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B → C FLAVOUR CHANGING DECAYS OF BARYONS CONTAINING A SINGLE HEAVY QUARK

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1. Introduction

Nowadays the investigation of semileptonic (s.l.) decays of baryons containing a single heavy quark has a great popularity [1]- [17]. First, it is a unique tool to determine of Cabibbo-Kabayashi- Maskawa matrix elements and also an original source to probe hadronic structure. One has to remark that the experimental programs concentrated mainly on charm baryons [1, 2]. For example, recently direct observations of the s.l. decay of Λ_c^+ in the decay channels $\Lambda_c^+ \to \Lambda e^+ X$ and $\Lambda_c^+ \to \Lambda \mu^+ X$ have made using the *ARGUS* detector at the *DORIS II* e^+e^- storage ring [17]. At the present time, the experimental research of s.l. decays of beauty baryons is planned.

Second, new theoretical ideas in heavy quark physics were borned [1]-[16]. Particularly, Isgur and Wise have discovered an additional kind of QCD spin-flavour symmetry [3]-[5]. The Isgur-Wise symmetry manifests itself in QCD when the mass of heavy quark m_Q goes to infinity ($m_Q \to \infty$). By exploiting this symmetry, model-independent predictions for some heavy-hadron weak form factors and relations between them have been obtained [3]-[5]. It was shown that form factors arising in heavy-meson decays with a pseudoscalar meson P in the initial state and pseudoscalar P or vector V ones in the final states are defined by a single universal function $\xi(\omega)$ of argument ω with $\xi(1) = 1$ [3]. Dimensionless quantity ω is the dot product of the four-velocities of the initial and final heavy-hadron states. The Isgur-Wise function $\xi(\omega)$ was calculated in the framework of the Valence Quark Model (QM) [5], Relativistic Oscilator Model (ROM) [10], QCD Sum Rules (QCD SR) [15] and Quark Confinement Model (QCM) [16]. It was found that the valence quark model gives the following result for the function $\xi(\omega)$:

$$\xi_{QM}(\omega) = \exp[-0.63(\omega - 1)]. \quad (1)$$

The ξ function obtained in the ref. [10] is

$$\xi_{ROM}(\omega) = \frac{2}{\omega + 1}\exp[-(2\rho^2 - 1)\frac{\omega - 1}{\omega + 1}], \quad \rho \simeq 1 \quad (2)$$

The explicit form of QCD SR form factor can be approximated by the formula

$$\xi_{QCD}(\omega) \approx \exp[-0.37\sqrt{\omega^2 - 1}] \quad (3)$$

And finally, the result of the Quark Confinement Model is

$$\xi_{QCM}(\omega) = 0.4 \cdot \Phi(\omega) + \frac{1.2}{1 + \omega}, \quad (4)$$

$$\Phi(\omega) = \frac{1}{\sqrt{\omega^2 - 1}} \ln[\omega + \sqrt{\omega^2 - 1}]$$

The s.l. decays of $\frac{1}{2}^+$ heavy baryons to $\frac{1}{2}^+$ and $\frac{3}{2}^+$ heavy baryon final states have been considered in [4] in the limit of large heavy quark masses. It was shown that the heavy-baryon weak form factors may be expressed in this limit through the three universal unknown functions $\xi(\omega)$, $\eta(\omega)$ and $\tau(\omega)$. The exclusive s.l. decays of heavy baryons have been worked out by J.G. Körner and collaborators (see,[2], [12]-[14]) in the relativistic spectator quark model.

In the papers [16],[18]-[25], we have developed the Quark Confinement Model (QCM) based on some assumptions about the hadronization and confinement of light quarks. By assuming hadrons to be colourless excitations of quark-gluon interactions, the transition to hadron variables in the QCD functional was made following [26]. The hadron interactions are described by quark diagrams averaged over vacuum gluon

backgrounds. The confinement hypothesis means that this averaging leads to that quarks do not appear in the observable hadron spectrum. Strong, weak and electromagnetic interactions of hadrons (both mesons and baryons) can be described in the QCM from a unified point of view. The calculations [18]-[23] of low-energy meson and baryon processes have shown that the model reproduces the quark structure of light hadrons quite accurately.

An extension of the QCM to heavy quark physics has been done in [16, 24]. It was based on that heavy quarks weakly interact with vacuum background fields, and therefore, they can be considered as the static Fermi particles with large constituent masses. At the same time, interactions of the light quarks are defined by the confinement forces. The form factors of the s.l. heavy meson decays were calculated. The heavy quark limit $m_Q \to \infty$ (Isgur-Wise symmetry) for these form factors was examined in paper [16].

In this paper, we calculate the weak form factors of s.l. decays $\Lambda_b^0 \to \Lambda_c^+$ and $\Sigma_b^+ \to \Sigma_c^{++}$ within the QCM. We obtain explicit expressions for these form factors and analyse them at heavy quark limit (Isgur-Wise symmetry). By making use these form factors we compute the decay rates Γ, differential q^2-distributions $d\Gamma/dq^2$ and lepton E_ℓ-spectra $d\Gamma/dE_\ell$. It is found out that the form factors obtained in the Isgur-Wise limit behave similar like to those in the explicit calculations. This confirms that the Isgur-Wise symmetry works well for the b-c transition. We compare our results with the calculations performed in the relativistic spectator quark model [12, 14] with various shapes of the form factors.

2. Kinematics of Semileptonic Decays of Heavy Baryons

In this Section we give the main kinematical values defining the semileptonic decays of heavy baryons with quantum numbers: spin $J = \frac{1}{2}$ and positive P-parity

$$B_b(p) \to B_c(p') + e(k_1) + \nu(k_2), \tag{5}$$

where $B_b(p)$ is a heavy baryon containing a b-quark, $B_c(p')$ is a heavy baryon containing a c-quark. The total momentum of a lepton pair is $q = k_1 + k_2$. We will neglect an electron mass in what follows. For masses of beauty and charm baryons we will use the following notations: m_{B_b} and m_{B_c}, respectively.

The corresponding matrix element has the following form

$$M(b \to c) = \frac{G_F}{\sqrt{2}} V_{bc} \bar{B}_c(p') \Lambda_\mu(p,p') B_b(p) \ell_\mu(q) \tag{6}$$

where $G_F = 1.1664 \cdot 10^{-5}\mathrm{GeV}^{-2}$ is the Fermi constant,

$\ell_\mu = \bar{u}_\nu \gamma_\mu (1 - \gamma_5) u_\ell$ is a lepton current,

V_{bc} is the Kobayashi-Mascawa matrix element.

The vertex function $\Lambda_\mu(p,p')$ can be decomposed into a combination of six relativistic form factors. Usually, two sets of heavy-baryon weak form factors are used

$$\begin{aligned} \Lambda_\mu(p,p') &= F_1(t)\gamma_\mu + F_2(t)v_\mu + F_3(t)v'_\mu \\ &+ G_1(t)\gamma_\mu\gamma_5 + G_2(t)v_\mu\gamma^5 + G_3(t)v'_\mu\gamma^5 \end{aligned} \tag{7}$$

where $v_\mu = p_\mu/m_{B_b}$, $v'_\mu = p'_\mu/m_{B_c}$ are the four-velocities of beauty and charm baryons, respectively, or

$$\begin{aligned} \Lambda_\mu(p,p') &= f_1(t)\gamma_\mu + f_2(t)i\sigma^{\mu\nu}q_\nu + f_3(t)q_\mu \\ &+ f_4(t)\gamma_\mu\gamma_5 + f_5(t)i\sigma^{\mu\nu}q_\nu\gamma_5 + f_6(t)q_\mu\gamma_5 \end{aligned} \tag{8}$$

The form factors F_i and G_i are related to $f_i, i = 1, ..., 6$ on mass-shell by the following way

$$\begin{aligned} f_1 &= F_1 + \frac{m_{B_b} + m_{B_c}}{2}\left(\frac{F_2}{m_{B_b}} + \frac{F_3}{m_{B_c}}\right), \\ f_2 &= \frac{1}{2}\left(\frac{F_2}{m_{B_b}} + \frac{F_3}{m_{B_c}}\right), \qquad f_3 = \frac{1}{2}\left(\frac{F_2}{m_{B_b}} - \frac{F_3}{m_{B_c}}\right), \\ f_4 &= G_1 - \frac{m_{B_b} - m_{B_c}}{2}\left(\frac{G_2}{m_{B_b}} + \frac{G_3}{m_{B_c}}\right), \\ f_5 &= \frac{1}{2}\left(\frac{G_2}{m_{B_b}} + \frac{G_3}{m_{B_c}}\right), \qquad f_6 = \frac{1}{2}\left(\frac{G_2}{m_{B_b}} - \frac{G_3}{m_{B_c}}\right). \end{aligned} \tag{9}$$

The s.l. decay widths are calculated according to the formula

$$\begin{aligned} \Gamma &= \int_0^{m_-^2} dt \frac{d\Gamma}{dt} \\ \frac{d\Gamma}{dt} &= \frac{1}{384\pi^3} \frac{G_F^2}{m_{B_b}^3} |V_{bc}|^2 R(t), \end{aligned} \tag{10}$$

where $m_\pm = m_{B_b} \pm m_{B_c}$. The function $R(t)$ is equal to

$$\begin{aligned} R(t) &= 2\sqrt{Q_+Q_-} \cdot [(m_-^2 - t) \cdot (f_1^2(t)(2t + m_+^2) \\ &+ f_2^2(t)(2m_+^2 + t)t - 6f_1(t)f_2(t)m_+t) \\ &+ (m_+^2 - t) \cdot (f_4^2(t)(2t + m_-^2) + f_5^2(t)(2m_-^2 + t)t \\ &+ 6f_4(t)f_5(t)m_-t)], \qquad Q_\pm = m_\pm^2 - t \end{aligned} \tag{11}$$

Here the product of the Q_+ and Q_- is the well-known triangle function $Q_+ \cdot Q_- = \lambda(m_{B_b}^2, m_{B_c}^2, t)$.

Differential electron E_ℓ-spectra $d\Gamma/dE_\ell$ for s.l. decays of baryons is defined as

$$\frac{d\Gamma}{dE_\ell} = \frac{1}{16\pi^3}\frac{G_F^2}{m_{B_b}^2}|V_{bc}|^2 \int\limits_0^{t_{max}} dt T(t,E_\ell), \tag{12}$$

where $E_\ell = pk_1/m_{B_b}$ is the lepton energy in parent rest frame ($\vec{p}=0$) and

$$\begin{aligned} T(t,E_\ell) &= f_1^2(t)[m_{B_b}(m_{B_b}-2E_\ell)(t_{max}-t)+\frac{t}{2}Q_-] \\ &+ f_2^2(t)t[-m_{B_b}(m_{B_b}-2E_\ell)(t_{max}-t)+\frac{t}{2}Q_- + \frac{1}{2}Q_+Q_-] \\ &+ f_4^2(t)[m_{B_b}(m_{B_b}-2E_\ell)(t_{max}-t)+\frac{t}{2}Q_+] \\ &+ f_5^2(t)t[-m_{B_b}(m_{B_b}-2E_\ell)(t_{max}-t)+\frac{t}{2}Q_+ + \frac{1}{2}Q_+Q_-] \\ &- f_1(t)f_2(t)tm_+Q_- + f_4(t)f_5(t)tm_-Q_+ \end{aligned} \tag{13}$$

Here

$$t_{max} = 4m_{B_b}E_\ell \cdot \frac{E_\ell^{max}-E_\ell}{m_{B_b}-2E_\ell}, \qquad E_\ell^{max} = \frac{m_{B_b}^2 - m_{B_c}^2}{2m_{B_b}}.$$

3. The Semileptonic Decays of Heavy Baryons in the QCM

The Quark Confinement Model (QCM), a relativistic quark model, is based on some assumptions about hadronization and quark confinement. A more detailed description of the QCM is given in papers [18]-[23]. In this section, main notions of the QCM for applications to physics of s.l. decays of heavy baryons will be given. We consider two typical processes $\Lambda_b^0 \to \Lambda_c^+ + e^- + \bar\nu$ and $\Sigma_b^+ \to \Sigma_c^{++} + e^- + \bar\nu$ as examples of heavy-baryon decays with spin of light quarks $S=0$ and $S=1$, respectively. The heavy baryons Λ_Q and Σ_Q contain antisymmetrical $(Q\{qq\}_A)$ and symmetrical $(Q\{qq\}_S)$ spin combinations of light quarks, respectively. Their masses are taken from the Particle Data Group [27]:

$$\begin{aligned} m_{\Lambda_c^+} &= 2.28 \text{ GeV}, \qquad m_{\Lambda_b^0} = 5.60 \text{ GeV} \\ m_{\Sigma_c^{++}} &= 2.45 \text{ GeV}, \qquad m_{\Sigma_b^+} = 5.73 \text{ GeV} \end{aligned} \tag{14}$$

Dynamical description of hadron processes in the QCM is based on interaction Lagrangians of hadrons with quarks. Particular interaction Lagrangians of Λ_Q and Σ_Q-baryons with quarks have the following form

$$L_{B_Q} = g_{B_Q}\bar{B}_Q J_{B_Q} + h.c. \tag{15}$$

where B_Q is the baryon field (Λ_Q or Σ_Q), g_{B_Q} is the coupling constant.

The J_{B_Q} are the three-quark currents with the quantum numbers of baryons Λ_Q and Σ_Q. It has been shown [19, 20] that there are two independent sets of three-quark currents for baryons with $J^P = \frac{1}{2}^+$. In particular, for Λ_Q and Σ_Q baryons we have

⋆ *Tensor variant*

$$\begin{aligned} J_{\Lambda_Q} &= \varepsilon^{abc}\cdot[Q^a(u^bC\gamma^5d^c) + \gamma^5Q^a(u^bCd^c)], \\ J_{\Sigma_Q} &= \varepsilon^{abc}\cdot\sigma^{\mu\nu}\gamma^5Q^a(u^bC\sigma^{\mu\nu}u^c), \end{aligned} \tag{16}$$

⋆ *Vector variant*

$$\begin{aligned} J_{\Lambda_Q} &= \varepsilon^{abc}\cdot[Q^a(u^bC\gamma^5d^c) - \gamma^5Q^a(u^bCd^c) - \frac{1}{2}\gamma^\mu Q^a(u^bC\gamma^\mu\gamma^5d^c)] \\ J_{\Sigma_Q} &= \varepsilon^{abc}\cdot\gamma^\mu\gamma^5Q^a(u^bC\gamma^\mu u^c), \end{aligned} \tag{17}$$

where a,b,c are colour indices, $C=\gamma^0\gamma^2$ is the matrix of charge conjugation; $Q=c$ or b quark.

S.l. decays of heavy baryons are described in the QCM by quark diagrams (Fig.1a). The heavy quarks were suggested to describe as usual Fermi particles [16, 24] because they weakly interact with external background fields whereas the behaviour of light quarks are governed by confinement mechanism. According to this assumption, a vertex function $\Lambda_\mu(p,p')$ corresponding to diagram Fig.1a is written as

$$\Lambda_\mu(p,p') = 6 l_f g_{B_b} g_{B_c} 3^4 i^2 \int d^4x_1 \int d^4x_2 \int d^4x_3 \delta(\sum_{i=1}^{3} x_i) e^{-ip(x_1-x_3)+p'(x_3-x_2)}$$

$$\sum_{\Gamma_1\Gamma_2} c_{\Gamma_1\Gamma_2} \Gamma_1 S_b(x_1-x_3) O_\mu S_c(x_3-x_2)\Gamma_2 \int d\sigma_{vac} \Pi^{\Gamma_1\Gamma_2}_{vac}(x_1,x_2|B_{vac}), \quad (18)$$

where

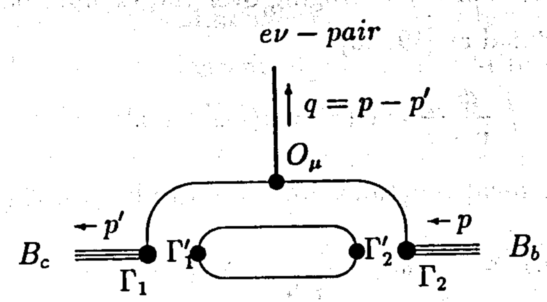
$$\Pi^{\Gamma_1\Gamma_2}_{vac}(x_1,x_2|B_{vac}) = \int d\sigma_{vac} \mathrm{tr}[\Gamma_1' S(x_1 x_2|B_{vac})\Gamma_2' S(x_2 x_1|B_{vac})].$$

The coupling constants g_{B_Q} in (18) are determined from the compositeness condition [18, 28]:

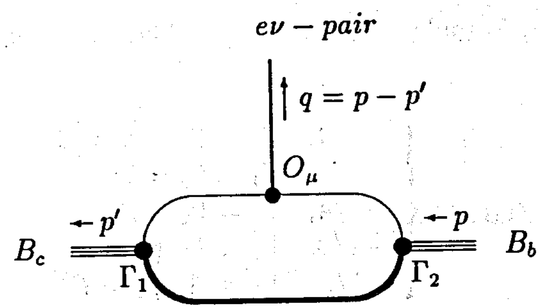
$$Z_{B_Q} = 1 + g^2_{B_Q}\Sigma'_{B_Q}(m_{B_Q}) = 0, \quad (19)$$

where Σ'_{B_Q} is the derivative of the heavy baryon mass operator which is defined as

$$\Sigma_{B_Q}(p) = 6 l_f \cdot 2^4 i \int d^4x_1 \int d^4x_2 \delta(x_1+x_2) e^{-ip(x_1-x_2)} \quad (20)$$
$$\cdot \sum_{\Gamma_1\Gamma_2} c_{\Gamma_1\Gamma_2}\Gamma_1 S_Q(x_1-x_2)\Gamma_2 \int d\sigma_{vac}\Pi^{\Gamma_1\Gamma_2}_{vac}(x_1,x_2|B_{vac}).$$



(a)



(b)

Fig.1 Heavy-Baryon Weak Form Factors in the QCM

Here

$$S_Q(x_1-x_2) = <0|T(Q(x)\bar{Q}(y))|0> = i(\not{p} - m_Q)^{-1}\delta(x-y), \quad (21)$$

is the heavy quark propagator with mass m_Q, and

$$S(x_1,x_2|B_{vac}) = <0|T(q(x)\bar{q}(y))|0> = i(\not{p} + \not{B}_{vac})^{-1}\delta(x-y), \quad (22)$$

is the light quark propagator in the external background field.

$d\sigma_{vac}$ is the measure of the averaging over the vacuum background field B^{μ}_{vac} which is defined as [19, 20]

$$\int \frac{d\sigma_v}{v-z} = G(z) = a(-z^2) + zb(-z^2), \qquad (23)$$

where the confinement functions are chosen to be [18]-[23]

$$a(u) = \int d\sigma_v \frac{v}{v^2+u} = 2\exp\{-u^2-u\}, \qquad (24)$$

$$b(u) = \int d\sigma_v \frac{1}{v^2+u} = 2\exp\{-u^2+0.4u\}. \qquad (25)$$

The Dirac matrices $\Gamma_{1(2)}$, $\Gamma'_{1(2)}$, the flavour coefficients l_f, and the coefficients $c_{\Gamma_1\Gamma_2}$ come from the interaction Lagrangians of heavy baryons with quarks (see, formula (16) and (17)). The coefficients $c_{\Gamma_1\Gamma_2}$ and l_f are equal to

$$c_{\Gamma_1\Gamma_2} = \begin{cases} 1, & \Gamma_1=\Gamma_2=S,P,T \\ -1, & \Gamma_1=\Gamma_2=V \\ -1/4, & \Gamma_1=\Gamma_2=A \\ -1/2, & \Gamma_1=P,\ \Gamma_2=A \\ 1/2, & \Gamma_1=A,\ \Gamma_2=P \end{cases}$$

and

$$l_f = \begin{cases} 1, & \text{for } \Lambda_Q \to \Lambda_{Q'} \text{ transition} \\ 2, & \text{for } \Sigma_Q \to \Sigma_{Q'} \text{ transition} \end{cases}$$

It was proposed in ref. [20] to use the *Quark-Diquark Approximation of the Three-Quark Structure of Baryons* under calcualtion of the integrals (18) and (20):

$$\int d\sigma_{vac} \Pi^{\Gamma_1\Gamma_2}_{vac}(x_1 x_2|B_{vac}) \to D(x_1-x_2) \qquad (26)$$

where $D(x_1-x_2)$ is considered to be the diquark propogator

$$D(x_1-x_2) = \int \frac{d^4k}{(2\pi)^4 i} \int d\sigma_v e^{-ik(x_1-x_2)} \frac{d^{\Gamma_1\Gamma_2}}{v^2\Lambda_D^2-k^2} \qquad (27)$$

This assumption essentially simplifies calculations because it allows one to replace two-loop quark diagrams drawn in Fig.1a to one-loop quark-diquark diagrams showed in Fig.1b. From the physical point of view this prescription may be justified by that the baryons containing the only heavy quark may be considered as two-particle systems Qd where d is a light diquark. This approximation can be realized for each of two three-quark currents (see, formula (16) and (17)) independently. It was shown [20] that the tensor current gives more best description of the experimental data. So in this paper we will work only with *T-variant* of the *Quark-Diquark Approximation of the Three-Quark Structure of Baryons.* The adjustable parameters for *T-variant* are turned out to be equal to $\Lambda_D = 827.7$ MeV and

$$d^{\Gamma_1\Gamma_2} = \begin{cases} 1, & \Gamma_1=\Gamma_2=S,P \\ 2\cdot g^{\mu\alpha}g^{\nu\beta}, & \Gamma_1=\Gamma_2=T \end{cases}$$

Using this approximation gives the following expressions for the vertex (18) and the self-energy (20) functions:

$$\Lambda_\mu(p,p') = 6 l_f g_{B_b} g_{B_c} \sum_{\Gamma_1\Gamma_2} c_{\Gamma_1\Gamma_2} d^{\Gamma_1\Gamma_2} \int \frac{d^4k}{(2\pi)^4 i} b(-k^2) \qquad (28)$$

$$\cdot\ \Gamma_1 \frac{1}{m_c - \not k - \not p'} O_\mu \frac{1}{m_b - \not k - \not p} \Gamma_2$$

and

$$\Sigma_{B_Q}(p) = 6 l_f \sum_{\Gamma_1\Gamma_2} c_{\Gamma_1\Gamma_2} d^{\Gamma_1\Gamma_2} \int \frac{d^4k}{(2\pi)^4 i} b(-k^2) \Gamma_1 \frac{1}{m_Q - \not k - \not p} \Gamma_2 \qquad (29)$$

Here and further, all masses and momenta are given in the units of the dimensional parameter Λ_D.

The typical four-dimensional integral in the expressions (28) and (29)

$$I(p^2) = \int \frac{d^4k}{\pi^2 i} \frac{b(-k^2)}{m_Q^2 - (k+p)^2} \quad (30)$$

is calculated in a standard manner (see, [16, 24]):

(i) The transition to the Euclidean region is performed for the internal momentum $k^0 \to ik_4$, $k^2 \to -k_E^2$ and external ones $p^0 \to ip_4$, $p^2 \to -p_E^2$;

(ii) The integration over sphere angles is carried out using the formula:

$$\int\limits_0^\pi d\theta \frac{\sin^2\theta}{r+\cos\theta} = \pi[r - \sqrt{r^2-1}] \qquad (r \geq 1); \quad (31)$$

(iii) The analytical continuation to the physical region over external momentum is fulfilled.

Finally we have

$$I(p^2) = \int\limits_0^\infty dub(u)C(u,p^2,m_Q^2), \quad (32)$$

where

$$C(u,x,z) = \frac{\sqrt{(u+z-x)^2+4ux} - (u+z-x)}{2x}.$$

It is to be remarked that in our approach baryon masses are not directly related to quark masses. We assume that the light quarks have no the fix masses at low energy. Their masses are smeared by a background field so that the quarks do not appear in the observable hadron spectrum. In other words we do not solve any equations on the bound states which could give us the relations between hadron and quark (or diquark) masses. The hadron masses are taken from the experiment. Generally speaking, the parameter $\Delta = m_{B_Q} - m_Q$ is not the binding energy from this point of view. To avoid the uncertainties connected with the choice

of this parameter, we suppose that the mass of heavy baryon containing the only heavy quark is equal to the mass of heavy quark $m_{B_Q} = m_Q$. It can be justified because the masses of baryons having the same heavy quark but the different combinations of light quarks are closed to each other (see, e.g. (14)).

Making use the compositeness condition (19) we get the following expressions for the quark-baryon coupling constants

$$g_{B_Q}^{-2} = \frac{3}{8\pi^2} l_B m_{B_Q}^2 R_0(m_{B_Q}^2), \quad (33)$$

where

$$R_0(x) = \int\limits_0^\infty dub(u) \frac{C_0(u,x)[1-C_0(u,x)]}{\sqrt{u^2+4ux}},$$

$$C_0(u,x) = C(u,x,x) = \frac{\sqrt{u^2+4ux}-u}{2x}.$$

The factor l_B being related to the flavour and T-product combinations is equal to

$$l_B = \begin{cases} 4, & \text{for } B = \Lambda_Q \\ 96, & \text{for } B = \Sigma_Q \end{cases}$$

By analogy the vertex function (28) is computed. We have

$$\begin{aligned} F\{G\}_i(t) &= \frac{3}{8\pi^2} l_B m_{B_b} m_{B_c} g_{B_c} g_{B_b} \int\limits_0^\infty dub(u) \int\limits_0^1 d\alpha \tilde{F}\{\tilde{G}\}_i(u, D(\alpha)) \\ &\cdot \frac{C_0(u,D(\alpha))}{\sqrt{u^2+4uD(\alpha)}} \\ &= \frac{1}{\sqrt{R_0(m_{B_b}^2) R_0(m_{B_c}^2)}} \int\limits_0^\infty dub(u) \int\limits_0^1 d\alpha \tilde{F}\{\tilde{G}\}_i(u, D(\alpha)) \\ &\cdot \frac{C_0(u,D(\alpha))}{\sqrt{u^2+4uD(\alpha)}} \end{aligned} \quad (34)$$

where $D(\alpha) = m_{B_b}^2\alpha + m_{B_c}^2(1-\alpha) - t\alpha(1-\alpha)$:

The functions $\tilde{F}\{\tilde{G}\}_i(u, D(\alpha))$ have the following form:

a. Decay $\Lambda_b^0 \to \Lambda_c^+$

$$\tilde{F}_1 = 1 - \frac{1}{2}\left(1 - \frac{\alpha}{r} - r(1-\alpha)\right)C_0(u, D(\alpha)),$$
$$\tilde{F}_2 = -\frac{\alpha}{r}C_0(u, D(\alpha)),$$
$$\tilde{F}_3 = -r(1-\alpha)C_0(u, D(\alpha)),$$
$$\tilde{G}_1 = -1 + \frac{1}{2}\left(1 + \frac{\alpha}{r} + r(1-\alpha)\right)C_0(u, D(\alpha))$$
$$\tilde{G}_2 = -\tilde{F}_2, \qquad \tilde{G}_3 = \tilde{F}_3$$

b. Decay $\Sigma_b^+ \to \Sigma_c^{++}$

$$\tilde{F}_1 = -\frac{1}{3}\left[1 - \frac{1}{2}\left(1 - \frac{\alpha}{r} - r(1-\alpha)\right)C_0(u, D(\alpha))\right],$$
$$\tilde{F}_2 = \frac{2}{3}\left[1 - \frac{\alpha}{2}\left(1 + 2r\right)C_0(u, D(\alpha))\right],$$
$$\tilde{F}_3 = \frac{2}{3}\left[1 - \frac{1-\alpha}{2}\left(1 + \frac{2}{r}\right)C_0(u, D(\alpha))\right],$$
$$\tilde{G}_1 = \frac{1}{3}\left[1 - \frac{1}{2}\left(1 + \frac{\alpha}{r} + r(1-\alpha)\right)C_0(u, D(\alpha))\right],$$
$$\tilde{G}_2 = \frac{2}{3}\left[1 - \frac{\alpha}{2}\left(1 - 2r\right)C_0(u, D(\alpha))\right],$$
$$\tilde{G}_3 = -\frac{2}{3}\left[1 + \frac{1-\alpha}{2}\left(1 - \frac{2}{r}\right)C_0(u, D(\alpha))\right].$$

Here $r = m_{B_c}/m_{B_b}$.

Table 1. Form Factors of $\Lambda_b^0 \to \Lambda_c^+$ Decay in the Heavy Quark Limit

Form Factors	Approaches		
	QCM	Isgur and Wise [4]	Körner [2]
F_1	$\Phi(\omega)$	$\xi(\omega)$	$F_\Lambda(\omega)$
F_2	0	0	0
F_3	0	0	0
G_1	$-\Phi(\omega)$	$-\xi(\omega)$	$-F_\Lambda(\omega)$
G_2	0	0	0
G_3	0	0	0

Let us discuss the heavy quark limit $m_Q = m_{B_Q} \to \infty$ in the form factors $F\{G\}_i (i = 1,2,3)$ obtained in the QCM. It is easily to get this limit in the typical integral

$$\int_0^\infty dub(u) \int_0^1 d\alpha \frac{C_0(u, D(\alpha))}{\sqrt{u^2 + 4uD(\alpha)}}$$
$$= \int_0^\infty dub(u) \int_0^1 d\alpha \frac{1}{2D(\alpha)}\left[1 + \sum_{n=0}^\infty (-)^{n+1}\left(\frac{u}{4D(\alpha)}\right)^{n+\frac{1}{2}} \frac{\Gamma(n+\frac{1}{2})}{\Gamma(n)\Gamma(\frac{1}{2})}\right]$$
$$= \int_0^\infty dub(u) \int_0^1 d\alpha \frac{1}{2D(\alpha)}\left[1 + O\left(\sqrt{\frac{u}{D(\alpha)}}\right)\right]$$
$$= \frac{\Phi(\omega)}{2m_b m_c}\{1 + O(1/m)\}$$

where

$$\Phi(\omega) = \frac{1}{\sqrt{\omega^2-1}} \ln\left(\omega + \sqrt{\omega^2-1}\right).$$

Here ω is the dot product of the four-velocities $v = p/m_b$ and $v' = p'/m_c$.

Table 2. Form Factors of $\Sigma_b^+ \to \Sigma_c^{++}$ Decay in the Heavy Quark Limit

Form Factors	Approaches		
	QCM	Isgur and Wise [4]	Körner [2]
F_1	$-\frac{1}{3}\Phi(\omega)$	$-\frac{1}{6}(2\omega\eta(\omega) + (\omega-1)\tau(\omega))$	$-\frac{1}{3}F_\Lambda(\omega)$
F_2	$\frac{2}{3}\Phi(\omega)$	$\frac{2}{3}\eta(\omega)$	$\frac{2}{3(\omega+1)}(F_L(\omega)+F_T(\omega))$
F_3	$\frac{2}{3}\Phi(\omega)$	$\frac{2}{3}\eta(\omega)$	$\frac{2}{3(\omega+1)}(F_L(\omega)+F_T(\omega))$
G_1	$\frac{1}{3}\Phi(\omega)$	$\frac{1}{6}(2\omega\eta(\omega) + (\omega-1)\tau(\omega))$	$\frac{1}{3}F_\Lambda(\omega)$
G_2	$\frac{2}{3}\Phi(\omega)$	$-\frac{2}{3}(\eta(\omega)+\tau(\omega))$	$-\frac{2}{3(\omega-1)}(F_L(\omega)-F_T(\omega))$
G_3	$-\frac{2}{3}\Phi(\omega)$	$\frac{2}{3}(\eta(\omega)+\tau(\omega))$	$\frac{2}{3(\omega-1)}(F_L(\omega)-F_T(\omega))$

Particularly the form factor F_1 in the limit $m_Q \to \infty$ looks as

$$F_1(t) = \begin{cases} \Phi(\omega), & \text{for } \Lambda_b^0 \to \Lambda_c^+ \text{ transition,} \\ -\frac{1}{3}\cdot\Phi(\omega), & \text{for } \Sigma_b^+ \to \Sigma_c^{++} \text{ transition} \end{cases}$$

Other form factors are shown in Tables 1 and 2. For comparison, we give the Isgur-Wise [4] and J.G. Körner [2] results. In the definition of the Isgur-Wise form factors we omit the coefficient function $C^{ji}(\omega)$ (see, ref. [4]). One has to remark that all form factors obtained in the QCM are expressed through the universal function $\Phi(\omega)$ which is independent on the model parameters. It is differed from the heavy meson form factors having the dependence on the integrals of the confinement functions $A_0 = \int_0^\infty du a(u)$ and $B_{1/2} = \int_0^\infty du\sqrt{u} b(u)$, see [16].

It is easily to express the Isgur-Wise and Körner form factors through the function Φ (see Table 1 and 2). We have

$$\xi(\omega) = \Phi(\omega), \qquad F_\Lambda(\omega) = \Phi(\omega)$$
$$\eta(\omega) = \Phi(\omega), \qquad F_L(\omega) = \Phi(\omega)$$
$$\tau(\omega) = -2\Phi(\omega), \qquad F_T = \omega\Phi(\omega).$$

4. Analysis

In this Section we discuss numerical results obtained within the QCM for basic characteristics of s.l. decays of heavy baryons corresponding to $b \to c$ flavour exchange: the dependence of weak form factor F_1 on the ω-variable, decay rates Γ, differential distributions $d\Gamma/dq^2$ and leptonic spectra $d\Gamma/dE_\ell$. In numerical calculations we will take for V_{bc} the value 0.045 as in ref. [12].

In Fig.2 we present the dependence of the QCM form factor F_1 arising in the process $\Lambda_b^0 \to \Lambda_c^+$. The explicit form factor and its mathematical asymptotics in the heavy quark limit are showed in the region $1 \le \omega \le (m_{B_b} - m_{B_c})^2$. Also we plot the form factors which are used in the spectator quark model [12]. In the paper [12] three different q^2 (or ω)-dependences of the F_1 form factor were considered:

(i) Dipole and monopole form factor

$$F_1(q^2) = F_1(0)\left[\frac{m_{FF}^2}{m_{FF}^2 - q^2}\right]^n, \quad n = 1, 2$$

(ii) Form factor dictated by the Generalized Vector Dominance Model (GVDM)

$$F_1(q^2) = F_1(0)\prod_{l=1}^{2}\frac{m_{FF}^{(l)2}}{m_{FF}^{(l)2} - q^2},$$

where $m_{FF} = 6.34$ GeV, $m_{FF}^{(l)} = m_{FF} + (l-1)\cdot 0.55$ GeV

It must be remarked the explicit form factor F_1 and its asymptotics Φ practically coincide between each other. So in $b \to c$ flavour transitions of heavy baryons Isgur-Wise symmetry manifests itself very clearly. Thus $1/m_Q$-corrections to the form factors arising in the leading order can be neglected. One can see that the QCM form factors go higher than

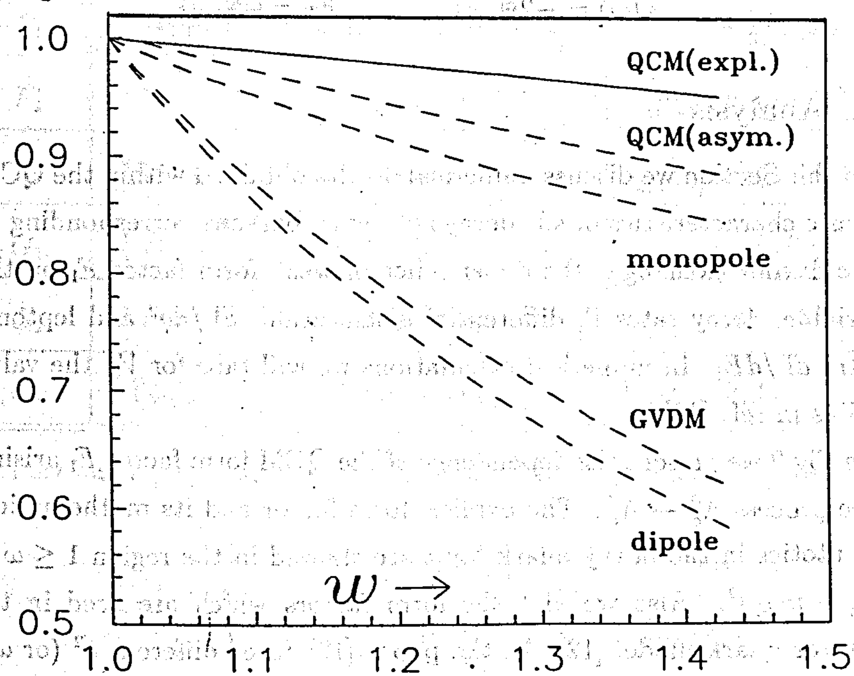


Fig.2

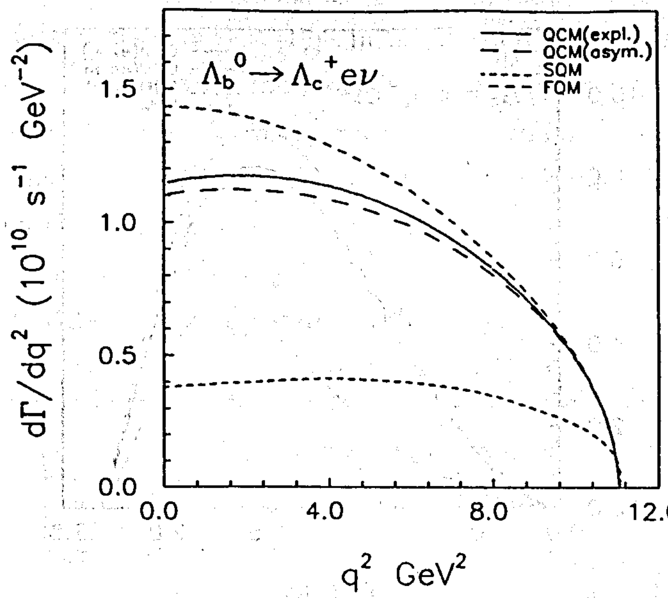


Fig.3

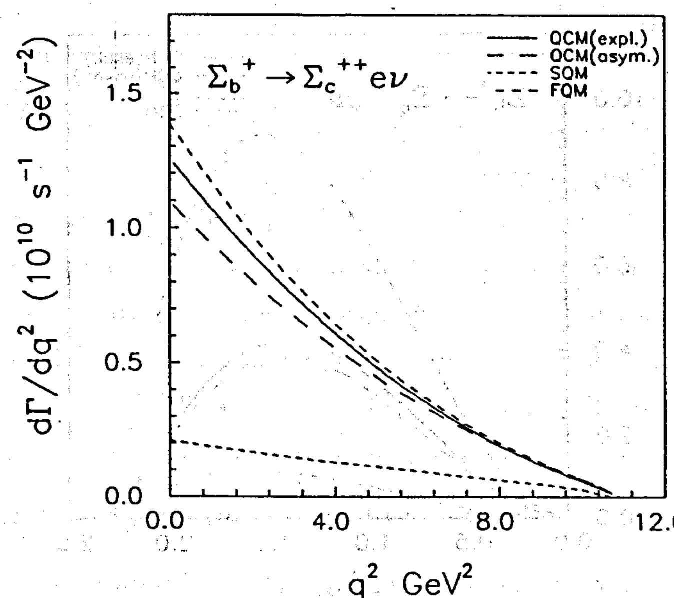


Fig.4

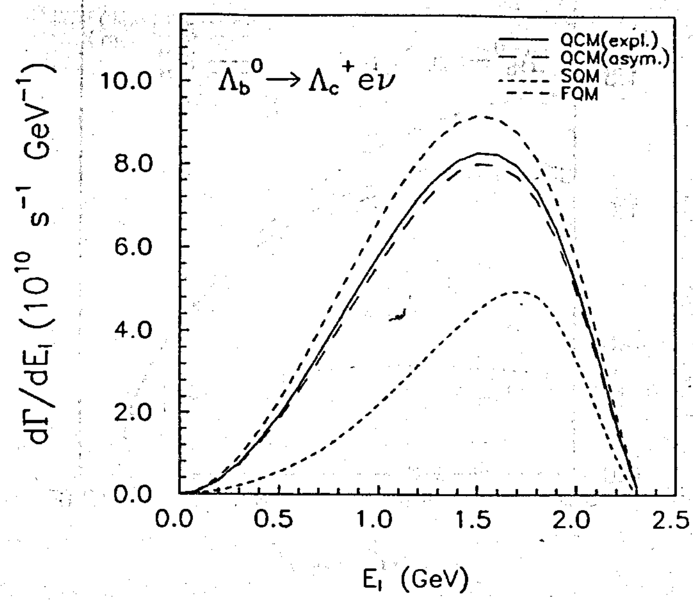


Fig.5

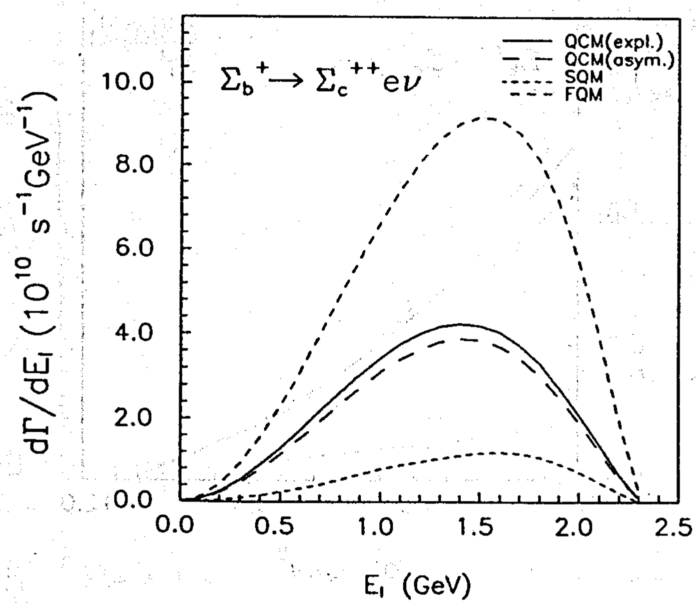


Fig.6

the form factors used in ref. [12]. In fact the QCM form factors are in qualitative agreement with the monopole form factor.

The values obtained for decay rates corresponding to the transitions $\Lambda_b^0 \to \Lambda_c^+$ and $\Sigma_b^+ \to \Sigma_c^{++}$ are given in Table 3. We make comparison of our results with ones obtained in the spectator quark model (SQM) [14] and the free quark model (FQM).

Table 3. Decay Rates of Semileptonic Decays $\Lambda_b^0 \to \Lambda_c^+$ and $\Sigma_b^+ \to \Sigma_c^{++}$

Process	Decay Rate, Γ $10^{10} sec^{-1}$			
	QCM		SQM [14]	FQM
	Explicit results	Heavy Quark Limit		
$\Lambda_b^0 \to \Lambda_c^+$	10.4	10	5.9	11.9
$\Sigma_b^+ \to \Sigma_{cc}^+$	5.4	4.9	4.3	5.7

Differential q^2-rates $d\Gamma/dq^2$ for s.l. processes $\Lambda_b^0 \to \Lambda_c^+$ and $\Sigma_b^+ \to \Sigma_c^{++}$ in the region of values q^2: $0 \le q^2 \le (m_{B_b} - m_{B_c})^2$ are drawn in Fig.3,4. The results for E_ℓ-spectra $d\Gamma/dE_\ell$ in the region of values E_ℓ: $0 \le E_\ell \le E_\ell^{max}$ are performed in Fig.5,6. For comparison, the results for differential distributions and leptonic spectra obtained by Körner and collaborators in ref. [12] and in the FQM are given. One can see that our results are more close to the free quark model.

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