2 + m^{(i)2}}{x^{(i)}} \right]^{-1} .$$

The function $\tilde g_0(P^{(123)};[p^{(i)}_+, \vec p^{(i)}_\perp])$ is defined under the conditions

$$0 < x^{(i)} < 1 ; \quad \sum_{i=1}^{3} x^{(i)} = 1 ; \quad \sum_{i=1}^{3} \vec p^{(i)}_\perp = \vec P^{(123)}_\perp .$$

The variables $x^{(i)}$ are introduced by the formula:

$$x^{(i)} = p^{(i)}_+ / P^{(123)}_+$$

Introducing now the inverse operator $\tilde G^{-1}$ by:

$$\int_0^{P^{(123)}_+} \prod_{i=1}^{3} dp^{(i)\prime}_+\, \delta(P^{(123)}_+ - \sum_{i=1}^{3} p^{(i)\prime}_+) \int \prod_{i=1}^{3} d\vec p^{(i)\prime}_\perp\, \delta^{(2)}(\vec P^{(123)}_\perp - \sum_{i=1}^{3} \vec p^{(i)\prime}_\perp) \times$$

$$\times\, \tilde G^{-1}(P^{(123)}; [p^{(i)}_+, \vec p^{(i)}_\perp]; [p^{(i)\prime}_+, \vec p^{(i)\prime}_\perp]) \times$$

$$\times \tilde{G}\,(P^{(123)};[p_+^{(i)\prime},\vec{p}_\perp^{(i)\prime}];[q_+^{(i)},\vec{q}_\perp^{(i)}]) =$$

$$= \prod_{i=1}^{3} \delta\,(p_+^{(i)} - q_+^{(i)})\,\delta\,(\vec{p}_\perp^{(i)} - \vec{q}_\perp^{(i)}) \times \tag{4}$$

$$\times\, \delta\,(P_+^{(123)} - \sum_{i=1}^{3} p_+^{(i)})\,\delta^{(2)}\,(\vec{P}_\perp^{(123)} - \sum_{i=1}^{3}\vec{p}_\perp^{(i)})$$

we obtain the following series for $\tilde{G}^{-1}$:

$$\tilde{G}^{-1} = \tilde{g}_0^{-1} - \tilde{g}_0^{-1} * \widetilde{g_0\, K\, g_0} * \tilde{g}_0^{-1} + \dots \tag{5}$$

The symbol $*$ in (5) has to be understood in the sense of integration according to the formula (4).

Defining the quasipotential $K^{(3)}$ in the three-particle system

$$\tilde{G}^{-1} = \tilde{g}^{-1} - (32\pi^2 i)^{-1}\,\tilde{K}^{(3)}$$

we have the following equation for $\tilde{G}$:

$$\tilde{G} = \tilde{g} + (32\pi^2 i)^{-1}\,\tilde{g} * \tilde{K}^{(3)} * \tilde{G}\,. \tag{6}$$

In general the quasipotential $\tilde{K}^{(3)}$ can be written in the form

$$\tilde{K}^{(3)} = \sum_{i=1}^{3} \tilde{K}_i + \tilde{K}_T\,,$$

where $\tilde{K}_i$ corresponds to the pair interaction of j-th and k-th particles, $\tilde{K}_T$ is an analogue at the three-particle potential of the nonrelativistic theory.

In what follows we consider the approximation of pair interactions. In this case

$$\tilde{K}^{(3)} = \sum_{i=1}^{3} \tilde{K}_i\,. \tag{7}$$

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*Here we choose the reference frame $\vec{P}_\perp^{(123)} = 0$.

Introduce the Green type functions $\tilde{g}_i$, which obey the following equations:

$$\tilde{g}_i = \tilde{g}_0 + (32\pi^2 i)^{-1}\,\tilde{g}_0 * \tilde{K}_i * \tilde{g}_i\,. \tag{8}$$

In the pair approximation /2/

$$\tilde{g}_i\,(P_-^{(123)},\dots) = 4\pi\,(p_+^{(i)})^{-1}\,\delta(\vec{p}_+^{(i)} - \vec{q}_+^{(i)})\,\delta^{(2)}(p_\perp^{(i)} - q_\perp^{(i)})\,g_i^{(2)} \times$$

$$\times\,(P_-^{(123)} - p_-^{(i)},\dots)$$

$$\tilde{K}_i\,(P^{(123)},\dots) = -p_+^{(i)}\,\delta(p_+^{(i)} - q_+^{(i)})\,\delta^{(2)}(\vec{p}_\perp^{(i)} - \vec{q}_\perp^{(i)})\,\tilde{K}_i^{(2)} \times$$

$$\times\,(P_-^{(123)} - p_-^{(i)},\dots).$$

Here $\tilde{g}_i^{(2)}$ and $\tilde{K}_i^{(2)}$ are two-particle "two-time" Green function and the two-particle quasipotential of j-th and k-th particles, respectively.

Define the scattering amplitudes $\tilde{T}_i$ and transition operators $\tilde{M}_{ik}$:

$$\tilde{g}_i = \tilde{g}_0 + (32\pi^2 i)^{-1}\,\tilde{g}_0 * \tilde{T}_i * \tilde{g}_0\,,$$

$$\tilde{G} = \tilde{g}_i + (32\pi^2 i)^{-1}\,\tilde{g}_i * \tilde{M}_{ik} * \tilde{g}_k\,. \tag{9}$$

Inserting (7) into equation (6) we get

$$\tilde{G} = \tilde{g}_0 + \sum_{i=1}^{3}\tilde{G}_i\,,$$

where

$$\tilde{G}_i = (32\pi^2 i)^{-1}\,\tilde{g}_0 * \tilde{K}_i * \tilde{G}\,. \tag{10}$$

Taking into account equations (8) and definitions (9) we obtain the following system of equations for the operators $\tilde G_i$:

$$\tilde G_i = (32\pi^2 i)^{-1} [\tilde g_0 * \tilde T_i * \tilde g_0 + + \tilde g_0 \tilde T_i * \sum_{k\neq i}^{3} \tilde G_k \quad . \tag{11}$$

Combining (11) with (10), we get the system of equations for the transition operators $\tilde M_{ik}$:

$$\tilde M_{ik} = \sum_{j\neq i}^{3} \tilde K_j + (32\pi^2 i)^{-1} \sum_{j\neq k}^{3} \tilde M_{ij} * \tilde g_j * \tilde K_j \, . \tag{12}$$

Amplitudes of particular physical processes in three-particle system can be expressed in a definite manner through transition operators M_{ik} and bound state wave functions. On the mass-shell they coincide with corresponding amplitudes of the four-dimensional formalism. For instance, the amplitude T_{31} of the process

$(12)+3 \to (23)+1$

looks as follows

$$T_{31} = \int_{-P_+^{(12)}/2}^{P_+^{(12)}/2} dp_+^{(12)} \int d\vec p_\perp^{(12)} \int_{-Q_+^{(23)}/2}^{Q_+^{(23)}/2} dq_+^{(23)} \int d\vec q_\perp^{(23)} \times$$

$$\times \Psi^{+(3)}_{P^{(12)}}(p_+^{(12)}, \vec p_\perp^{(12)})\, \tilde M_{31}(P^{(12)}, p_+^{(12)}, \vec p_\perp^{(12)}, p_+^{(3)}, \vec p_\perp^{(3)} ; \tag{13}$$

$$Q^{(23)}, q_+^{(23)}, \vec q_\perp^{(23)}, \vec q_+^{(1)}, q_\perp^{(1)})\, \Psi^{(1)}_{Q^{(23)}}(q_+^{(23)}, \vec q_\perp^{(23)}) .$$

Two-particle bound state wave functions obey the quasipotential type equations /19/ with two particle quasipotentials $\tilde K_i^{(2)}$:

$$[P^{(jk)^2} - \frac{\vec p_\perp^{(jk)^2} + m^{(j)^2}}{x^{(jk)}} - \frac{\vec p_\perp^{(jk)^2} + m^{(k)^2}}{1-x^{(jk)}}]\, \Phi^{(j)}_{P^{(jk)}}(x^{(jk)}, \vec p_\perp^{(jk)}) =$$

$$= \int_0^1 \frac{dx^{(jk)'}}{x^{(jk)'}(1-x^{(jk)'})} \int d\vec p_\perp^{(jk)'} \tilde K_i^{(2)}(P^{(jk)}; x^{(jk)}, \vec p_\perp^{(jk)};$$

$$x^{(jk)'}, \vec p^{(jk)'})\, \Phi^{(i)}_{P^{(jk)}}(x^{(jk)}, \vec p_\perp^{(jk)}) ;$$

$$\Phi^{(i)}_{P^{(jk)}} = P_+^{(jk)} x^{(jk)} (1-x^{(jk)})\, \Psi^{(i)}_{P^{(jk)}} \quad . \tag{14}$$

In (13) and (14) relative momenta

$p^{(jk)} = p^{(j)} - p^{(k)}$

of two-particle subsystems and variables

$$x^{(jk)} = \frac{1}{2} + \frac{p_+^{(jk)}}{P_+^{(jk)}}$$

are introduced.

Analysis of eq. (14) in the framework of field-theoretic models and problems of the asymptotic behaviour of the two- and three-body form factors will be considered elsewhere. Note, that the asymptotic behaviour of vertex functions from the point of view of anomalous dimensions has been considered in refs. /32,33/. Form factors in the Bethe-Salpeter formalism have been studied in ref. /34/.

In order to find the amplitudes of three-particle processes one has to solve the system (12). In particular, some problems of the eikonal approximation in the scattering on the bound system have been considered in ref. /35/. Here we study only some simple models for transition operators and obtain expressions for the corresponding amplitudes.

3. In *Fig. 1* model of interaction via the exchange of some intermediate particle (1a) and constituent interchange model (1b, 1c) are shown.

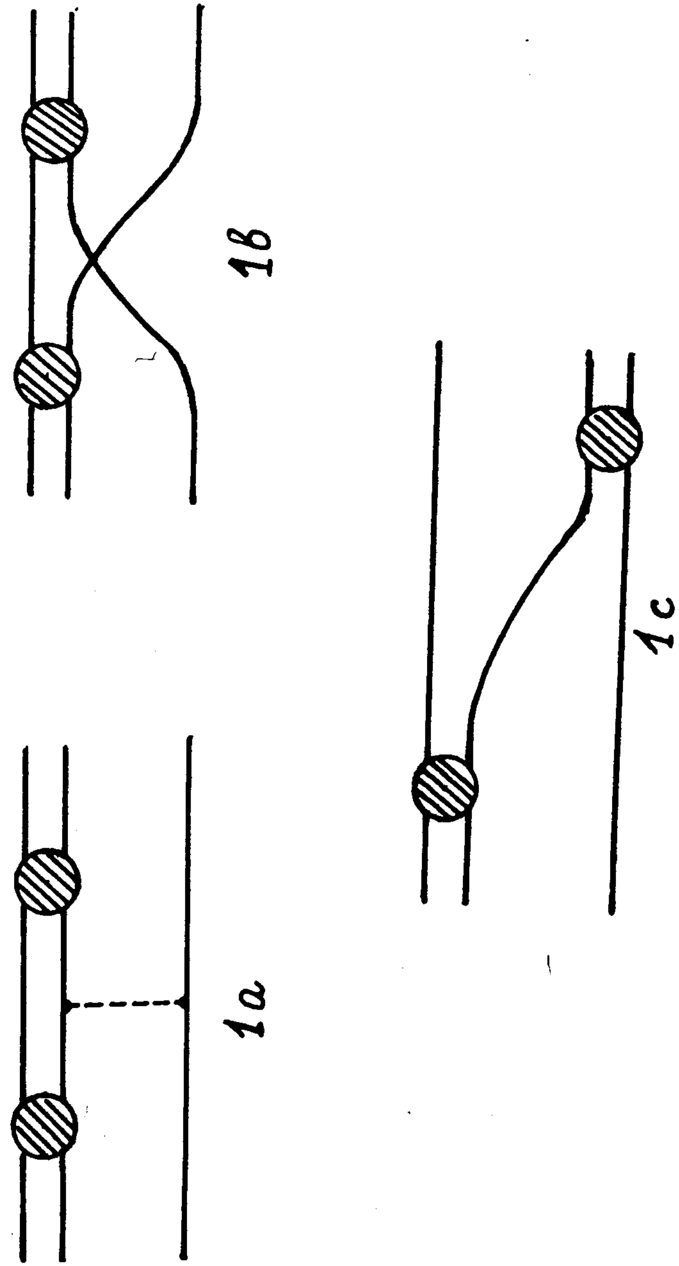

Fig. 1

It can be shown that in the case 1a the amplitude of "deuteron-nucleon" scattering looks as follows

$$T_{33} = 2.(2\pi)^4 \frac{g^2}{t-\mu^2} F_{12}(t) ,$$

where $F_{12}(t)$ is the form-factor of the bound state of two particles. Its expressions through the wave function of the two-body system can be found in ref. /19/, (see, also /36/). Note, that if particles entering the reaction possess their own form factors, this fact leads to the appropriate modification of the scattering amplitude. In particular, assuming that the form factors of all the three particles are of one and the same form $f(t)$, we get

$$T_{33} = 2(2\pi) \frac{g^2}{t-\mu^2} F_{12}(t)\, f^2(t).$$

Considering the models 1b and 1c we obtain the following expressions for corresponding amplitudes:

$$T'_{33} = -(4\pi)^{-3}\, i\, (P_+^{(12)})^2 \int_0^1 dx^{(12)} (x^{(12)})^{-1} \times$$

$$\times \int d\vec{p}_\perp^{(12)}\, \Phi^{+(3)}_{\vec{P}_\perp^{(12)}=0} (x^{(12)}, \vec{p}_\perp^{(12)} - x^{(12)} \vec{\Delta}_\perp^{(t)})\, \Phi^{(3)}_{\vec{P}_\perp^{(12)}=0} (x^{(12)}, \vec{p}_\perp^{(12)});$$

$$t = -\vec{\Delta}_\perp^{(t)2}$$

and

$$T_{31} = -(4\pi)^{-3}\, i\, (P_+^{(12)})^2 \int_0^1 dx^{(23)} (x^{(23)})^{-1} \times$$

$$\times \int d\vec{p}_\perp^{(23)}\, \Phi^{+(1)}_{\vec{Q}_\perp^{(23)}=0} (x^{(23)}, \vec{p}_\perp^{(23)} - (1-x^{(23)}) \vec{\Delta}_\perp^{(u)})\, \Phi^{(3)}_{\vec{P}_\perp^{(12)}=0} (x^{(23)}, \vec{p}_\perp^{(23)});$$

$$u = -\vec{\Delta}_\perp^{(u)2} .$$

In the framework of some special assumptions for the bound state wave functions [21,22] we get following asymptotic behaviour for T'_{33} and T_{31} (mod. log.);

$$T'_{33} \underset{|t|\to\infty}{\sim} |t|^{-N_{12}}$$

$$T_{31} \underset{|u|\to\infty}{\sim} |u|^{-N_{12}}$$

The parameter N_{12} enters the definition of the wave function in the following manner:

$$\Phi(x,\vec{p}_\perp) = \phi(x)\left[\frac{m^{(1)2} + \vec{p}_\perp^{\,2}}{1-x} + \frac{m^{(2)2} + \vec{p}_\perp^{\,2}}{x}\right]^{-N_{12}} .$$

Note, that in the framework of the formalism developed here it is possible to consider the processes with desintegration of composite particles, to study various inclusive distributions and other many-body problems.

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