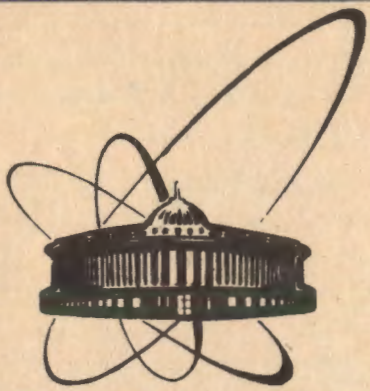

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INTERACTION OF THE COULOMB BOUND STATE OF TWO ELEMENTARY PARTICLES WITH ATOMS

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1. Introduction

Study of atom-atom interactions at high velocities has a long history. An interest to theoretical study of this problem resumes after publishing the papers where production of Coulomb bound states of elementary particles (elementary atoms) is predicted for decays $\pi^0 \to A_{2e} + \gamma$ [1], $K_L^0 \to A_{\pi\mu} + \nu$ [2] and for inclusive processes $p + p \to A_{ab} + X$ [3]. Here A_{2e} denotes a positronium atom, $A_{\pi\mu}$ denotes an atom consisting of π and μ mesons (with the opposite sign), A_{ab} denotes dimeson atoms consisting of meson pairs $\pi^+\pi^-$, πK, K^+K^-. The relativistic atoms A_{2e} [4-6] and $A_{\pi\mu}$ [7-9] have already been observed.

Knowledge of cross sections is very important for observation of dimeson atoms because of a method of its recording based on ionization (breaking up) while passing through a target. So it is necessary to calculate a total ionization probability in subsequent acts of interaction. Therefore one must know total, ionization and excitation cross sections for different initial states of elementary atoms.

A detailed analysis of previous cross section calculations for atom-atom interaction can be found in Ref.[10-13]. So I briefly recount the main results obtained in the recent papers [10-13]. In these papers the total and excitation cross sections were calculated in the first Born approximation for the ground state of elementary atoms.

In Ref.[10-12] formulas for the cross sections of elementary atoms with arbitrary masses of particles were given. For description of target atoms the Thomas-Fermi-Molier parameterization [14] was used. The authors allowed for only elastic interactions of target atoms (coherent scattering of projectile atoms).

The more accurate parameterization of target atom form factors based on the self-consistent field method of Hartree-Fock [15] (H.F.) was applied in Ref.[13]. Moreover the cross sections involving inelastic interaction of target atoms (incoherent cross sections) were calculated.

The relativistic corrections to a cross section of elementary atoms were obtained in Ref.[11-12]. These values are equal to or less than 1%.

In the paper [16] the eikonal approximation was applied to cross section calculations instead of the Born one. This approach gives a possibility of considering the contribution of all multi-photon exchanges

for an atom-atom interaction. The accuracy of this approximation for heavy atoms is about 0.2%, compared with 10% for the Born one. The total, elastic and inelastic coherent cross sections were obtained for the interaction of $A_{2\pi}$ with Ta (Z=73).

In the present paper the total, excitation and ionization cross sections are calculated in first Born approximation with the most possible accuracy.

2. Formulas

The well-known formulas of the first Born approximation were used for cross section calculations (see Ref.[13,17]). These formulas were changed to consider elementary atoms with arbitrary masses of components according to Ref.[10]. Then the cross section of atom-atom interaction is given by

$$\sigma_i^{f,k} = 8\pi \frac{\alpha^2}{\beta^2} \int_{q_{min}}^{q_{max}} | F_i^f(\eta\vec{q}) - F_i^f(\xi\vec{q}) |^2 \cdot | F_k(\vec{q}) |^2 \frac{dq}{q^3} . \qquad (1)$$

Here i and f are the initial and final state of the elementary atom, k is the final state of the target atom (initially target atoms are in the ground state), q is the transfer momentum, α is the fine structure constant, β is the velocity of a projectile elementary atom. The first factor under integral describes an elementary atom and the second one relates to a target atom.

The form factor of the elementary atom $F_i^f(\vec{q})$ is expressed as

$$F_i^f(q) = < f \; | exp(i\vec{q}\vec{r}) | \; i > . \qquad (2)$$

The variables ξ and η depend on the masses of elementary atom components:

$$\xi = m_1 / (m_1 + m_2), \quad \eta = -m_2 / (m_1 + m_2).$$

The target atom form factor $F_k(\vec{q})$ is written as

$$F_k(q) = < k \; | Z - \sum_{l=1}^{Z} exp(i\vec{q}\vec{r}_l) | \; 0 > .$$

Here summation is done over all atom electrons. If $k \neq 0$, the term Z has no

contribution because of orthogonality of wave functions. If $k=0$, the expression of the elastic form factor $F_0(q)$ in terms of the atomic form factor $F(q,Z)$ is obtained

$$F_0(q) = Z - \langle 0 \mid \sum_{l=1}^{Z} exp(i\vec{q}\vec{r}_l) \mid 0 \rangle = Z - F(q,Z).$$

At high velocity of a projectile atom the limits of integration in Eq.(1) approach their bound values [10,17]: $q_{min} \to 0, q_{max} \to \infty$.

After summation of Eq.(1) over all excited states of the target atom $k \neq 0$ the incoherent cross section (inc) is

$$\sigma_i^{f,inc} = 8\pi \frac{\alpha^2}{\beta^2} \int_0^{\infty} \mid F_i^f(\eta\vec{q}) - F_i^f(\xi\vec{q}) \mid^2 \cdot S_{inc}(q) \frac{dq}{q^3} . \qquad (3)$$

Here $S_{inc}(q)$ is the incoherent scattering function

$$S_{inc}(q) = \sum_{k \neq 0} \mid F_k(q) \mid^2 .$$

The coherent cross section (elastic for a target atom) is

$$\sigma_i^{f,coh} = 8\pi \frac{\alpha^2}{\beta^2} \int_0^{\infty} \mid F_i^f(\eta\vec{q}) - F_i^f(\xi\vec{q}) \mid^2 \cdot \mid F_0(q) \mid^2 \frac{dq}{q^3} . \qquad (4)$$

To obtain the total cross section the sum rule for a complete set of elementary atom final states is used in the form [10]

$$\sum_f \mid F_i^f(\eta\vec{q}) - F_i^f(\xi\vec{q}) \mid^2 = 2 - 2 \cdot F_i^i(\vec{q}) .$$

Here $F_i^i(\vec{q})$ is the atomic form factor of the initial state. So after summation of Eqs.(3) and (4) over all f the total cross section is written as

$$\sigma_{tot} = 16\pi \frac{\alpha^2}{\beta^2} \int_0^{\infty} \left(1 - F_i^i(\vec{q})\right) \cdot \left(\mid F_0(q) \mid^2 + S_{inc}(q) \right) \frac{dq}{q^3} . \qquad (5)$$

Therefore the σ_{tot} and other cross sections discussed below consist of two parts: the coherent cross section σ_{coh} and the incoherent one σ_{inc}.

In this approach the cross sections of elastic scattering of elementary atoms (σ_{el}) are calculated as any transition (excitation)

cross sections. Inelastic cross sections (σ_{inel}) and ionization ones (σ_{ion}) are calculated by subtracting the σ_{el} and the sum of excitation cross sections (σ_{ex}) from the correspondent total one [13,18].

$$\sigma_{inel} = \sigma_{tot} - \sigma_{el}$$
$$\sigma_{ion} = \sigma_{tot} - \sigma_{el} - \sigma_{ex} . \qquad (6)$$

Direct calculation of the σ_{ion} is difficult because the exact Coulomb wave functions of continuum are expressed through infinite series.

In the Born approximation all cross sections depend on a projectile atom velocity as $1/\beta^2$. In this paper the asymptotic cross sections at $\beta=1$ are calculated.

3. Form factors of elementary atoms

Exact wave functions of elementary (hydrogen-like) atoms are well known. Nevertheless form factors of these atoms for various initial and final states are not available (for the author). Therefore the exact analytic calculations of the form factors of interest were done using the algebraic programming system REDUCE (see Ref.[19]).

The form factor of the elementary atom Eq.(2) may be written

$$F_i^f(\vec{q}) = \int d\vec{r}\ exp(i\vec{q}\vec{r})\ \varphi_f^*(\vec{r})\ \varphi_i(\vec{r}) .$$

Here $\varphi_i(\vec{r})$ and $\varphi_f(\vec{r})$ are the wave functions. Choosing the quantization axis along the transfer momentum and denoting the initial and final states by the sets of quantum numbers (n,l,m) and (n',l',m') respectively one has

$$F_{nlm}^{n'l'm'}(\vec{q}) = \int_0^{\infty} dr\ r^2 \int_0^{\pi} d\theta\ Sin\theta\ exp(i\vec{q}\vec{r}) \int_0^{2\pi} d\phi\ \varphi_{n'l'm'}^*(\vec{r})\ \varphi_{nlm}(\vec{r}) \qquad (7)$$

The integration over ϕ leads to the selection rule of permitted transfers in the first Born approximation $m - m' = 0$. The integration over θ reduces to integrals of functions Sin and Cos in integer powers. The improper integral over r is evaluated using the formula (see, e.g., Ref.[20])

$$\int_0^\infty x^n e^{-ax} \cdot \begin{Bmatrix} Sin(bx) \\ Cos(bx) \end{Bmatrix} dx = n! \left[\frac{a^2}{a^2+b^2}\right]^{n+1} \sum_{k=0}^{[(n+1-\delta)/2]} (-1)^k \begin{bmatrix} n+1 \\ 2k+\delta \end{bmatrix} \cdot \left[\frac{b}{a}\right]^{2k+\delta}.$$

Here $\delta = \begin{Bmatrix} 1 \\ 0 \end{Bmatrix}$ corresponds to $\begin{Bmatrix} Sin(bx) \\ Cos(bx) \end{Bmatrix}$. This integration was done with the help of substitution rules.

In this way the exact analytic expressions of elementary atom form factors for any discrete-discrete transition from initial s-states were obtained. This method can be easy applied to an arbitrary initial state. Some of the form factors are given in the Appendix.

4. Form factors and incoherent scattering functions of target atoms

The hydrogen atom is described by exact formulas as elementary atom. For other atoms parameterization of H.F. was applied. There are tables of $F(q,Z)$ and $S_{inc}(q,Z)$ [15] obtained in this way practically for all chemical elements.

The transfer momentum q in Ref.[15] is expressed in terms of frequently used parameter x: $q=4\pi x$ [$Å^{-1}$]. To compute the cross section the interpolation of the $F(x,Z)$ and $S_{inc}(x,Z)$ tables over x was done. In the range $0<x<0.005$ $Å^{-1}$ the square interpolation was applied. In the range $x>10$ $Å^{-1}$ the asymptotic Bethe-Levinger formula for $F(x,Z)$ was used as in Ref.[15]. The $S_{inc}(x,Z)$ also has the asymptotic value (Z) when the value of x is higher than Z $Å^{-1}$. In the interval of x from 0.005 $Å^{-1}$ to the corresponding asymptotic boundary the $F(x,Z)$ and $S_{inc}(x,Z)$ tables were interpolated using the log-log cubic spline. Additional requirement for spline was smoothness of the first derivatives at the edges of the interpolation intervals taking into account the function behavior outside.

5. Numerical calculations and discussion

Numerical integration of Eqs.(3)-(5) gives coherent and incoherent parts of excitation and total cross sections. The accuracy of the computation was 10^{-4}. The finite upper limit of integration q_{max} was used, for atoms containing mesons it was $2\cdot10^4$ $Å^{-1}$ (≈39MeV/c) and $1.5\cdot10^3$ $Å^{-1}$(≈3MeV/c) for A_{2e}. This value defines the range of the transfer momenta significant for the interaction.

Table 1. Cross sections of A_{2e} in the 1s state. Explanations are given in the text.

	1 H		6 C		7 N		18 Ar		73 Ta	
tot	8.01-20		5.91-19		6.59-19		2.36-18		1.57-17	
f	coh	inc	coh	inc	coh	inc	coh	inc	coh	inc
tot	2.15-01	7.85-01	6.43-01	3.57-01	6.56-01	3.44-01	8.23-01	1.77-01	9.37-01	6.32-02
ex	2.57-02	2.61-01	7.51-02	1.08-01	6.23-02	9.99-02	7.55-02	5.08-02	6.98-02	2.03-02
ion	1.89-01	5.24-01	5.68-01	2.49-01	5.94-01	2.44-01	7.48-01	1.26-01	8.67-01	4.29-02
2p	1.66-02	1.82-01	4.85-02	7.49-02	4.00-02	6.92-02	4.85-02	3.53-02	4.53-02	1.43-02
3p	4.46-03	4.06-02	1.31-02	1.69-02	1.09-02	1.57-02	1.32-02	7.96-03	1.21-02	3.11-03
4p	1.82-03	1.57-02	5.33-03	6.51-03	4.48-03	6.06-03	5.42-03	3.08-03	4.90-03	1.19-03

Table 2. Cross sections of A_{2e} in the 2s state

	1 H		6 C		7 N		18 Ar		73 Ta	
tot	1.80-19		9.67-19		1.03-18		3.23-18		1.97-17	
f	coh	inc	coh	inc	coh	inc	coh	inc	coh	inc
tot	1.18-01	8.82-01	4.84-01	5.16-01	4.98-01	5.02-01	7.06-01	2.94-01	8.66-01	1.34-01
ex	1.23-02	4.73-01	4.91-02	2.64-01	3.94-02	2.48-01	5.41-02	1.46-01	6.80-02	7.67-02
ion	1.06-01	4.08-01	4.35-01	2.53-01	4.58-01	2.54-01	6.52-01	1.48-01	7.98-01	5.75-02
2p	9.17-03	3.68-01	3.66-02	2.05-01	2.92-02	1.93-01	4.01-02	1.13-01	5.11-02	5.97-02
3p	8.83-04	4.41-02	3.52-03	2.46-02	2.91-03	2.29-02	3.97-03	1.35-02	4.77-03	7.41-03
4p	3.12-04	1.32-02	1.25-03	7.39-03	1.03-03	6.91-03	1.41-03	4.07-03	1.67-03	2.20-03
4f	4.89-04	1.15-02	1.95-03	6.43-03	1.56-03	6.12-03	2.14-03	3.58-03	2.65-03	1.74-03

Table 3. Cross sections of A_{2e} in the 3s state

	1 H		6 C		7 N		18 Ar		73 Ta	
tot	2.41-19		1.16-18		1.21-18		3.60-18		2.11-17	
f	coh	inc	coh	inc	coh	inc	coh	inc	coh	inc
tot	9.01-02	9.10-01	4.11-01	5.89-01	4.29-01	5.71-01	6.44-01	3.56-01	8.20-01	1.80-01
ex	5.22-03	5.11-01	2.32-02	3.14-01	1.90-02	2.93-01	2.72-02	1.84-01	3.88-02	1.08-01
ion	8.48-02	3.99-01	3.88-01	2.75-01	4.10-01	2.78-01	6.17-01	1.72-01	7.81-01	7.24-02
2p	9.80-04	1.54-02	4.36-03	9.66-03	3.62-03	9.41-03	5.29-03	5.81-03	6.37-03	2.74-03
3p	1.61-03	3.29-01	7.13-03	2.06-01	6.04-03	1.91-01	8.25-03	1.21-01	1.33-02	7.24-02
4p	2.27-04	3.78-02	1.01-03	1.84-02	8.70-04	1.67-02	1.19-03	1.06-02	1.80-03	6.29-03
4f	1.33-03	7.95-02	5.93-03	4.92-02	4.69-03	4.70-02	6.84-03	2.93-02	9.75-03	1.65-02
5f	3.66-04	1.34-02	1.63-03	8.30-03	1.30-03	7.98-03	1.89-03	4.96-03	2.59-03	2.65-03

Table 4. Cross sections of $A_{\pi\mu}$ in the 1s state

	1 H		6 C		7 N		18 Ar		73 Ta	
tot	2.72-23		4.57-22		5.98-22		3.28-21		4.47-20	
f	coh	inc	coh	inc	coh	inc	coh	inc	coh	inc
tot	4.58-01	5.42-01	8.33-01	1.67-01	8.53-01	1.47-01	9.38-01	6.23-02	9.85-01	1.52-02
el	7.59-04	7.59-04	1.62-03	2.70-04	1.69-03	2.41-04	2.02-03	1.13-04	2.40-03	3.33-05
inel	4.58-01	5.41-01	8.31-01	1.67-01	8.51-01	1.47-01	9.36-01	6.22-02	9.82-01	1.52-02
ex	2.98-01	3.58-01	5.32-01	1.09-01	5.43-01	9.58-02	5.97-01	4.01-02	6.09-01	9.56-03
ion	1.60-01	1.83-01	2.99-01	5.80-02	3.08-01	5.12-02	3.39-01	2.21-02	3.73-01	5.61-03
2p	2.27-01	2.73-01	4.03-01	8.29-02	4.11-01	7.28-02	4.46-01	3.04-02	4.59-01	7.22-03
3p	3.87-02	4.60-02	6.95-02	1.41-02	7.11-02	1.24-02	8.10-02	5.23-03	8.08-02	1.26-03
4p	1.37-02	1.63-02	2.47-02	5.00-03	2.53-02	4.40-03	2.89-02	1.86-03	2.89-02	4.49-04
5p	6.49-03	7.69-03	1.17-02	2.37-03	1.20-02	2.08-03	1.37-02	8.80-04	1.37-02	2.13-04

Table 5. Cross sections of $A_{\pi\mu}$ in the 2s state

	1 H		6 C		7 N		18 Ar		73 Ta	
tot	2.90-22		4.51-21		5.85-21		3.08-20		3.93-19	
f	coh	inc	coh	inc	coh	inc	coh	inc	coh	inc
tot	4.46-01	5.54-01	8.23-01	1.77-01	8.44-01	1.56-01	9.33-01	6.67-02	9.84-01	1.61-02
el	1.28-03	1.29-03	2.92-03	4.94-04	3.05-03	4.44-04	3.73-03	2.14-04	4.57-03	6.43-05
inel	4.44-01	5.53-01	8.20-01	1.77-01	8.41-01	1.56-01	9.30-01	6.65-02	9.79-01	1.60-02
ex	4.05-01	5.07-01	7.40-01	1.61-01	7.58-01	1.42-01	8.70-01	6.03-02	8.70-01	1.44-02
ion	3.98-02	4.61-02	7.96-02	1.56-02	8.24-02	1.39-02	5.99-02	6.22-03	1.09-01	1.66-03
2p	2.86-01	3.56-01	5.28-01	1.14-01	5.41-01	1.00-01	6.34-01	4.28-02	6.29-01	1.03-02
3p	8.22-02	1.07-01	1.44-01	3.29-02	1.47-01	2.89-02	1.58-01	1.20-02	1.58-01	2.72-03
4p	1.62-02	2.04-02	2.93-02	6.45-03	2.99-02	5.68-03	3.27-02	2.40-03	3.37-02	5.62-04
5p	6.14-03	7.70-03	1.12-02	2.45-03	1.15-02	2.16-03	1.26-02	9.15-04	1.31-02	2.17-04

Table 6. Cross sections of $A_{\pi\mu}$ in the 3s state

	1 H		6 C		7 N		18 Ar		73 Ta	
tot	1.14-21		1.64-20		2.11-20		1.06-19		1.27-18	
f	coh	inc	coh	inc	coh	inc	coh	inc	coh	inc
tot	4.32-01	5.68-01	8.12-01	1.88-01	8.34-01	1.66-01	9.28-01	7.17-02	9.83-01	1.72-02
el	1.71-03	1.71-03	4.00-03	7.05-04	4.17-03	6.36-04	5.18-03	3.12-04	6.45-03	9.24-05
inel	4.30-01	5.66-01	8.08-01	1.88-01	8.29-01	1.66-01	9.23-01	7.14-02	9.76-01	1.71-02
ex	4.11-01	5.44-01	7.65-01	1.79-01	7.84-01	1.58-01	8.69-01	6.82-02	9.17-01	1.62-02
ion	1.96-02	2.24-02	4.29-02	8.29-03	4.49-02	7.45-03	5.45-02	3.16-03	5.93-02	8.95-04
2p	1.95-03	2.53-03	3.74-03	8.48-04	3.85-03	7.51-04	4.37-03	3.27-04	4.76-03	8.12-05
3p	3.32-01	4.38-01	6.19-01	1.45-01	6.34-01	1.28-01	7.03-01	5.48-02	7.36-01	1.30-02
4p	4.70-02	6.65-02	8.10-02	2.08-02	8.25-02	1.83-02	8.70-02	7.89-03	9.09-02	1.76-03
5p	9.93-03	1.33-02	1.82-02	4.34-03	1.86-02	3.83-03	2.04-02	1.61-03	2.07-02	3.79-04
4f	6.80-03	6.80-03	1.69-02	2.84-03	1.78-02	2.57-03	2.25-02	1.30-03	2.96-02	4.16-04

Table 7. Cross sections of $A_{2\pi}$ in the 1s state

	1 H		6 C		7 N		18 Ar		73 Ta	
tot	2.08-23		3.52-22		4.60-22		2.54-21		3.48-20	
f	coh	inc	coh	inc	coh	inc	coh	inc	coh	inc
tot	4.60-01	5.40-01	8.34-01	1.66-01	8.54-01	1.46-01	9.38-01	6.19-02	9.85-01	1.51-02
ex	3.01-01	3.59-01	5.37-01	1.09-01	5.48-01	9.58-02	6.22-01	4.02-02	6.16-01	9.61-03
ion	1.59-01	1.82-01	2.97-01	5.73-02	3.05-01	5.05-02	3.16-01	2.18-02	3.69-01	5.52-03
2p	2.29-01	2.74-01	4.07-01	8.31-02	4.16-01	7.30-02	4.72-01	3.05-02	4.65-01	7.27-03
3p	3.90-02	4.62-02	7.01-02	1.41-02	7.17-02	1.24-02	8.16-02	5.23-03	8.15-02	1.27-03
4p	1.38-02	1.63-02	2.49-02	5.00-03	2.55-02	4.40-03	2.90-02	1.85-03	2.91-02	4.51-04
5p	6.54-03	7.71-03	1.18-02	2.37-03	1.21-02	2.08-03	1.38-02	8.79-04	1.39-02	2.14-04

Table 8. Cross sections of $A_{2\pi}$ in the 2s state

	1 H		6 C		7 N		18 Ar		73 Ta	
tot	2.24-22		3.52-21		4.58-21		2.42-20		3.13-19	
f	coh	inc	coh	inc	coh	inc	coh	inc	coh	inc
tot	4.48-01	5.52-01	8.24-01	1.76-01	8.45-01	1.55-01	9.34-01	6.60-02	9.84-01	1.59-02
ex	4.08-01	5.07-01	7.46-01	1.60-01	7.64-01	1.41-01	8.41-01	6.00-02	8.78-01	1.43-02
ion	3.93-02	4.54-02	7.80-02	1.53-02	8.08-02	1.35-02	9.27-02	6.03-03	1.06-01	1.60-03
2p	2.89-01	3.56-01	5.33-01	1.13-01	5.46-01	1.00-01	6.03-01	4.26-02	6.35-01	1.03-02
3p	8.40-02	1.07-01	1.48-01	3.31-02	1.51-01	2.91-02	1.63-01	1.21-02	1.63-01	2.77-03
4p	1.64-02	2.05-02	2.98-02	6.46-03	3.05-02	5.69-03	3.33-02	2.40-03	3.44-02	5.66-04
5p	6.23-03	7.73-03	1.14-02	2.45-03	1.17-02	2.16-03	1.36-02	9.16-04	1.34-02	2.18-04

Table 9. Cross sections of $A_{2\pi}$ in the 3s state

	1 H		6 C		7 N		18 Ar		73 Ta	
tot	8.89-22		1.30-20		1.68-20		8.53-20		1.03-18	
f	coh	inc	coh	inc	coh	inc	coh	inc	coh	inc
tot	4.35-01	5.65-01	8.14-01	1.86-01	8.36-01	1.64-01	9.29-01	7.05-02	9.83-01	1.70-02
ex	4.18-01	5.45-01	7.78-01	1.79-01	7.98-01	1.58-01	8.84-01	6.75-02	9.33-01	1.62-02
ion	1.70-02	1.97-02	3.62-02	7.15-03	3.79-02	6.36-03	4.55-02	3.02-03	4.97-02	8.37-04
2p	2.00-03	2.55-03	3.82-03	8.50-04	3.93-03	7.52-04	4.46-03	3.27-04	4.89-03	8.19-05
3p	3.37-01	4.38-01	6.27-01	1.44-01	6.43-01	1.27-01	7.13-01	5.44-02	7.53-01	1.30-02
4p	5.05-02	6.92-02	8.89-02	2.18-02	9.07-02	1.92-02	9.71-02	7.93-03	9.64-02	1.80-03
5p	1.02-02	1.34-02	1.87-02	4.35-03	1.91-02	3.84-03	2.10-02	1.63-03	2.19-02	3.85-04
4f	6.72-03	6.73-03	1.65-02	2.76-03	1.73-02	2.49-03	2.19-02	1.25-03	2.88-02	4.03-04

Table 10. Cross sections of $A_{\pi K}$ in the 1s state

	1 H		6 C		7 N		18 Ar		73 Ta	
tot	9.28-24		1.60-22		2.10-22		1.17-21		1.63-20	
f	coh	inc	coh	inc	coh	inc	coh	inc	coh	inc
tot	4.63-01	5.37-01	8.36-01	1.64-01	8.56-01	1.44-01	9.39-01	6.10-02	9.85-01	1.50-02
el	1.03-02	1.03-02	2.15-02	3.58-03	2.23-02	3.18-03	2.64-02	1.47-03	3.10-02	4.27-04
inel	4.53-01	5.27-01	8.14-01	1.61-01	8.33-01	1.41-01	9.13-01	5.95-02	9.54-01	1.45-02
ex	2.92-01	3.45-01	5.16-01	1.04-01	5.27-01	9.11-02	5.94-01	3.80-02	5.87-01	9.08-03
ion	1.61-01	1.82-01	2.98-01	5.68-02	3.06-01	5.00-02	3.18-01	2.15-02	3.67-01	5.45-03
2p	2.20-01	2.61-01	3.86-01	7.82-02	3.94-01	6.86-02	4.44-01	2.85-02	4.35-01	6.76-03
3p	3.72-02	4.38-02	6.61-02	1.32-02	6.76-02	1.16-02	7.63-02	4.86-03	7.58-02	1.17-03
4p	1.32-02	1.55-02	2.35-02	4.68-03	2.40-02	4.11-03	2.71-02	1.72-03	2.70-02	4.16-04
5p	6.23-03	7.31-03	1.11-02	2.21-03	1.14-02	1.94-03	1.28-02	8.15-04	1.28-02	1.97-04

Table 11. Cross sections of $A_{\pi K}$ in the 2s state

	1 H		6 C		7 N		18 Ar		73 Ta	
tot	1.02-22		1.66-21		2.17-21		1.17-20		1.55-19	
f	coh	inc	coh	inc	coh	inc	coh	inc	coh	inc
tot	4.53-01	5.47-01	8.28-01	1.72-01	8.49-01	1.51-01	9.36-01	6.42-02	9.84-01	1.56-02
el	1.64-02	1.64-02	3.61-02	6.04-03	3.76-02	5.41-03	4.56-02	2.57-03	5.48-02	7.66-04
inel	4.36-01	5.31-01	7.92-01	1.66-01	8.11-01	1.46-01	8.90-01	6.16-02	9.30-01	1.48-02
ex	3.97-01	4.86-01	7.15-01	1.51-01	7.31-01	1.33-01	8.09-01	5.58-02	8.28-01	1.33-02
ion	3.96-02	4.51-02	7.72-02	1.49-02	7.99-02	1.31-02	8.15-02	5.81-03	1.02-01	1.53-03
2p	2.72-01	3.33-01	4.91-01	1.03-01	5.02-01	9.09-02	5.48-01	3.83-02	5.69-01	9.11-03
3p	7.89-02	1.00-01	1.36-01	3.01-02	1.39-01	2.64-02	1.58-01	1.09-02	1.47-01	2.45-03
4p	1.53-02	1.90-02	2.71-02	5.82-03	2.76-02	5.11-03	2.98-02	2.13-03	3.03-02	4.94-04
5p	5.78-03	7.12-03	1.03-02	2.20-03	1.05-02	1.93-03	1.14-02	8.08-04	1.17-02	1.89-04
3d	7.93-03	7.94-03	1.73-02	2.92-03	1.80-02	2.61-03	2.15-02	1.23-03	2.55-02	3.58-04

Table 12. Cross sections of $A_{\pi K}$ in the 3s state

	1 H		6 C		7 N		18 Ar		73 Ta	
tot	4.16-22		6.38-21		8.27-21		4.31-20		5.43-19	
f	coh	inc	coh	inc	coh	inc	coh	inc	coh	inc
tot	4.43-01	5.57-01	8.21-01	1.79-01	8.42-01	1.58-01	9.32-01	6.77-02	9.84-01	1.63-02
el	2.08-02	2.08-02	4.76-02	8.11-03	4.96-02	7.29-03	6.08-02	3.53-03	7.48-02	1.05-03
inel	4.22-01	5.36-01	7.73-01	1.71-01	7.92-01	1.51-01	8.71-01	6.41-02	9.09-01	1.52-02
ex	4.04-01	5.15-01	7.34-01	1.64-01	7.51-01	1.44-01	8.23-01	6.10-02	8.51-01	1.44-02
ion	1.87-02	2.11-02	3.92-02	7.45-03	4.08-02	6.65-03	4.83-02	3.10-03	5.82-02	8.43-04
2p	1.68-03	2.17-03	3.04-03	6.85-04	3.10-03	6.03-04	3.39-03	2.54-04	3.48-03	5.93-05
3p	3.14-01	4.03-01	5.66-01	1.27-01	5.79-01	1.12-01	6.32-01	4.73-02	6.48-01	1.10-02
4p	4.66-02	6.30-02	7.93-02	1.92-02	8.07-02	1.68-02	8.49-02	6.82-03	8.09-02	1.51-03
3d	5.80-03	5.81-03	1.32-02	2.26-03	1.37-02	2.03-03	1.67-02	9.78-04	2.04-02	2.88-04
4d	5.66-03	5.68-03	1.25-02	2.20-03	1.30-02	1.97-03	1.56-02	9.35-04	1.86-02	2.65-04
4f	4.42-03	4.42-03	1.03-02	1.73-03	1.08-02	1.56-03	1.34-02	7.65-04	1.69-02	2.36-04

Table 13. Coefficients of atomic form factors given in Eq.(8).

Initial state is 1s, f is the final state

f	1s	2s	3s	4s	5s	6s	7s
C	2^4	$2^{1/2}\ 2^8$	$3^{1/2}\ 2^4\ 3^3$	2^{13}	$5^{1/2}2^4 5^4$	$6^{1/2}2^8 3^3$	$7^{1/2}\ 2^4\ 7^4$
j	0	2	2	2	2	2	2
d_0	2^2	3^2	2^4	5^2	$2^2\ 3^2$	7^2	2^6
d_1	1	2^2	3^2	2^4	5^2	$2^2\ 3^2$	7^2
k	2	3	4	5	6	7	8
b_0	1	1	2^4	5^3	$2^8\ 3^3$	$5^3\ 7^5$	$2^{22}\ 3^3$
b_1			3^3	$2^5\ 11$	$2^4 3^2 5\cdot 37$	$2^4 3^4 7^3 23$	$2^{16} 3^2 7^2\ 23$
b_2				2^8	$2^3\ 5^4\ 7$	$2^5 3^7 271$	$2^{10} 7^3 11\cdot 409$
b_3					5^6	$2^8 3^6 5\cdot 17$	$2^4\ 7^6\ 1009$
b_4						$2^8\ 3^9$	$2^3\ 5^2\ 7^8$
b_5							7^{10}

f	2p	3p	4p	5p	6p	7p
C	$2^{1/2}2^7 3$	$2^{1/2}2^5 3^3$	$5^{1/2}2^{11}3$	$10^{1/2}2^5 3\cdot 5^3$	$70^{1/2}2^7 3^3$	$7^{1/2}2^6 3\cdot 7^3$
j	1	1	1	1	1	1
d_0	3^2	2^4	5^2	$2^2 3^2$	7^2	2^6
d_1	2^2	3^2	2^4	5^2	$2^2 3^2$	7^2
k	3	4	5	6	7	8
b_0	1	2^4	5^3	$2^8 3^3$	$5^3 7^5$	$2^{22} 3^3$
b_1		3^3	$2^5 11$	$2^4 3^2 5\cdot 37$	$2^4 3^4 7^3 23$	$2^{16} 3^2 7^2 23$
b_2			2^8	$2^3 5^4 7$	$2^5 3^7 271$	$2^{10} 7^3 11\cdot 409$
b_3				5^6	$2^8 3^6 5\cdot 17$	$2^4 7^6 1009$
b_4					$2^8 3^9$	$2^3 5^2 7^8$
b_5						7^{10}

Table 14. Coefficients of atomic form factors given in Eq.(8).

Initial state is 1s, f is the final state

f	3d	4d	5d	6d	7d
C	$6^{1/2}2^8 3^3$	2^{16}	$14^{1/2}2^8 5^4 7^{-1}$	$21^{1/2}2^{13}3^3 7^{-1}$	$21^{1/2}2^9 3^{-1}7^4$
j	2	2	2	2	2
d_0	2^4	5^2	$2^2 3^2$	7^2	2^6
d_1	3^2	2^4	5^2	$2^2 3^2$	7^2
k	4	5	6	7	8
b_0	1	5^2	$2^4 3^3 7$	$5^2 7^5$	$2^{17}3^4$
b_1		2^5	$2^5 3^2 5^2$	$2^6 3^3 7^2 17$	$2^{12}3^3 7\cdot 61$
b_2			$5^4 7$	$2^4 3^6 11\cdot 13$	$2^5 5\cdot 7^3 1451$
b_3				$2^7 3^6 7$	$2^2 3\cdot 7^5 13\cdot 23$
b_4					$3\cdot 7^8$

f	4f	5f	6f	7f
C	$5^{1/2}2^{18}$	$10^{1/2}2^9 5^4$	$5^{1/2}2^{14}3^6$	$6^{1/2}2^9 3^{-1}5^5$
j	3	3	3	3
d_0	5^2	$2^2 3^2$	7^2	2^6
d_1	2^4	5^2	$2^2 3^2$	7^2
k	5	6	7	8
b_0	1	$2^2 3^3$	$3\cdot 7^3$	$2^{14}3^4$
b_1		5^3	$2^3 7\cdot 41$	$2^9 7^2 173$
b_2			$2^4 3^4$	$2^2 7^5 71$
b_3				$3\cdot 5\cdot 7^6$

Table 15. Coefficients of atomic form factors given in Eq.(8).

Initial state is 2s, f is the final state

f	2s	3s	4s	5s	6s	7s
C	1	$6^{1/2}2^8 3^3$	$2^{1/2}2^{11}$	$10^{1/2}2^8 5^4$	$3^{1/2}3^3$	$14^{1/2}2^8 7^4$
j	0	2	2	2	2	2
d_0	1	5^2	3^2	7^2	2^2	3^4
d_1	1	$2^2 3^2$	2^4	$2^2 5^2$	3^2	$2^2 7^2$
k	4	5	6	7	8	9
b_0	1	$5^3 23$	$3^3 11$	$7^5 71$	$2^6 13$	$3^{13}5^3 11\cdot 3$
b_1	-3	$-2^3 3^2 97$	$2^4 3^2 19$	$2^4 5\cdot 7^3 631$	$2^4 3^2 109$	$2^3 3^{10}7^2 17\cdot 1663$
b_2	2	$2^4 3^5$	$-2^8 29$	$2^5 5^3 11239$	$2^2 3^6 31$	$2^4 3^6 7^3 59\cdot 23027$
b_3			2^{12}	$-2^8 5^6 43$	$3^6 83$	$2^8 7^5 3833087$
b_4				$2^8 5^8$	$-3^8 5\cdot 17$	$-2^8 3\cdot 7^8 1571$
b_5					$2\cdot 3^{11}$	$-2^{11}7^{10}137$
b_6						$2^{12}7^12$

f	2p	3p	4p	5p	6p	7p
C	3	$2^{10}3^3$	$10^{1/2}2^9 3$	$5^{1/2}2^{10}3\cdot 5^3$	$35^{1/2}3^3$	$14^{1/2}2^{10}3\cdot 7^3$
j	1	1	1	1	1	1
d_0	1	5^2	3^2	7^2	2^2	3^4
d_1	1	$2^2 3^2$	2^4	$2^2 5^2$	3^2	$2^2 7^2$
k	4	5	6	7	8	9
b_0	-1	5^4	3^4	$3\cdot 7^6$	2^7	$3^{15}5^4$
b_1	1	$-2^4 3^2 5\cdot 7$	$2^4 3^3$	$2^3 5\cdot 7^4 41$	$2^5 3^2 7$	$2^5 3^{12}5\cdot 7^2 43$
b_2		$2^4 3^5$	$-2^8 3\cdot 7$	$-2^6 3^2 5^3 7\cdot 11$	$2^3 3^6$	$2^4 3^8 7^3 67\cdot 439$
b_3			2^{12}	$-2^7 5^6 61$	$-2\cdot 3^6 31$	$-2^9 3^2 7^5 11\cdot 13\cdot 73$
b_4				$2^8 5^8$	$-3^8 29$	$-2^8 7^8 17551$
b_5					3^{11}	$-2^{14}7^{10}11$
b_6						$2^{12}7^{12}$

Table 16. Coefficients of atomic form factors given in Eq.(8).

Initial state is 2s, f is the final state

f	3d	4d	5d	6d	7d
C	$3^{1/2}2^{16}3^3$	$2^{1/2}2^{14}$	$7^{-1/2}2^{16}5^4$	$42^{1/2}2^{1}3^{3}7^{-1}$	$42^{1/2}2^{16}3^{-1}7^4$
j	2	2	2	2	2
d_0	5^2	3^2	7^2	2^2	3^4
d_1	$2^2 3^2$	2^4	$2^2 5^2$	3^2	$2^2 7^2$
k	5	6	7	8	9
b_0	-5^2	-3^3	-7^5	$-2^5 7$	$-3^1 3_5^2$
b_1	$2\cdot 3^2$	$-2^4 3^2 5$	$-2\cdot 5^2 7^2 317$	$-2^7 3^2 13$	$-2\cdot 3^{10} 7\cdot 61\cdot 71$
b_2		2^9	$-2^6 5^5 23$	$-2\cdot 3^5 7\cdot 47$	$-2^6 3^5 7^4 1427$
b_3			$2^5 5^6 7$	$-3^6 19\cdot 31$	$-2^6 7^5 263597$
b_4				$2^3 3^8 7$	$-2^8 3^2 7^7 227$
b_5					$2^9 3\cdot 7^{10}$

f	4f	5f	6f	7f
C	$10^{1/2}2^{16}$	$5^{1/2}2^{18}5^4$	$10^{1/2}2^{1}3^{6}$	$3^{1/2}2^{18}3^{-1}7^5$
j	3	3	3	3
d_0	3^2	7^2	2^2	3^4
d_1	2^4	$2^2 5^2$	3^2	$2^2 7^2$
k	6	7	8	9
b_0	-3^3	$-7^3 29$	$-2^4 3$	$-3^{12}5\cdot 53$
b_1	2^4	$-2^3 3\cdot 5^2 7\cdot 19$	$-2^3 83$	$-2^4 3^6 7^3 659$
b_2		$2^4 5^5$	$-3^2 5\cdot 47$	$-2^5 3^3 7^4 11\cdot 19\cdot 37$
b_3			$2\cdot 3^6$	$-2^8 7^6 937$
b_4				$2^8 3\cdot 5\cdot 7^8$

Table 17. Coefficients of atomic form factors given in Eq.(8).

Initial state is 3s, f is the final state.

f	3s	4s	5s	6s
C	2^4	$3^{1/2}2^{13}3^3$	$15^{1/2}2^4 3^3 5^4$	$2^{17/2}3^{-11}$
j	0	2	2	2
d_0	2^2	7^2	2^6	1
d_1	3^2	$2^4 3^2$	$3^2 5^2$	2^2
k	6	7	8	9
b_0	2^8	$7^5 2161$	$2^{22}17\cdot 23$	$3\cdot 523$
b_1	$-2^8 3\cdot 7$	$-2^8 3^3 7^3 235$	$2^{16}3^2 5\cdot 14519$	$2^3 3^2 5^3 7$
b_2	$2^7 3^5$	$2^9 3^5 20117$	$-2^{10}3^6 5^3 8603$	$2^4 3^2 17\cdot 71$
b_3	$-2^6 3^6$	$-2^{15}3^6 139$	$2^4 3^6 5^5 97289$	$-2^8 43387$
b_4	3^9	$2^{16}3^9$	$-2^3 3^8 5^8 223$	$2^8 3^4 43\cdot 61$
b_5			$3^{11}5^{10}$	$-2^{11}3^5 143$
b_6				$2^{12}3^8$

f	2p	3p	4p	5p	6p
C	$6^{1/2}2^7 3^3$	$6^{1/2}2^5 3$	$15^{1/2}2^{11}3^3$	$30^{1/2}2^5 3^3 5^3$	$210^{1/2}2^7 3^{-11}$
j	1	1	1	1	1
d_0	5^2	2^2	7^2	2^6	1
d_1	$2^2 3^2$	3^2	$2^4 3^2$	$3^2 5^2$	2^2
k	5	6	7	8	9
b_0	5^4	-2^7	$5\cdot 7^6 17$	$2^{24}37$	$3\cdot 5^2$
b_1	$-2^3 3^3 65$	$2^4 3\cdot 29$	$-2^8 3^2 7^4 85$	$2^{18}3^4 5\cdot 37$	$2^3 3^2 5^2$
b_2	$2^4 3^6$	$-2^4 3^5$	$2^9 3^4 84443$	$-2^{12}3^5 5^3 5563$	$-2^4 3^2 17\cdot 19$
b_3		3^7	$-2^{15}3^7 107$	$2^6 3^6 5^5 35123$	$-2^8 61^2$
b_4			$2^{16}3^{10}$	$-2^3 3^9 5^8 173$	$2^8 3\cdot 5\cdot 2251$
b_5				$3^{12}5^{10}$	$-2^{11}3^5 37$
b_6					$2^{12}3^7$

Table 18. Coefficients of atomic form factors given in Eq.(8).

Initial state is 3s, f is the final state

f	3d	4d	5d	6d	7d
C	$2^{1/2}2^8 3^1$	$3^{1/2}2^{16}3^5$	$42^{1/2}2^8 3^5 5^4 7^{-1}$	$7^{-1/2}2^{13}3^{-11}$	$7^{1/2}2^9 3^6 7^4$
j	2	2	2	2	2
d_0	2^2	7^2	2^6	1	$2^2 5^2$
d_1	3^2	$2^4 3^2$	$3^2 5^2$	2^2	$3^2 7^2$
k	6	7	8	9	10
b_0	$2^4 5$	$-7^5 43$	$-2^{19}7\cdot 13$	$-3\cdot 7^2$	$-2^{13}5^8 19$
b_1	$-2^4 3^3$	$2^6 3^2 7^2 383$	$-2^{14}3^2 5^2 727$	$-2^3 3^2 167$	$-2^{11}3^2 5^6 21007$
b_2	3^5	$-2^8 3^5 571$	$2^7 3^4 5^4 5717$	$-2^5 3\cdot 1129$	$-2^9 3^6 5^5 7^3 319$
b_3		$2^{13}3^7$	$-2^5 3^7 5^6 101$	$2^8 9091$	$2^6 3^6 5^3 7^5 677$
b_4			$3^9 5^8 7$	$-2^8 3^3 17\cdot 61$	$2^4 3^8 5^3 7^7 1199$
b_5				$2^{11}3^5 7$	$-2^2 3^{11}5\cdot 7^9 313$
b_6					$3^{13}7^{12}$

f	4f	5f	6f	7f
C	$15^{1/2}2^{18}3$	$30^{1/2}2^9 3^7 5^4$	$15^{1/2}2^{14}3^{-12}$	$2^{1/2}2^9 3^8 7^5$
j	3	3	3	3
d_0	7^2	2^6	1	$2^2 5^2$
d_1	$2^4 3^2$	$3^2 5^2$	2^2	$3^2 7^2$
k	7	8	9	10
b_0	$7^3 101$	$-2^{14}131$	$-3^2 19$	$-2^{10}5^6 1109$
b_1	$-2^5 3^2 7\cdot 67$	$2^9 3^3 5^2 17\cdot 19$	$2^7 5$	$-2^8 3^2 5^5 7^2 863$
b_2	$2^8 3^5$	$-2^2 3^4 5^4 31\cdot 61$	$2^5 3\cdot 1231$	$2^{10}3^7 5^3 7^4 13$
b_3		$3^7 5^7$	$-2^9 3^2 7\cdot 13$	$2^6 3^6 5\cdot 7^6 13\cdot 383$
b_4			$2^8 3^6$	$-2^2 3^8 7^8 19\cdot 193$
b_5				$3^{11}5\cdot 7^{10}$

To calculate the sum of excitation cross sections it was taken into account that expression $|F_i^f(n\vec{q}) - F_i^f(\xi\vec{q})|^2$ decreases as n'^{-3} for the fixed l' [17,18]. The cross sections were computed up to $n'\leq 10$ and $l'\leq 4$. Then the infinite series were approximated.

The cross sections calculated using Eqs.(3)-(6) and $F(x,Z)$, $S_{inc}(x,Z)$ function tables [15] are presented in Tables 1-12. The total cross sections are given in cm^2, others are given as the ratio to the corresponding total cross section. The initial state and the type of elementary atoms are mentioned in table headers, final states f are denoted by common symbols 1s, 2p, 3d etc. For all targets (H, C, N, Ar, Ta) there is a coherent (coh) part of the cross section in a left column and an incoherent (inc) one in a right column. The tables contain cross sections of only those transitions whose contribution to the total cross section is not smaller than 10^{-2} at least for one of the targets. The numbers in the tables should be read as for instance: $1.32\text{-}2=1.32\cdot 10^{-2}$.

For symmetric atoms A_{2e} and $A_{2\pi}$ in the first Born approximation the transitions with the even $l-l'$ number are inhibited [10,18]. Therefore elastic scattering is also inhibited and $\sigma_{inel}=\sigma_{tot}$.

The total cross section of A_{2e} interacting with carbon measured in Ref.[5] $\sigma_{tot}=(16^{+16}_{-6})\cdot 10^{-19}$ cm^2/atom is consistent with the value given in Table 1 $\sigma_{tot}=5.91\cdot 10^{-19} cm^2$. For more detailed discussion see Ref.[5].

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Appendix

The transition form factors of elementary atoms $F_i^f(q)$ evaluated for Eq.(7) can be written as

$$F_i^f(q) = \frac{C\cdot q^j}{(d_0 + d_1\cdot q^2)^k}\cdot\sum_{n=0}^{N} b_n\cdot q^{2n} \qquad (8)$$

Here q is given in atomic units i.e. $q=q[MeV/c]/\alpha\mu$, μ is the atomic reduced mass. For various initial and final states the constants C, j, d_0, d_1, k, b_n are presented in Tables 13-18. Initial states i are notified in table headers, final states f are given in a common way as 1s, 2p, 3d etc. Numbers in the tables should be read as products of prime

number powers, though not all numbers are completely factorized to get shorter records. The hydrogen wave functions given in Ref.[21] were used for the calculations, another choice of the wave function phases might lead to an insignificant common factor -1 or i.

References

1. Nemenov L.L. - Yad.Fiz., 1972, v.15, p.1047.
2. Nemenov L.L. - Yad.Fiz., 1972, v.16, p.125.
3. Nemenov L.L. - Yad.Fiz., 1985, v.41, p.980.
4. Alekseev G.D. et al. - Yad.Fiz., 1984, v.40, p.139.
5. Afanasyev L.G. et al. - Yad.Fiz., 1989, v.50, p.7.
6. Afanasyev L.G. et al. - Phys.Lett., 1990, v.236B, p.116.
7. Coomdes R. et al. - Phys.Rev.Lett., 1976, v.37, p.249.
8. Aronson S.H. et al. - Phys.Rev.Lett., 1982, v.48, p.1078.
9. Aronson S.H. et al. - Phys.Rev., 1986, v.D33, p.3180.
10. Mrowczynshi St. - Phys.Rev., 1986, v.A33, p.1549.
11. Mrowczynshi St. - Phys.Rev., 1987, v.D36, p.1520.
12. Denisenko K.G., Mrowczynshi St. - Phys.Rev., 1987, v.D36, p.1529.
13. Kuptsov A.V., Pak A.S., Saakya S.B. - Yad.Fiz., 1989, v.50, p.936.
14. Moliere G. - Z.Naturforsch. 1947, v.2A, 133.
15. Hubbell J.H. et al. J.Phys.Chem.Ref.Data, 1975, v.4, p.471.
16. Tarasov A.V., Christova I.U. - JINR Communication, P2-91-10, Dubna 1991.
17. Gillespie G.H. - Phys.Rev. 1978, v.A18, p.1967.
18. Pak A.S., Tarasov A.V. - JINR Preprint, P2-85-903, Dubna 1985.
19. Reduce User's Manual, The Rand Corporation, Rand Publication CP78
20. Prudnikov A.P., Brychkov Yu.A., Marichev O,I. - Integrals and Series, Nauka Publ., Moscow, 1981, p.446.
21. Landau L.D., Lifshitz E.M. Quantum Mechanics, Nauka, Moscow, 1974.