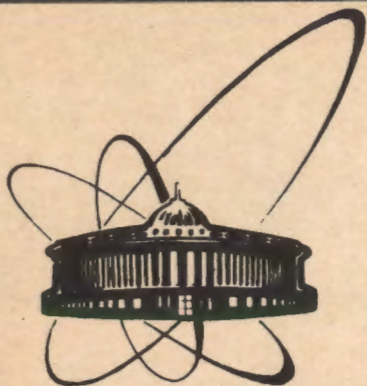


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COHERENT STATE APPROACH FOR THE
 ϕ^6 -LATTICE MODEL AND PHASE TRANSITIONS

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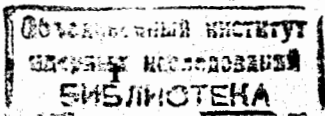
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Introduction

In the past years a number of remarkable efforts were taken aimed at extracting physical information from quantum field theory by studying its semiclassical versions. Nonlinear models which support kinks, bubbles as elementary excitations, such as sine-Gordon, ϕ^4 and ϕ^6 ones, play an important role in various branches of physics and chemistry. The model ϕ^6 admits the nontopological soliton solution in the gravitational background. This scalar field then would be stable and can form a scalar soliton star [1]. The well known fact of spontaneous symmetry breaking in a scalar field theory spurred the interest in getting searching for classical solutions in those models which provide the topological and nontopological types of solitons. Standard semiclassical methods give the link between the quantum field theory and nonlinear phenomena and allow for physical interpretation of the results obtained.

Recently, Masperi et al [2] and [3], constructed a lattice version of the ϕ^6 -Klein-Gordon model in 1+1 dimension. If in this quantum lattice version, the lateral walls of the potential are deeper than the central one, a bubble-like soliton appears in addition to kink solutions. Applying perturbation treatment and renormalization techniques the authors obtained a phase diagram for ordered and disordered states, where the bubbles and kink solitons play a great role in phase transitions.

On the other hand, in the work [4] the method of Generalized Coherent States (GCS) developed in the work [5] and [6] have been successfully employed for investigating the Heisenberg lattice model of ferromagnetism. It is well known that the GCS are states which minimize the Heisenberg uncertainty principle. Although the conclusions



of [2] are seen to be sure enough, we need to improve our understanding of the symmetry breaking phenomena via the coherent states method. This method would allow us for a clearly understanding of how there is appearing the phase transitions in the systems with different kinds of symmetry.

In this paper we investigate the phase transitions in the lattice model of ϕ^6 -field theory with the aid of coherent state on the $SU(3)/SU(2) \otimes U(1)$ coset space. We will obtain a phase diagram by means of computer experiments.

1 The Hamiltonian Model

We study the Φ^6 -theory as one of the simplest example of nonlinear field theory. Its Lagrangian density is

$$L = \frac{1}{2} \partial_\mu \Phi \partial^\mu \Phi - U(\Phi)$$

where

$$U(\Phi) = a_1 \Phi^6 + a_2 \Phi^4 + a_3 \Phi^2$$

The lattice version of its Hamiltonian assumes the following form

$$H = \lambda \sum_{j=1} \left[\frac{1}{2} \pi_j^2 + Q(\Phi_j) - \Phi_j \Phi_{j+1} \right] \quad (1)$$

and

$$Q(\Phi_j) = U(\Phi_j) + \Phi_j^2$$

For our purpose we must choose the signs of coefficients in $U(\Phi)$ in such a way that $a_1, a_3 > 0$ and $a_2 < 0$. In this case the potential $Q(\Phi_j)$ reveals three wells. This sort of assumptions leads us to consider a quantum mechanical problem here. The problem of quantum mechanics for the single site Hamiltonian may be approximated by investigation of the following square-wall potentials, see *fig.1*

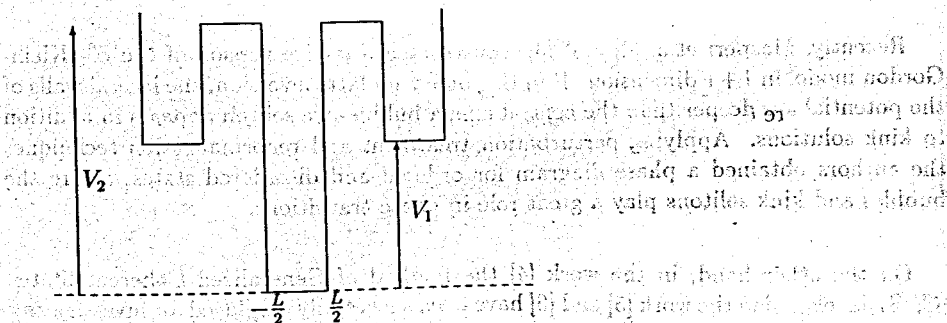


fig.1

According [2] we assume that $V_2 \gg V_1$ and L are small enough to discard all levels except the lowest one in each wall. When $E_0 = -V_2$ the hamiltonian for one site in

the basis of the lowest-energy eigenvalues without overlapping takes the form

$$H_{ss} = \begin{pmatrix} E_0 + V_1 & 0 & 0 \\ 0 & E_0 & 0 \\ 0 & 0 & E_0 + V_1 \end{pmatrix} \quad (2)$$

Quantum mechanical problems reduce to solution of the system

$$H_{ss}|1\rangle = (E_0 + V_1)|1\rangle$$

$$H_{ss}|2\rangle = (E_0)|2\rangle$$

$$H_{ss}|3\rangle = (E_0 + V_1)|3\rangle$$

where

$$|j\rangle = \begin{pmatrix} \delta_{3j} \\ \delta_{2j} \\ \delta_{1j} \end{pmatrix}$$

Taking into account interacting overlapping terms in (2), the hamiltonian will take the form

$$H_{ss} = \begin{pmatrix} E_0 + V_1 & V & 0 \\ V & E_0 & V \\ 0 & V & E_0 + V_1 \end{pmatrix} \quad (3)$$

For this kind of the hamiltonian we naturally have the eigenvalue equation

$$H_{ss}|\psi\rangle = \lambda|\psi\rangle$$

Resolving it we obtain that

$$\lambda_0 = V_1$$

$$\lambda_{\pm} = \frac{1}{2}(V_1 \pm \sqrt{V_1^2 + 8V^2})$$

Representing the eigenfunction in the form

$$|\psi\rangle_{\lambda} = a|1\rangle + b|2\rangle + c|3\rangle$$

for the first eigenvalue, $\lambda_0 = V_1$ one finds

$$|\psi\rangle_{\lambda_0} = \frac{1}{\sqrt{2}}(|1\rangle - |3\rangle)$$

For others eigenvalues, λ_{\pm} after some algebra one can see that

$$c_{\pm} = a_{\pm}$$

$$a_{\pm}^2 = \frac{\lambda_{\mp}^2}{4V^2 + 2\lambda_{\mp}^2}$$

$$b_{\pm}^2 = \frac{4V^2}{4V^2 + 2\lambda_{\mp}^2}$$

As is seen from the above calculation the system has three energy levels

$$E_1 = \lambda_-, \quad E_2 = \lambda_0, \quad E_3 = \lambda_+ \quad (4)$$

If we employ the following generators of faithful representation of the group $SU(3)$

$$T_1^\pm = \begin{pmatrix} 0 & 1 & 0 \\ 1 & 0 & 0 \\ 0 & 0 & 0 \end{pmatrix}, \quad T_2^\pm = \begin{pmatrix} 0 & 0 & 1 \\ 0 & 0 & 0 \\ 1 & 0 & 0 \end{pmatrix}, \quad T_3^\pm = \begin{pmatrix} 0 & 0 & 0 \\ 0 & 0 & 1 \\ 0 & 1 & 0 \end{pmatrix}$$

we can replace the function Φ_j with the following operation

$$\Phi_j \rightarrow \sqrt{\delta}(T_1^\pm + \alpha T_3^\pm)$$

where

$$\sqrt{\delta} = \langle \lambda_+ | \Phi_j | \lambda_0 \rangle$$

$$\alpha \sqrt{\delta} = \langle \lambda_- | \Phi_j | \lambda_0 \rangle$$

$\alpha = \left(\frac{\epsilon}{\kappa - \epsilon}\right)^{\frac{1}{2}}$ and $\epsilon = \lambda_+ - \lambda_0$; $\kappa = \lambda_+ - \lambda_-$. The further procedures are similar to those of Masperi [2]. Then we can rewrite the hamiltonian as

$$H = \sum_j \left[\begin{pmatrix} 0 & 0 & 0 \\ 0 & -\epsilon & 0 \\ 0 & 0 & -\kappa \end{pmatrix}_j - \delta \begin{pmatrix} 0 & 1 & 0 \\ 1 & 0 & \alpha \\ 0 & \alpha & 0 \end{pmatrix}_j \begin{pmatrix} 0 & 1 & 0 \\ 1 & 0 & \alpha \\ 0 & \alpha & 0 \end{pmatrix}_{j+1} \right] \quad (5)$$

2 Phase Transitions

To analyze the phase transitions in this model qualitatively it is more convenient to work with the wall potential representing three energy levels (4) see fig2 [2]. Neglecting overlapping integral the potential wall is represented in fig1.

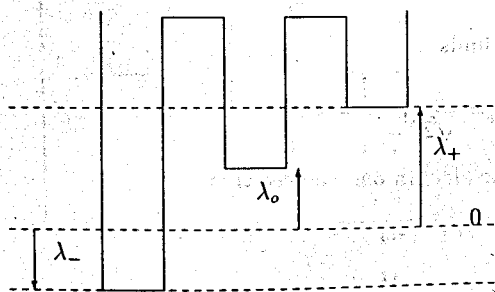


fig.2

In this problem we deal with two parameters V_1 and V . We will be approaching to the point V_1 from both semispace ($V_1 > 0$ and $V_1 < 0$). Then we have two cases:

First case

corresponds the semispace $V_1 > 0$ In this region without overlapping integral $V = 0$ we have

$$\lambda_- = 0 \quad \lambda_0 = V_1 \quad \lambda_+ = V_1$$

and the corresponding picture is

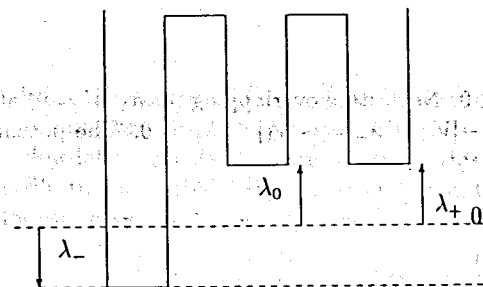


fig.3

When $V_1 \rightarrow 0$ the parameters $\lambda_+, \lambda_-, \lambda_0 \rightarrow 0$, the potential walls transform to the one given in fig.4.

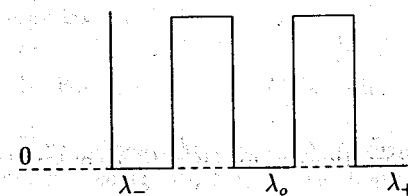


fig.4

If the interaction has been introduced ($V \neq 0$), the fig.4 changes to fig.5 and one can observe that $\lambda_+ = \sqrt{2}V$, $\lambda_- = -\sqrt{2}V$, $\lambda = 0$

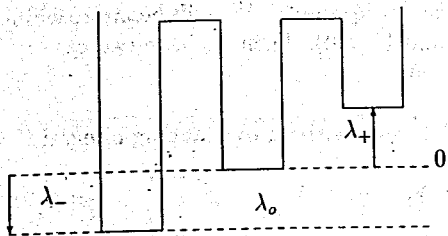


fig.5

Second case.

We now start in the semispace $V_1 < 0$. Neglecting overlapping terms ($V = 0$) after some bit of calculation one finds $\lambda_0 = -|V_1|$, $\lambda_- = -|V_1|$, $\lambda_+ = 0$. The potential wall transforms to fig.6

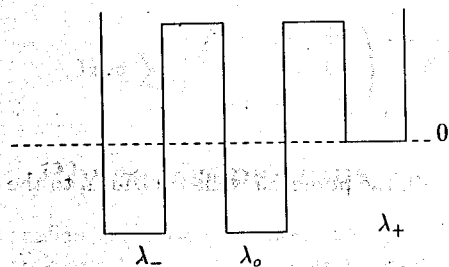


fig.6

In the same way taking into account the interactions terms ($V \neq 0$) we obtain that

$$\begin{aligned} \lambda_+|_{V_1 < 0} &= -\lambda_-|_{V_1 > 0} \\ \lambda_-|_{V_1 < 0} &= -\lambda_+|_{V_1 > 0} \\ \lambda|_{V_1 < 0} &= -\lambda|_{V_1 > 0} \end{aligned} \quad (6a)$$

the picture will be the fig.7

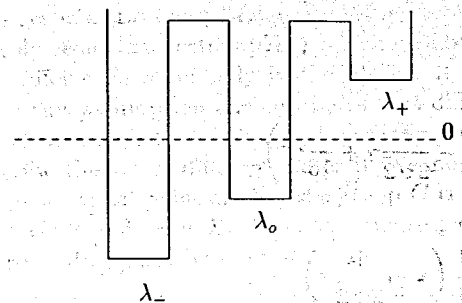


fig.7

From the expression (6a) one can see that the parameters λ_i in the semispace $V_1 > 0$ are the mirror map with opposite sign.

Now let us analyze phenomenologically this problem in the framework of the density distribution of probabilities. As it seen from the plots presented above the system has a lowest energy λ_- for all cases. For these lowest levels the wave function has the form

$$|\psi \rangle_{\lambda_-} = A|1 \rangle + B|2 \rangle + C|3 \rangle$$

Where $A = a_-$, $B = b_-$, $C = c_-$

It is obvious that the system would be situated in that configuration with lowest energy. Consider two limiting cases:

- (I) $V_1 \gg V$

The values for the coefficients of the wave vector in the both sectors of the lowest energy level will take the form

I) For $V_1 > 0$, $A^2 = C^2 \approx (\frac{V}{V_1})^2$, $B \approx 1$

II) For $V_1 < 0$, $A^2 = C^2 \approx 1/2(1 - 2\frac{V^2}{V_1})$, $B \approx 2\frac{V^2}{V_1}$

If we present the probabilities corresponding to the state 1,2 and 3 we obtains the pictures (see fig. 10) in which the height of the lateral bar correspond to the values of A^2 and C^2 respectively. The central one is for B^2 . We can see that the system undergoes a transition of first order since the change of values A and B occurs continuously so in the critical point $V_1 = V$, the both "phases" (see fig. 10a and 10b) will acquire the same probability. This means that the transition just considered is connected to the change of the probabilities rather than the energy changing.

Now let us consider the opposite case when

(2). $V_1 \ll V$

I) For $V_1 > 0$,

$$\lambda_- = -\sqrt{2}V \left(1 - \frac{1}{2\sqrt{2}}\gamma + \frac{1}{16}\gamma^2 \right)$$

Where $\gamma = |V_1|/V$.

Then

$$A^2 = C^2 \approx \frac{1}{4} \left(1 - \frac{1}{2\sqrt{2}}\gamma \right)$$

$$B^2 \approx \frac{1}{2} \left(1 + \frac{1}{2\sqrt{2}}\gamma \right)$$

II) For $V_1 < 0$,

$$\lambda_- = -\sqrt{2}V \left(1 + \frac{1}{2\sqrt{2}}\gamma + \frac{1}{16}\gamma^2 \right)$$

for which

$$A^2 = C^2 \approx \frac{1}{4} \left(1 + \frac{1}{2\sqrt{2}}\gamma \right)$$

$$B^2 \approx \frac{1}{2} \left(1 - \frac{1}{2\sqrt{2}}\gamma \right)$$

Now, if we proceed analogously as above in (1) we can observe that in this region there is no phase transition. (see fig. 10c and 10d). Note that from the energy point of view one-site hamiltonian does not undergo any transition, may be except the point $\epsilon = 1$. The account for the interaction of nearest neighbouring sites in the chain will be made by the following

3 Coherent States Approach

Now let us try to describe some features of the hamiltonian (3) using the generalized coherent states approach.

In the view of a better understanding we outline some basic ideas of the coherent state techniques, (for more details see [5].)

Let G be a Lie group and T its irreducible unitary representation, acting in a Hilbert space H . Let us denote through $|\Psi\rangle$, a vector of this space. The scalar multiplication will be $\langle \Phi | \Psi \rangle$ and the projection operator on to Ψ as $|\Psi\rangle\langle\Psi|$.

We fix some vector $|\Psi_0\rangle \in H$. We consider a set of vectors $\{|\Psi(g)\rangle\}$, such that $|\Psi(g)\rangle = T(g)|\Psi_0\rangle$ and g spans all the group G . Vectors which define the same state i.e. different only in the phase will be collected in a class of equivalence ($|\Psi(g_1)\rangle \sim |\Psi(g_2)\rangle$). It is possible only if $|\Psi(g_1)\rangle = \exp i\theta|\Psi(g_2)\rangle$ then $T(g_2^{-1} \times g_1)|\Psi(g_0)\rangle = \exp i\theta|\Psi(g_1)\rangle$

Let $H = \{h\}$ be a set of elements of the group G satisfying

$$T(h)|\Psi(g_0)\rangle = \exp i\theta(h)|\Psi(g_0)\rangle$$

This set is a stationary subgroup of the vector $|\Psi_0\rangle$. From the above assumptions it is easily seen that vectors $|\Psi(g)\rangle$ being embedded in the left adjoint class $g_1h \in g_1H$ will differ each other only in the phase. It means that they define the same state. From this assumption one concludes that different vectors (states) will correspond to the elements g_m that belong to the factor space $M = G/H$. In this way in order to describe the set of different states it is enough to take one element of each class. In the geometrical point of view the group G is treated as fibre-bundle space with a base $M = G/H$ and layer H . Then the choosing of g_m corresponds to some section of this fibre-bundle space. The set of vectors

$$|\Psi_m\rangle = T(g_m)|\Psi_0\rangle$$

with $g_m \in G/H$ we call a system of the generalized coherent states on the group G with a referent vector $|\Psi_0\rangle$. These coherent states are closest to classical in the sense that they minimize the dispersion

$$\Delta C_2 = \langle \Psi_g | C_2 | \Psi_g \rangle - g^{jk} \langle \Psi_g | X_j | \Psi_g \rangle \langle \Psi_g | X_k | \Psi_g \rangle$$

Where, $C_2 = g^{jk} X_j X_k$ is the quadratic Casimir operator, X_j - generators of the Lie algebra, and g^{jk} is the Cartan-Killing metric tensor.

A further development of this method was made by Debergh N. and Beckers J. [7] and also B. W. Fatyga, V. Alan Kostelecký et al. [8] up to supersymmetric quantum mechanics. It is also interesting to note in this sense works of S. De Bièvre [9], which had constructed coherent states for the Euclidean group $E(n)$. The generalized procedure to construct generalized coherent states presented in the cited paper works in a number of cases where the Perelomov's procedure fails.

As a ground state (referent) in the system described by the hamiltonian (5) we take

$$|o\rangle = \begin{pmatrix} 0 \\ 0 \\ 1 \end{pmatrix}$$

The generalized coherent states in our case correspond, according to the description given above, to the point of the factor space $SU(2s+1)/(SU(2s) \otimes U(1))$, namely the complex projective space CP^2 . For this space the system of coherent states have been constructed (for more details see [4]) looking like

$$|\psi\rangle = \exp \left\{ \sum_j^{2s} \left\{ \xi_j \hat{T}_j^+ - \bar{\xi}_j \hat{T}_j^- \right\} \right\} |0\rangle$$

or in the faithful representation

$$|\psi\rangle = (1 + |\psi|^2)^{-1/2} \left\{ |0\rangle + \sum_j^{2s} \psi_j |j\rangle \right\}$$

Where $|\psi|^2 = \sum_j^{2s} |\psi_j|^2$ and \hat{T}_j^{\pm} are the generators of faithful representations of the group $SU(2s+1)$

Or in a more explicit form

$$|\Psi\rangle = \frac{\begin{pmatrix} 0 \\ 0 \\ 1 \end{pmatrix} + \psi_1 \begin{pmatrix} 0 \\ 1 \\ 0 \end{pmatrix} + \psi_2 \begin{pmatrix} 1 \\ 0 \\ 0 \end{pmatrix}}{\sqrt{1 + |\psi_1|^2 + |\psi_2|^2}} \quad (6)$$

Now the classical hamiltonian we obtain by averaging the quantum hamiltonian (5) over the above coherent states (6). This procedure gives the following result:

$$H = U^J + V$$

$$U = - \int dx \frac{\alpha_0 \delta}{2} (\partial_x \langle T_x \rangle)^2$$

where

$$\langle T_x \rangle = \langle \Psi | T_1 + \alpha T_3 | \Psi \rangle$$

$$V = - \int dx \left[\frac{\kappa + \epsilon |\psi_1|^2}{1 + |\psi_1|^2 + |\psi_2|^2} + \delta \frac{(\alpha(\psi_1 + \bar{\psi}) + \psi_1 \bar{\psi}_2 + \psi_2 \bar{\psi}_1)^2}{(1 + |\psi_1|^2 + |\psi_2|^2)^2} \right] \quad (7)$$

All information about the "thermodynamic" feature of the system is contained in the potential V . When ψ_j can be represented as $\psi_j = R_j e^{i\phi_j}$, it is evident that the hamiltonian achieves the minimum when $\phi = 0$. Then the expression (7) transforms to

$$V = - \int dx \left[\frac{1 + \sigma R_1^2}{1 + R_1^2 + R_2^2} + 4\Delta R_1^2 \frac{(\alpha + R_2)^2}{(1 + R_1^2 + R_2^2)^2} \right] \quad (8)$$

Where $\sigma = (\epsilon/\kappa)$ and $\Delta = (\delta/\kappa)$

4 Numerical Calculation

The main purpose of this paper has been to acquire some general experience in applying the coherent states approach in Φ^6 - field theory.

Now let us consider it for a physically reliable range of the system parameters, σ and Δ . Looking at σ we can see that it may vary between 0 and 1. Δ is only > 0 . The problem for $\alpha = 1$ has been totally resolved in [10]. The variables R_1 and R_2 are positive by definition. With the aid of the Physical Analysis Workstation (PAW) we have performed investigations of the integrand expression in formula (8) starting with $\sigma = 0.1$, and $\Delta = 0.01$. Then fixing some value of σ and varying Δ with the step=0.01 we have looked for the appearance of additional minima, besides the ground one and studied the manner they have appeared.

The results of our experiment are depicted in the *fig.8*. The solid line corresponds to a phase transition of the first order. The dashed curve represents a second order phase transition. All in the sense that in the $\sigma > 0.24$ interval the complementary minimum of first appears very close to the ground, then it goes apart getting growing deeper while the ground minimum disappears. For $\sigma < 0.24$ the additional minimum

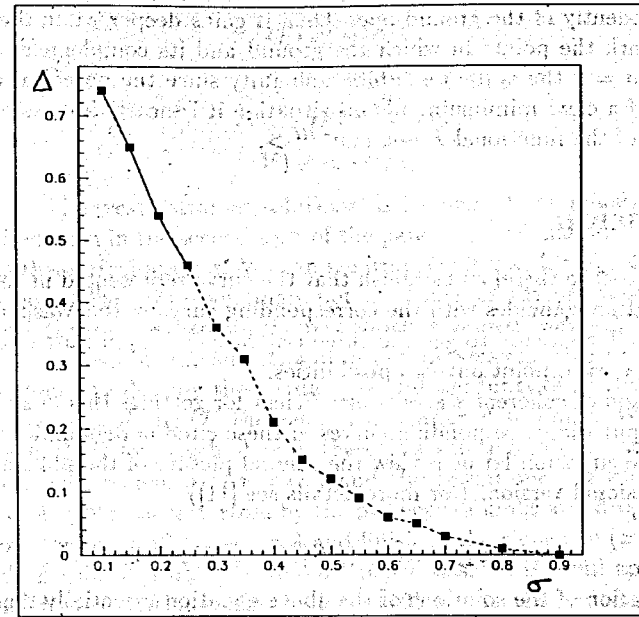


fig. 8

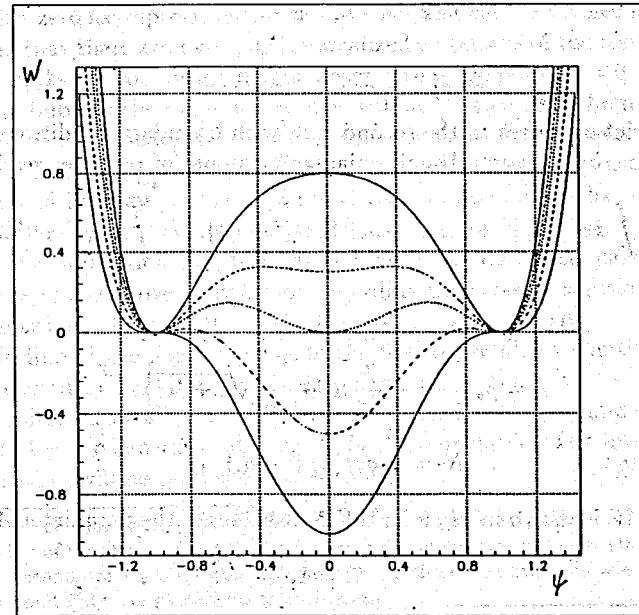


fig. 9

appears independently of the ground one. Then it gains deeper while the ground one reduces. We mark the points in which the ground and its complementary minimum coincide. Near $\sigma = 0$ the system exhibits ambiguity since the potential display wide valley, instead of a clear minimum. In this situation it is nearly impossible to predict the exact value of the functional $F \sim \langle \exp^{-\beta H} \rangle$.

5 Discussion

In conclusion, we have therefore establish that the curve represented in the *fig.8* with good approximation coincides with the corresponding curve in the Masperi's works [2] and [3], but all efforts made to get the three critical point by means of our methods failed. This allow us to point out two possibilities:

- 1.- The method of coherent states is insufficient for gettings this in such models or
- 2.- One can put the corresponding curves in these cited papers under doubt¹

To clarify the situation let us review the general picture of the phi-six (ϕ^6) theory in its one dimensional version. (for more details see [11])

$$i\psi_t + \psi_{xx} + \alpha\psi + (|\psi|^2 - |\psi|^4)\psi = 0 \quad (9)$$

The investigation of the solutions of the above equation essentially depends on the asymptotics of $\psi(x)$. We shall consider two cases which are interesting from a physical point of view

$$\psi(x) \longrightarrow 0, \quad x \rightarrow \infty \quad (10)$$

$$\psi(x) \longrightarrow \text{const}, \quad x \rightarrow \infty \quad (11)$$

Now we restrict ourselves to the second case with boundary conditions of the condensate type. The corresponding Hamiltonian and number of particles read

$$E = \int dx \{ |\psi_x|^2 + (|\psi|^2 - \rho_0)^2 (|\psi|^2 - A) \} = \int dx \{ T + W \} \quad (12)$$

where

$$W = (|\psi|^2 - \rho_0)^2 (|\psi|^2 - A) \quad T = |\psi_x|^2$$

$$A = \rho_0 \left\{ -2 - \frac{3}{4\alpha} \pm \frac{3}{4|\alpha|} \sqrt{1 + 4\alpha} \right\}$$

$$N = \int dx (|\psi|^2 - \rho_0) \quad (13)$$

The function W is shown in *fig.9* for different values of the parameter A and $\rho_0 = 1$.

$$I) A = 1,$$

¹It looks like this curve is related to the transition which is connected with the changes of probabilities. One point of this curve at $\Delta = 0$ was found earlier.

$$II) 1 > A > 0,$$

$$III) A = 0,$$

$$IV) -1/2 < A < 0,$$

$$V) A < -1/2,$$

For $A > 1$, the investigation of solutions' behaviour of the phi-six model will be given in detailed form in the second part of the paper, for this has still remained at the frontier of the theoretical physics. We can see from *fig.9* that for (I) $A = 1$ there is no localized solutions. The correspondingly potential is of oscillatory type. In the region (II) $0 < A < 1$ there is a soliton in the rest which is named "bubble" [12]. This static bubble solution is unstable for any A , and in any dimension. Due to their instability these bubbles tends to destroy the condensate and the function W transforms to that as is represented by the line (I) the easier the nearer A to unity. When the system reaches the form (III) for which $A = 0$, there is a solution which connects two stable vacuum states: the condensate $|\psi|^2 = 1$ and the trivial one $|\psi|^2 = 0$. The sector (IV), $-1/2 < A < 0$ is characterized by the presence of kinks and drop solutions. The first start from the left wall ($x \rightarrow -\infty$) and finish in the right one ($x \rightarrow \infty$) and vice versa. The drop solutions have remarkable properties. These will appear essentially when the boundary conditions take the form (10) and in the region (IV). Then the point $A_0 = 0$ divides the domains where the solutions are quite different. On the right of A_0 at $A \in (0, 1)$ soliton-like solutions are bubbles with the condensate asymptotics. On the left from A_0 , there are kinks and also appear particle-like solutions which are drops with zero asymptotics which are stables and can be stables in this "gaseous" state. We see that there are two types of localized excitations of the condensate (liquid): bubbles and kinks. In conclusion we can observe that the point $A = 0$ is critical. Along the kinks and bubbles the phase transitions will occur also with the presence of drops. The last type of solutions will be responsible for the pass of the system to the condensed state at $A \approx 0$. Finally, in the sector (V), $A < -1/2$ there exist only the kink type solutions defined by the paths going from the top of one hill to that of another in the picture of the mechanical analogy relation.

We are now in position to interpret our results of numerical calculations within the framework of the above explanation regarding the types of solutions and definition of phase transition:

The solid line of the *fig.8* represents the first order phase transition between the states represented by (II) and (V) {see *fig.9*}²

The other line would represent the phase transitions of the second order, which arise at the point $A = 1$ when the stable phase become unstable. The investigation of this kind of transition will be published elsewhere.

²As usual as the first order phase transition we can consider only those of the transitions when below and higher of the critical point the both phases can simultaneously exist (though) with different probabilities. Second order phase transition we will call those transitions in which below and higher of the critical point only one phase lives in each sector.

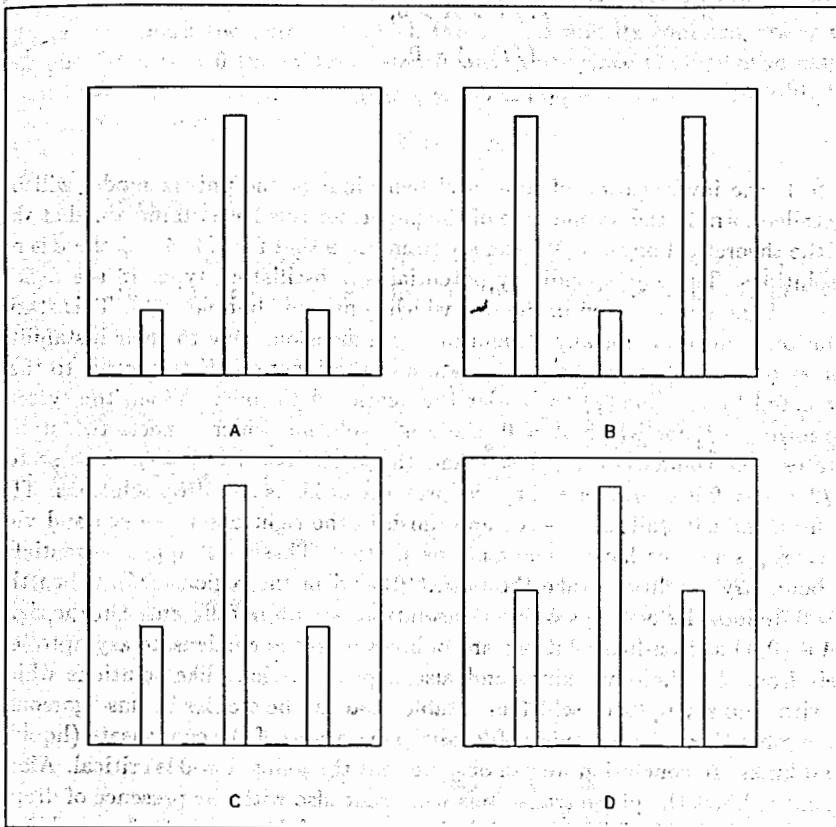


fig. 10

6 Acknowledgment

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