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OF THE MULTI-PARTICLE PRODUCTION
AND HADRON NUCLEUS INTERACTIONS
AT HIGH ENERGIES**

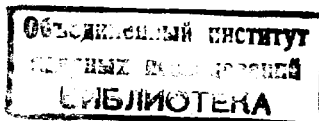
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1. Introduction

The multi-particle production is the main process at high energies accompanying the collision of hadrons. However, there is no so far a common viewpoint as to its development. Abundance of theoretical models for its interpretation is explained at least by two reasons: the absence of the consistent theory of strong interactions, and essential multi-particle character of a production act. Thus, at the present stage it is important to select the most realistic models. The study of the development of the multi-particle production in the hadron-nucleus interactions is of importance too. The main and qualitatively new fact in this case is the space-time proximity between the acts of generation and rescattering of the produced system on the nuclear nucleons. One may hope that as far as the models differ in describing the initial stage of the process, they will lead to different results for the multi-particle production

of hadron-nucleus interactions. These hopes are grounded; the investigations of multi-particle production in nuclear matter have already resulted in critical situation with abundance of contradictions between traditional representations based on the cascade schemes and observations.

These discrepancies have been mentioned in a large number of papers (see, e.g. ^{1,2}). The most serious facts are the following:

- i) It appears that the mean value of the inelasticity coefficient $\langle K \rangle$ for the multi-particle production on the hydrogen target and on complex nuclei slightly differs and is practically independent of the number of slow particles (n_k) accompanying the process ^{3,4} .
- ii) A surprising fact is the approximate invariance of the mean transverse momentum of produced particles $\langle p_T \rangle$ with respect to the nucleus size ⁵ .
- iii) The absence of the dependence of the ratio of mean multiplicities on nuclei and hydrogen target on the energy ($E \gg 100$ GeV) is determined. ²
- iv) The ratio $D^2 / \langle n_s \rangle$ is also independent of the number of k -particles. In the models based on the assumption that the multi-particle production in nuclear matter consists of a number of independent acts, this function must fall rapidly ⁶ .
- v) The mean multiplicity depends weakly on the atomic number $\langle n_s \rangle \sim A^\alpha$, α of the order ⁷ 0.15 ± 0.06
- vi) The data on the angular distribution of particles produced on a nuclear target are very important. Their number in the forward cone corresponding to $\theta_{1/2}^{(s)}$ in the elementary act does not depend on the energy ($\sim n_k$) transferred to nucleus ⁴ .
- vii) Finally, the ratio of multiplicities at different but sufficiently high energies is approximately independent of the energy transferred to nucleus and equals that for the multi-particle production on the hydrogen target at the same energies ^{4,8} .

From these facts it follows that as far as the cascade schemes are based on the assumption of the instantaneous production of real secondaries, their difficulties fail to favour

one-stage models of the elementary act. This is just a positive result of a great amount of papers of this trend.

Thus, the previous investigations of the hadron-nucleus interactions lead to the necessity of taking into account the space-time peculiarities of the process development.

Apparently, some first papers pointing out the importance of consideration of the creation delay are ^{9,10} . Then ¹ , under the assumption of the diffraction mechanism of hadron excitation and its decay beyond the nucleus, estimates have been made. These estimates showed that in this case the dependence of multiplicity on the energy and the mass number of nucleus-target is considerably improved as compared with the results of cascade schemes. The authors of the paper ¹¹ made a further step: the diffractively excited hadron may increase its excitation energy in subsequent interactions with nuclear nucleons. Both in ¹¹ and in ¹¹ the cross-section of the resonance-nucleon inelastic interaction was assumed to be equal to the known value for usual hadrons $\sigma_{\pi N}^{in} \approx 20$ mb or $\sigma_{NN}^{in} \approx 30$ mb.

The results of these papers inspite of their qualitative preliminary character (they relate to the fragmentation region only) are very interesting since they certainly demonstrate the necessity to take into account the space-time relations when describing the production in nuclear matter.

However, to interpret the above presented facts, the considerations of the mentioned papers are insufficient. Most probably that the model of one-dimensional "cascading" energy flux, widely discussed recently (e.g., allowing a number of independent production acts in matter, it encounters the difficulty of explanation iv) ^{2,6}), is, also, insufficient.

Below, we shall briefly describe the picture developed in ¹²⁻¹⁸ which is as a whole consistent with experimental data. We shall, also, make some conclusions.

2. On the Formulation of the Model

According to this picture, one should give up the assumption that the "leading" particle (or rather the system) arising in the average act of multiple production in subsequent collisions with nuclear nucleons can induce the multiple process with usual intensity. Otherwise it is difficult to explain the facts mentioned in the introduction (i, iii - vi). One should consider that "the leading" particle just after the multiple generation act becomes unable to interact actively with nucleons: the nucleus for it becomes, practically, transparent.

Note, that even before the majority of the pointed out facts were stated the idea of possible sharp change of the leading system properties has been proposed by a number of authors^{/19-21/}. For such a state, the terms "bare", "cut" are used. This state is assumed to be, generally speaking, excited. The time necessary for the "cut" hadron to restore its proper equilibrium field slows down (is retarded) by the Lorentz γ factor (which is usually, very high). Thus, as the first approximation in ^{/12-18/} it is assumed that the inelastic interaction of a leading particle (system) with nucleons may be neglected.

To describe most of the produced particles (pionization part of a spectrum), we keep to the following considerations. In the elementary act there occurs high energy release in the volume not exceeding the Lorentz contracted nucleon volume. The produced hadron system is at first expanding with the velocity close to the light velocity, and after reaching the definite energy density it disintegrates into separate particles. Most probably, that in the expanding phase due to the presence of a strong interaction in the system, these particles cannot be separated (this assumption is a basic one in some concrete models of multi-particle production ^{/22/}). Thus, over a period of expanding it should be considered as a whole. As most of the produced particles are π -mesons, then the decay radius of the system should be approximately equal to

$$r_d \approx \frac{\hbar}{\mu_\pi c} \langle n_s \rangle^{1/3} \quad (1)$$

where μ_π is the π -meson mass, and $\langle n_s \rangle$ is the mean multiplicity in the elementary act. The state of the hadron system in its expanding phase we call a cluster.

Henceforth taking into consideration the comparison of the observed data corresponding to the incoherent processes with large energy transfer to the nuclear matter (up to 3.5-4.0 GeV), we shall describe the cluster-nucleon interaction in terms of the cross sections, but not amplitudes.

Further, we should expect that like in the collisions of the known hadron, the mean-transverse momentum transferred in the cluster-nucleon interaction is much smaller than the initial one. Thus, the motion of center of cluster mass may be considered to be rectilinear. Since the de Broglie wave length of cluster is much smaller than the internucleonic distances, this motion can be determined by classical equation.

At high energies the nucleon wave length in the cluster rest frame is also very small ($\lambda \ll r_0$). Consequently, one can calculate the cross-section of the cluster-nucleon interaction in classical limit.

Let us proceed to the equation system of the cluster motion in matter. They can be presented in the dimensionless form:

$$\begin{cases} d\eta/d\xi = - \left[S_0 + \int_0^\xi \frac{d\xi'}{\sqrt{\gamma^2(\xi') - 1}} \right]^2 & (2a) \\ d\eta/d\xi \approx L \left(\sqrt{\eta^2 + 2\eta \frac{mc^2}{E_0(0)}} - \eta - \frac{mc^2}{E_0(0)} \right) \left[\xi + \int_0^\xi \frac{d\xi'}{\sqrt{\gamma^2(\xi') - 1}} \right]^2 & (2b) \end{cases}$$

where

$$\begin{aligned} \eta(\xi) &= E(\xi)/E_0(0), \quad \eta(0) = E_0(\xi)/E_0(0), \\ \xi &= (z - z_0) \sqrt[3]{\pi \rho \frac{E}{E_0(0)}}, \quad S_0 = z_0 \sqrt[3]{\pi \rho \frac{E}{E_0(0)}}, \end{aligned} \quad (3)$$

and

$$L = \frac{E_o(o)}{\bar{E}} \left(\langle K \rangle \frac{\sigma^{in}}{\sigma^{tot}} \right)_{K,N}$$

Besides

$$\eta(\xi) = \mathcal{H}(\xi) \cdot \gamma(\xi), \quad (4)$$

i.e., (4) connects the total cluster energy with its mass.

The meaning of the equations and notations are as follows.

E is the total cluster energy, E_o is the inner cluster energy, γ is the Lorentz-factor of its motion in the laboratory system. ξ_o is the point of the production act, x is the coordinate along the cluster motion (see Fig.1),

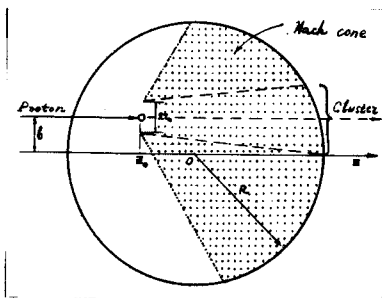


Fig.1.

ρ is the nuclear matter density, \bar{E} is the mean energy of recoil nucleon after its collision with the cluster, r_o is the range of nuclear forces in the nucleon-cluster system in the moment of cluster production ($r_o \approx \hbar/\mu v$).

The motion of a cluster in a nucleus.

The equation (2a) connects the energy loss of the cluster per unit path with the cross section $\sigma_{K,N}$ of its interaction with the nucleon ($\sigma_{K,N}$ is proportional right hand side 2a). The second term in brackets represents the change of the transverse cluster sizes as a result of its expanding with near light velocity (see /23/, p.75), taking account of relativistic retardation down of time (by analogy with the consideration in /20/).

In the second equation (2b) $\langle K \rangle$, σ^{in} , σ^{tot} are the mean value of the inelasticity coefficient, the total inelastic cross section, the total cross-section, respectively, for the cluster-nucleon interaction. The combination of these quantities, put in round paranthesis we shall call the

inelasticity parameter. The expression put in paranthesis in (2b) and multiplied by $E_o(o)$ is the kinetic energy of the cluster collision with the nucleon in the o.m.s.

Thus, the second equation describes the change of the cluster internal energy (which can be identified with mass up to rather high energies) per unit path, caused by its successive inelastic collisions with nucleons. This change is proportional to the number of collisions and to the mean energy release in each of them.

1. In fig.1 the cluster production is localized. It means that the space region of its production is much smaller of the nucleus sizes. Let us estimate the scale of this region. The mean energy (in o.m.s.) for the cluster production $\approx \langle K_{NN} \rangle \sqrt{2mc^2 E_L}$. We obtain from the uncertainty relation that it is released within the time interval $\Delta t \approx \hbar / \langle K_{NN} \rangle \sqrt{2mc^2 E_L}$ and, consequently, the dimension of the region is of the order $\Delta z \approx c \Delta t \approx \hbar c / \langle K_{NN} \rangle \sqrt{2mc^2}$. On the other hand, the distance between the nucleons (in the same system) is of the order $r_o / \gamma \approx r_o / \sqrt{E_L / 2mc^2}$ order. The ratio of these quantities is

$$\Delta z / (r_o / \gamma) \approx \frac{\hbar c}{\langle K_{NN} \rangle r_o 2mc^2} \approx 0.1 \ll 1, \quad (\langle K_{NN} \rangle \approx 0.4-0.5) \quad (5)$$

Hence it follows the possibility of localization of cluster production in the nucleus. It may be considered that it is produced under the collision of the primary hadron with one of the nucleons (from (5) is seen that at very small values of K the localization is impossible and the process is developed in the range of nucleus size order).

2. From fig.1, illustrating the motion of the cluster produced near ξ_o it is evidently assumed (as well as the equations (2a,b) that the cluster decays beyond the nucleus. What energies does it hold for?

It is reasonable to define the time of cluster decay as $\Delta \tau_L \approx r_d \gamma_{c.m.s.} / c$. During this time its path equals $x - x_o \approx \Delta \tau_L \cdot c \approx r_d \gamma_{c.m.s.}$. The mean decay radius r_d is estimated by the relation (1). Since the mean multiplicity $\langle n_s \rangle \approx \langle K_{NN} \rangle \sqrt{2mc^2 E_L} / \langle E_T \rangle$

$$\langle E_{\pi} \rangle_{c.m.s.} \approx 0.45 - 0.50 \text{ GeV}$$

$$z - z_0 = l \approx z_0 \frac{E_L^{2/3}}{(2mc^2)^{1/3}}; \quad (\langle K_{NN} \rangle \approx 0.5) \quad (6)$$

At $E_L = 10 \text{ GeV}$ from (6) we obtain $z - z_0 \approx 5.2 \text{ fm}$. Taking into account that in the act, the cluster is produced at the distance $\Lambda \approx 1/\rho \sigma_{NN}^{in}$ from the forward boundary of the nucleus, then $\Lambda + (z - z_0) \approx \frac{4}{3} R$, where R is the radius of photoemulsion heavy nuclei. In fact, such an approach at $E_L = 10 \text{ GeV}$ is also justified for the paths $z - z_0 \approx 2R$. This is due to the nuclear matter reaction to the cluster moving inside it which prevents its rapid decay (see /15/). Thus, the lower limit of applicability of equations (2a,b) is near $E_L = 10 \text{ GeV}$. It is yet difficult to estimate the upper limit.

One may only suppose that the given scheme is valid without considerable changes up to energies at which the pionization part of a spectrum may be (or effectively) described at least approximately in the framework of one cluster production.

3. The equation (2b) is written under the assumption that the energy released during each collision of the cluster with nucleons increases its internal energy E_0 . In favour of such a solution we give the following arguments. When discussing the possibility of the mechanism of hadron excitation in paper /24/ the conditions of its realization have been formulated:

$$[(E_0^*)^2 - E_0^2] (2E_L)^{-1} < \mu_{\pi} \quad (7)$$

where $E_0^* - E_0$ is the hadron excitation energy as a result of inelastic collision. The estimate with the inelasticity parameter for the known hadrons shows that the condition (7) is fulfilled in most of the collisions of the cluster with the nucleon.

The previous argument explains the reason of energy release localization in one of the colliding partners, but gives no preference to the cluster over the nucleon. However, in favour

of localization in cluster one may give an argument based on the analysis of the experiments on the 60 GeV meson-nucleon interactions /24/. It shows that in the cases which may be interpreted as excitation, π -meson is excited with probability by an order higher than the nucleon. In the mean nucleon-nucleon interaction ($\langle K_{NN} \rangle \approx 0.4 - 0.5$), there are mainly produced the pionization clusters, the excited "hot" systems in the continuous spectrum. It is reasonable to assume that relative probability of their excitation in the cluster-nucleon collision is not smaller.

Thus, besides a natural wish to simplify the problem one may give objective arguments allowing to describe the cluster-nucleon interaction as the two-particle one.

Equation system (2a,b) includes only two essential parameters: \bar{E} is the mean energy of recoil nucleons, and $(\langle K \rangle \frac{\sigma_{in}}{\sigma_{tot}})_{K,N}$ is the inelasticity parameter of the cluster-nucleon interaction. They were defined in /12,14/. The parameter $\bar{E} \approx (0.12 - 0.14) \text{ GeV}$ corresponding to the mean energy of g -particles (it is almost independent of the initial hadron energy). The quantity $(\langle K \rangle \frac{\sigma_{in}}{\sigma_{tot}})_{K,N} \approx 0.2 - 0.25$. This value results in satisfactory agreement in the $\langle n_g \rangle - n_s$ correlation at $E_p = 22 \text{ GeV}$. In all further calculations both the parameters are supposed to be known and equal to the pointed out values.

3. Comparison with Data of the Multiple Production in the Hadron-Nucleus Interactions

The validity of the presented picture should be verified by comparison with the experiment.

We will not discuss the details of the solution of equations, its connection with the observed quantities (n_s, n_g and so on) the averaging over the impact parameter and the cluster production coordinate. They are explained in refs. /12-18/.

3.1. The interpretation of facts stated in Introduction

Let us begin with discussing the above mentioned facts. The absence of considerable differences in the average value of inelasticity coefficient for the process on the hydrogen target and on complex nuclei (1) is trivial from the viewpoint of accepted hypotheses. Really, the incident particle initiating the multiple generation act on one of the nuclear nucleons in the same way as on the hydrogen-target creates "leading particle" which interacts weakly with the nucleus. Thus the mean value of energy for the production of new particles should be approximately the same as in the case of hydrogen target.

A direct consequence of initial assumptions is also the invariance of mean value of the transverse momentum of produced particles in the elementary act and in nuclear target (11).

It is very well known that $\langle p_{\perp} \rangle$ is practically unchanged in a very large energy range of colliding particles, including accelerating and cosmic regions. From the viewpoint of the accepted picture, this fact is completely determined by the cluster decay dynamics and is independent of its mass. Thus, it is quite natural that the cluster produced and "overgrown" inside the nucleus, will obey this rule as usual when decaying beyond the nucleus. This fact which defies description from the viewpoint of models assuming instantaneous production of secondaries (due to the effect of multiple scattering) is very simple in the considered picture.

The comparison of calculation with data for the R_{Em} ratio of the mean multiplicity of particles n_s produced in the nucleus and in the elementary act at high E_p is given in fig.2. There is no dependence of the ratio on energy

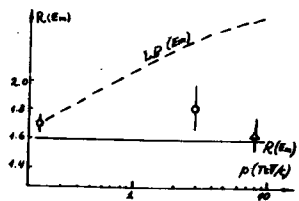


Fig.2. The ratio $R_{Em}(E_p)$.

E_p . In this case the solution of the system of equations (2a,b) permits a simple qualitative analysis as at high γ_0 it is "uncoupled". (One may neglect the expansion terms). Then from (2b) neglecting the term $mc^2/E_0(0)$

using the definition of its quantities, and passing from γ to κ , we obtain

$$d\delta/dz \approx \rho \pi r_0^2 \langle \kappa \rangle \frac{\sigma^{in}}{\sigma^{tot}} \Big|_{\kappa, N} \left(\sqrt{\delta^2 + 1} / \langle \kappa_{NN} \rangle - \delta \right). \quad (8)$$

Since the quantities of (8) are practically independent of the energy E_p up to this accuracy we have

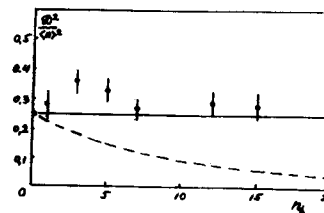
$$\langle n_s \rangle_A / \langle n_s \rangle_{PP} \approx \langle n_s \rangle_{PP} \cdot \langle \delta \rangle / \langle n_s \rangle_{PP} = \text{const}(E_p), \quad (9)$$

where $\langle \delta \rangle$ - is the function δ , averaged over the impact parameters and the points Z_0 of cluster production in the nucleus.

Henceforth, using the relation between n_s and $n_{s,PP}$ (21) in [14] and the definition of dispersion, one may show that in the framework of the accepted picture the following relation

$$\frac{D^2}{\langle n_s \rangle^2} \approx \frac{D_0^2}{\langle n_s \rangle_{PP}^2} \quad (D_0 \text{ is the dispersion in the elementary act}) \quad (10)$$

holds, i.e., the function $D^2/\langle n_s \rangle^2$ is independent of magnitude of cluster path in nucleus, consequently, of the number of n_h (iv). This conclusion is correct if the leading hadron interaction is neglected. Fig.3 presents the results for the function $D^2/\langle n_s \rangle^2$.



By points we denote the experimental data, a solid line corresponds to relation (10), and the dashed line is the result of calculation of this function by the Gottfried model [2]. Earlier [6] it was emphasized that these data are

Fig.3. $D^2/\langle n_s \rangle^2$ as a function of n_h .

difficult to explain if one keeps to the models assuming the presence of a number of independent production acts.

The A - dependence of the mean multiplicity $\langle n_s \rangle_A$ (v) can be obtained by direct calculations, then approximating the results by the function of the form $\sim A^\alpha$. Fig.4 presents such an approximation with $\alpha = 0.12$. The experimental points are taken from ref. /25/ . The initial protons had

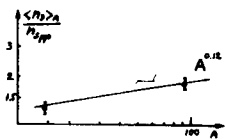


Fig.4. A-dependence of $\langle n_s \rangle$.

the energy $E_p = 70$ GeV. Close results are obtained by evaluation of experimental data at $E_p = 200$ GeV /26/ .

The absence of the dependence of the number of s - particles (vi) in the forward cone on n_h (i.e., on the cluster-path) is

easily explained in the framework of the developed model. Firstly, the contribution to the forward cone is given by fast particles produced during the decay of "the leading system" which interacts weakly with the nuclear nucleons. Secondly, the integral contribution of particles (the cluster decay products) to this cone is also independent of n_h (see /13,14/). The latter can be qualitatively explained by approximate compensation of two effects. On the one hand the "overgrowing" of the cluster results in decreasing of its δ (i.e., increases $\theta_{1/2}^{(s)}$). On the other hand, "overgrowing" leads to the increasing number of s - particles, in the changed angular interval.

At last, consider the relation (vii). It is reproduced by the numerical results. However, an approximate result for this relation is possible in this case also (see ref. /14/). At high energies, the following equation from the system (2a,b) may be obtained

$$\frac{1}{\langle n_s \rangle_{pp}} \frac{dn_s}{dAE} \approx \frac{\langle K \rangle \frac{\sigma_{in}}{\sigma_{tot} K, N}}{\bar{E}} \left(\sqrt{\delta^2 + 1 / \langle K_{NN} \rangle} - \delta - \frac{mc^2}{E_0} \right) \quad (11)$$

The right-hand side of (11) is the same at different energies with an accuracy up to small term mc^2/E_0 .

Thus we have:

$$\frac{dn_s(E_{p_2})}{dn_s(E_{p_1})} = \frac{\langle n_s \rangle_{pp}(E_{p_2})}{\langle n_s \rangle_{pp}(E_{p_1})}, \quad (12)$$

hence it follows the relation:

$$n_s(E_{p_2}) / n_s(E_{p_1}) = \langle n_s \rangle_{pp}(E_{p_2}) / \langle n_s \rangle_{pp}(E_{p_1}), \quad (13)$$

(The additive integration constant is equal to zero). This relation has been experimentally determined for $E_{p_1} = 200$ GeV, $E_{p_2} = 70$ GeV /8/ and $E_{p_1} = 300$ GeV, $E_{p_2} = 200$ GeV in ref. /4/ .

3.2. Differential characteristics of relativistic particles

The considered model does not pretend to describe the dynamics of cluster state decay into separate particles. It only assumes that, e.g., the distribution of $F(n_s)$ over the number of relativistic particles corresponds to that of produced clusters over masses. Thus, the distribution of $F_A(n_s)$ in the hadron-nucleus interaction is determined by the distribution of $F_0(n_s)$ in the elementary act /17/ :

$$F_A(n_s) = \left[\int dV_\ell y(\ell) \right]^{-1} \int \frac{dV_\ell}{R(\ell)} y(\ell) F_0\left(\frac{n_s}{R(\ell)}\right) \quad (14)$$

The quantity $R(\ell)$ is the factor of growing multiplicity with increasing cluster mass, dV_ℓ is the element of the nuclear volume corresponding to equal cluster paths $y(\ell)$ is the factor of "shading" of ℓ -layers of the nucleus. All these are determined by (21), (22) and (23) in ref. /14/ .

The distributions of $F_{C,N,0}(n_s)$ on light nuclei and of $F_{A,B,C}(n_s)$ on heavy nuclei of photoemulsion at $E_p = 200$ GeV is presented on fig.5. The experimental data are also presented in fig.5 /26/ . Obviously, at small n_s , (14) cannot be used as the production mechanism is non-cluster here, but the

It is interesting to consider the behaviour of $F(y)$ in the range of small angles. It was investigated in refs. ^{130, 131}. Fig. 8 presents the experimental results for the ratio

$Z(y_{min}) = \int_{y_{min}}^{\infty} F_A(y) dy / \int_{y_{min}}^{\infty} F_0(y) dy$ for two nuclear groups of photoemulsion at $E_p = 200$ GeV ¹³⁰. A rough theoretical evaluation of the function $Z(y_{min})$ under the assumption of absence of the leading system interaction is presented by smooth curves. Such results are obtained for the $\pi^- + Em$ -interaction ($E_{\pi^-} = 60$ GeV) ¹³¹.

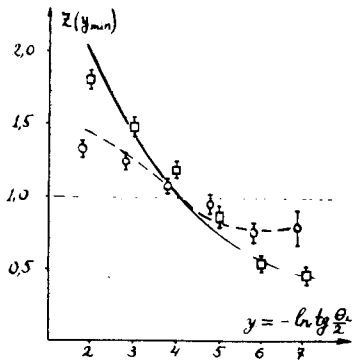


Fig. 8. The distribution $Z(y_{min})$.

3.3. The correlation relations and the nucleus-target response

Obviously, the characteristics connected with nuclear response to the development of multi-particle production in it are the important points in testing the validity of the model. To such characteristics we may refer the data on the correlation between the number of s - and g - particles, the distributions over the numbers n_g , the dependence of $\langle n_g \rangle$ on A and E since g -particles are the nucleons being directly collided with the cluster. The model allows one to find also these quantities by solving system (2a,b). Thus, by analogy with derivation of relations (14) and (15), it is not difficult to obtain for the $\langle n_g \rangle - n_s$ -correlation the formula

Note, that for y_{min} , approximately corresponding to half-angle, $Z \approx 1$. This reflects the fact (vi) which has been considered above.

$$\langle n_g \rangle \approx \frac{Z}{A E_p} \left[\int F_0 \left(\frac{n_s}{R(E)} \right) y(e) dV_e \right]^{-1} \int F_0 \left(\frac{n_s}{R(E)} \right) y(e) \Delta E [g(e)] dV_e. \quad (18)$$

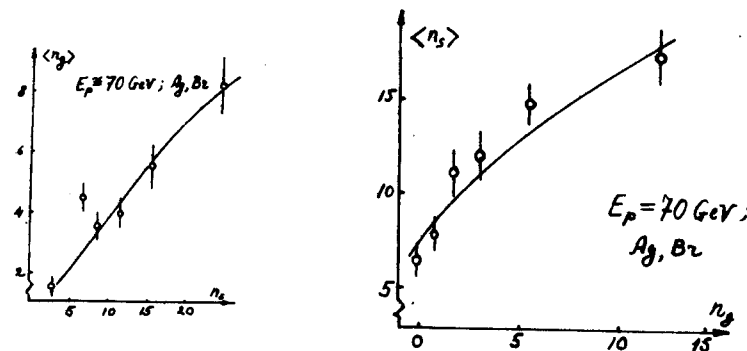


Fig. 9. The $\langle n_g \rangle - n_s$ correlation. Fig. 10. The $\langle n_s \rangle - n_g$ correlation.

Figs. 9 and 10 present the experimental ¹²⁵ and theoretical results for the correlations $\langle n_g \rangle - n_s$ and $\langle n_s \rangle - n_g$ at $E_p = 70$ GeV. A satisfactory description of these characteristics is also obtained for the process at $E_p = 200$ GeV ¹²⁶.

Fig. 11 a, b illustrates the correspondence of theory with experimental data on the distribution of $F_A(n_g)$ at $E_p = 200$ GeV in the case of light (fig. a) and heavy (fig. b) nuclei of photoemulsion ¹²⁶. The dashed curve in fig. 11a corresponds to $\pi^- + C^{12}$ at $E_{\pi^-} = 40$ GeV, and, also, gives a satisfactory description of experimental data, obtained when irradiating the propane chamber ¹¹⁶.

Hence, there follow two interesting peculiarities of the process on nuclei. The first one is seen from the data in fig. 11a: the mean value $\langle n_g \rangle$ is practically constant in a wide energy interval. The second peculiarity is the dependence of $\langle n_g \rangle$ on the mass number of A -target at the same energy. The comparison of the data in fig. 11a, b) leads to the following approximate dependence:

$$\langle n_g \rangle \sim A^{0.6} \approx A^{2/3}$$

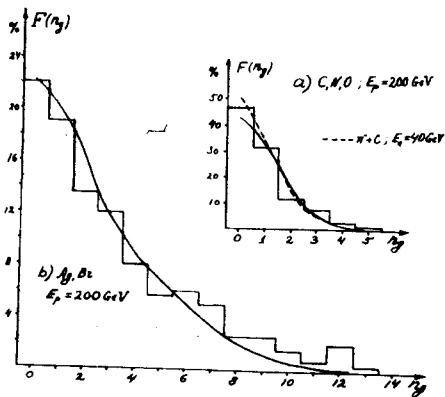


Fig.11. The distribution $F(n_g)$.

eration model. In the cases of production of the large number of h -particles ($n_h = n_g + n_e$), when the nucleus is almost completely destroyed (this phenomenon is investigated in the experiments ^[32,33]), the evaporation model is not applicable. In ref. ^[15] the disintegration of nuclei has been studied on the basis of the considered model with the only addition: in the nucleus the collective excitation of the shock wave type becomes possible at sufficiently large part of the cluster. In the framework of such an approach one may explain all the basic features of the effect.

In this connection we should like to point out another interesting fact shown in fig.12 ^[4]. This is an approximate invariance of the integral distribution $F(n_h)$ in a wide energy interval. Taking into account the results of ref. ^[15] we assume in the first approximation, that h -particles are mainly nucleons in the Mach cone (see fig.1) in which the shock wave front is propagated. Its angle is practically

In conclusion we shall comment the final stage of nuclear response to the multi-particle production in it: the emission of b -particles. Naturally, in the average act of interaction (only several b -particles correspond to it) the connection between the nucleus excitation and b -particle emission is very complicated. This stage is usually described by the evapo-

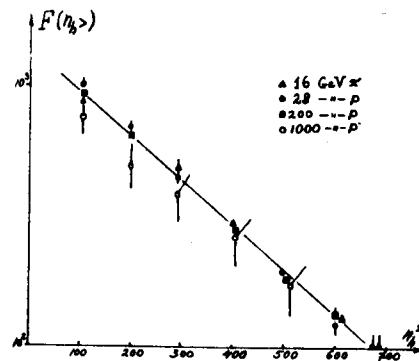


Fig.12. The integral spectrum $F(n_h)$.

independent of energy (see ref. ^[15]) and that leads to the invariance of $F(n_h)$. And what is more, actual calculation performed in this approximation, reproduces correctly the form of $F(n_h)$ (the solid line in fig.12). Thus, the evaporation mechanism is most likely the correction one to the basic collective mechanism of nucleus disintegration.

4. Conclusion

In Sect.3 by a large number of examples we have compared the experiment with the model (Sect.2) which main elements are constructed by taking into account the space-time development of multi-particle production. A wide range of qualitatively different manifestations of the process (integral, differential including the nuclear response) is in agreement with it. Now we formulate briefly the most interesting results.

4.1. A good correspondence to experiment testifies the validity of the hypothesis accepted in this model:

a) The majority of the produced particles is determined by the decay of intermediate state: the cluster. The independence of a number of results of the cluster mass explains evidently the absence of definite isolation of particles in the given state of the process.

b) The leading particle (system) produced in the average act of multiple production interacts weakly with the nucleons during the time interval of nuclear order.

4.2. As far as the hypothesis on the hadron cluster production does not contradict the fact,

a) the models of the elementary act assuming the development of the process in two stages have the advantages compared to the single-stage ones.

b) Among the models of this class those are less grounded which do not take into account the expansion of the system in the preintegrating phase /34/.

4.3. And what is more, further application of the hadron-nucleus interactions in the discussed trend can give more valuable information necessary to improve and define more accurately the models.

To illustrate this, we indicate some interesting points.

a) In one of the first attempts to obtain actually the data on the $\tilde{\sigma}^{in}$ cross-section of the leading particle (system) inelastic interaction with the nucleons /18/ in $\pi^- + C^{12}$ ($E_{\pi^-} = 40$ GeV) we have obtained the estimate:

$$\tilde{\sigma}^{in} \leq \frac{1}{4} \sigma_{\pi N}^{in} \quad (19)$$

If this result is not disproved it could indicate that the leading hadron is not only "cut" (in configuration sense), but loses, to a great extent, its proper field.

Note that this effect may be explained in terms of the parton model also /35/.

b) All the results of section 3 have been obtained at the same value of inelasticity parameter

$$\langle K \rangle \frac{\sigma^{in}}{\sigma^{tot}} \Big|_{K,N} = \text{const} (E_p)$$

A qualitative analysis of this peculiarity also leads to some interesting properties of the cluster hadron matter. For this purpose we consider the clusters produced in the average act at two sharply different energies of the initial proton, e.g., $E_p = 20$ GeV and $E_p = 200$ GeV. Their initial masses differ, roughly speaking, by a factor of three.

$E_0 (E_p = 200 \text{ GeV}) \approx 3E_0 (E_p = 20 \text{ GeV})$. Let us consider the cluster-nucleon interaction in the rest frame of the cluster. Then, in the case of the same radii, these clusters are characterized by different density.

The constancy of inelasticity parameter of the $K-N$ interaction means, in terms of the optical model, that both clusters "absorb" the incident nucleon flux with the same probability. Hence, the specific "absorbing" ability (per unit mass of the cluster) of a more heavy cluster is by about a factor of three smaller than that of the light one. Thus, the hadron matter of the cluster with increasing energy, at which it is produced, is "lightening": the probability of inelastic processes at the collision of the nucleon with the cluster mass element decreases.

Thus, one may hope that the study of multi-particle production in nuclear matter will be useful for understanding some important aspects of the dynamics of strong interactions.

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