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AHARONOV-CASHER EFFECT FOR THE TOROIDAL SOLENOIDS

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At first we repeat well-known arguments /1/ using the cylindrical solenoid (CS) as an example. Consider charged particle (CP) in the field of the resting infinite CS. The term in Lagrangian describing their interaction is

$$\frac{e}{c}\vec{v}_e \cdot \vec{A}(\vec{r}_e - \vec{r}_s). \tag{I}$$

Here $\vec{r}_e$ and $\vec{v}_e$ are the radius-vector and velocity of the CP resp., $\vec{r}_s$ is the radius-vector of CS; $\vec{A}$ is the vector potential (VP) produced by CS at the position of CP. The Galilean invariance leads to the following modification of Eq.(I) when both the CP and CS are in motion

$$\frac{e}{c}(\vec{v}_e - \vec{v}_s)\cdot\vec{A}(\vec{r}_e - \vec{r}_s). \tag{2}$$

Here $\vec{v}_s$ is the velocity of CS. As was shown in /1/ the added term

$$-\frac{e}{c}\vec{v}_s\vec{A}(\vec{r}_e - \vec{r}_s) \tag{3}$$

corresponds to the scattering of neutral particles with the magnetic dipole moment by the infinite charged filament. The experiment in which the neutrons were scattered by such a filament was performed in 1989 /2/. It has confirmed the existence of AC effect.

The interaction of CP with resting toroidal solenoid (TS) is described by the same Eq.(I) where under $\vec{A}$ one should understand VP of TS. This VP was obtained in ref. /3/, its properties were discussed in /4/. The excellent experiment in which the electrons were scattered by the magnetic field of TS was performed by Tonomura et al. /5/. Their theoretical description may be found in refs. /6/. The same considerations of the Galilean invariance oblige us to choose Eq.(2) as an interaction when both CP and TS are in motion. It is our goal to derive and interpret the added term (3). Let TS with poloidal current $\vec{j}$ (fig.1) move with velocity $\vec{v}_s$. According to Special Relativity this induces charge density $\rho = (\vec{v}_s \cdot \vec{j})\gamma/c^2$. As we limit ourselves to the Galilean invariant theory, so factor γ can be discarded. The interaction of moving TS with an external electrical field $\vec{E} = -\mathrm{grad}\,\varphi$ is given by

$$U = \int \rho\varphi\, dV = \frac{1}{c^2}\int \varphi(\vec{v}_s\cdot\vec{j})\,dV. \tag{4}$$

We change the current by the equivalent magnetization $\vec{M}$ ($\vec{j} = c\,\mathrm{rot}\,\vec{M}$) and integrate by parts

$$U = \frac{1}{c}\vec{v}_s\int \vec{E}\times\vec{M}\,dV. \tag{5}$$

At large distances (comparing to the dimensions of TS) from the source of $\vec{E}$ the latter can be developed into the series

$$\vec{E}(\vec{r}_s-\vec{r}_e) = \vec{E}(\vec{r}_0-\vec{r}_e) + (\vec{r}\vec{\nabla}_0)\cdot\vec{E}(\vec{r}_0-\vec{r}_e). \tag{6}$$

Here $\vec{r}_0$ refers to some point at the neighbourhood of TS (e.g., its centre-of-mass). Substitute (6) into (5)

$$U = \frac{1}{c}\vec{v}_s\cdot(\vec{E}\times\vec{\mu}_d) - \frac{1}{2c}(\vec{v}_s\cdot\vec{\nabla}_0)(\vec{E}\cdot\vec{\mu}_t). \tag{7}$$

Here $\vec{\mu}_d = \int\vec{M}\,dV$ and $\vec{\mu}_t = \int(\vec{r}\times\vec{M})\,dV$ are the magnetic dipole and toroidal moments (TM) of the TS, resp. For the TS $\vec{\mu}_d = 0$ and TM $\vec{\mu}_t$ is directed along TS symmetry axis (fig.1). It has the magnitude $\mu_t = \frac{1}{2}\pi g d R^2$. (Here $g = \Phi/[2\pi(d-\sqrt{d^2-R^2})]$, Φ is the magnetic flux inside TS, d,R are geometrical parameters of TS $(\rho-d)^2+z^2=R^2$). As expansion (6) holds

outside the source of electric field, the term containing div $\vec{E}$ was omitted in the derivation of (7). Thus

$$U = -\frac{1}{2c}(\vec{v}_s\cdot\vec{\nabla}_0)(\vec{E}\cdot\vec{\mu}_t). \tag{8}$$

We ask now: what electric field $\vec{E}$ should be substituted into (8) to obtain term (3)? By comparing Eqs. (3) and (8) we get

$$e\vec{v}_s\cdot\vec{A}(\vec{r}_0-\vec{r}_e) = -\frac{1}{2}(\vec{v}_s\vec{\nabla}_0)(\vec{E}(\vec{r}_0-\vec{r}_e)\cdot\vec{\mu}_t). \tag{9}$$

The sign minus arises here because the potential energy enters into the Lagrangian with negative sign; it is taken into account also that VP of TS is an even function of coordinates /7/ (contrary to the CS case). Equating coefficients at the particular cartesian component of v_s we arrive to

$$eA_i(\vec{r}_0-\vec{r}_e) = -\frac{1}{2}(\mu_t)_k\frac{\partial E_k(\vec{r}_0-\vec{r}_e)}{\partial x_{0i}}. \tag{10}$$

Without loss of generality we assume that symmetry axes of TS in both sides of this Eq. are parallel to the z axis. This gives $\mu_{tz} = \frac{1}{2}\pi g d R^2$ /8/. As an expansion (6) holds for the large separations of TS and electric field source, one should use in the LHS of (10) the asymptotic values of $\vec{A}$ /3,4/:
$A_z \sim \frac{1}{8}\pi g d R^2(1+3\cos 2\theta)/r^3$, $A_\rho \sim \frac{3}{8}\pi g d R^2\sin 2\theta/r^3$. This gives

$$3e\frac{xz}{r^5} = -\frac{\partial E_z}{\partial x},\quad 3e\frac{yz}{r^5} = -\frac{\partial E_z}{\partial y},\quad \frac{1}{2}e\frac{r^2+3(x^2-y^2)}{r^5} = -\frac{\partial E_z}{\partial z}$$

($x = x_0 - x_e$, etc.).

It is easy to check that these Eqs. are satisfied if $E_z = ez/r^3$. But this is just z component of $\vec{E} = e\vec{r}/r^3$. This means that term (3) restoring Galilean symmetry at large distances corresponds to the motion of TS in the Coulomb field. To find the electric field at finite distances we turn to Eq.(4) where expansion of $\vec{E}$ was not yet performed. Again we require the coincidence of Eqs. (3) and (4): $e\vec{v}_s\vec{A} = \frac{1}{c}\int\varphi(\vec{v}_s\cdot\vec{j})dV$. We remind that φ is the scalar potential of the electrical field to be defined. Compare coefficients at $\vec{v}_s$

$$e\vec{A}(\vec{r}) = \frac{1}{c}\int\varphi(\vec{r}-\vec{r}\,')\cdot\vec{j}(\vec{r}\,')dV'. \tag{11}$$

We keep in mind that VP of TS satisfies Poisson Eq.

$$\Delta\vec{A} = -\frac{4\pi}{c}\vec{j}$$

Its solution is

$$\vec{A} = \frac{1}{c}\int\frac{1}{|\vec{r}-\vec{r}\,'|}\vec{j}(\vec{r}\,')dV'. \tag{12}$$

By comparing Eqs. (11) and (12) we get $\varphi = e/|\vec{r}-\vec{r}\,'|$ that corresponds to the scalar electric potential of the point charge. Thus, term (3) describes the motion of TS in the Coulomb field. There is no classical scattering as Eqs. $\dot{\vec{v}}_e = \dot{\vec{v}}_s = 0$ follow from Lagrangian $L = \frac{1}{2}m_e v_e^2 + \frac{1}{2}m_s v_s^2 + \frac{e}{c}(\vec{v}_e-\vec{v}_s)\vec{A}(\vec{r}_e-\vec{r}_s)$. Fixing the position of the Coulomb center ($\vec{v}_e = \vec{r}_e = 0$, $\vec{v}_s = \vec{v}$, $\vec{r}_s = \vec{r}$, $m_s = m$). we obtain Lagrangian describing the scattering of TS by the resting Coulomb charge: $L = \frac{1}{2}mv^2 - \frac{e}{c}\vec{v}\cdot\vec{A}(\vec{r})$. For the infinitely small TS this reduces to $L = \frac{1}{2}mv^2 + \frac{1}{2c}(\vec{v}\,\vec{\nabla})(\vec{E}\,\vec{\mu}_t)$, $\vec{E} = e\vec{r}/r^3$ that corresponds to the scattering of toroidal dipole moments by the Coulomb field. Again there is no classical scattering $(\dot{\vec{v}} = 0)$. The corresponding Schroedinger Eqs. are

$$-\frac{\hbar^2}{2m}\left(\vec{\nabla} + \frac{ie}{\hbar c}\vec{A}\right)^2\psi = \mathcal{E}\psi,$$

$$-\frac{\hbar^2}{2m}\left(\vec{\nabla} - \frac{i}{2\hbar c}\vec{\nabla}(\vec{E}\cdot\vec{\mu}_t)\right)^2\psi = \mathcal{E}\psi, \quad \vec{E} = \frac{e\vec{r}}{r^3}. \tag{13}$$

They describe the scattering of TS and TM by the Coulomb field. The question arises: how to verify the existence of AC effect for TM? One should find the neutral particles having the nonvanishing TM (and zero magnetic dipole moment). It was claimed in ref./9/ that Majorana neutrinos are just such particles. The second way is to study the scattering of ferromagnetic microparticles (which according to /10/ carry nonzero toroidal moment) by the Coulomb field.

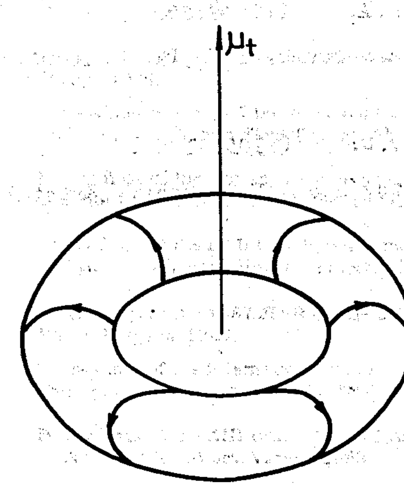

Fig.1. There are shown poloidal current on the surface of torus and the associated toroidal moment.

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