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NONCOMPACT EXTENSION OF ONE-DIMENSIONAL SUPERSYMMETRY AND SPINNING PARTICLE

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1 Introduction

1. Usually only scalar representations of N-extended supersymmetry algebra are used for the description of spinning particle models. The space-time coordinates x^μ transform as scalars of the isomorphism group $SO(N)$ of N-extended supersymmetry algebra and all x^μ belong to different irreducible representations of supersymmetry algebra. Therefore, to describe a D-dimensional spinning particle, we have to use D copies of a scalar representation which form a D-dimensional Lorentz vector. The Grassmann components of these representations,describing half-integer spin after quantization, transform as Lorentz and $SO(N)$ vectors. The resulting spin of the particle is $\frac{N}{2}$[1,2].

On the other hand,spinning particle models are the models of Supersymmetric Quantum Mechanics (SQM) and it is known that other representations can be used for the description of a D-dimensional SQM for some values of D [3,4]. For example, in $N = 4$ case the isomorphism group is $SO(4) = SU(2) \times SU(2)$ and three bosonic components of one definite irreducible representation of $N = 4$ supersymmetry algebra play the role of space coordinates and transform as a vector of one $SU(2) = SO(3)$ subgroup. Four Grassmann components of the representation transform as a complex spinor of this $SO(3)$ subgroup. One additional bosonic component is auxiliary. Thus, the $SO(3)$ subgroup of the isomorphism group plays the role of rotational group in 3-dimensional Euclidean space and all coordinates of the space are included in one irreducible representation of the supersymmetry algebra.

In the case of spinning particle there is additional time-like bosonic coordinate and rotational group in space-time is pseudoorthogonal group $SO(D-1,1)$. If all space-time coordinates belong to the same irreducible representation of extended supersymmetry then the isomorphisms group must include this pseudoorthogonal group $SO(D-1,1)$ as a subgroup. It means that supersymmetry algebra is not ordinary N-extended algebra. Instead, it must have the form

$$\{Q_a, Q_b\} = \omega_{ab} H \tag{1.1}$$

with some tensor ω_{ab} which has m positive and n negative eigenvalues.It should be noted that there is only one momentum operator H in this algebra playing the role of system's Hamiltonian which generates translations in proper time τ. The algebra (1.1) is (m,n)-extended one-dimensional supersymmetry algebra and it must not be confused with the (m,n) supersymmetry algebra of spinning string which acts in two dimensions. The isomorphism group of the algebra (1.1) is $SO(m,n)$ group.

In this paper we consider the case of (4,4)-extended supersymmetry algebra. In the second part of the paper we describe some representations of this algebra among which is one containing four coordinates of usual Minkowski space-time. The Lagrangian is constructed in the third part of the paper. In the forth part we describe the algebra of constraints and quantization procedure used in the paper. The whole spectrum of the model contains only two physical states, describing massless particle with helicities +1 and −1, i.e. massless spin one photon-like particle. It is rather interesting, because the total number of supercharges is 8 and in conventional approach spinning particle has spin 4 [1,2].

2 The algebra and some representations

The algebra (1.1) under consideration is generated by 8 real supercharges Q_a with 4 positive and 4 negative eigenvalues of tensor ω_{ab}. It is convenient for us to combine 8 real supercharges into 4 complex ones $Q_{i\alpha}$ which are Weyl spinors ($\alpha=1,2$) of Lorentz group and form also a doublet ($i=1,2$) of internal $SU(2)$ group. The whole $SO(4,4)$ isomorphysm group contains $SO(3,1)\times SU(2)$ as a subgroup. In terms of $Q_{i\alpha}$ and $\overline{Q}^{i}_{\dot\alpha}=-(Q_{i\alpha})^*$ the superalgebra has the form[1]

$$
\begin{aligned}
\{Q_{i\alpha},Q_{k\beta}\} &= \varepsilon_{ik}\varepsilon_{\alpha\beta}H,\\
\{\overline{Q}^{i}_{\dot\alpha},\overline{Q}^{k}_{\dot\beta}\} &= \varepsilon^{ik}\varepsilon_{\dot\alpha\dot\beta}H,\\
\{Q_{i\alpha},\overline{Q}^{k}_{\dot\beta}\} &= 0.
\end{aligned}
\tag{2.1}
$$

The algebra (2.1) has several irreducible representations. One of them contains Lorentz vector X_μ,two Grassmannian spinors $\theta^{i\alpha}$, $\overline{\theta}^{\dot\alpha}_{i}\equiv(\theta^{i\alpha})^*$ and auxiliary fields $G^k_i=(G^i_k)^*$. Its transformation law with supersym-

metry parameters $\epsilon^{i\alpha}$ has the following form[1]

$$
\begin{aligned}
\delta X_\mu &= i\epsilon^{i\alpha}(\sigma_\mu)_{\alpha\dot\beta}\overline{\theta}^{\dot\beta}_i - i\theta^{i\alpha}(\sigma_\mu)_{\alpha\dot\beta}\overline{\epsilon}^{\dot\beta}_i,\\
\delta\theta^{i\alpha} &= \frac{1}{2}\varepsilon^{\alpha\beta}\dot X_{\beta\dot\beta}\overline{\epsilon}^{i\dot\beta} + i\epsilon^{k\alpha}G^i_k,\\
\delta\overline{\theta}^{\dot\alpha}_i &= -\frac{1}{2}\varepsilon^{\dot\alpha\dot\beta}\dot X_{\beta\dot\beta}\epsilon^{\beta}_i - i\overline{\epsilon}^{\dot\alpha}_k G^k_i,\\
\delta G^i_k &= -\epsilon^{\beta}_k\dot\theta^i_\beta - \overline{\epsilon}^{i\dot\beta}\dot{\overline{\theta}}_{k\dot\beta}.
\end{aligned}
\tag{2.2}
$$

There are also more complicated representations of the algebra (2.1). For example, one of them contains fields $D,\psi_{k\alpha},\overline{\psi}^{k}_{\dot\alpha}, M_{\alpha\beta},\overline{M}_{\dot\alpha\dot\beta},K^i_{k\alpha\dot\beta},\phi^k_{\alpha\beta\dot\beta}$, $\overline{\phi}_{k\beta\dot\beta\dot\alpha},H_{\alpha\beta\dot\alpha\dot\beta}$ which are subject to the constraints: $\overline{\psi}^{k}_{\dot\alpha}=-(\psi_{k\alpha})^*, K^k_{i\beta\dot\alpha}=(K^i_{k\alpha\dot\beta})^*, M_{\alpha\beta}=M_{\beta\alpha},\overline{M}_{\dot\alpha\dot\beta}=(M_{\alpha\beta})^*,\overline{\phi}_{k\alpha\dot\beta\dot\gamma}=(\phi^k_{\gamma\beta\dot\alpha})^*=(\phi^k_{\beta\gamma\dot\alpha})^*, H_{\alpha\beta\dot\gamma\dot\delta}=H_{\beta\alpha\dot\gamma\dot\delta}=H_{\alpha\beta\dot\delta\dot\gamma}=(H_{\gamma\delta\dot\alpha\dot\beta})^*$ and have following transformation laws:

$$
\begin{aligned}
\delta D &= \epsilon^{k\alpha}\dot\psi_{k\alpha} - \dot{\overline{\psi}}^{k}_{\dot\alpha}\overline{\epsilon}^{\dot\alpha}_k,\\
\delta\psi_{k\alpha} &= \frac{i}{2}\epsilon_{k\alpha}D + M_{\alpha\beta}\epsilon^{\beta}_k + \overline{\epsilon}^{\dot\beta}_i K^i_{k\alpha\dot\beta},\\
\delta M_{\alpha\beta} &= -\frac{i}{2}\epsilon^k_\alpha\psi_{k\beta} - \frac{i}{2}\epsilon^k_\beta\psi_{k\alpha} + \phi^k_{\alpha\beta\dot\beta}\overline{\epsilon}^{\dot\beta}_k,\\
\delta K^i_{k\alpha\dot\beta} &= -\frac{i}{2}\overline{\epsilon}^i_{\dot\beta}\psi_{k\alpha} - \frac{i}{2}\epsilon_{k\alpha}\overline{\psi}^i_{\dot\beta} + \epsilon^{\beta}_k\dot\phi^i_{\alpha\beta\dot\beta} + \overline{\epsilon}^{i\dot\gamma}\dot{\overline{\phi}}_{k\alpha\dot\beta\dot\gamma},\\
\delta\phi^i_{\alpha\beta\dot\beta} &= -\frac{i}{2}\epsilon^k_\alpha K^i_{k\beta\dot\beta} - \frac{i}{2}\epsilon^k_\beta K^i_{k\alpha\dot\beta} + \frac{i}{2}\overline{\epsilon}^i_{\dot\beta}M_{\alpha\beta} - \frac{1}{32}\overline{\epsilon}^{i\dot\gamma}\dot H_{\alpha\beta\dot\beta\dot\gamma},\\
\delta H_{\alpha\beta\dot\alpha\dot\beta} &= -4i\overline{\epsilon}_{i\dot\alpha}\dot\phi^i_{\alpha\beta\dot\beta} - 4i\overline{\epsilon}_{i\dot\beta}\dot\phi^i_{\alpha\beta\dot\alpha} - 4i\epsilon^i_\alpha\dot{\overline{\phi}}_{i\beta\dot\alpha\dot\beta} - 4i\epsilon^i_\beta\dot{\overline{\phi}}_{i\alpha\dot\alpha\dot\beta}.
\end{aligned}
\tag{2.3}
$$

Another representation contains einbein and gravitino fields and will be used in the following section to construct the locally supersymmetric action for the spinning particle.

[1] Our conventions for spinors are as follows: $\theta^i_\beta=\theta^{i\alpha}\varepsilon_{\alpha\beta}$, $\theta^\alpha_k=\theta^{i\alpha}\varepsilon_{ik}$, $\theta_{k\beta}=\theta^{i\alpha}\varepsilon_{ik}\varepsilon_{\alpha\beta}$, $\overline{\theta}_{i\dot\beta}=(\theta^i_\beta)^*$, $\overline{\theta}^{\dot\alpha}_i=(\theta^{i\alpha})^*$, $\overline{\theta}^{k\dot\alpha}=\varepsilon^{ki}\overline{\theta}^{\dot\alpha}_i=-(\theta^\alpha_k)^*$, $\overline{\theta}^k_{\dot\beta}=\varepsilon^{ki}\varepsilon_{\dot\alpha\dot\beta}\overline{\theta}^{\dot\alpha}_i=-(\theta_{k\beta})^*$, $(\sigma_\mu)_{\alpha\dot\beta}=(1,\sigma)_{\alpha\dot\beta}$.

3 The action

In the flat Minkowski space with the metric tensor $g_{\mu\nu} = diag(1,-1,-1,-1)$ the action

$$S_0 = \int d\tau \{-\frac{1}{2} g_{\mu\nu}\dot{x}^\mu \dot{x}^\nu - i\dot{\theta}^{i\alpha}\theta_{i\alpha} - i\overline{\theta}^{\dot\alpha}_i \dot{\overline{\theta}}^i_{\dot\alpha} + G^i_k G^n_m \varepsilon_{in}\varepsilon^{km}\} \qquad (3.1)$$

is invariant under global supersymmetry transformations (2.1). There is no local reparameterization and supersymmetry invariances in the action(3.1). As a consequence, such theory is not free from ghosts. To remove ghosts,we introduce einbein, one dimensional gravitino and other auxiliary fields.The resulting action looks similar to the action for the N-extended spinning particle[2]:

$$S = \int d\tau \{-\frac{1}{2e} g_{\mu\nu} r^\mu r^\nu + i\dot{\overline{\theta}}^k_{\dot\beta}\overline{\theta}^{\dot\beta}_k - i\dot{\theta}^{i\alpha}\theta_{i\alpha} + 2\overline{S}^i_k \overline{\theta}_{i\dot\beta}\overline{\theta}^{k\dot\beta} - 2S^k_i \theta^\beta_k \theta^i_\beta\}, \qquad (3.2)$$

where $r^\mu = \dot{X}^\mu + \frac{1}{2}\psi^{k\gamma}(\sigma^\mu)_{\gamma\dot\delta}\overline{\theta}^{\dot\delta}_k + \frac{1}{2}\theta^{k\gamma}(\sigma^\mu)_{\gamma\dot\delta}\overline{\psi}^{\dot\delta}_k$. The functions $e(\tau), \psi^{k\alpha}(\tau)$ and $S^i_k(\tau)$, $\overline{S}(\tau)^i_k = (S(\tau)^k_i)^*$ are einbein,gravitino and internal symmetry gauge fields in one dimension respectively and belong to some irreducible representation of the algebra (2.1).

The action (3.2),besides the local reparameterization invariance, is invariant under the following local $N = (4,4)$ supersymmetry transformations:

$$\begin{aligned}
\delta X_\mu &= i\epsilon^{i\alpha}(\sigma_\mu)_{\alpha\dot\beta}\overline{\theta}^{\dot\beta}_i - i\theta^{i\alpha}(\sigma_\mu)_{\alpha\dot\beta}\overline{\epsilon}^{\dot\beta}_i, \\
\delta\theta_{i\alpha} &= \frac{1}{2e} r^\mu (\sigma_\mu)_{\alpha\dot\beta}\overline{\epsilon}^{\dot\beta}_i, \\
\delta\overline{\theta}^k_{\dot\beta} &= -\frac{1}{2e}\epsilon^{k\beta}(\sigma_\mu)_{\beta\dot\beta} r^\mu, \\
\delta\psi_{k\alpha} &= -2i\dot{\epsilon}_{k\alpha} + 2iS^i_k\epsilon_{i\alpha}, \\
\delta\overline{\psi}^k_{\dot\alpha} &= 2i\dot{\overline{\epsilon}}^k_{\dot\alpha} - 2i\overline{S}^k_i\overline{\epsilon}^i_{\dot\alpha}, \\
\delta e &= \frac{1}{2}\psi_{k\alpha}\epsilon^{k\alpha} - \frac{1}{2}\overline{\epsilon}^{\dot\alpha}_k\overline{\psi}^k_{\dot\alpha}, \\
\delta S^i_k &= \delta\overline{S}^i_k = 0.
\end{aligned} \qquad (3.3)$$

In addition, the action (3.2) is invariant under the local internal symmetry transformations:

$$\delta\theta^{i\alpha} = 2i\Lambda^i_k\theta^{k\alpha}, \quad \delta S^i_k = \dot{\Lambda}^i_k \qquad (3.4)$$

where $S^i_k = (\overline{S}^k_i)^*, S^i_i = 0$ are corresponding gauge fields.

With the help of enumerated local transformations we can choose the gauge in which $e(\tau) = 1, \psi_{k\alpha} = S^i_k = 0$ and equations of motion for dynamical variables have the form:

$$\ddot{X}^\mu = \dot{\theta}^{k\alpha} = \dot{\overline{\theta}}_{k\dot\alpha} = 0.$$

The remaining equations of motion for the fields $e(\tau), \psi_{k\alpha}(\tau)$ and $S^i_k(\tau)$ are constraints in the phase space and they remove all ghost states from the space of physical states in quantum theory.

4 Quantization

As it follows from the definitions of momenta conjugated to Grassmann coordinates:

$$\Pi_{k\alpha} = \frac{\partial L}{\partial\dot{\theta}^{k\alpha}} = -i\theta_{k\alpha}, \quad \overline{\Pi}^k_{\dot\alpha} = \frac{\partial L}{\partial\dot{\overline{\theta}}^{\dot\alpha}_k} = i\overline{\theta}^k_{\dot\alpha}$$

the second class constraints take place in the model. Usually one overcomes this difficulty by introducing Dirac brackets. In our case the only nonvanishing Dirac brackets are:$\{\theta_{k\alpha}, \theta_{i\beta}\}^* = -\frac{i}{2}\varepsilon_{\alpha\beta}\varepsilon_{ki}$, $\{\overline{\theta}^k_{\dot\alpha}, \overline{\theta}^i_{\dot\beta}\}^* = -\frac{i}{2}\varepsilon_{\dot\alpha\dot\beta}\varepsilon^{ki}, \{X^\mu, P_\nu\}^* = \delta^\mu_\nu$ which after quantization become

$$\{\theta_{i\alpha}, \theta_{k\beta}\} = \frac{1}{2}\epsilon_{\alpha\beta}\epsilon_{ik}, \quad \{\overline{\theta}^k_{\dot\alpha}, \overline{\theta}^i_{\dot\beta}\} = \frac{1}{2}\epsilon^{ki}\epsilon_{\dot\alpha\dot\beta}, \quad [X^\mu, P_\nu] = i\delta^\mu_\nu. \qquad (4.1)$$

To diagonalize anticommutators we introduce four pairs of creation and annigilation operators

$$\begin{aligned}
a^i &= \theta^{i1} + \overline{\theta}^{i2} \qquad & b^i &= \theta^{i1} - \overline{\theta}^{i2} \\
a^\dagger_i &= \overline{\theta}^1_i - \theta^2_i \qquad & b^\dagger_i &= \overline{\theta}^1_i + \theta^2_i
\end{aligned} \qquad (4.2)$$

In terms of new operators the commutation relations (4.1) are

$$\{a^i, a^\dagger_k\} = \delta^i_k, \quad \{b^i, b^\dagger_k\} = -\delta^i_k, \quad \{a^i, b^k\} = \{a^i, b^\dagger_k\} = 0. \qquad (4.3)$$

It should be noted that b-type operators correspond to ghost degrees of freedom and we have to remove all negative norm states from the Fock space constructed with the help of $a_i^\dagger$ and $b_k^\dagger$ creation operators.

To do this we will find all constraints in the theory which play the role of generators of gauge transformations. We have three types of constraints which correspond to the three types of local invariances - the proper time reparameterization, extended worldline supersymmetry and internal symmetry. The first constraint is just the condition of masslessness of the particle

$$H = -\frac{1}{2}P^2 = 0 \qquad (4.4)$$

The generators of supersymmetry transformations

$$Q_{i\alpha} = iP^\mu(\sigma_\mu)_{\alpha\dot\beta}\overline{\theta}_i^{\dot\beta}, \qquad (4.5)$$
$$\overline{Q}_{i\dot\alpha} = i\theta_i^\beta(\sigma_\mu)_{\beta\dot\alpha}P^\mu \qquad (4.6)$$

and internal symmetry transformations

$$I^{ik} = \frac{\delta L}{\delta S_{ik}} = \theta^{i\alpha}\theta_\alpha^k + \theta^{k\alpha}\theta_\alpha^i, \qquad (4.7)$$
$$\overline{I}_{ik} = -\frac{\delta L}{\delta \overline{S}^{ik}} = -\overline{\theta}_i^{\dot\alpha}\overline{\theta}_{k\dot\alpha} - \overline{\theta}_k^{\dot\alpha}\overline{\theta}_{i\dot\alpha} \qquad (4.8)$$

commute with H and have the following commutation relations among themselves in addition to the algebra (2.1):

$$\begin{aligned}
[I^{ik}, I^{mn}] &= \varepsilon^{km}I^{in} + \varepsilon^{im}I^{kn} + \varepsilon^{kn}I^{im} + \varepsilon^{in}I^{km}, &(4.9)\\
[\overline{I}_{ik}, \overline{I}_{mn}] &= -\varepsilon_{km}\overline{I}_{in} - \varepsilon_{im}\overline{I}_{kn} - \varepsilon_{kn}\overline{I}_{im} - \varepsilon_{in}\overline{I}_{km}, &(4.10)\\
[I^{ik}, \overline{Q}_{m\dot\alpha}] &= -\delta_m^k\overline{Q}_{\dot\alpha}^i - \delta_m^i\overline{Q}_{\dot\alpha}^k, &(4.11)\\
[\overline{I}_{ik}, Q_\alpha^m] &= -\delta_k^m Q_{i\alpha} - \delta_i^m Q_{k\alpha}, &(4.12)\\
[I^{ik}, Q_\alpha^m] &= [\overline{I}_{mn}, \overline{Q}_{i\alpha}] = [I^{ik}, \overline{I}_{mn}] = 0. &(4.13)
\end{aligned}$$

In classical theory the expressions (4.4)-(4.8) are equal to zero. According to the Dirac quantization procedure [5] the physical states $|Phys\rangle$ in quantum theory satisfy the following conditions

$$\Omega_r|Phys\rangle = 0, \qquad (4.14)$$

where Ω_r represent all constraints. In the case of the second class constraints the equations (4.14) are inconsistent and weaker conditions due to Gupta [6] and Bleuler [7]

$$\langle Phys|\Omega_r|Phys\rangle = 0 \qquad (4.15)$$

must be imposed on physical states. In spite of the fact that all constraints in our case are of the first class, the conditions (4.14) are too strong as well and only zero norm physical states survive. It means that we must use equations (4.15) instead of (4.14). The method of solving these equations is as follows [8]. We divide all constraints on three groups. The first group Ω^0 contains only hermitean first class constraints. The second group Ω^+ contains constraints which are conjugated to the corresponding constraints of the third group Ω^-. In addition the generators (Ω_s^0, Ω_t^-) and (Ω_s^0, Ω_t^+) form two different first class subalgebras in the space of all constraints. The requirements

$$\Omega_s^0|Phys-\rangle = \Omega_t^-|Phys-\rangle = 0 \qquad (4.16)$$

or

$$\Omega_s^0|Phys+\rangle = \Omega_t^+|Phys+\rangle = 0, \qquad (4.17)$$

lead to the equations $\langle -Phys|\Omega_t^+ = 0$ or $\langle +Phys|\Omega_t^- = 0$ and, as a consequence, to the equations

$$\langle \pm Phys|\Omega_r|Phys\pm\rangle = 0 \qquad (4.18)$$

valid for all constraints in accordance with the correspondence principle. Moreover all matrix elements $\langle \pm Phys|\Omega_r|Phys\mp\rangle$ in our case are zero as well.

5 The spectrum of physical states

The structure of commutation relations (4.9)-(4.13) suggests the following subdivision of constraints:

$$\begin{aligned}
\Omega_s^0 &= (-\frac{1}{2}P^2), &(5.1)\\
\Omega_t^+ &= (I^{ik}, \overline{Q}_{m\dot\alpha}), &(5.2)\\
\Omega_t^- &= (\overline{I}_{ik}, Q_\alpha^i). &(5.3)
\end{aligned}$$

According to the condition $P^2|Phys\pm\rangle = 0$, the particle is massless and we can choose the light cone system $P_\mu = (P_0, 0, 0, P_0)$ in which the expressions for remaining constraints are very simple in terms of a and b operators:

$$
\begin{aligned}
I^{11} &= -(a_2^+ - b_2^+)(a^1 - b^1), \\
I^{22} &= (a_1^+ - b_1^+)(a^2 + b^2), \\
I^{12} &= \frac{1}{2}(a_1^+ - b_1^+)(a^1 + b^1) - \frac{1}{2}(a_2^+ - b_2^+)(a^2 + b^2), \qquad (5.4) \\
Q_1^i &= 0, \\
Q_2^i &= iP_0(a^i - b^i), \\
\overline{I}_{11} &= -(a_1^+ - b_1^+)(a^2 - b^2), \\
\overline{I}_{22} &= (a_2^+ + b_2^+)(a^1 - b^1), \\
\overline{I}_{12} &= \frac{1}{2}(a_1^+ + b_1^+)(a^1 - b^1) - \frac{1}{2}(a_2^+ + b_2^+)(a^2 - b^2), \qquad (5.5) \\
\overline{Q}_{i1} &= 0, \\
\overline{Q}_{i2} &= -iP_0(a_i^+ - b_i^+).
\end{aligned}
$$

With the help of these expressions we can find the following solutions to the equations (4.16)-(4.17):

$$
\begin{aligned}
|Phys+\rangle &= |0\rangle, \qquad (5.6) \\
|Phys1+\rangle &= (a_1^+ - b_1^+)(a_2^+ - b_2^+)|0\rangle, \\
|Phys1-\rangle &= (a_1^+ - b_1^+)(a_2^+ - b_2^+)|0\rangle, \\
|Phys-\rangle &= a_1^+ a_2^+ b_1^+ b_2^+ |0\rangle. \qquad (5.7)
\end{aligned}
$$

The states $|Phys1\pm\rangle$ coincide and have zero norm. Two other physical states $|Phys\pm\rangle$ are orthonormal and satisfy the Gupta-Bleuler conditions. These both states are not scalars of 4-dimensional Lorentz group. Though $\theta^{i\alpha}$ transform as the doublet of Weyl spinors, the operators a^i and b^i have more complicated transformation law and the Fock space vacuum $|0\rangle$ must not be a scalar with respect to the Lorentz group. To study the transformation properties of all states we have to construct the Lorentz generators and find the Pauli-Lubanski vector which is proportional to the momentum of the particle in the massless case:

$$W_\mu = \frac{1}{2}\varepsilon_{\mu\nu\rho\sigma}P^\nu L^{\rho\sigma} = \lambda P_\mu. \qquad (5.8)$$

where the coefficient λ is the helicity of the state.

The action (3.2) is invariant under the Lorentz transformations with generators

$$L_{\mu\nu} = M_{\mu\nu} + J_{\mu\nu},$$

where $M_{\mu\nu}$ and $J_{\mu\nu}$ are orbital and spin parts of these generators respectively. Only the spin part of Lorentz generators gives contribution to W_μ. We can find $J_{\mu\nu}$ using the Noether procedure. The result is as follows:

$$J_{\mu\nu} = \frac{1}{4}((\sigma_{\mu\nu})_{\alpha\beta}J^{\alpha\beta} + (\overline{\sigma}_{\mu\nu})_{\dot\alpha\dot\beta}\overline{J}^{\dot\alpha\dot\beta}), \qquad (5.9)$$

where

$$
\begin{aligned}
J^{\alpha\beta} &= \theta^{i\alpha}\theta_i^\alpha + \theta^{i\beta}\theta_i^\alpha, \qquad (5.10) \\
\overline{J}^{\dot\alpha\dot\beta} &= \overline{\theta}_i^{\dot\alpha}\overline{\theta}^{i\dot\beta} + \overline{\theta}_i^{\dot\beta}\overline{\theta}^{i\dot\alpha}, \qquad (5.11)
\end{aligned}
$$

and

$$
\begin{aligned}
(\sigma_{\mu\nu})_{\alpha\beta} &= \frac{1}{2i}((\sigma_\mu)_{\alpha\dot\gamma}(\sigma_\nu)_{\beta\dot\delta} - (\sigma_\nu)_{\alpha\dot\gamma}(\sigma_\mu)_{\beta\dot\delta})\varepsilon^{\dot\gamma\dot\delta}, \\
\overline{\sigma}_{\mu\nu} &= (\sigma_{\mu\nu})^+.
\end{aligned}
$$

According to definition of spinor

$$
\begin{aligned}
[J_{\mu\nu}, \theta_{i\alpha}] &= \frac{1}{2}(\sigma_{\mu\nu})_\alpha^{\ \beta}\theta_{i\beta}, \qquad (5.12) \\
[J_{\mu\nu}, \overline{\theta}_{i\dot\alpha}] &= \frac{1}{2}(\overline{\sigma}_{\mu\nu})_{\dot\alpha}^{\ \dot\beta}\overline{\theta}_{i\dot\beta}. \qquad (5.13)
\end{aligned}
$$

The expressions (5.9)-(5.11) lead to the following form of the helicity operator λ in terms of a and b operators:

$$\lambda = -\frac{1}{2}(J^{12} + \overline{J}^{\dot 1\dot 2}) = -\frac{1}{2}(a_1^+a^1 - a^2a_2^+ - b_1^+b^1 + b^2b_2^+) = 1 - \frac{1}{2}a_i^+a^i + \frac{1}{2}b_i^+b^i. \qquad (5.14)$$

The physical states are eigenstates of helicity operator with eigenvalues ± 1:

$$\lambda|Phys\pm\rangle = \pm|Phys\pm\rangle. \qquad (5.15)$$

It means that the action (3.2) describes free massless particle with spin one in four dimensions.

6 Conclusion

We have started from the extended supersymmetry algebra in one dimension with 8 real supercharges four of which have squares opposite in sign to the squares of others. It means that we can realize this algebra only in the space with non-positive-definite scalar product, i.e. with ghosts. We have shown that if this supersymmetry algebra is gauge, all ghosts are cancelled and physical states have positive-definite norm. They describe massless vector particle in four dimensional space-time.

Analogous description is possible in three dimensional space-time as well. In this case we use the $(2,2)$-extended supersymmetry algebra. The representations of this algebra are derived from the representations of $(4,4)$-extended algebra with the help of some reality conditions. For example, conditions:

$$\begin{aligned}(X_{\alpha\beta})^* &= X_{\alpha\beta} = X_{\beta\alpha}, \\ (\theta^{i\alpha})^* &= \overline{\theta}_i^{\dot{\alpha}} = \theta^{i\alpha}, \\ G_i^k &= i\varepsilon^{ik}G,\ G^* = G\end{aligned} \tag{6.1}$$

lead to supermultiplet with three space-time coordinates. There is no difference between dotted and undotted spinor indices in three dimensions. The action (3.2) subjected to constraints (6.1) describes the spinning particle in three dimensions with one time and two space coordinates. However, the quantization of such action results in a single physical state with zero norm.

In general we can use the (m,n)-extended superalgebra (1.1) with space-time coordinates transforming as a vector and Grassmann coordinates as a spinor of some subalgebra $SO(p,q)$ of the whole isomorphism algebra $SO(m,n)$. In this case the spase-time in which spinning particle lives has p spase and q time coordinates. The numbers p and q have to be related with the numbers m and n and it would be interesting to enumerate all space-times in which considered description of spinning particle is possible. Perhaps there are also such numbers p and q that the possibility of analogous description of spinning string exists.

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