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V.V.Nesterenko, N.R.Shvets

THE CASIMIR ENERGY OF THE RIGID STRING WITH MASSIVE ENDS

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1 Introduction

The Casimir energy in string models turns out to be up to a positive constant equal to the squared mass of the ground state [1]. Therefore after calculating the Casimir energy one can conclude with confidence whether the ground state in the string spectrum is a tachyon or not. The tachyon problem in its turn is tightly connected with nonphysical dimension of the space-time in the quantum theory of the Nambu-Goto string. In the preceding paper of one of the authors [2] the investigation of the "improved" (more realistic) string model of hadrons has been started. As the first step the nonzero quark mass was taken into consideration by calculating the Casimir energy. It was shown that there are values of the quark mass for which the Casimir energy is definitely positive. The second step in improving the hadron string model is allowance for the thickness of the flux tube described by the string. For this purpose one has to go from the Nambu-Goto string to the rigid string [3, 4].

The aim of this paper is the calculation of the Casimir energy for the rigid string with massive ends. The contribution of the quark mass to this energy turns out to be the same as in the Nambu-Goto string with massive ends. The string stiffness results in an additional positive contribution to the Casimir energy. The allowance for the string thickness alone i. e. consideration of the rigid string with free massless ends does not enable one to make the Casimir energy positive.

The layout of the paper is as follows. In the second Section, the action is introduced for the rigid string, the ends of which are supplied wiyh point-like masses (quarks). For this action the harmonic approximation linearizing the equations of motion and the boundary conditions is constructed. General solutions to these equations are obtained. It turns out that the string dynamics is described by two independent modes of oscillations, one of which corresponds to the Nambu-Goto string with massive ends and the other takes into account the string rigidity. The third section is devoted to the canonical quantization of the model under consideration. In the fourth section the finite renormalized Casimir energy is calculated. In the conclusion (the fifth section) the obtained results are discussed and a possible way for solving the problem of the negative energy related with the rigid oscillations of the string is considered.

2 Harmonic approximation for the rigid string

The action of the hadronic string model taking into account the finite thickness of the gluonic tube and nonzero quark masses reads as follows

$$S = -\rho_0 c \iint d^2u \sqrt{-g}\,(\gamma - \frac{\alpha}{2} r_s^2\, \Delta x^\mu\, \Delta x_\mu\,) - \sum_{a=1}^{2} m_a \int_{C_a} ds\,, \quad (2.1)$$

where ρ_0 is the linear mass density of the flux tube (of the string), r_s is the radius of this tube, c is the velocity of light, m_a, $a = 1,2$ are the masses of quarks attached to the string ends, $x^\mu(u^0,\ u^1)$, $\mu = 0,1,\ldots,D-1$ are the string coordinates in the D-dimensional space-time. The metric of the space-time has the signature $(+,-,\ldots,-)$. The internal geometry on the string world sheet is defined by the induced metric $g_{ij}(u) = \partial_i x^\mu \partial_j x_\mu$, $i,j = 0,1$, $g = \det(g_{ij})$, $g < 0$. The Laplace-Beltrami operator corresponding to this metric is defined by

$$\Delta = \frac{1}{\sqrt{-g}} \frac{\partial}{\partial u^i} \left(\sqrt{-g}\, g^{ij} \frac{\partial}{\partial u^j} \right), \quad g_{ij}\, g^{jk} = \delta_i^k\,. \quad (2.2)$$

For the curvilinear coordinates u^0 and u^1 on the world sheet of the string another more ordinary notation will be used also $u^0 = \tau$ and $u^1 = \sigma$. In the action (2.1) α is a numerical parameter. Its specific value depends on the concrete mechanism generating the flux tube. In the abelian gauge model with the simplest Higgs potential (the Nielsen-Olesen vortex model for the relativistic string) the parameter α proves to be ~20 [3].

We shall use the physically preferable time-like gauge

$$x^0(\tau, \sigma) \equiv ct(\tau, \sigma) = c\tau. \tag{2.3}$$

In addition to (2.3) one can impose one more gauge condition, namely the transversality condition for the string oscillations

$$\dot{\mathbf{x}}\mathbf{x}' = 0, \tag{2.4}$$

where $\mathbf{x}(t, \sigma)$ is the space-like part of $x^\mu(t, \sigma)$. The dot means differentiation with respect to τ; and the prime, with respect to σ.

As we shall be interested only in the zero point oscillations in the model (2.1) one can assume as it has been done in [2] that the string length L does not alter in time. In this case the parameter σ can be related with the string length by the formula

$$\mathbf{x}'^2(t, \sigma) = (L/\pi)^2. \tag{2.5}$$

In addition it is natural to employ here the harmonic approximation in the action (2.1) that linearizes the equations of motion and the boundary conditions. In this approximation we have

$$-g = c^2\mathbf{x}'^2, \quad g^{00} = -\frac{1}{c^2}, \quad g^{01} = g^{10} = 0, \quad g^{11} = \frac{1}{\mathbf{x}'^2}. \tag{2.6}$$

Actually the same approximation is used in calculating the Casimir energy by the effective potential method [5].

Finally after dropping out the unharmonic terms the action (2.1) takes the form

$$S = \frac{\rho_0 L}{2\pi}\int dt \int_0^\pi d\sigma \left[\xi(\sigma)\,\dot{\mathbf{x}}^2 - a^2\mathbf{x}'^2 - \varepsilon a^2(a^{-2}\ddot{\mathbf{x}} - \mathbf{x}'')^2\right], \tag{2.7}$$

where $\mathbf{x}(t, \sigma)$ is the transverse displacement of the string, $\xi(\sigma)$ is the weight function

$$\xi(\sigma) = 1 + q^{-1}[\delta(\sigma) + \delta(\sigma - \pi)], \tag{2.8}$$

$$q = \frac{\rho_0 L}{\pi m}, \quad a = \frac{\pi c}{\lambda}, \quad \varepsilon = \alpha\left(\frac{\pi r_s}{L}\right)^2. \tag{2.9}$$

Here one assumes that the string ends are loaded with equal masses $m_1 = m_2 = m$.

As mentioned in the Introduction, we shall be interested in the dependence of the Casimir energy on the quark mass (the parameter q) and on the transverse dimensions of the gluonic tube (the parameter ε).

The action (2.7) entails the following equations of motion

$$(1 + \varepsilon\,\Box)\,\Box\,\mathbf{x} = 0, \tag{2.10}$$

where $\Box$ is the two-dimensional D'Alembert operator

$$\Box \equiv \frac{1}{a^2}\frac{\partial^2}{\partial t^2} - \frac{\partial^2}{\partial \sigma^2}, \tag{2.11}$$

and the boundary conditions

$$\begin{aligned} (1 + \varepsilon\,\Box)\,\mathbf{x}' &= g^{-1}\ddot{\mathbf{x}}, \quad \sigma = 0, \\ (1 + \varepsilon\,\Box)\,\mathbf{x}' &= -g^{-1}\ddot{\mathbf{x}}, \quad \sigma = \pi, \end{aligned} \tag{2.12}$$

$$g = \rho_0 c^2/(Lm),$$

$$\Box\mathbf{x} = 0, \quad \sigma = 0, \pi. \tag{2.13}$$

The Lagrangian in (2.7) depends on the first and second derivatives of the string coordinates, therefore the number of obtained boundary conditions is twice that in the Nambu-Goto string. The boundary value problem (2.10), (2.13) reduces to the two independent boundary problems. Indeed the left hand side of the equations of motion (2.10) stands for the product of two commuting differential operators $(1 + \varepsilon\,\Box)$ and $\Box$. Hence the general solution to this equation can be represented as a sum of two terms

$$\mathbf{x}(t, \sigma) = \mathbf{x}_1(t, \sigma) + \mathbf{x}_2(t, \sigma), \tag{2.14}$$

where

$$\Box\,\mathbf{x}_1 = 0, \tag{2.15}$$

$$(1 + \varepsilon\,\Box)\,\mathbf{x}_2 = 0. \tag{2.16}$$

With allowance for (2.15) – (2.16) the initial edge conditions are rewritten for $\mathbf{x}_1$ and $\mathbf{x}_2$ separately

$$\mathbf{x}'_1 = g^{-1}\ddot{\mathbf{x}}_1, \quad \sigma = 0, \qquad \mathbf{x}'_1 = -g^{-1}\ddot{\mathbf{x}}_1, \quad \sigma = \pi, \tag{2.17}$$

$$\mathbf{x}_2(t,0) = \mathbf{x}_2(t,\pi) = 0. \tag{2.18}$$

Thus $\mathbf{x}_1(t,\sigma)$ is the solution for the Nambu-Goto string with point-like masses at ends [2]. All the dependence on the string rigidity is taken into account by the second term $\mathbf{x}_2(t,\sigma)$.

The general solution to (2.15) and (2.16) obeying boundary conditions (2.17), (2.18) can be represented as two series of corresponding eigenfunctions

$$x_1^j(t,\sigma) = Q^j + \frac{P^j t}{\rho_0 L + 2m} + i\sqrt{\frac{\hbar}{2\rho_0 c}}\sum_{n\neq 0}\exp\left(-ia\overset{(1)}{\omega}_n t\right)\frac{\alpha_n^j}{\overset{(1)}{\omega}_n}u_n(\sigma), \tag{2.19}$$

$$x_2^j(t,\sigma) = -i\sqrt{\frac{\hbar}{2\rho_0 c}}\sum_{n\neq 0}\exp\left(ia\overset{(2)}{\omega}_n t\right)\frac{\beta_n^j}{\overset{(2)}{\omega}_n}v_n(\sigma), \tag{2.20}$$

$$j = 1,2,\ldots,D-2.$$

Here $u_n(\sigma)$ and $\overset{(1)}{\omega}_n$ are the eigenfunctions and eigenfrequencies in the Nambu-Goto string with massive ends [2]. The boundary problem (2.16), (2.18) entails the eigenfunctions

$$v_n(\sigma) = v_{-n}(\sigma) = \sqrt{\frac{2}{\pi}}\sin n\sigma, \quad n = 1,2,\ldots, \tag{2.21}$$

and natural frequencies

$$\overset{(2)}{\omega}_n = -\overset{(2)}{\omega}_{-n} = \sqrt{n^2+\varepsilon^{-1}}, \quad n = 1,2,\ldots \tag{2.22}$$

The amplitudes α_n and β_n in (2.19) and (2.20) obey the common rules of complex conjugation

$$\alpha_n^* = \alpha_{-n}, \quad \beta_n^* = \beta_{-n}. \tag{2.23}$$

3 Quantum theory

To quantize the model under consideration, we have to cast it into the Hamiltonian form. According to Ostrogradskii [6, 7] the canonical variables are introduced as follows

$$q_1^j = x^j, \quad q_2^j = \dot{x}^j,$$

$$p_1^j = \frac{\partial\mathcal{L}}{\partial\dot{x}^j} - \dot{p}_2^j, \quad p_2^j = \frac{\partial\mathcal{L}}{\partial\ddot{x}^j}, \tag{3.1}$$

$$j = 1,2,\ldots,D-2.$$

Here $\mathcal{L}$ is the Lagrange density in the action (2.7). Substituting $\mathcal{L}$ and (2.14) into (3.1) and taking into account the equations of motion (2.15) and (2.17) one gets

$$\mathbf{q}_1 = \mathbf{x}_1 + \mathbf{x}_2,$$

$$\mathbf{q}_2 = \dot{\mathbf{x}}_1 + \dot{\mathbf{x}}_2, \tag{3.2}$$

$$\mathbf{p}_1 = \frac{\rho_0 L}{\pi}(1+\varepsilon\,\Box)\dot{\mathbf{x}} = \frac{\rho_0 L}{\pi}\mathbf{x}_1, \quad \mathbf{p}_2 = -\varepsilon\frac{\rho_0 L}{\pi}\Box\mathbf{x} = \frac{\rho_0 L}{\pi}\mathbf{x}_2.$$

The canonical Hamiltonian is

$$H = \int_0^\pi d\sigma(\mathbf{p}_1\dot{\mathbf{q}}_1 + \mathbf{p}_2\dot{\mathbf{q}}_2 - \mathcal{L}). \tag{3.3}$$

We shall not express the Hamiltonian in terms of the canonical variables $\mathbf{q}_a, \mathbf{p}_a$, $a = 1,2$ but at once calculate it with the equations of motion in terms of the amplitudes $\vec{\alpha}_n$ and $\vec{\beta}_n$. Substituting (3.2) into (3.3) we obtain

$$H = \frac{\rho_0 L}{2\pi}\int_0^\pi d\sigma\left[\xi(\sigma)\dot{\mathbf{x}}_1^2 + a^2\mathbf{x}_1'^2 + \mathbf{x}_2\ddot{\mathbf{x}}_2 - \dot{\mathbf{x}}_2^2\right]. \tag{3.4}$$

In terms of the Fourier-amplitudes the Hamiltonian reads

$$H = \frac{\mathbf{P}^2}{2M} + \frac{a\hbar}{2}\sum_{n=1}^{\infty}(\vec{\alpha}_n\vec{\alpha}_n^+ + \vec{\alpha}_n^+\vec{\alpha}_n) - \frac{a\hbar}{2}\sum_{n=1}^{\infty}(\vec{\beta}_n\vec{\beta}_n^+ + \vec{\beta}_n^+\vec{\beta}_n), \tag{3.5}$$

where M is the total mass of the string $M = \rho_0 L + 2m$, $\mathbf{P}$ is the conserved total momentum of the string

$$P^j = \int_0^\pi d\sigma\, p^j(t,\sigma). \tag{3.6}$$

Thus the second mode of oscillations caused by the string rigidity entails *the negative contribution* to the energy of the system.

The quantum theory is based on the following commutation relations

$$[\,x^i_a(t,\,\sigma),\,p^j_b(t,\,\sigma')\,] = i\hbar\delta_{ab}\,\delta^{ij}\,\delta(\sigma-\sigma')\,, \qquad (3.7)$$

$$a,\,b = 1,2, \quad i,\,j = 1,2,\ldots,D-2.$$

The amplitudes $\vec{\alpha}_n$ and $\vec{\beta}_n$in the series (2.19) and (2.20) are defined by

$$\alpha^j_n = \frac{\exp(i\overset{(1)}{\omega}_n t)}{\sqrt{2\hbar\rho_0 c}}\left\{\int\limits_0^\pi d\sigma\, u_n(\sigma)\,p^j_1(\sigma) - \right.$$

$$\left. -i\,\frac{\rho_0 c}{\overset{(1)}{\omega}_n}\int\limits_o^\pi d\sigma\, u'_n(\sigma)\left[q'^j_1(\sigma) - \frac{\pi}{\rho_0 L}\,p'^j_2(\sigma)\right]\right\},$$

$$\alpha^j_{-n} = \alpha^{*j}_n, \quad n = 1,2,\ldots,$$

$$\beta^j_n = \exp(-ia\overset{(2)}{\omega}_n t)\sqrt{\frac{\rho_0 c}{2\hbar}}\int\limits_0^\pi d\sigma\, v_n(\sigma)\times \qquad (3.8)$$

$$\times\left[\frac{1}{a}\left(q_2(\sigma) - \frac{\pi}{\rho_0 L}\,p_1(\sigma)\right) + i\,\overset{(2)}{\omega}_n\,\frac{\pi}{\rho_0 L}\,p_2(\sigma)\right],$$

$$\beta^j_{-n} = \beta^{*j}_n, \quad n = 1,2,\ldots, \quad j = 1,2,\ldots,D-2.$$

From here we obtain the commutators for α_n and β_n

$$[\,Q^i,\,P^j\,] = i\hbar\,\delta_{ij}, \quad [\,\alpha^i_n,\,\alpha^j_n\,] = \overset{(1)}{\omega}_n\,\delta^{ij}\,\delta_{n+m,0}\,,$$

$$[\,\beta^i_n,\,\beta^j_m\,] = \overset{(2)}{\omega}_n\,\delta^{ij}\,\delta_{n+m,0}\,, \quad n,m = \pm1,\pm2,\ldots. \qquad (3.9)$$

The annihilation and creation operators are introduced in a standard manner

$$\alpha^i_n = \sqrt{\overset{(1)}{\omega}_n}\,a^i_n, \quad \alpha^i_{-n} = \alpha^{+i}_n = \sqrt{\overset{(1)}{\omega}_n}\,a^{+i}_n\,,$$

$$\beta^i_n = \sqrt{\overset{(2)}{\omega}_n}\,b^i_n, \quad \beta^i_{-n} = \beta^{+i}_n = \sqrt{\overset{(2)}{\omega}_n}\,b^{+i}_n\,, \qquad (3.10)$$

$$[\,a^i_n,\,a^{+j}_m\,] = [\,b^i_n,\,b^{+j}_m\,] = \delta^{ij}\,\delta_{nm}\,,$$

$$n,\,m = 1,2,\ldots, \quad i,\,j = 1,2,\ldots,D-2.$$

Finally the Hamilton operator acquires the form

$$H = \frac{\mathbf{P}^2}{2M} + a\,\hbar\sum_{n=1}^{\infty}\overset{(1)}{\omega}_n\sum_{i=1}^{D-2}a^{+i}_n\,a^i_n - a\,\hbar\sum_{n=1}^{\infty}\overset{(2)}{\omega}_n\sum_{i=1}^{D-2}b^{+i}_n\,b^i_n +$$

$$+\,a\hbar\,\frac{D-2}{2}\sum_{n=1}^{\infty}\overset{(1)}{\omega}_n - a\hbar\,\frac{D-2}{2}\sum_{n=1}^{\infty}\overset{(2)}{\omega}_n\,. \qquad (3.11)$$

The last two terms in (3.11) define the Casimir energy in the model under consideration. It is important that the second oscillation mode caused by the string rigidity gives the contribution to the Casimir energy with opposite sign as compared to the oscillation of the basic first mode. It follows directly from the classical expression for the energy (3.5). In paper [5] the Casimir energy in the theory of the relativistic membranes with rigidity has been investigated. Contribution of the zero point oscillations from the rigid mode turns out to have the same sign as an analogous contribution from the basic mode. But it is possible only in the case when one takes as the vacuum wave function for the corresponding oscillators a function like this $\exp\,(q_2^2/2)$. Obviously this state vector is not normalizable and the quantum theory based on such a vacuum state should be unstable.

4 Casimir energy

As in paper [2] we shall investigate the dimensionless Casimir energy per one transverse degree of freedom

$$E(q,\,\varepsilon) = \frac{1}{2}\sum_{n=1}^{\infty}\overset{(1)}{\omega}_n - \frac{1}{2}\sum_{n=1}^{\infty}\overset{(2)}{\omega}_n\,. \qquad (4.1)$$

A finite value for the first sum has been obtained in the preceding paper [2]

$$E_1(q) = \frac{1}{2}\sum_{n=1}^{\infty} \overset{(1)}{\omega}_n(q) = -\frac{1}{24} + \frac{1}{2\pi}\int_0^{\infty} dx \ln\left[1 + \frac{1}{(x/q)^2 + 2(x/q)\coth(\pi x)}\right]. \tag{4.2}$$

Here we consider in detail the evaluation of the second sum in (4.1)

$$E_2(\varepsilon) = -\frac{1}{2}\sum_{n=1}^{\infty} \overset{(2)}{\omega}_n(\varepsilon) = -\frac{1}{2}\sum_{n=1}^{\infty}\sqrt{n^2 + \varepsilon^{-1}}. \tag{4.3}$$

For regularization of the divergent sum in (4.3) we introduce into consideration the function

$$S(s, \alpha) = \sum_{n=1}^{\infty}(n^2 + \alpha)^{-s}; \tag{4.4}$$

defined at $\mathrm{Re}\, s > \frac{1}{2}$ and $\alpha > 0$. This function can be analytically continued to the point $s = -\frac{1}{2}$. For this purpose we apply at first the following integral representation

$$\frac{1}{(n^2+\alpha)^s} = \frac{1}{\Gamma(s)}\int_0^{\infty} t^{s-1} e^{-(n^2+\alpha)t} dt,$$

where $\Gamma(s)$ is the Euler gamma function. Equation (4.4) acquires the form

$$S(s, \alpha) = -\frac{1}{2\alpha^s} + \frac{1}{2\Gamma(s)}\int_0^{\infty} t^{s-1} e^{-\alpha t}\sum_{n=-\infty}^{+\infty} e^{-n^2 t}. \tag{4.5}$$

The sum in (4.5) can be rewritten as

$$\sum_{n=-\infty}^{+\infty} e^{-n^2 t} = \sqrt{\frac{\pi}{t}}\sum_{n=-\infty}^{+\infty} e^{-\frac{\pi^2}{t}n^2}. \tag{4.6}$$

This equality is a direct consequence of the following property of the Jacobi ϑ_3 function [8]

$$\vartheta_3\left(\frac{z}{\tau}\middle| -\frac{1}{\tau}\right) = \sqrt{\frac{\pi}{i}}\exp\left(i\frac{\pi z^2}{\tau}\right)\vartheta_3(z|\tau), \tag{4.7}$$

where

$$\vartheta_3(z|\tau) = \sum_{n=-\infty}^{+\infty} q^{n^2}\left(e^{i\pi z}\right)^{2n}, \quad q = e^{i\pi\tau}, \quad \mathrm{Im}\,\tau > 0. \tag{4.8}$$

Upon substituting (4.6) into (4.5) and integrating over dt one obtains

$$S(s, \alpha) = -\frac{1}{2\alpha^s} + \frac{\sqrt{\pi}}{2}\frac{\Gamma\left(s-\frac{1}{2}\right)\alpha^{-s+1/2}}{\Gamma(s)} + \frac{2\pi^s\alpha^{-s/2+1/4}}{\Gamma(s)}\sum_{n=1}^{\infty} n^{s-1/2}K_{s-1/2}(2n\pi\sqrt{\alpha}), \tag{4.9}$$

where $K_\nu(z)$ is the MacDonald function

$$K_\nu(2\sqrt{\beta\gamma}) = \frac{1}{2}\left(\frac{\gamma}{\beta}\right)^{\frac{\nu}{2}}\int_0^{\infty} x^{\nu-1}e^{-\frac{\beta}{x}-\gamma x}dx,$$

$$K_{-\nu}(z) = K_\nu(z), \quad \mathrm{Re}\,\beta > 0,\ \mathrm{Re}\,\gamma > 0.$$

As the regularized Casimir energy $E_2(\varepsilon)$ in (4.3) we take the expression

$$E_2^{\mathrm{reg}}(\varepsilon) = -\frac{1}{2}S\left(-\frac{1}{2}, \frac{1}{\varepsilon}\right) =$$

$$= \frac{1}{4\sqrt{\varepsilon}} + \frac{\Gamma(-1)}{8\varepsilon} + \frac{1}{2\pi\sqrt{\varepsilon}}\sum_{n=1}^{\infty}\frac{1}{n}K_1\left(\frac{2\pi n}{\sqrt{\varepsilon}}\right). \tag{4.10}$$

The renormalized observable of the Casimir energy is obtained by subtracting from (4.10) E_2^{reg} calculated for the infinite rigid string. This removes the second term in (4.10) with a pole singularity due to $\Gamma(-1)$. The first term in (4.10) after multiplying by the dimensional coefficient $a\hbar$ becomes independent of the string length L. Therefore this term can be omitted too. As a result, the renormalized Casimir energy E_2 turns out to be

$$E_2(\varepsilon) = \frac{1}{2\pi\sqrt{\varepsilon}}\sum_{n=1}^{\infty}\frac{1}{n}K_1\left(\frac{2\pi n}{\sqrt{\varepsilon}}\right). \tag{4.11}$$

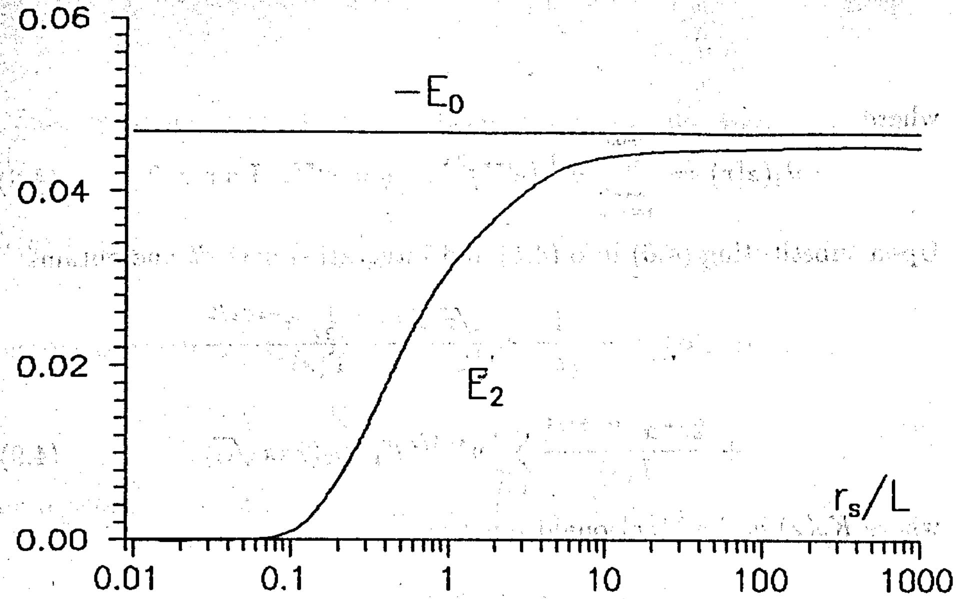

Fig. 1. *Dependence of dimensenless Casimir energy E_2 on the thickness of the gluonic tube $\delta = r_s/L$. The straight line is the Casimir energy $-E_0 = 1/24$ in the Nambu-Goto string.*

Thus the allowance for the transverse dimension of the flux tube (the string stiffness) entails the positive contribution to the Casimir energy. When the thickness of the string tends to zero

$$\delta = \frac{r_s}{L} \to 0, \quad \varepsilon = \alpha\pi^2\delta^2, \; \alpha \sim 20 \tag{4.12}$$

the energy E_2 vanishes exponentially

$$E_2 \underset{\delta\to 0}{\longrightarrow} \frac{\exp\left[-2/(\alpha^{1/2}\delta)\right]}{\pi^{3/2}\alpha^{1/4}\delta^{1/2}}\,. \tag{4.13}$$

In Fig. 1 the dependence of the energy E_2 on the dimensionless parameter $\delta = r_s/L$ is represented. The region $\delta \geq 1$ has obviously no meaning in the framework of the flux tube model, but in the theory of the fundamental rigid string [10] it can be considered. When $\delta \to \infty$, then $E_2 \to 1/24$. This can be seen directly from the definition (4.3). Hence the allowance for the string thickness only does not compensate for the negative Casimir energy $E_0 = -1/24$ in the Nambu-Goto string

with free ends and remove the tachyon from the string spectrum. This can be attained by taking into consideration the quark masses (the first term in (4.1), see also the plot of the function $E_1(q)$ in paper [2]).

5 Conclusion

The results obtained above show that the situation with tachyon in a more realistic hadron string model taking into account the quark mass and flux tube thickness is not so hopeless as in the original Nambu-Goto string. There are such values of the model parameters (the quark mass and transverse dimension of the flux tube) at which the Casimir energy is strictly positive. Hence in this case the tachyon in the string spectrum should be absent.

In our consideration we have assumed that the parameter α in the action (2.1) is positive, i. e. the Nambu-Goto action and the rigidity term have opposite signs. Only in this case the frequencies $\overset{(2)}{\omega}_n$ will be real numbers and as a consequence the solution $\mathbf{x}_2(t, \sigma)$ will be stable. In the opposite case ($\alpha < 0$ that means the negative Young's modulu s for the flux tube material) the solution $\mathbf{x}_2(t, \sigma)$ will increase exponentially in time. This can be interpreted as the flux tube decay.

In the model under consideration there remains an open problem of the negative energy generated by the second rigid mode of oscillation (the term with operators $\mathbf{b}_n^+\mathbf{b}_n$ in the Hamiltonian (3.11)). This defect is customary in all the theories with higher derivatives [11, 12]. So far there is no acceptable solution of this problem. Omitting these solution as it is suggested in a recent paper [13] seems to us not quite consistent at least in the framework of the gluonic tube model. Obviously the solution $\mathbf{x}_2(t, \sigma)$ describes an additional internal degree of freedom of the flux tube. Therefore one could try to obtain the energy of this tube in the first order formalism without higher derivatives by taking into account this additional degree of freedom exactly. Here the analogy with vibration of rods and beams described by the Timoshenko equation [14, 15] may be useful.

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