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ON THE ENERGY-MOMENTUM TENSORS FOR FIELD THEORIES IN SPACES WITH AFFINE CONNECTION AND METRIC.

Conditions for Existence of Energy-Momentum Tensors

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I. INTRODUCTION

The purpose of this paper is the relations between the energy-momentum tensors to be found as well as the connections between them and the covariant Euler-Lagrange equations. In Sec. I. conditions for the existence of the symmetric energy-momentum tensor as a local conserved quantity will be considered as well as relations between them and the covariant Euler-Lagrange equations. In Sec. II. connections between the symmetric energy-momentum tensor of Belinfante and that of Hilbert (introduced by means of the functional variation of the Lagrangian density along the components of the metric tensor g_{ij}) will be found. In Sec. III. the basic relations between all elements of the structure of the Lagrangian system connected with the energy-momentum tensors will be given in a scheme. As an example of all these relations, Einstein's theory of gravitation will be investigated from a more general point of view for existence of energy-momentum tensors for the gravitational field in vacuum. At the end some conclusions will be drawn about the construction of field theories on the basis of their structure, connected with the energy-momentum tensors.

II. ENERGY-MOMENTUM TENSORS AND COVARIANT EULER-LAGRANGE EQUATIONS

1. The covariant Euler-Lagrange equations (CELE) for the field $V^A{}_B$ can be found by means of the functional variation $\delta\mathcal{L}$ of the Lagrangian density $\mathcal{L}$ under the condition

$$\delta\mathcal{L} = 0 , \qquad (2.1)$$

$$\delta g_{ij} = 0 , \quad \delta . E_i = E_i . \delta , \quad \delta\Gamma^i_{jk} = 0 , \qquad (2.1a)$$

in the form

$$\delta L = \frac{\partial L}{\partial v^A{}_B}.\delta v^A{}_B + \frac{\partial L}{\partial v^A{}_{B/i}}.\delta(v^A{}_{B/i}) + \frac{\partial L}{\partial v^A{}_{B/i/j}}.\delta(v^A{}_{B/i/j}) = 0 . \quad (2.2)$$

The prerequisites (2.1-2.1a) are sufficient conditions for the commutation between the variation δ and the covariant derivative $_{/i}$ along the basic vector field E_i, i.e.

$$\delta(v^A{}_{B/i}) = (\delta v^A{}_B)_{/i} . \quad (2.3)$$

Using (2.3), δL can be written in the form:

$$\delta L = \frac{\delta L}{\delta v^A{}_B}.\delta v^A{}_B + j^i{}_{/i} , \quad (2.4)$$

where

$$j^i = [\frac{\partial L}{\partial v^A{}_{B/i}} - (\frac{\partial L}{\partial v^A{}_{B/i/j}} + \frac{\partial L}{\partial v^A{}_{B/j/i}})_{/j}].\delta v^A{}_B + $$

$$+ (\frac{\partial L}{\partial v^A{}_{B/j/i}}.\delta v^A{}_B)_{/j} \quad (2.5)$$

Under the conditions

$$\delta v^A{}_B{}_{/(V^n)} = 0 , \delta(v^A{}_{B/j})_{/(V^n)} = 0 \text{ (or } (\delta v^A{}_B)_{,j}{}_{/(V^n)} = 0) \quad (2.6)$$

for the variation of $v^A{}_B$ on the boundary of the volume V^n, for which the action

$$S = \int_{V^n} L.d\omega = \int_{V^n} \mathcal{L} \, d^{(n)}x$$

is defined, the CELE follow.

2. The connections between the CELE for a given Lagrangian density $\mathcal{L}$ and the energy-momentum tensors can be investigated on the ground of the GCBI. At that, the following propositions can be proved:

Proposition 1. The covariant Euler-Lagrange equations are sufficient conditions for the equality between $\Theta_i{}^j$ and ${}_sT_i{}^j$.

Proof: From CELE and the expression (Part I - 3.8) for $Q_i{}^j$, it follows that $Q_i{}^j = 0$. From the identity (I-3.30) and $Q_i{}^j = 0$, it follows $\Theta_i{}^j = {}_sT_i{}^j$.

Proposition 2. The necessary and sufficient conditions ($V^A{}_B \neq \varphi$) for the existence of ${}_sT_i{}^j$ as a local conserved quantity, i.e.

$$ {}_sT_i{}^j{}_{/j} = 0 \ , \qquad (2.7) $$

are the conditions

$$ Q_i{}^j{}_{/j} + F_i = 0 \ . \qquad (2.8) $$

Proof: a) Necessity: from (2.7) and the identity (I-3.29) and (I-3.30), it follows (2.8).

b) Sufficiency: from (2.8) and the identities (I-3.29) and (I-3.30), it follows (2.7).

Proposition 2.1. The necessary and sufficient conditions for the existence of ${}_sT_i{}^j$ as a local conserved quantity (2.7) for Lagrangian densities, constructed only of scalar fields (and their first and second covariant derivatives), i.e. for $V^A{}_B = \varphi$, are the conditions

$$ F_i = 0 \quad \text{or} \quad \frac{\delta L}{\delta \varphi} . \varphi_{/i} + W_i = 0 \ . \qquad (2.9) $$

Proof: a) Necessity: from (2.7) and the identities (I-3.29) and (I-3.30) under the condition $Q_i{}^j = 0$ for scalar fields, follows (2.9).

b) Sufficiency: from (2.9) and the identities (I-3.29) and (I-3.30) under the condition $Q_i{}^j = 0$, follows (2.7)

Proposition 2.2. The necessary and sufficient conditions for the existence of $\Theta_i{}^j$ as a local conserved quantity, i.e.

$$ \Theta_i{}^j{}_{/j} = 0 \ , \qquad (2.10a) $$

are the conditions

$$ F_i = 0 \ . \qquad (2.10b) $$

Proof: It follows directly from the identity (I-3.29).

Proposition 3. Sufficient conditions for the existence of ${}_sT_i{}^j$ as a local conserved quantities (2.7) are the conditions

$$ \frac{\delta L}{\delta V^A{}_B} = 0 \ , \quad W_i = 0 \ . \qquad (2.11) $$

Proof: From the identities (I-3.29) and (I-3.30) and the condition (2.11), one obtains (2.7)

Proposition 4. Sufficient conditions for the existence of ${}_sT_i{}^j$ as a local conserved quantity (2.7) are the conditions

$$Q_i{}^j = 0 \ , \ F_i = 0 \ . \qquad (2.12)$$

Proof: From the identities (I-3.29) and (I-3.30) and the conditions (2.12), follows (2.7).

III. SYMMETRIC ENERGY-MOMENTUM TENSOR OF BELINFANTE AND SYMMETRIC ENERGY-MOMENTUM TENSOR OF HILBERT

1. The symmetric energy-momentum tensor ${}_sT_i{}^j$, defined in (I-3.16), is equal to $\Theta_i{}^j$ (s. (I-3.13)) if the covariant Euler-Lagrange equations are fulfilled. In this case ${}_sT_i{}^j$ can be considered as a tensor constructed on the basis of the canonical tensor $t_i{}^j$ by means of adding the term $K_i{}^j$ (I-3.3)(when L is depending on the second covariant derivative of $V^A{}_B$) and the term $W_i{}^{jk}{}_k$. The last one is analogous to the composition of Belinfante terms /1-3/, proposed for symmetrization of $t_i{}^j$ and obtaining ${}_sT_i{}^j$ (if $K_i{}^j = 0$) in V_n-spaces /5/ for Lagrangian densities depending on $V^A{}_B$ and their first (and second) partial derivatives with respect to the coordinates /1,2,7,10/. This method for finding ${}_sT_i{}^j$ from $t_i{}^j$ by means of $W_i{}^{jk}{}_k$ (= $W_i{}^{jk}{}_{/k}$ in V_n-spaces) has been called Belinfante method; and the tensor ${}_sT_i{}^j$, symmetric energy-momentum tensor of Belinfante. In this method the GCBI from which in a natural way the structure of $\Theta_i{}^j$ and ${}_sT_i{}^j$ appeare, are not considered.

2. In Einstein's theory of gravitation (ETG) the symmetric energy-momentum tensor for material distribution is defined by means of the functional variation of the Lagrangian density for material fields $V^A{}_B$ with respect to the components of the metric tensor g_{ij} and its first (and second) partial derivatives

with respect to the coordinates /2,3,9,12,13/, i.e.

$$ {}_s\bar{T}^{ij} := - \frac{2}{\sqrt{-g}} . \frac{\delta \varkappa}{\delta g_{ij}} \quad \text{or} \quad {}_s\bar{T}_i{}^j := - \frac{2}{\sqrt{-g}} . \frac{\delta \varkappa}{\delta g_{jk}} . g_{ik} , \qquad (3.1) $$

where

$$ \frac{\delta \varkappa}{\delta g_{ij}} = \frac{\partial \varkappa}{\partial g_{ij}} - \left(\frac{\partial \varkappa}{\partial g_{ij,k}}\right)_{,k} + \left(\frac{\partial \varkappa}{\partial g_{ij,kl}}\right)_{,lk} = $$

$$ = \sqrt{-g}\left[\frac{\partial L}{\partial g_{ij}} + \frac{1}{2} . g^{ij} . L - \left(\frac{\partial L}{\partial g_{ij,k}}\right)_{,k} + \left(\frac{\partial L}{\partial g_{ij,kl}}\right)_{,lk}\right] . \qquad (3.2) $$

In this way ${}_s\bar{T}_i{}^j$ can be written in the form

$$ {}_s\bar{T}_i{}^j = - 2 . \frac{\partial L}{\partial g_{jk}} . g_{ik} - g_i^j . L + 2\left(\frac{\partial L}{\partial g_{jk,l}}\right)_{,l} . g_{ik} - 2\left(\frac{\partial L}{\partial g_{jk,lm}}\right)_{,ml} . g_{ik} . \qquad (3.3) $$

The method for obtaining the symmetric energy-momentum tensor ${}_s\bar{T}_i{}^j$ by means of variation of $\varkappa$ with respect to g_{ij} is called Hilbert method and ${}_s\bar{T}_i{}^j$ – symmetric energy-momentum tensor of Hilbert /2/.

3. The symmetric tensor ${}_sT_i{}^j$ which has the form (I-3.16) for the Lagrangian density (I-0.2) (where $V^A{}_B \neq g_{ij}$) can be rewritten using the equality

$$ 2 . \frac{\partial L}{\partial g_{jk}} . g_{ik} + g_i^j . L = \frac{2}{\sqrt{-g}} . \frac{\delta \varkappa}{\delta g_{jk}} . g_{ik} , \qquad (3.4) $$

where

$$ \frac{\partial L}{\partial g_{jk/l}} = 0 , \quad \frac{\partial L}{\partial g_{jk/l/m}} = 0 , $$

in the form

$$ {}_sT_i{}^j = \mathcal{T}_i{}^j - \frac{2}{\sqrt{-g}} . \frac{\delta \varkappa}{\delta g_{jk}} . g_{ik} = \mathcal{T}_i{}^j + {}_s\bar{T}_i{}^j . \qquad (3.5) $$

In this case ${}_sT_i{}^j$ will be equal to ${}_s\bar{T}_i{}^j$, defined by the expression

$$ {}_s\bar{T}_i{}^j = - \frac{2}{\sqrt{-g}} . \frac{\delta \varkappa}{\delta g_{jk}} . g_{ik} , \qquad (3.6) $$

if $\mathcal{T}_i{}^j = 0$.

For more general cases, when L depends not only on the components g_{ij} but also on their first (and second) covariant derivatives, i.e.

$$L = L(g_{ij}, g_{ij/k}, g_{ij/k/l}, V^A{}_B, V^A{}_{B/i}, V^A{}_{B/i/j}) , \quad (3.7)$$

and the functional variation of $\mathscr{L} = \sqrt{-g}.L$ with respect to g_{ij} will have the form

$$\frac{\delta\mathscr{L}}{\delta g_{ij}} = \frac{\partial\mathscr{L}}{\partial g_{ij}} - \left(\frac{\partial\mathscr{L}}{\partial g_{ij/k}}\right)_{/k} + \left(\frac{\partial\mathscr{L}}{\partial g_{ij/k/l}}\right)_{/l/k} \quad (3.8)$$

and

$${}_s\bar{T}_i{}^j = -\frac{2}{\sqrt{-g}}.\frac{\delta\mathscr{L}}{\delta g_{jk}}.g_{ik} \quad , \quad (3.9)$$

${}_sT_i{}^j$ can be written in the form

$${}_sT_i{}^j = -\frac{2}{\sqrt{-g}}.\frac{\delta\mathscr{L}}{\delta g_{jk}}.g_{ik} + \bar{\mathcal{T}}_i{}^j = {}_s\bar{T}_i{}^j + \bar{\mathcal{T}}_i{}^j \quad , \quad (3.10)$$

where

$$\bar{\mathcal{T}}_i{}^j = \mathcal{T}_i{}^j - \frac{2}{\sqrt{-g}}\left[\left(\frac{\partial\mathscr{L}}{\partial g_{jk/l}}\right)_{/l} - \left(\frac{\partial\mathscr{L}}{\partial g_{jk/l/m}}\right)_{/m/l}\right] g_{ik} \quad . \quad (3.11)$$

Using (3.10), the following proposition can be proved in a very standart way:

Proposition 5. The necessary and sufficient conditions for the equality of the symmetric energy-momentum tensor of Belinfante, defined in (I-3.16) and the symmetric energy-momentum tensor of Hilbert, defined in (3.9) or in (3.6), are the conditions

$$\bar{\mathcal{T}}_i{}^j = 0 \quad . \quad (3.12)$$

Example: In the case of Lagrangian density for the electromagnetic field in V_n-space

$$\mathscr{L}_{el} := \sqrt{-g}.F_{ij}F^{ij} = \sqrt{-g}.L_{el} \quad , \quad (3.13)$$

where

$$F_{ij} := A_{j/i} - A_{i/j} = A_{j,i} - A_{i,j} \quad ,$$

$$\bar{\mathcal{T}}_i{}^j = 0 \text{ and } {}_sT_i{}^j = {}_s\bar{T}_i{}^j \quad .$$

Proposition 5.1. For the Lagrangian density of the type

$$\mathscr{L} := \sqrt{-g}.L(g_{ij}, V^A{}_B) \quad (3.14)$$

the condition

$${}_sT_i{}^j = {}_s\bar{T}_i{}^j \quad (3.15)$$

is always valid.

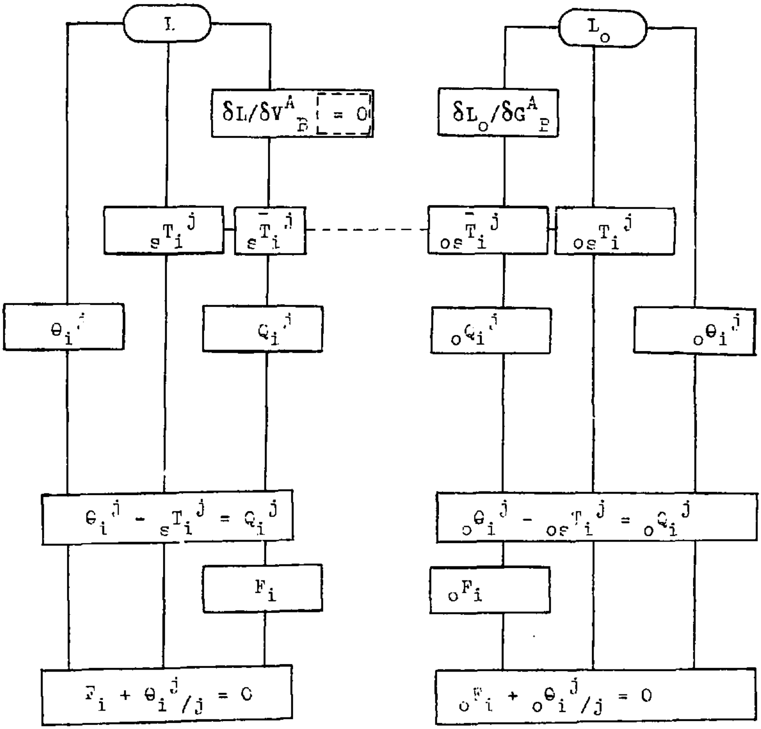

Fig.

Elements and connections of the classical Lagrangian theory

Proof: From (3.11), (I-3.16-3.18, 3.25-3.27) and (I-3.4-3.7), it follows that $\mathcal{T}_i^{\ j} = 0$. From (3.5) and $\mathcal{T}_i^{\ j} = 0$ follows (3.15)

<u>Remark:</u> In the case of (3.14) an important role plays the fact that in the structure of $\mathcal{T}_i^{\ j}$ only the covariant derivatives of $V^A{}_B$ appear and not $V^A{}_B$ themselves. Ther fore, if L does not depend on the covariant derivatives of $V^A{}_B$, then $\mathcal{T}_i^{\ j} = 0$.

In the general case of the Lagrangian density (I-0.2) or (3.7) the SEMT(B) ${}_sT_i^{\ j}$ differs from the SEMT(H) ${}_s\bar{T}_i^{\ j}$ by the symmetric part $\mathcal{T}_i^{\ j}$ (or $\bar{\mathcal{T}}_i^{\ j}$) in contrast to the case of Lagrangian densities (I-0.1)(without the second partial derivatives of $V^A{}_B$ with respect to the coordinates), for which both tensors appear to be identical /1-3/.

III. APPLICATION OF THE ENERGY-MOMENTUM TENSORS IN FIELD THEORIES

1. The covariant Euler-Lagrange equations and the energy-momentum tensor are essential characteristics in the structure of the field theory. On the basis of the found connections between them, a rough scheme can be drawn describing the elements and the connections of the classical Lagrangian theory with respect to the CELE and the energy-momentum tensors (s.the Figure). If one has two Lagrangian systems (with Lagrangian densities $\mathcal{L} = \sqrt{-g}.L$ and $\mathcal{L}_o = \sqrt{-g}.L_o$ respectivly), which can be considered as a joint system with Lagrangian density $\mathcal{L}_t = \mathcal{L} + \mathcal{L}_o = \sqrt{-g}.L_t = \sqrt{-g}(L + L_o)$, then different relations between the elements of both schemes for L and L_o can be defined. They would describe on the one hand the field equations as well as the relations between the energy-momentum tensors, and on the other hand the relations determining the character of the interaction between the individual fields.

In most field theories in V_n-spaces, the validity of the

Euler-Lagrange equations is assumed /1,2/. That leads automatically to the equivalence between ${}_sT_i{}^j$ and $\Theta_i{}^j$, to the disappearence of $Q_i{}^j$ and to the existence of ${}_sT_i{}^j$ as a local conserved quantity.

2. On the basis of the proposed scheme for the Lagrangian theory in L_n-spaces, one can consider Einstein's theory of gravitation (ETG) from a more general point of view.

In L_n-space (n = 4) a Lagrangian density $\mathcal{L}_g$ of the type /1,4,11/

$$\mathcal{L}_g = \sqrt{-g}.L_g(g_{ij}, R^i{}_{jkl}) , \qquad (4.1)$$

can be defined, where

$$V^A{}_B := R^i{}_{jkl}$$

are the components of the curvature tensor

$$[R(E_k,E_l)]E_j = \nabla_{E_k}\nabla_{E_l}E_j - \nabla_{E_l}\nabla_{E_k}E_j - \nabla_{[E_k,E_l]}E_j = R^i{}_{jkl}E_i . \qquad (4.2)$$

The connection Γ in L_n-space is not connected with the metric g, so that g_{ij} and $R^i{}_{jkl}$ can be considered as independent to each other functions, i.e.

$$\frac{\partial R^i{}_{jkl}}{\partial g_{mn}} = 0 , \quad \frac{\partial g_{mn}}{\partial R^i{}_{jkl}} = 0 . \qquad (4.3)$$

Using the functional variation and (3.8), one can find

$$\frac{\delta L_g}{\delta R^i{}_{jkl}} = \frac{\partial L_g}{\partial R^i{}_{jkl}} , \quad \frac{\delta L_g}{\delta g_{ij}} = \frac{\partial L_g}{\partial g_{ij}} , \qquad (4.4)$$

and for ${}_gQ_i{}^j$ it follows from (I-3.8)

$${}_gQ_i{}^j = \frac{\partial L_g}{\partial R^i{}_{kmn}}.R^j{}_{kmn} - \frac{\partial L_g}{\partial R^k{}_{jmn}}.R^k{}_{imn} - 2.\frac{\partial L_g}{\partial R^k{}_{mnj}}.R^k{}_{mni} . \qquad (4.5)$$

On the other hand, for L_g, independent of the covariant derivatives of $R^i{}_{jkl}$, and of the type (4.1), one can find easily for $K_i{}^j$ in (I-3.3), $V_i{}^{kj}{}_l$ in (I-3.4) and $\mathcal{T}_i{}^j$ in (I-3.17) that they all vanish, i.e.

$$K_i{}^j = 0\ , \quad V_i{}^{kj}{}_l = 0, \quad \mathcal{T}_i{}^j = 0, \quad W_i{}^{jk}{}_k = 0, \quad W_i{}^{jk} = 0. \quad (4.6)$$

The energy-momentum tensors ${}_g t_i{}^j$, ${}_g\Theta_i{}^j$, ${}_{sg}T_i{}^j$ and ${}_gQ_i{}^j$ (s. (4.5.-4.8)) have the form

$${}_g t_i{}^j = -\, g_i^j.L_g = \Theta_i{}^j\ , \quad (4.7)$$

$${}_{sg}T_i{}^j = -\,2.\frac{\partial L_g}{\partial g_{jk}}.g_{ik} - g_i^j.L_g\ , \quad (4.8)$$

$${}_gQ_i{}^j = 2.\frac{\partial L_g}{\partial g_{jk}}.g_{ik}\ ,$$

respectively.

From the identity (I-3.30) and the form of ${}_g\Theta_i{}^j$, ${}_{sg}T_i{}^j$ and ${}_gQ_i{}^j$ the identity for L_g follows

$${}_gQ_i{}^j = 2.\frac{\partial L_g}{\partial g_{jk}}.g_{ik} = \frac{\partial L_g}{\partial R^i{}_{kmn}}.R^j{}_{kmn} - \frac{\partial L_g}{\partial R^k{}_{jmn}}.R^k{}_{imn} -$$

$$-\, 2.\frac{\partial L_g}{\partial R^k{}_{mnj}}.R^k{}_{mni} \quad (4.9)$$

For W_i and F_i we have from (I-3.22) and (I-3.21):

$$W_i = \frac{\partial L_g}{\partial g_{jk}}.g_{jk/i}\ , \quad (4.10)$$

$$F_i = \frac{\partial L_g}{\partial R^k{}_{mnl}}.R^k{}_{mnl/i} + \frac{\partial L_g}{\partial g_{jk}}.g_{jk/i}\ . \quad (4.11)$$

The functional variation of $\mathcal{L}_g$ with respect to $R^i{}_{jkl}$ is

$$\frac{\delta L_g}{\delta R^i{}_{jkl}} = \frac{\partial L_g}{\partial R^i{}_{jkl}}\ , \quad \frac{\delta \mathcal{L}_g}{\delta R^i{}_{jkl}} = \sqrt{-g}.\frac{\partial L_g}{\partial R^i{}_{jkl}}\ . \quad (4.12)$$

The functional variation of $\mathcal{L}_g$ with respect to g_{ij} will have the form

$$\frac{\delta \mathcal{L}_g}{\delta g_{ij}} = \frac{\partial \mathcal{L}_g}{\partial g_{ij}} = \sqrt{-g}\left(\frac{\partial L_g}{\partial g_{ij}} + \frac{1}{2}.g^{ij}.L_g\right)\ . \quad (4.13)$$

The covariant Euler-Lagrange equations for the field variables g_{ij} are

$$\frac{\delta \mathscr{L}_g}{\delta g_{ij}} = 0 \text{ or } \sqrt{-g}\left(\frac{\partial L_g}{\partial g_{ij}} + \frac{1}{2}.g^{ij}.L_g\right) = 0 \text{ , i.e.} \tag{4.14}$$

$$\frac{\partial L_g}{\partial g_{jk}}.g_{ik} + \frac{1}{2}.g_i^j.L_g = 0 \text{ .}$$

The last expression is equal up to a factor -2, to the expression (4.8) for ${}_{sg}T_i{}^j$, i.e.

$${}_{sg}T_i{}^j = -\frac{2}{\sqrt{-g}}.\frac{\delta \mathscr{L}_g}{\delta g_{jk}}.g_{ik} \quad , \tag{4.15}$$

and from (4.14) it follows that ${}_{sg}T_i{}^j = 0$.

Therefore for a Lagrangian density of the type (4.1) (in accordance with proposition 5.1 the equality

$${}_{sg}T_i{}^j = {}_{sg}\overline{T}_i{}^j$$

is valid. If the CELE (4.14) are also fulfilled, then the symmetric energy-momentum tensor ${}_{sg}T_i{}^j$ is equal to zero and it is identical in its form with the CELE for g_{ij}.

3. When $\mathscr{L}_g$ of type (4.1) has the special form

$$\mathscr{L}_g = -\frac{1}{2\varkappa}.\sqrt{-g}(R - \lambda) = \sqrt{-g}.L_g \text{ ,} \tag{4.16}$$

where

$$L_g = -\frac{1}{2\varkappa}(R - \lambda) \text{ ,} \quad \varkappa, \lambda = \text{const., } R = g^{kl}R_{kl} \text{ ,} \tag{4.17}$$

$$R_{kl} = g_i^j.R^i{}_{klj} \text{ ,} \tag{4.18}$$

in this case

$$\frac{\partial L_g}{\partial g_{jk}}.g_{ik} = \frac{1}{4\varkappa}(R_{ik}g^{kj} + g^{jk}R_{ki}) = \frac{1}{4\varkappa}(R_i{}^j + R^j{}_i) \text{ ,} \tag{4.19}$$

$$\frac{\delta \mathscr{L}_g}{\delta g_{jk}}.g_{ik} = \sqrt{-g}\left(\frac{\partial L_g}{\partial g_{jk}}.g_{ik} + \frac{1}{2}.g_i^j.L_g\right) = \frac{1}{2\varkappa}\sqrt{-g}\left[\frac{1}{2}(R_i{}^j + R^j{}_i) - \right.$$

$$\left. - \frac{1}{2}.g_i^j(R - \lambda)\right] \text{ ,} \tag{4.20}$$

$${}_{sg}T_i{}^j = -\frac{1}{\varkappa}\left[\frac{1}{2}(R_i{}^j + R^j{}_i) - \frac{1}{2}.g_i^j(R - \lambda)\right] \text{ .} \tag{4.21}$$

The CELE for g_{ij} will have the form

$$\frac{1}{2}(R_{ij} + R_{ji}) - \frac{1}{2}.g_{ij}(R - \lambda) = 0 \text{ ,} \tag{4.22}$$

or

$$\frac{1}{2}\cdot g^k_l(R^l{}_{ijk} + R^l{}_{jik}) - \frac{1}{2}\cdot g_{ij}(g^{kl}g^n_m R^m{}_{kln} - \lambda) = 0.$$

If the components $R^i{}_{jkl}$ of the curvature tensor are given (in some way independent of g_{ij}, that means Γ is not a metric connection) and considered as the well-known functions of the coordinates x^k, then (4.22) appear as algebraic equations for g_{ij}. There exists, however, a possibility of choosing Γ^i_{jk} as components of the Riemannian connection Γ, i.e.

$$\Gamma^i_{jk} = \Gamma^i_{kj} = \frac{1}{2}\cdot g^{il}(g_{kl,j} + g_{jl,k} - g_{jk,l}) \qquad (4.23)$$

and $g_{ij/k} = 0$. That would mean that the equations (4.22) considered as algebraic equations for g_{ij} in L_n-spaces (n=4) with given Γ^i_{jk} can be reconsidered as differential equations for g_{ij} in V_n-space (n=4) without torsion and with a given Riemannian connection, depending on g_{ij}, and their first partial derivatives with respect to the coordinates. In the last case $R_{ij} = R_{ji}$ and (4.22) receives the form of Einstein's equations for the gravitational field in vacuum (with cosmological term λ)

$$G_{ij} = R_{ij} - \frac{1}{2}(R - \lambda)g_{ij} = 0 \quad \text{or} \quad R_{ij} = \frac{1}{2}\cdot\lambda\cdot g_{ij} \qquad (4.24)$$

for which

$$_{sg}T_{ij} = -\frac{1}{\varkappa}\left[R_{ij} - \frac{1}{2}(R - \lambda)g_{ij}\right] = -\frac{1}{\varkappa}\cdot G_{ij} = 0\ . \qquad (4.25)$$

The fact that Einstein's tensor G_{ij} coincides in its form with the symmetric energy-momentum tensor $_{sg}T_{ij}$ is well-known /2,14,15/ and leads to the important conclusion that in ETG for gravitational field in vacuum, one cannot define a nonzero symmetric (or canonical) energy-momentum tensor because of the equality between the Einstein's tensor in the field equations for the gravitational field and $_{sg}T_i{}^j$, which vanishes outside the gravitational sources, considered as a Lagrangian system with Lagrangian density of the type (0.1) ($V_E \neq g_{ij}$) or (0.2) and with an energy-momentum tensor $_s\bar{T}_i{}^j$, different in the general case from $_sT_i{}^j$. This conclusion is the reason for construction of gravitational theories other than ETG. If one considers in the Figure L_o as L_g and L as L_M of material distribution, then the connections between L_g and L_M reflecting the structure of ETG in the Figure are given with dotted lines.

IV. CONCLUSION

1. In contrast to the case in V_n-spaces in U_n- and L_n-spaces ${}_sT_i{}^j$ is not a local conserved quantity, when the covariant Euler-Lagrange equations are valid for the Lagrangian system.

2. The construction of affine-metric theories for the gravitational interaction on the basis of models, using more general spaces than V_n-spaces, requires the determination not only of the field equations but also the energy-momentum tensors as local conserved quantities. Moreover, in the general case there exists a difference between the symmetric energy-momentum tensor of Belinfante and that, introduced by Hilbert.

3. In considering ETG with respect to the existence of a symmetric energy-momentum tensor for the gravitational field, one can show that such a tensor of Belinfante, identical with that of Hilbert can be found by means of MLCD. This tensor is unfortunatelly equal to Einstein's tensor, identical to the left side of the field equations for the gravitational field in ETG. For the gravitational field in vacuum it vanishes ($\lambda = 0$). This leads to the critical conclusions of Logunov /2,14,15/ with respect to ETG, found on the ground of other methods for obtaining local conserved quantities in V_n-spaces /16/.

4. The constructed scheme for description of the elements and the connections for the Lagrangian system concerning the CELE and the energy-momentum tensors reveals a possibility of finding other field equations than the CELE or other gravitational equations than Einstein's equations in ETG, or its variants in U_n- or L_n-spaces.

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