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1. *Introduction*

Recent experiments on production of particles with large transverse momenta in hadron - hadron collisions at high energies have revealed a definite change in cross section behaviour as compared with that in the small transverse momentum region [1,2]. Some specific features of the processes in question are as follows: A steep decrease of cross sections with growing $p_\perp$ at fixed s, the increase of cross sections with energy at large fixed transverse momenta $p_\perp$, the appearance of appreciable correlations between particles with large $p_\perp$ and other secondaries, etc. Here we especially note experimental indications [3,4] of the appearance of essential dependence of the so-called associated multiplicity on the transverse momentum at $p_\perp \gtrsim 1$ *GeV/c*.

By modern ideas on particle interactions at high energies [5] the hadron production at large transverse momenta corresponds to the region of small space-time distances. A smooth power behaviour of cross sections of elastic and inclusive processes with growing $p_\perp$ can be treated as an argument in favour of a composite (quark) structure of hadrons. This is supported, in particular, by agreement of predictions of the automodelity hypothesis and dimensional quark analysis [6] with the experimental data on two hadron collisions at large transverse momenta.

In this paper we consider the dependence of increasing mean multiplicity of charged secondaries on the transverse momentum on the basis of the assumption on the automodel character of the behaviour of semi-inclusive spectra. Besides, we use also some results of investigation of multiple production processes deduced in the framework of the straight-line path method [7] and coherent state model [8].

2.

A study of correlation dependences of average characteristics of hadron production may indicate only existence of certain inter-relations between secondaries. The next step in understanding the mechanism of multiparticle production is to investigate the behaviour of the one particle distributions as a function of the multiplicity of secondaries. There arises the question: What restrictions on the shape and character of dependence of the single-particle distributions on n and $\vec{p}$ result from correlations between the average multiplicity and magnitude of the transverse momentum or momentum transfer $\bar{n} = \bar{n}(\vec{p})$?

Consideration of such effects is convenient to be carried out in terms of characteristics of the so-called semi-inclusive processes:

$$ab \to c \text{ (particle with large } \vec{p}_{\perp}) + n \text{ charged particles} + \text{any number of neutral particles}, \qquad /1/$$

i.e., here in the reaction, where only one of the secondaries which after interaction receives a large transverse momentum is considered inclusively.

The average number of charged secondaries at fixed transverse momentum of the detected particle, i.e. the associated multiplicity is defined as follows

$$\langle n(\vec{p})\rangle = \frac{\sum\limits_n n F_n(\vec{p}, s)}{\sum\limits_n F_n(\vec{p}, s)} . \qquad /2/$$

Here $F_n(\vec{p}, s)$ is the differential one-particle distribution of the detected particle at a given number of additional charged particles n :

$$F_n(\vec{p}, s) \approx \frac{d\sigma_n}{d\vec{p}_{\perp}} , \quad d\vec{p} = \frac{d^3 p}{E} . \qquad /3/$$

In formula (3) the variable $p_{||}$ is fixed.
It would be recalled here that summation of (3) over the number of all charged particles results, by definition, in the one-particle inclusive distribution [9]

$$\frac{d\sigma}{dp_{\perp}^2}(a + b \to c_{p_{\perp}} + \text{anything}) = \sum F_n(p_{\perp}, s) . \qquad /4/$$

One can also introduce the equivalent to (2) definition of the associated multiplicity which clearly demonstrates the correlation character of this quantity

$$\langle n(p_{\perp})\rangle = \int d\vec{q}_{\perp} \frac{d\sigma}{d\vec{p}_{\perp}\, d\vec{q}_{\perp}} \Big/ \frac{d\sigma}{d\vec{p}_{\perp}} . \qquad /5/$$

From formula (5) it is seen, in particular, that if there are correlations between particles with momenta $\vec{p}$ and $\vec{q}$, the associated multiplicity for the inclusive production of the particle with momentum $\vec{q}$ does not depend on $\vec{p}$, i.e. $\langle n(p_{\perp})\rangle = \langle n\rangle_{tot.} - 1$.

Note that in accordance with the total momentum conservation a large transverse momentum $\vec{p}_{\perp}$ of the detected particles is compensated by the whole transverse momentum of the group of other particles that causes a strong correlation between them.

When choosing a concrete form of dependence of the average number of particles on the transverse momentum one should take into consideration the mechanism of multi-particle production. Proceeding from the assumption on the coherent excitation of particles colliding at high energies one can obtain that the average number of secondaries increases linearly with the squared transverse momentum transferred [8]:

$$\langle n(p_{\perp})\rangle = a + bp_{\perp}^2 . \qquad /6/$$

Within the framework of the straight-line path method, this result has been derived for the diffractive production of secondaries in papers /7, 10/. Such behaviour is in qualitative agreement with experimental data on pp -collisions at the lab. momentum of the incident proton $p_{lab.} \approx$ ~ 30 *GeV/c* (see *Fig. 1*). Analogous phenomenon follows also from the hypothesis of limiting fragmentation where the growth of $<n>$ with $p_\perp$ arises due to the impossibility of giving large transverse momentum to a hadron without its break up. Note that in the multiperipheral model /12/ the average multiplicity decreases logarithmically with growing $p_\perp$ *. . This point, apparently, reflects the fact that the multiperipheral model corresponds mainly to the mechanism of secondary production connected with the appearance of hadron clusters in a central region, while the results of the coherent state model, the straight-line path method and fragmentation principle correspond to the mechanism of diffractive dissociation of colliding particles.

A direct experimental examination of dependence of the mean (associated) multiplicity on the particle transverse momentum is thus of great interest for testing theoretical models.

Furthermore, proceeding from considerations of physical similarity which reveals itself in a number of observed properties of particle interactions at high energies, one can assume that shape of the dependence $<n(\vec{p})> = f(\vec{p})$ will affect the character of asymptotic behaviour of cross sections of the semi-inclusive processes.

Let us assume, for instance, that the semi-inclusive cross sections obey the similarity relation

$$\frac{d\sigma_n}{d\vec{p}_\perp} = A(p_\perp^2)\ \psi(n/f(\vec{p}_\perp)) . \qquad /7/$$

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*At the same time, within the multiperipheral scheme it is possible to reproduce the growth of spectra with energy and their power decrease $p_\perp^{-8}$ at large transverse momenta /13/.

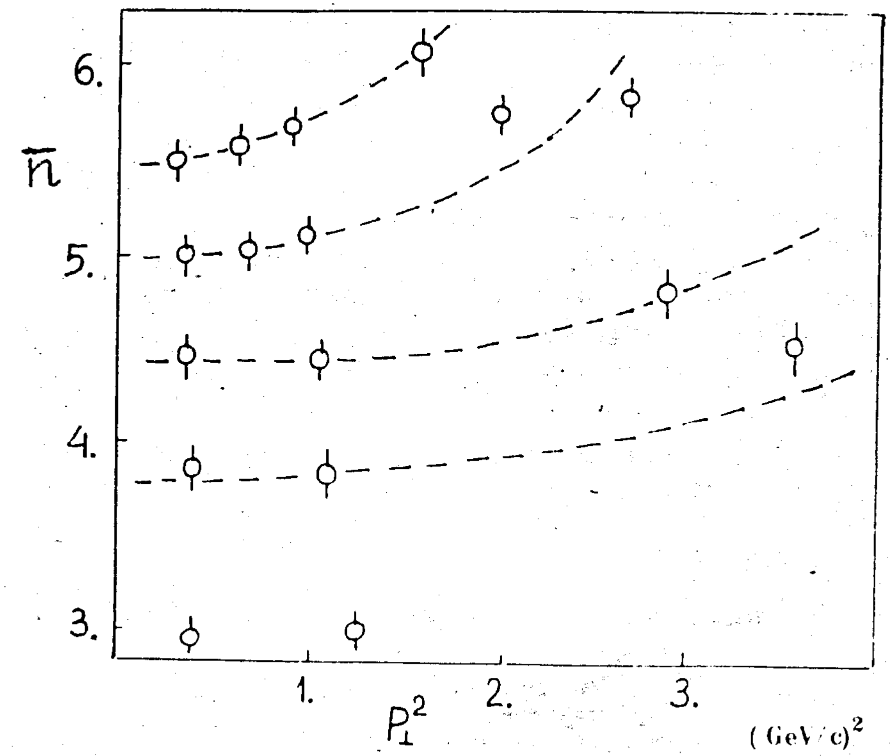

Fig. 1. Dependence of the average multiplicity of charged particles on the momentum transfer squared at $p_{lab.} \approx$ ≈ 30 *GeV/c* /4/. *Dashed lines are drawn by hand and correspond to different values of MM.*

Substituting this relation into formula /2/ for the associated multiplicity and passing over from summation to integration we get

$$\langle n(\vec{p}_\perp)\rangle = \frac{\sum_n^{N_s} n F(p_\perp, s)}{\sum_n^{N_s} F(p_\perp, s)} \approx \frac{\int^{N_s} n\, dn\, \psi(n/f(\vec{p}_\perp))}{\int^{N_s} dn\, \psi(n/f(\vec{p}_\perp))} = f(p_\perp)\cdot g(N_s/f(p_\perp)), \qquad /8/$$

where $N_s \sim \sqrt{s}$.

Thus, the function $f(\vec{p}_\perp)$ really represents the dependence of the associative multiplicity $\langle n(\vec{p}_\perp)\rangle$ on momentum provided that:

$$g(N_s / f(\vec{p}_\perp)) \to 1 \quad \text{for} \quad s \to \infty,\ p_\perp\text{-fixed}. \qquad /9/$$

A deviation from the asymptotic limit /9/ may appear only in the region, where

$$f(\vec{p}_\perp) / \sqrt{s} \sim 1. \qquad /10/$$

If the function $f_{p_\perp}$ has the power asymptotic form

$$f_{p_\perp} \sim p_\perp^{\alpha}, \qquad /11/$$

then the condition /10/ corresponds to relatively small transverse momenta

$$p_\perp \sim s^{1/2\alpha}, \qquad /12/$$

i.e. to the value of the parameter $x_\perp = 2p_\perp / \sqrt{s}$ tending to zero with increasing s.

Note further that the function $A(p_\perp^2)$ defined by /7/ can be ralated to the inclusive cross section

$$\frac{d\sigma}{d\vec{p}_\perp} = \sum_n \frac{d\sigma_n}{d\vec{p}_\perp} \approx A(\vec{p}_\perp^{\,2})\, f(\vec{p}_\perp). \qquad /13/$$

Making use of formulae /7/, /8/ and /13/, one can easily establish the validity of the following relation*

$$\langle n(\vec{p}_\perp)\rangle \frac{d\sigma_n}{d\vec{p}_\perp} \Big/ \frac{d\sigma}{d\vec{p}_\perp} = \psi(n/\langle n(\vec{p}_\perp)\rangle). \qquad /14/$$

The similarity relation /14/ is the basic result of this paper. This relation being analogous to the KNO - scaling [14] is based only on the general considerations of physical similarity and does not employ, in particular, the assumption of Feynman scaling.

Therefore, the present relation can be considered as a particular manifestation of automodelity specific for a wide class of phenomena in particle interactions at high energies.

3.

As an illustration we consider the concrete function $\psi(z)$ obtained in models of the diffractive type [15]:

$$\psi(z) = z^{-2} e^{-c/z}, \quad z = n/p_\perp^{2\alpha}. \qquad /15/$$

The relevant semi-inclusive cross section /7/ obeying the automodelity law /14/ is, in general, a function of two-variables n and $p_\perp$ and defines, as a matter of fact, two physical projections at fixed values of one of the variables (see Figs. 2,3,4).

Note that the topological distributions (at fixed values of n) at large $p_\perp$ are characterized by "flattening" of a curve describing them, with growing multiplicity (i.e., broadening of the distribution). The inclusive cross sec-

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*See Preprint JINR P2-8670, Dubna (1975).

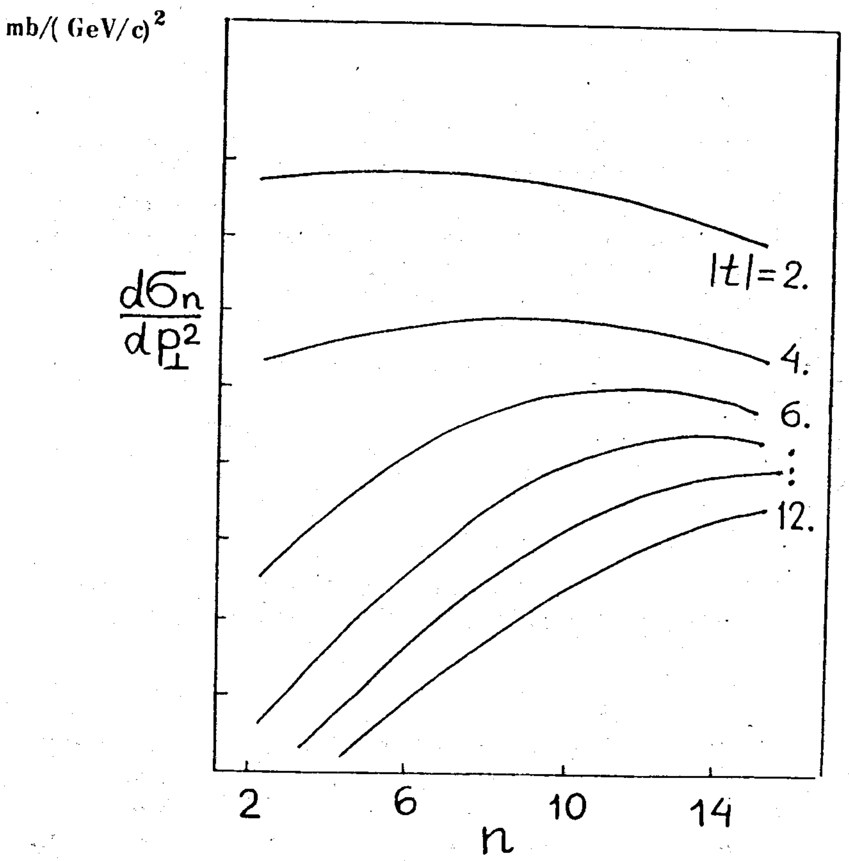

Fig. 2. The semi-inclusive cross section as a function of the multiplicity at different fixed values of $|t| = p_\perp^2$.

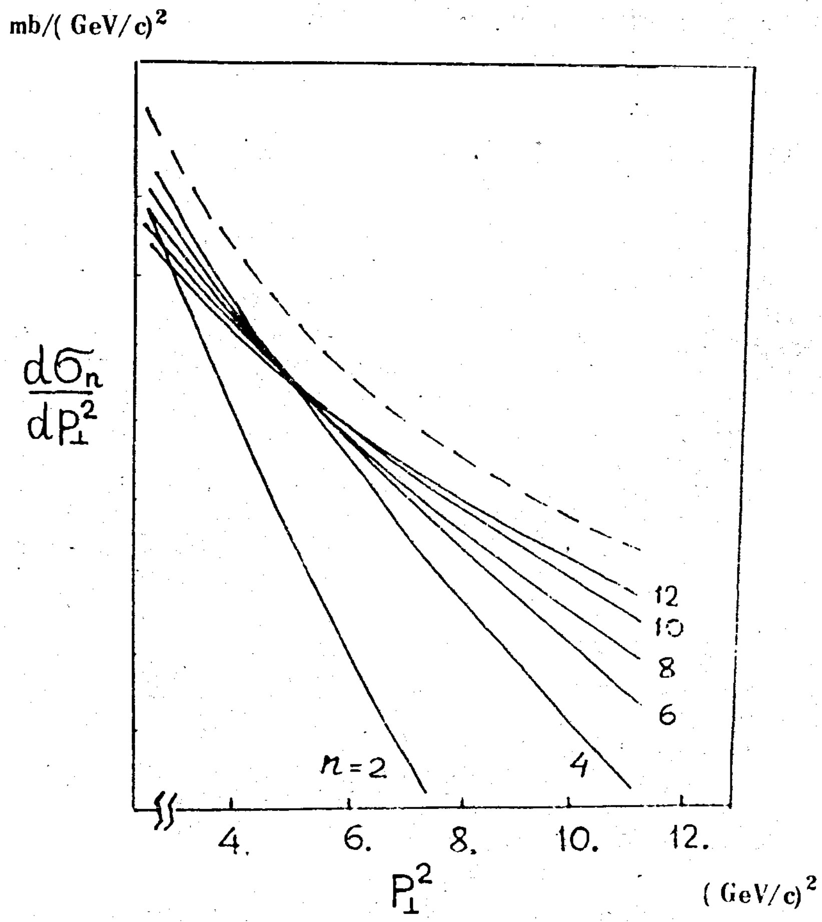

Fig. 3. Semi-inclusive spectra for different n *versus transfer momentum squared in the region of large* $p_\perp$. *The dashed line stand for the summed (inclusive) distribution.*

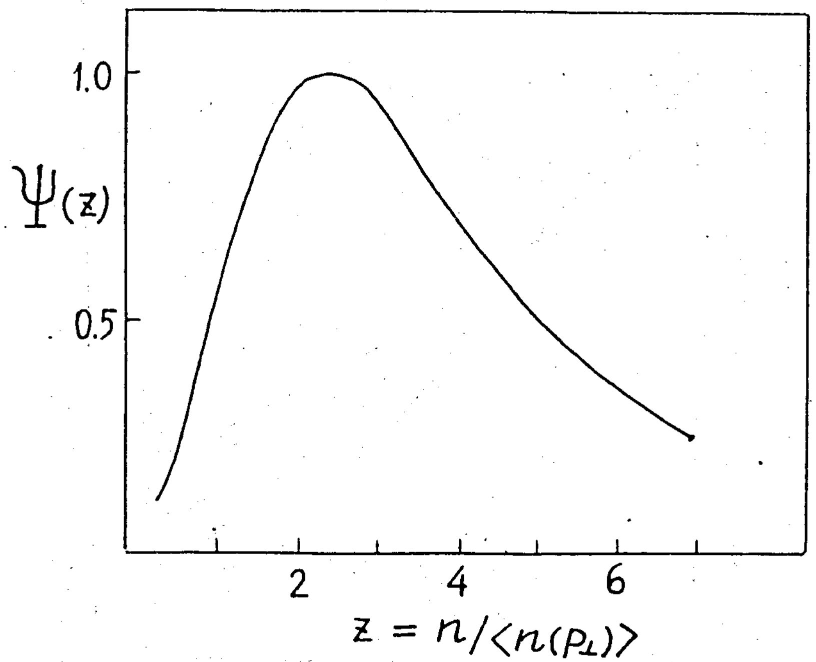

Fig. 4. Depedence of the model function $\psi(n/\langle n_{p_\perp}\rangle) = \langle n_{p_\perp}\rangle \frac{d\sigma_n}{d\vec{p}} \Big/ \frac{d\sigma}{d\vec{p}}$ *on the variable* $z = \frac{n}{\langle n_{p_\perp}\rangle}$.

tions corresponding to such topological distributions are consistent with the power asymptotic behaviour of the form

$$\frac{d\sigma}{dp_\perp^2} \sim \frac{1}{(p_\perp^2)^{2+a}}\left[\exp\left(-\frac{cp_\perp^{2a}}{\sqrt{s}}\right) - \exp\left(-\frac{cp_\perp^{2a}}{2}\right)\right]. \qquad /16/$$

We emphasize here that the given by /7/ distributions at small $p_\perp$, with increasing multiplicity, show a "shrinkage" /16/ which corresponds to associated multiplicities weakly dependent on $p_\perp$ /17/; $\langle n(p_\perp)\rangle \approx \mathrm{const}$. This fact points out the change in the behaviour of the observed cross sections in going over a certain critical value $p_\perp^{(0)} \approx 1$ *GeV/c*.

In models of the diffractive type the associated multiplicity at transverse momenta $p_\perp \gtrsim p_\perp^{(0)}$ approximately has the power dependence

$$\langle n(p_\perp)\rangle \approx (ap_\perp)^{2a}. \qquad /17/$$

In this connection note the fact that the assumptions, made in the framework of our consideration, make it possible to establish a relation between the effective degree of fall for the inclusive cross-sections at large $p_\perp$ and the increasing character of the associated multiplicity relative to $p_\perp$. This correlation depends on the range of $x_\perp$. There are experimental indications /18/ on such an effective dependence of power decrease on the interval of variables $x_\perp$. It is possible to describe the inclusive spectra at large $p_\perp$ not by a single term of the type /16/, but by their superposition with various N. Note that the appearance of an effective $x_\perp$ dependence of the degree of the value N, may be interpreted as a result of the competition of several different dynamical mechanisms. The considered example shows that a detailed study of specific phenomenological schemes on the basis of coherent excitation nature together with the automodelity hypothesis for large transverse momenta is an interesting aspect for investigation of the multi-particle production mechanism at high energies.

4.

Fig. 5 shows a distribution of the experimental quantity corresponding to the left-hand side of eq. /14/ obtained by analysing the semi-inclusive characteristics of $\pi^{\pm}$-mesons of $\pi^- p$-interaction at $p = 40$ *GeV/c*. The data are obtained on the basis of the processing of about 6000 inelastic $\pi^- p$-events detected in two-meter propane chamber of the JINR irradiated by (40.00 ± 0.24) *GeV/c* π^--mesons at the Serpukhov accelerator.

We stress here that the experimental points corresponding to the two-dimensional distributions $\frac{d\sigma_n}{dp_\perp} = f(n, p_\perp)$ with different values of charged particle multiplicity $n = 2 \div 12$ and to the whole measured range of $p_\perp$ in the scale $z = \frac{n}{\langle n(p_\perp)\rangle}$ according to formula /14/ are on the same universal curve. It is interesting to note that according to the consideration analogous to that given in 2 one can easily obtain a similarity relation for the semi-inclusive characteristics depending on the rapidity y

$$\langle n(y)\rangle \frac{d\sigma_n}{dy} / \frac{d\sigma}{dy} = \psi\left(\frac{n}{\langle n(y)\rangle}\right). \qquad /18/$$

It is just sufficient to assume the correlation between the associated multiplicity and the rapidity of the detected particle C.

It is seen from fig. 6 that this relation is confirmed by the data obtained in the $\pi^- p$-collisions at 40 *GeV/c*.

As has been mentioned, to the decreasing character of the associated multiplicity there corresponds a "shrinkage" of the semi-inclusive distributions, i.e., at small $p_\perp$ the probabilities of production of a large number of particles drop much faster than those for small multi-

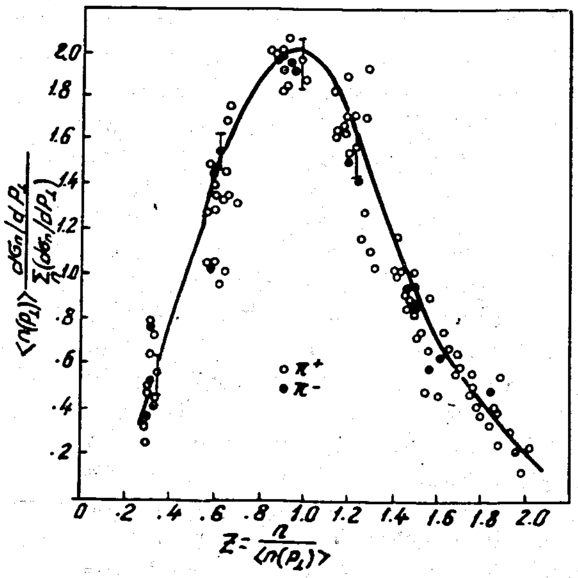

Fig. 5. *The dependence of* $\langle n(p_\perp)\rangle \frac{d\sigma}{d\vec{p}_\perp} / \frac{d\sigma}{d\vec{p}_\perp}$ *on* $\frac{n}{\langle n(p_\perp)\rangle}$ *due to the data of the* $\pi^- p$ *-interaction* $(\pi^- p \to \pi^\pm X_{ch} \ldots)$ *at* $p = 40$ *GeV/c (the collaboration of the 2-meter propane chamber of JINR, IHEP accelera- -tor).*

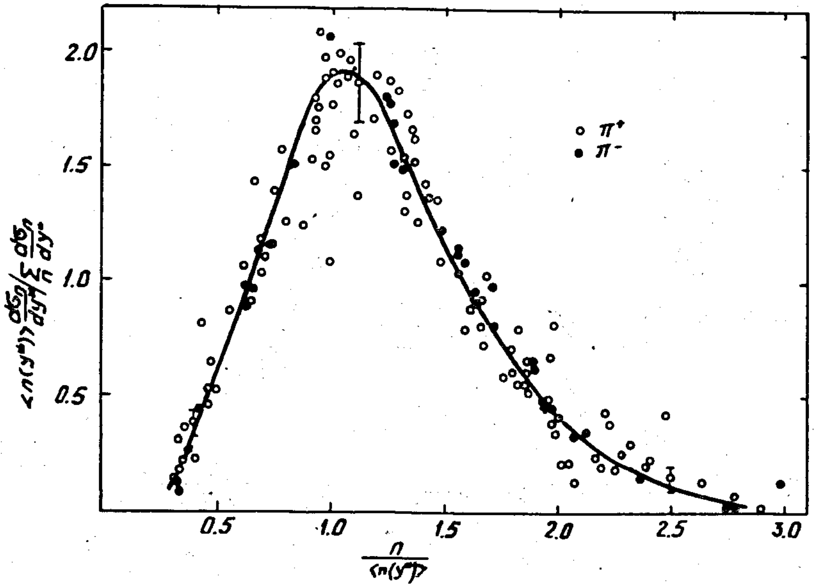

Fig. 6. The dependence of $\langle n(y^*)\rangle \frac{d\sigma}{dy^*} \Big/ \frac{d\sigma}{dy^*}$ *on* $\frac{n}{\langle n(y^*)\rangle}$ *due to the data of the* $\pi^- p$ *-interaction* ($\pi^- p \to \pi^\pm x_{ch} \ldots$) *at* $p =$ *40 GeV/c (The collaboration of the 2-meter propane chamber of JINR, IHEP accelerator).*

plicities. Further, the smallness on $n \leftrightarrow p_\perp$ correlations for $p_\perp \approx p_\perp^{(0)}$ i.e., an approximate constancy $\langle n(p_\perp)\rangle \approx$ const, means that a degree of fall of the cross sections for small and large multiplicities with growing $p_\perp \to p_\perp^{(0)}$ becomes the same.

On the other hand, the growth of $\langle n(p_\perp)\rangle$ as a function of $p_\perp$ corresponds to the transition to a new regime: at increasing $p_\perp$ the cross sections with large n become more flat than for small n (the so-called "broadening" of distributions).

Thus, the regions of small and large $p_\perp$ are clearly separated by essentially different regimes of behaviour both for the inclusive and semi-inclusive cross sections and for the moments of these distributions. A relation between the semi-inclusive distributions and associated multiplicities in definite combination (14) with an essentially different behaviour at small and large transverse momenta indicate a certain universality of the similarity law obtained for semi-inclusive spectra /14/.

To conclude we note the following two points. The analysis of the behaviour of the associated multiplicities reveals that the growth of $\langle n(p_\perp)\rangle$ is due to the particles emitted in the hemisphere opposite to fixed particles with large $p_\perp$, and in an "accompanying" hemisphere $\langle n(p_\perp)\rangle$ is a decreasing quantity. Thus, it is, in general, necessary to perform a separation of events into those the "same" and "opposite" to a detected particle since without this selection the distributions may mix and give average effects. And finally, we note that in order to study the transverse distributions in a wide range of $p_\perp$ it is necessary to analyse a multi-component description which requires a joint consideration of production both of soft particles corresponding to the statistical mechanism of hadron production in the central region, and of hadron clusters due to the mechanism of particle coherent excitation at high energies.

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