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The Lorentz invariance postulate which is unavoidable in describing the production processes of the elementary particles seems to be not enough exploited from physical point of view. The requirement of the Lorentz invariance both with respect to the variables and with respect to the n-point functions used for describing reactions with n-particles $n>4$ has been mathematically investigated in detail[1]. The representation of the spaces of n-point Lorentz invariant differentiable functions (and distributions) onto appropriate spaces of Lorentz invariant variables are discussed[2] and the conditions for validity of (1) is stated.

Let us denote a set of n four-vectors $p_1,\dots,p_n$ by p and let π be the mapping which carries any set p in the set of Minkowski scalar products $p_i p_j = p_i^o p_j^o - \vec{p}_i \vec{p}_j$. Let $f(p)$ be a differentiable function invariant under the orthochronous Lorentz group $\mathfrak{L}$ then $f(p)$ is uniquely represented[2] by the functions $F(\pi(p))$ and $\tilde{F}_{i_1 i_2 i_3 i_4}(\pi(p))$(the arguments of which are scalar products) by

$$f(p) = F(\pi(p)) + \sum_{1\le i_1<i_2<i_3<i_4\le n} \Delta_{i_1 i_2 i_3 i_4} \tilde{F}_{i_1 i_2 i_3 i_4}(\pi(p)) \quad (1)$$

in which besides simple invariants like scalar products there appear the variables

$\Delta_{i_1 i_2 i_3 i_4}$ each of them means the determinant of the components of the four vectors p_{i_1}, p_{i_2}, p_{i_3} and p_{i_4}. These variables are not all independent [3]. Let us recall some properties. The determinants are skew symmetric coefficients of the exterior form of degree four of the vectors p_{i_1}, p_{i_2}, p_{i_3} and p_{i_4}. These four vectors are linearly independent if and only if $\Delta_{i_1 i_2 i_3 i_4} \neq 0$. This property is invariant to $GL(4,R)$ which is the automorphism group of the affine vector space R^4. The linear forms admit as a group of symmetry the holoedric extension of $GL(4,R)$. An exterior form is regular if the rank is equal to the degree, otherwise it is called singular. $\Delta_{i_1 i_2 i_3 i_4} \neq 0$ is a canonical regular form of degree four which preserves the value under the special linear group $SL(4,R)$. If one chooses $SL(4,R)$ as the group of automorphism then R^4 is called unimodular vector space. One calls R^4 the Minkowski space when one chooses the Lorentz group as the automorphism group. Actually the Minkowski space is a geometrization of the indefinite quadratic form (the real-valued function on the vectors p_i of R^4 linear in p_i) with the signature 2.

Now for a given process the minimum and maximum values of any Δ (sometimes we suppress indices) depend only on the energy of the colliding particles. These variables are pseudoscalars, and they are invariants of $SL(4,R)$. They are connected exclusively with the multiparticle production reactions. Therefore we have proposed [4] to use experimental data for getting new empirical information about the multiple production mechanism from the different reactions and at different energies, utilizing the histograms with respect to Δ.

As a starting step consider the process $a+b \to a_1+a_2+a_3+a_4$ or $a \to a_1+a_2+a_3+a_4$ for any particles which satisfy the conservation laws. Shortly speaking after averaging over the other variables which characterize the process let only $\Delta_{1234} = \det\{p_i^k\}_{i=1,2,3,4;\,k=0,1,2,3}$ be left. In the center of mass system of colliding particles, from the energy momentum conservation and from the property of tensor product to be distributive with respect to sum we have $\Delta = \sqrt{s}\,(\vec{p}_{i_1} \times \vec{p}_{i_2}) \cdot \vec{p}_{i_3}$, $1 \le i_1 < i_2 < i_3 \le 4$, where $\sqrt{s}$ as usual is denoted c.m.s. energy. Now for a fixed s the variable we consider is the value of the $\det\{p_i^k\}_{i=i_1,i_2,i_3;\,k=1,2,3}$ which is the measure of the volume of the parallelepiped built on $\vec{p}_{i_1}$, $\vec{p}_{i_2}$ and $\vec{p}_{i_3}$. For any choice of i_1, i_2, i_3 one gets the same value for Δ_{1234}. The experimental distributions on Δ_{1234} for $p+p \to p+p+\pi^+ + \pi^-$ have been performed for ten values of the energy between 4.0 GeV and 24.8 GeV. A strong shrinkage ([5], fig. 1) with respect to the energy is obtained when they are compared with the phase space distributions, for details see [5]. For $\pi^- + p \to \pi^- + \pi^- + \pi^+ + p$ the results are similar but we have only two values of energy [6].

The shape of the histograms is of the form $\phi(s,\Delta) = A(s) - \Delta^2 B(s)$. The dependence on s is connected immediately with the extremum of the Δ for a given energy s. The modulo of $f(p)$ in (1) for some restrictions of F and F_{1234} can approximate the shape of the experimental histograms. But for doing it accurately the spins of particles should be accounted. Then besides invariant functions the covariant polynoms enter in the expression of a spinor amplitude $A_\sigma(p)=$ $=\sum_{i=1}^{N}\mathcal{P}_{\sigma i}(p)\,f_i(p)$ where $f_i(p)$ are N Lorentz invariant functions (distributions) and $\mathcal{P}_{\sigma i}(p)$ are N Lorentz covariant polynoms[7], where p belongs to physical domain generated by n four-momenta $p_1,\dots,p_n$ of the n particles which enter in the process and σ states for the spins of the particles (two four-momenta parallel are excluded).

But the aim of this note is not to approximate the Δ histograms by using (1). The point is to emphasize the new regularities with respect to Δ which has to be investigated using the experimental data. Actually we have mentioned (1) to indicate one of the reasons to think of Δ as a variable which can be appropriate for getting some empirical information of the production mechanism of elementary particles and for connecting them with the mathematical structure of the orbit space[2] (of the Lorentz group acting on the product of n Minkowski spaces) and the n-point Lorentz invariant functions defined on it. Or more precisely the new constraints of invariant functions valid for high energy.

In order to do this it seems to us that we have first to draw from experimental data the answers to the following questions:

a. Do Δ distributions as functions of energy, i.e., $\phi(s,\Delta)$ depend on the nature of colliding particle or final particles (neutrino-production, foto-production, etc.,)?

b. For four inclusive reactions (four exclusive particles) one can divide the interval of the energy of the four particles in their center of mass system $(p_1+p_2+p_3+p_4)^2$ into intervals of the "fixed" energy. For the events corresponding to every certain energy to get the Δ histogram. Will be the shrinkage the same with respect to energy?

c. Important is to know from the reactions with more then four particles in final state for which a few independent determinants exist if they are simultaneously going to zero for a given event. (There are $n(n-1)(n-2)(n-3)/24$ determinants but not all independent[3] since $\Delta_{i_1 i_2 i_3 i_4}\Delta_{k_1 k_2 k_3 k_4}=$ $=\det\{p_i\, p_k\}_{i=i_1,i_2,i_3,i_4;\,k=k_1,k_2,k_3,k_4}$.

d. Finally Δ being a pseudoscalar it is interesting to investigate if there exists any asymmetry in the Δ distribution with respect to $\Delta=0$.

It is possible to perform it for the channels where the four particles in final state are different thus they can be uniquely labelled, and the ordering of them permits one to introduce the orientation of the space.

To summarize, the variable Δ_{1234} in the case of four particles with fixed energy is reduced to the volume of a parallelepiped which is a scalar density of weight -1. This variable equal to zero defines the singular domain of the physical region of the vectors p_1, p_2, p_3 and p_4. For encreasing energy this region becomes prevalent. If we have n particles in final state the n 4-momenta may be written in the form of $n\times 4$ matrix. The problem is to investigate the rank of this matrix using experimental data for different processes $a+b \to a_1+...+a_n$. We stipulate that for increasing energy the rank is not four. That means that the dimensionality of the space generated by n 4-momenta is <4 . Thus the production of n particles at high energy is realised in a most"economic"way - the minimum number of degrees of freedom. This statement has to be true independent of the nature of the particles which are involved in the production processes and independent of the number n. If it is so then the support of the amplitude in (1) has to be in the singular kinematical region[1,2].

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