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M.Majewski*

A MODEL FOR GLUEBALL MIXING AND THE PROBLEM OF THETA MESON

*Permanent address: Institute of Physics, University of Łódź, ul.Pomorska 149/153, 90-236 Łódź, Poland

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1. INTRODUCTION

The Exotic Commutator Mixing Model (ECMM) has been formulated as a means to describe mixing of the $q\bar{q}$ nonet states with an extra singlet state of the flavor SU(3) symmetry [1,2]. The best known candidate for the additional state is a glueball. The mixing enlarges the multiplet of the physical mesons to the decuplet $8 \oplus 1 \oplus 1$. The isoscalar physical states belonging to the decuplet are superpositions of the exact symmetry states: octet isosinglet, quark singlet and the additional singlet. These superpositions are defined by mixing matrix. Having known the elements of this matrix we can predict the flavor properties of the isosinglet physical states, e.g., their electromagnetic and hadronic decay rates.

The mixing matrix has the dimension 3x3 and is parametrized by 3 angles. The ECMM in its original formulation determines two of the angles but only up to the signs of the functions sin and cos of these angles. The third angle remains unknown [2,3]. However, the multiplet expected to include the glueball may be poorly known and therefore it is useful to supplement the model with an additional constraint making possible full determination of the mixing matrix.

The additional constraint we use below is known as flavor independency of the glueball [4]. It has been already used to reduce the number of unknown elements of the mass operator matrix in related model of the decuplet mixing [5,6]. We shall refer to the mixing of an extra singlet as flavor-independent or flavor-dependent depending on whether this supplementary condition is used or not. If the mixing is flavor-independent, then all elements of the mixing matrix are unambiguously determined.

ECMM imposes on decuplet states a set of constraints which are independent of the supplementary condition. They forbid to form the decuplets of 0^{-+} and 1^{--} mesons including the ground-state nonet and higher-lying singlet. Fortunately, a decuplet of 2^{++} mesons including the well investigated tensor meson nonet is allowed. This sets up to the tensor mesons a special role in verifying ECMM and examining its properties.

In the second section, after short recalling the main features of ECMM, we define its flavor-independent and flavor-dependent versions. In the third section we apply it to tensor mesons. In investigating hadronic decays of the mesons we avoid simplifying assumptions on purpose to reduce the number of unknown coupling constants.

2. THE FLAVOR-INDEPENDENT AND FLAVOR-DEPENDENT VERSIONS OF ECMM

2.1. Foundation of the ECMM

We begin with recalling the main relations of ECMM.

1^0 ECMM rests on requirement of vanishing of the three exotic commutators [7]:

$$[T_a, T_b^{(k)}] = 0 \quad / \ k = 1,2,3 \ / \tag{1}$$

where T_a - is a generator of flavor SU(3) group; $T_b^{(k)} = \dfrac{d^k T_b}{dt^k}$ - is its k-th time derivative, (a,b) - is an exotic combination of indices.

The constraints (1) determine three matrix elements:

$$\langle z_8 | (m^2)^k | z_8 \rangle = \frac{a^k}{3} + \frac{2}{3} b^k \quad / \ k = 1,2,3 \ / \tag{2}$$

where z_8 - is the isoscalar octet state, a - is the isotriplet meson mass squared;

$$b = 2K - a \tag{3}$$

- is the $s\bar{s}$- state mass squared, K - and other particle symbols are their mass squared [1].

2^0 The z_8-state may be represented as a linear combination of physical isoscalar states z_j

$$|z_8\rangle = \sum_j \lambda_j |z_j\rangle \tag{4}$$

where the sum runs over all physical isoscalar states z_j and coefficients λ_j obey the conditions:

$$\lambda_j^2 \geq 0, \quad \sum_j \lambda_j^2 = 1 \tag{5}$$

Using $|z_j\rangle$ as complete system of intermediate states on the left-hand side of Eq.(2), we obtain, by virtue of Eqs (2) and (4), the system of equations with respect to λ_j^2 [2]:

$$\lambda_1^2 z_1^k + \lambda_2^2 z_2^k + \lambda_3^2 z_3^k = \frac{1}{3} a^k + \frac{2}{3} b^k, \quad / \ k=0,1,2,3 \ / \tag{6}$$

where the second Eq.(5) is already included (for k=0).

The system of Eqs.(6) is overdetermined. Its consistency condition is the mass formula

$$F(a) + 2F(b) = 0, \tag{7}$$

where $F(x)$ is characteristic polynomial of the mass operator:

$$F(x) = (x - z_1)(x - z_2)(x - z_3). \tag{8}$$

The solution of the Eqs.(6) is :

$$\lambda_1^2 = \frac{1}{3}\frac{(a-z_2)(a-z_3) + 2(b-z_2)(b-z_3)}{(z_1-z_2)(z_1-z_3)}$$

$$\lambda_2^2 = \frac{1}{3}\frac{(a-z_3)(a-z_1) + 2(b-z_3)(b-z_1)}{(z_2-z_3)(z_2-z_1)} \tag{9}$$

$$\lambda_3^2 = \frac{1}{3}\frac{(a-z_1)(a-z_2) + 2(b-z_1)(b-z_2)}{(z_3-z_1)(z_3-z_2)}$$

Some properties of this solution should be mentioned:

(a) If $\lambda_3^2 = 0$, then

$$\lambda_1^2 = \frac{1}{3}, \quad \lambda_2^2 = \frac{2}{3}, \tag{10}$$

$$z_1 = a, \quad z_2 = b \tag{11}$$

and z_3 is arbitrary [2,3]. Such a decuplet is degenerate - it reduces to the nonet and disconnected singlet. If

$$\lambda_1\lambda_2 < 0, \tag{12}$$

then this nonet is ideal:

$$|z_1\rangle = |N\rangle, \quad |S\rangle = |s\bar{s}\rangle, \tag{13}$$

where

$$|N\rangle = |(u\bar{u} + d\bar{d})/\sqrt{2}\rangle, \quad |S\rangle = |s\bar{s}\rangle. \tag{14}$$

The solution (10),(11) remains true, if z_3 does not appear at all, i.e. for the nonet [1]. Thus the decuplet mixing and ideal mixing of the nonet follow in ECMM from the same dynamical constraints given by Eqs.(1).

(b) If the decuplet is not degenerate, the masses obey inequalities

$$z_1 < a < z_2 < b < z_3 \tag{15}$$

and the λ^2's are confined within the intervals:

$$0 < \lambda_1^2 < 1/3, \quad 0 < \lambda_2^2 < 1, \quad 0 < \lambda_3^2 < 2/3 \tag{16}$$

3^0 The wave functions of z_j are defined by the mixing matrix U

$$\begin{bmatrix} z_1 \\ z_2 \\ z_3 \end{bmatrix} = U \begin{bmatrix} z_8 \\ z_0 \\ G \end{bmatrix} \tag{17}$$

where z_8, z_0, G are wave functions of the octet isoscalar, the quark singlet and the glueball respectively. U may be chosen as 3-dimensional rotation matrix

$$U = \begin{bmatrix} c_1 & -s_1c_2 & s_1s_2 \\ s_1c_3 & c_1c_2c_3 - s_2s_3 & -c_1s_2c_3 - c_2s_3 \\ s_1s_3 & c_1c_2s_3 + s_2c_3 & -c_1s_2s_3 + c_2c_3 \end{bmatrix} \tag{18}$$

Here $c_j = \cos\vartheta_j$, $s_j = \sin\vartheta_j$ and ϑ_j are Euler angles: $0 \le \vartheta_1 < \pi$; $0 \le (\vartheta_2, \vartheta_3) < 2\pi$. The elements of the first column of the matrix U are equal to λ_j's:

$$\lambda_1 = c_1, \quad \lambda_2 = s_1c_3, \quad \lambda_3 = s_1s_3. \tag{19}$$

The coefficients λ_j represent octet contents of z_j states. The relations (9) and (19) determine the angles ϑ_1 and ϑ_3 in terms of masses, but not uniquely, as Eqs. (9) do not fix the signs of λ^2's. The angle ϑ_2 remains undetermined. It can be found with the help of an additional constraint or left arbitrary. Depending on the way the ϑ_2 is determined several versions of the ECMM are possible. We explain below how it can be determined and how to interprete its arbitrariness.

The physical isoscalar states z_j are usually expressed in the basis of the ideal states N,S,G. We shall refer to the mixing matrix in this basis as V and call its elements as follows:

$$V = \begin{bmatrix} x_1 & y_1 & u_1 \\ x_2 & y_2 & u_2 \\ x_3 & y_3 & u_3 \end{bmatrix} \tag{20}$$

The explicit form of this matrix is

$$V = \begin{vmatrix} \sqrt{\tfrac{1}{3}}\, c_1 - \sqrt{\tfrac{2}{3}} s_1c_2 & -\sqrt{\tfrac{2}{3}} c_1 - \sqrt{\tfrac{1}{3}} s_1c_2 & s_1s_2 \\ \sqrt{\tfrac{1}{3}} s_1c_3 + \sqrt{\tfrac{2}{3}}(c_1c_2c_3 - s_2s_3) & -\sqrt{\tfrac{2}{3}} s_1c_3 + \sqrt{\tfrac{1}{3}}(c_1c_2c_3 - s_2s_3) & -c_1s_2c_3 - c_2s_3 \\ \sqrt{\tfrac{1}{3}} s_1s_3 + \sqrt{\tfrac{1}{3}}(c_1c_2s_3 + s_2c_3) & -\sqrt{\tfrac{2}{3}} s_1c_3 + \sqrt{\tfrac{1}{3}}(c_1c_2s_3 + s_2c_3) & -c_1s_2s_3 + c_2c_3 \end{vmatrix} \tag{24}$$

2.2. The flavor-independent version of ECMM

The flavor-independent version of the ECMM is defined by including the condition of the glueball flavor independence [4]:

$$\langle G\, |m^2|\, z_8 \rangle = 0 \tag{22}$$

This condition defines the angle ϑ_2 (not changing the restrictions on masses and angles ϑ_1 and ϑ_3):

$$\operatorname{tg}\vartheta_2 = \frac{c_3s_3}{c_1} \, \frac{z_3 - z_2}{(z_3 - z_1) - (z_3 - z_2)\, c_3^2} \tag{23}$$

Thus the angle ϑ_2 is also expressed by the masses.

To have the elements of the mixing matrix V completely determined we still have to fix the signs of c_j, s_j. This can be done unambiguously. We first present the result and then comment on it:

(i) $s_1 > 0$, as $0 \le \vartheta_1 < \pi$

(ii) $c_2 < 0$, $s_2 > 0$, $c_3 < 0$, because of the choice: $x_1, u_1, u_3 > 0$

(iii) For c_1^2 not too small:

(a) $c_1 > 0$, if z_1 is mostly N-state,

(b) $c_1 < 0$, if z_1 is mostly S-state

(iv) sign s_3 = sign c_1, from (23)

The comment is in order.

The matrix of 3-dimensional orthogonal transformation includes 5 elements whose signs may be arbitrarily chosen. This follows from the sign ambiguity of initial and final state-vectors (there are 5 such elements, since changing signs of all state-vectors does not affect matrix elements). If the matrix is unimodular, then the ambiguity comprises only 4 elements. This is the case of U and V matrices.

Some of the c_j, s_j signs are fixed by the choice of signs of the V matrix elements. Notice that $\vartheta_1, \vartheta_2, \vartheta_3$ are Euler angles of the U matrix, not the V one. The latter matrix includes an additional rotation transforming the basis N,S,G into z_8, z_0,G. The two rotations may follow the same or opposite "direction". Therefore, only 3 of unknown c_j, s_j signs are determined by the choice of matrix element signs. These are the signs of c_2, s_2, c_3 (see (ii)).

The sign of c_1 cannot be determined this way. Choosing it positive or negative we make the matrix element x_1 big or small (as the signs of s_1 and c_2 are already fixed (cf. (21))); changing the sign of c_1 affects the state of z_1 meson. Therefore,

we need an additional information about the structure of z_1 state to determine the sign of c_1. The choice (iii) is possible, if c_1^2 is not too small. If it is small ($\lambda_1^2 < \lambda_2^2$, $\lambda_1^2 < \lambda_3^2$) but nonvanishing, then the sign of c_1 may be fixed using information about the properties of z_2 or z_3 meson. The sign of c_1 cannot be determined only in the case of vanishigly small value of λ_1^2.

The sign of s_3 can be only determined from Eq.(23).

Concluding this point, we stressthat in the flavor-independent version of ECMM all elements of the mixing matrix V are completely determined basing merely on the decuplet masses and one qualitative information which is necessary to fix the sign of c_1.

2.3. The Flavor-dependent versions of ECMM

The flavor-dependent version is obtained, if the constraint (22) is abandoned. Consequently, Eq.(23) determining angle ϑ_2 and sign of s_3 (cf.(iv)) is not included here. However, c_j, s_j signs defined in (i) – (iii) may be left unchanged. The restrictions on the masses and λ^2's remain unchanged. The free parameters of that model may be fitted from experimental data (notice that $\pi/2 < \vartheta_2 < \pi$, since $c_2 < 0$, $s_2 > 0$ (ii)). On the other hand, this freedom leaves room for other versions of ECMM defining these quantities in different way.

3. DECUPLET OF 2^{++} MESONS

We apply the flavor-independent version of ECMM to describe decuplet of tensor mesons. The decuplet includes old-standing nonet mesons: $a_2(1320)$, $K_2^*(1430)$, $f_2(1270)$, $f_2'(1525)$ and $\Theta/f_2(1720)$ as the tenth one. Fiting procedure may be split into few steps:

1) Establishing the decuplet assignment

The masses of the decuplet members have to obey inequalities (15) and mass formula (7). If these constraints are not obeyed then the ten particles do not form the decuplet.

2) Fitting $\gamma\gamma$-decay widths

Calculating widths of $\gamma\gamma$-decay requires only knowledge of mixing matrix [5]. All elements of this matrix are determined by the masses and no new parameter is introduced. The agreement with data is achieved by manipulating with mass values.

3) Fitting widths of hadronic decays

The fit requires 4 coupling constants. As the mixing matrix is fully determined, we are able to perform analysis of these decays without neglecting OZI-suppressed contribution.

Besides of the well established mesons quoted above, few other isoscalar mesons ("waiting confirmation") were reported [8]: $f_2(1430)$, $f_2(1565)$, $f_2(1640)$. For these mesons, treated as decuplet candidate replacing the $\Theta/f_2(1720)$, ECMM gives the following assignments:

- the meson $f_2(1430)$ does not satisfy constraints (15),
- the meson $f_2(1565)$ fits (15) but not the mass formula (7).

These two mesons cannot complete the decuplet.

- the meson $f_2(1640)$ fits (15) and (7). Its properties should be similar to the properties of the $\Theta/f_2(1720)$. We do not discuss this meson in detail.

The mass formula may be verified by solving it with respect to some mass. This mass, being calculated from Eq.(7), as well as λ_j^2's which are calculated from Eqs.(9), depend on many input masses. It is useful to realize to which masses these quantities are most sensitive.

We begin with observation that λ_3^2 is small ($\lambda_3^2 \le 0.03$), if the mass of the meson z_3 is bigger than 1700 MeV. At such values all properties of the decuplet mesons depend weekly on the mass of this meson. We solve the mass formula with respect to z_2. It follows from Eq.(7) that the limit $z_3 \to \infty$ defines an upper bound on z_2:

$$z_2(a,b,z_1,z_3) \le z_2(a,b,z_1,\infty) = z_2^\infty \tag{24}$$

The value z_2 corresponding to the mass of the $\Theta/f_2(1720)$ (treated as z_3 meson) is only a little lower of z_2^∞. Assuming the differences $a - z_1$ and $b - z_2$ small and considering only small changes of the masses in the vicinity of their central experimental values, we find from Eq.(7)

$$z_2^\infty \simeq b - \sqrt{\frac{a}{b}}\,\Delta \tag{25}$$

where

$$\Delta = m_{a_2} - m_{z_1} \tag{26}$$

It follows that z_2 depends mainly on b and Δ, if $z_3 > (1.7\ \mathrm{GeV})^2$. Its additional dependence on a, not covered by b and Δ, is much weeker.

However the most important in what follows is that the main mass dependence of V matrix elements is still simpler: they depend only on Δ.

3.1. Decays into $\gamma\gamma$

We use the following expressions for $\gamma\gamma$-decay widths [5]:

$$\frac{\Gamma_{\gamma\gamma}(f_2^{(j)})}{\Gamma_{\gamma\gamma}(a_2)} = \mathrm{ph.\ space} \times (x_j + \frac{\sqrt{2}}{5} y_j)^2 \tag{27}$$

where $f_2^{(1)} = f_2(1270)$, $f_2^{(2)} = f_2(1525)$, $f_2^{(3)} = \Theta/f_2(1720)$. The ratio (27) depends only on the elements of V. Thus we have three relations to determine Δ. We use the following data [8]:

$$\Gamma_{\gamma\gamma}(a_2) = 0.90 \pm 0.10\ \mathrm{keV} \tag{28}$$

$$\Gamma_{\gamma\gamma}(f_2^{(1)}) = 2.76 \pm 0.14\ \mathrm{keV} \tag{29}$$

$$\Gamma_{\gamma\gamma}(f_2^{(2)})\ B(f_2^{(2)} \to K\bar{K}) = (0.11 \pm 0.02)\ \mathrm{keV} \tag{30}$$

$$\Gamma_{\gamma\gamma}(f_2^{(3)})\ B(f_2^{(3)} \to K\bar{K}) < 0.11\ \mathrm{keV} \tag{31}$$

$$\Gamma_{K\bar{K}}(f_2^{(2)}) = (61.0 \pm 5)\ \mathrm{MeV} \tag{32}$$

$$\Gamma_{tot}(f_2^{(2)}) = (76 \pm 10)\ \mathrm{MeV} \tag{33}$$

$$B(f_2^{(3)} \to K\bar{K}) = 0.38^{+0.09}_{-0.19} \tag{34}$$

The Fig.1 shows the dependence of the ratios (27) on Δ and experimental limits for them following from Eqs. (29) – (33). Assuming $\Gamma_{\gamma\gamma}(a_2) = 1$ keV, we find restrictions on Δ:

- $\Delta < 19$ MeV – from data on $f_2^{(1)}$ meson decay,
- 12 MeV $< \Delta <$ 25 MeV – from data on $f_2^{(2)}$ decay,
- $\Delta < 28$ MeV – from data on $f_2^{(3)}$ decay,

and therefore

$$12\ \mathrm{MeV} < \Delta < 19\ \mathrm{MeV}. \tag{35}$$

In view of Eq.(28) and anticipating the calculated branching ratio $B(f_2^{(2)} \to K\bar{K})$ smaller than experimental value following from Eqs.(32) and (33) we choose

$$\Delta^{inp} = 16\ \mathrm{MeV} \tag{36}$$

Then Eq.(7) requires

$$1300\ \mathrm{MeV} \le m_a \le 1305\ \mathrm{MeV}$$

The value of Δ^{inp} is about 3 times smaller than observed $\Delta^{exp} = 44$ MeV. We shall comment on this problem in the discussion.

For further calculations we accept the following masses as an input:

$$a = (1.302\ \text{GeV})^2,\ K = (1.425\ \text{GeV})^2,$$
$$f_2^{(1)} = (1.286\ \text{GeV})^2,\ f_2^{(3)} = (1.713\ \text{GeV})^2. \qquad (37)$$

We calculate $f_2^{(2)} = (1.525\ \text{GeV})^2$ from the mass formula (7) and $\lambda_1^2 = 0.2851$, $\lambda_2^2 = 0.7017$, $\lambda_3^2 = 0.0132$ from Eqs.(9). The mixing matrix is

$$V = \begin{vmatrix} 0.9819 & 0.0404 & 0.1851 \\ -0.0865 & 0.9648 & 0.2484 \\ -0.1686 & -0.2599 & 0.9508 \end{vmatrix} \qquad (38)$$

and the ratios (27) are:

$$\frac{\Gamma_{\gamma\gamma}(f_2^{(j)})}{\Gamma_{\gamma\gamma}(a_2)} = \begin{cases} 2.64 \\ 0.16 \\ 0.37 \end{cases} \qquad (39)$$

for $f_2(1270)$, $f_2'(1525)$, $\Theta/f_2(1720)$, respectively.

3.2. Two-hadron decays

Two-hadron decays are calculated using formula [9]:

$$\Gamma_{mn}(k) = \frac{p^5}{M_k} |\langle mn|k\rangle|^2 \qquad (40)$$

where k – is decaying particle, M_k – is its mass; m,n – are decay products, p –is their c.m. momentum; $\langle mn|k\rangle$ – is SU(3) factor.

There are two kinds of tensor meson decays: T → VP (into vector and pseudoscalar meson) described by single coupling constant g_V and T → PP (into two pseudoscalar mesons) described by three coupling constants – g_8, g_0, g_G, corresponding to $T_{octet} \to PP$, $T_{q\bar{q}\ singl} \to PP$, G → PP, respectively.

Our main purpose is to describe decays of isoscalar tensor mesons, since only these mesons may include glueball component. These decays are the T → PP ones. As all elements of the mixing matrix are determined, it is simple to fit all coupling constants. However, we should also verify consistency of the Δ^{inp} with strong decay data. Therefore, we discuss also T → VP decays. These processes provide an independent test, as they depend on separate coupling constant and their widths strongly depend on the mass of the decaying meson.

We begin with the latter problem. The constant g_V may be chosen to fit K_2 decay partial widths. The same value of the g_V fits also $a_2 \to \rho K$ partial width. So these data do not contradict the input masses (37) and consequently the value of Δ^{inp}. The decays $K_2 \to PP$ and $a_2 \to PP$ are also described by single coupling constant g_8 which can be determined from similar fit. Again we find the Δ^{inp} consistent with data. This way we find:

$$g_V^2 = 2.020,\quad g_8 = 0.710 \qquad (41)$$

Using the value of g_V^2 we predict also the widths of the $f_2'(1525)$, $\Theta/f_2(1720) \to K^*\bar{K} + \bar{K}^*K$ decays.

The $\pi\pi$, $K\bar{K}$, $\eta\eta$ decay rates of isoscalar tensor mesons are given by the formulae:

$$\Gamma_{\pi\pi}(f_2^{(j)}) = \frac{3}{2}\frac{p^5}{M_j}\left[\sqrt{\frac{2}{3}}\left(g_0+\frac{1}{\sqrt{3}}g_8\right)x_j+\frac{1}{\sqrt{3}}\left(g_0-\frac{2}{\sqrt{3}}g_8\right)y_j+g_Gu_j\right]^2 \quad (42)$$

$$\Gamma_{K\bar{K}}(f_2^{(j)}) = 2\frac{p^5}{M_j}\left[\sqrt{\frac{2}{3}}\left(g_0-\frac{1}{2\sqrt{3}}g_8\right)x_j+\frac{1}{\sqrt{3}}\left(g_0+\frac{1}{\sqrt{3}}g_8\right)y_j+g_Gu_j\right]^2 \quad (43)$$

$$\Gamma_{\eta\eta}(f_2^{(j)}) = \frac{p^5}{M_j}\left[\sqrt{\frac{2}{3}}\left(g_0-\frac{1}{\sqrt{3}}g_8\right)x_j+\frac{1}{\sqrt{3}}\left(g_0+\frac{2}{\sqrt{3}}g_8\right)y_j+g_Gu_j\right]^2\cos^4\theta_P \quad (44)$$

where θ_P - is mixing angle of the pseudoscalar mesons.

In these expressions the only unknown quantities are g_0 and g_G. It has been verified by calculation that experimental results on $f_2, f_2' \to \pi\pi$, $K\bar{K}$, $\eta\eta$ decays and on the ratios $\Gamma_{\pi\pi}/\Gamma_{K\bar{K}}$, $\Gamma_{\eta\eta}/\Gamma_{K\bar{K}}$ for $\Theta/f_2(1720)$ are well reproduced, if these constants are chosen such that [8]:

$$\Gamma_{\pi\pi}(f_2(1270)) = 157 \text{ MeV} \quad (45)$$

$$\Gamma_{\pi\pi}(f_2'(1525))/\Gamma_{K\bar{K}}(f_2'(1525)) = 0.0115 \quad (46)$$

We find

$$g_0 = 1.007, \quad g_G = 0.187. \quad (47)$$

The predicted widths are compared with the observed ones in the Table 1. We use the following values for the masses of the decay products:

m_π = 139.6 MeV, m_K = 495.6 MeV, m_η = 548.8 MeV,

m_ρ = 768.3 MeV, m_{K^*} = 894.0 MeV, m_ω = 781.95 MeV,

where the masses of K and K^* mesons are arithmetical means of charged and neutral components. We use $\theta_P = -10^\circ$ for mixing angle of pseudoscalar mesons and 35° - for vector mesons.

The Table 1 shows that for that mesons which are assigned to the nonet the predicted widths well agree with data. For $\Theta/f_2(1720)$ meson, in contrast, the ratios of the calculated widths agree with the data, but the widths themselves are far below 138 MeV quoted for total width [8] (cf.e.g.[10]):

$$\Gamma_{\pi\pi}(\Theta) + \Gamma_{K\bar{K}}(\Theta) + \Gamma_{\eta\eta}(\Theta) + \Gamma_{K^*\bar{K}+c.c.}(\Theta) < 10 \text{ MeV}. \quad (48)$$

4. DISCUSSION

a) The value of Δ^{inp}

The value of Δ^{inp} required by radiative decays contradicts the experimental values of the $a_2(1320)$ and $f_2(1270)$ meson masses. The Fig.1 shows that the upper bound for Δ is shifted up to about 25 MeV (the upper buond for $\Gamma_{\gamma\gamma}(f_2'))$, if $\Gamma_{\gamma\gamma}(a_2) \approx 1.1$ keV. However, higher values of Δ^{inp} are not allowed, since $\Gamma_{\gamma\gamma}(f_2')$ would become too low and $\Gamma_{\gamma\gamma}(\Theta)$ probably too high.

The value of Δ^{inp} is also controlled by the mass formula and by the hadronic widths.

The mass formula (7) is not obeyed by the central experimental values of the masses. Some of the masses must be changed a little (in a favoured case) beyond the experimental bounds [8]. Reduction of Δ acts in good direction, but the accepted value of Δ^{inp} imposes the change of the masses remarkably exceeding experimental error.

Also the Δ^{inp} does not contradict data on the hadronic decays. This is seen from the widths of $a_2, K_2 \to VP$, PP decays. These decays depend on single coupling constant and the ratio of $a_2 \to VP$ decay width to the $K_2 \to VP$ one depend on the ratio of a_2 and K_2 masses. The same is true for the PP decays. It follows from the Table 1 that the accepted value of Δ^{inp} does not contradict data.

Consequently, we conclude that the discrepancy between Δ^{inp} and the observed masses of $a_2(1320)$, $f_2(1270)$ mesons is something real and calls for independent explanation.

b) Suppression of the glueball decay

The coupling constant of the OZI suppressed decay $S \to \pi\pi$ is simply connected with g_0 and g_8:

$$g_{S\pi\pi} = \frac{1}{\sqrt{3}} \left(g_0 - \frac{2}{\sqrt{3}} g_8\right) \tag{49}$$

It is interesting that the fit requires:

$$g_{S\pi\pi} \approx \frac{1}{\sqrt{3}} g_G \tag{50}$$

Actually, the widths quoted in the Table 1 are calculated assuming the Eq. (50) exact. We thus find that decays of the glueball into the pair of quarkonia are suppressed. This result suggests that the mechanism of such glueball decays is closely related with the mechanism of the OZI-suppressed decay $S \to \pi\pi$.

c) Nonet mesons

Small g_G implies week influence of the glueball component on the hadronic widths of these mesons which are mostly quarkonium states. Therefore, we may expect difficult to detect their glueball contents by investigating only decay processes of that mesons which are assigned to the nonet (cf. [11]). Let us now discuss shortly the nonet mesons.

It has been shown [3] that the mass sum rule and the mixing angle of the non-ideal nonet are obtained as $z_3 \to \infty$ limit of the decuplet ones. Therefore, we may use the decuplet computing program choosing sufficiently large value of the mass of z_3 meson. We put $z_3 = (1000\ \text{GeV})^2$. For such z_3 we find $\lambda_3^2 = 5\cdot 10^{-20}$ and $\Theta/f_2(1720)$ is disconnected. The calculated widths are given in the last column of the Table 1. The data are well fitted with a slightly changed values of masses and coupling constants:

$a = (1.310 \text{ GeV})^2$, $K = (1.428 \text{ GeV})^2$, $f_2^{(1)} = (1.275 \text{ GeV})^2$,
$f_2^{(2)} = (1.525 \text{ GeV})^2$, $g_8 = 0.690$, $g_0 = 1.065$.
For mixing angle we obtain 30.3^{o}.

In a sense this fit is better than the decuplet one, since the value of Δ (cf. Eq.(26)) is less restricted and the mass of a_2 may be higher. However, also the nonet is not out of this trouble. It accepts Δ larger than decuplet, but still smaller than observed.

5. CONCLUSSION

The present analysis does not give definite answer to the main question: is $\Theta/f_2(1720)$ the glueball, or not. According to ECMM the $\Theta/f_2(1720)$ may be a meson including about 95% of the glueball state in the amplitude provided the discrepancy between the input and real masses of the $a_2(1320)$ and $f_2(1270)$ mesons has independent explanation and hadronic two-body decay widths of $\Theta/f_2(1720)$ are sufficiently narrow.

The coupling constant of the glueball decay into two pseudoscalar $q\bar{q}$ mesons would be simply related to the OZI - suppressed decay coupling constant: $g_{S\pi\pi} \approx \frac{1}{\sqrt{3}} \cdot g_G$.

Suppression of the hadronic glueball decays makes the decays of that isoscalar physical mesons which are asssigned to the nonet insensitive to the glueball admixture. On the other hand the $q\bar{q}$ component of the mostly glueball meson would be emphasised not only in its hadronic decays, but in the $\gamma\gamma$ decays as well (cf.(36)).

This makes the features of $\Theta/f_2(1720)$ distinct from what we expect for pure glueball. To understand them, we must know the glueball mixing.

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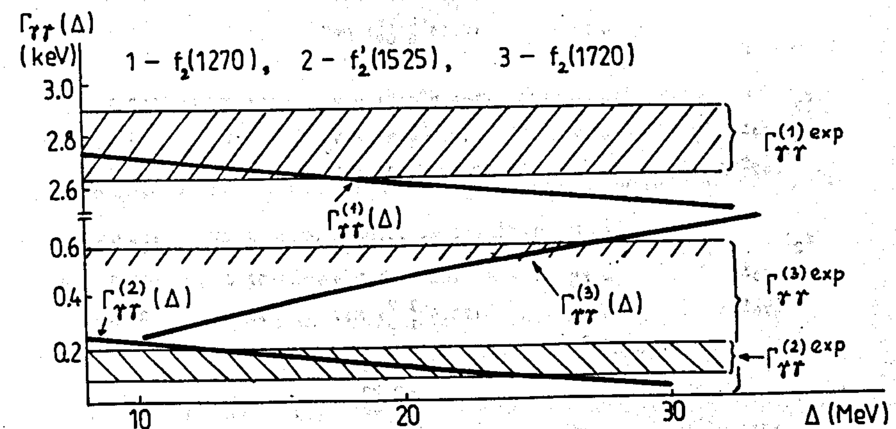

Fig.1. The $\gamma\gamma$ widths of isoscalar tensor mesons as functions of Δ (mass difference of the $a_2(1320)$ and $f_2(1270)$ mesons). The curves 1,2,3 correspond to $f_2(1270)$, $f_2'(1525)$ and $\Theta/f_2(1720)$, respectively. (1),(2),(3) are experimentally allowed domains of these widths.

Table 1. Hadron decays of tensor mesons

Particle M, Γ_{tot} (MeV)	Decay mode VP	Decay mode PP	Width: Experim. VP	Width: Experim. PP	Calculated (MeV) Decuplet	Calculated (MeV) Nonet
a_2	$\to \rho\pi$		(70.1 ± 2.7)%		70.7	73.0
1318.4 ± 0.7		$\to K\bar{K}$		(4.9 ± 0.8)%	5.2	5.2
102.7 ± 2.2		$\to \eta\pi$		(14.5 ± 1.2)%	9.9	9.8
		$\to \eta'\pi$		< 1 %	0.01	0.01
K_2	$\to \rho K$		(8.7 ± 0.8) MeV		7.5	7.6
1425.4 ± 1.3	$\to K^*\pi$		(24.8 ± 1.7) MeV		26.5	26.4
98.4 ± 2.4	$\to \omega K$		(2.9 ± 0.8) MeV		2.4	2.4
		$\to K\pi$		(48.9 ± 1.7) MeV	47.7	45.6
		$\to K\eta$		$(0.14^{+0.28}_{-0.09})$ MeV	1.5	1.5
f_2		$\to \pi\pi$		$(156.7^{+3.0}_{-1.3})$ MeV	156.9	158.7
1274 ± 5		$\to K\bar{K}$		(8.6 ± 0.9) MeV	9.1	9.8
185 ± 20		$\to \eta\eta$		(0.83 ± 0.19) MeV	0.95	1.1
f_2'		$\to \pi\pi$		(0.70 ± 0.14) MeV	0.60	0.60
1525 ± 5		$\to KK$		(61 ± 5) MeV	52.0	52.1
76 ± 10		$\to \eta\eta$		$(23.9^{+2.2}_{-1.3})$ MeV	26.8	26.8
	$\to K^*\bar{K}+\bar{K}^*K$				14.3	14.7
Θ		$\to \pi\pi$		$(3.90^{+0.20}_{-2.40})$ %	0.77	
$1713.2^{+1.9}_{-4.5}$		$\to K\bar{K}$		(38^{+9}_{-19}) %	4.1	
138^{+12}_{-9}		$\to \eta\eta$		$(18^{+2.0}_{-13.0})$ %	2.2	
	$\to K^*\bar{K}+\bar{K}^*K$				2.6	

For the decay products we assume:

P: $m_\pi = 139.6$ MeV, $m_K = 495.6$ MeV, $m_\eta = 548.8$ MeV, $\Theta_P = -10^\circ$;

V: $m_\rho = 768.3$ MeV, $m_{K^*} = 894.0$ MeV, $m_\omega = 781.95$ MeV, $\Theta_V = 35^\circ$

Input for the decuplet:

$a_2 = (1.302\ \text{GeV})^2$, $K_2 = (1.425\ \text{GeV})^2$,

$f_2^{(1)} = (1.286\ \text{GeV})^2$, $f_2^{(3)} = (1.713\ \text{GeV})^2$;

$g_V^2 = 2.020$, $g_8 = 0.710$, $g_0 = 1.007$, $g_G = 0.187$.

Input for the nonet:

$a_2 = (1.310\ \text{GeV})^2$, $K = (1.428\ \text{GeV})^2$, $f_2^{(1)} = (1.275\ \text{GeV})^2$,

$g_V^2 = 2.020$, $g_8 = 0.690$, $g_0 = 1.065$.

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