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CLASSICAL MECHANICS OF CONFINED PARTICLES

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In modern elementary particle physics, the problem of quark confinement is one among the fundamental and unsolved troubles [1]. All attempts to explain the strange behaviour of quarks which never appear as free particles failed.

Usually, the problem of quark confinement is attacked in the framework of quantum mechanics or quantum field theory because quarks are quantum objects. In spite of that in the present paper we propose a new purely classical approach to this problem. We start with the very foundation of classical mechanics because we strongly believe that the heart of the problem is here. We shall show that there exists a particular kind of classical mechanics of N-particle systems in which the constituent particles cannot realize themselves as individual mechanical objects. We are strongly convinced that this kind of classical mechanics is a right prototype for any quantum theory of confined particles.

Our approach to classical mechanics of confined particles is based on a particular extension to many-particle systems of the recently developed new mathematical formalism of the Galilean covariant dynamics of a single particle [2]. The essence of this new formalism may be recapitulated in two items:

i) all mechanical quantities which describe motion of particles satisfy simple evolution equations;

ii) the interaction of each particle with its environment is described by two vector-valued measures which also should be determined from their own evolution equations.

It is to be noticed that the property listed initem i) allows one to divide all mechanical equations into kinematical and dynamical equations of motion, equations of balance and equations of the environment which replace the customary force laws. Among these equations the kinematical equations of motion and the equations of balance have universal and standard forms while the dynamical equation of motion and the equations of the environment are new in treating mechanics. The latter have a particular meaning for each type of mechanical interaction. They are necessary to complete the theory and to guarantee its Galilean covariance. The equations of environment acquire their beautiful form only after increasing the number of vector-valued measures of mechanical interaction mentioned in item ii). Apart from the

usual measure called the force, the new formalism uses a second measure called the influence. The introduction of influence opens new possibilities of extending the range of applicability of classical mechanics to new hypothetical types of systems. One of such possibilities will be described in the present paper. For all necessary details we send the interested readers to Ref. /2/.

It is straightforward to extend the new Galilean covariant dynamics described in /2/ to mechanical systems which contain more than one particle. We shall describe this extension in two steps.

The first step is common to all passages from one-particle to many-particle systems and consists in introducing separate, individual kinematics for each particle of the considered system since otherwise we could not speak about many-particle systems at all. This means that for N-particle systems we have to consider $6N$ kinematical equations of motion

$$\dot{\vec{x}}_j(t) = \vec{v}_j(t) \tag{1}$$

$$\dot{\vec{v}}_j(t) = \vec{a}_j(t) \tag{2}$$

and $3N$ dynamical equations of motion:

$$\dot{\vec{Q}}_j(t) = \vec{I}_j(t) \tag{3}$$

where $j = 1,2,\ldots N$, and $\vec{x}_j(t)$, $\vec{v}_j(t)$ and $\vec{a}_j(t)$ are the usual trajectory function, velocity function and the acceleration function of the jth particle, respectively. The quantities $\vec{I}_j(t)$, called influences, represent one of the vector-valued measures of mechanical interaction of the jth particle with its environment. We would like to stress here the conceptual difference between the new notion of the influence $\vec{I}_j(t)$ and the standard notion of the force $\vec{F}_j(t)$. The former is a measure of non-uniformity of the motion while the latter measures the violation of conservation laws during the motion. Both $\vec{I}_j(t)$ and $\vec{F}_j(t)$ provide the complete description of the dynamical action of the environment of the particle on the particle itself.

The second step in passing from one-particle to many-particle systems consists in introducing a corresponding dynamics. The new formalism described in Ref. /2/, allows one to implement this step either in a standard or in a non-standard way and this is one of the advantages of the new formalism.

In the standard way we introduce for each particle the notion

of its momentum $\vec{p}_j(t)$ and the notion of the force $\vec{F}_j(t)$ acting on the jth particle. The dynamics of each particle is then described by the Newton equations of balance

$$\dot{\vec{p}}_j(t) = \vec{F}_j(t). \tag{4}$$

The inertial properties of each particle appear in double face /3/ since the Galilean transformation rules for the momenta

$$\vec{p}_j(t) \to \vec{p}_j'(t') = \mathcal{R}\,\vec{p}_j(t) + m_j\vec{u} \tag{5}$$

introduces the Galilean masses m_j of the particles while the second law of dynamics

$$\vec{F}_j(t) = M_j\vec{a}_j(t) \tag{6}$$

introduces the inertial masses M_j of the particles. The standard mechanics works with the additional assumption that

$$m_j = M_j \tag{7}$$

but an alternative case of violation of equalities (7) may be considered as well /4/.

The non-standard approach to many-particle systems we shall follow below consists in the following: In contradistinction to kinematics we do not introduce separate dynamical characteristics of individual particles at all and consider only global dynamical description of the system as a whole. This means that the dynamics of the N-particle system is collectively described by one total momentum $\vec{P}(t)$ which cannot be represented as a sum of individual momenta of the particles because the latter do not existand by one total force $\vec{F}(t)$ which also cannot be treated as a resulting force of the individual forces $\vec{F}_j(t)$ because the particles do not feel any force at all. The total momentum $\vec{P}(t)$ and the total force $\vec{F}(t)$ satisfy the Newton equation of balance

$$\dot{\vec{P}}(t) = \vec{F}(t) \tag{8}$$

and the motions of individual particles are solely governed by the dynamical equations of motion (3). The individual influences $\vec{I}_j(t)$ are related in a certain way, we shall discuss below, to the total force $\vec{F}(t)$.

We shall not vizualize the total force $\vec{F}(t)$ as a three-dimensional vector acting on a particular point in space and therefore we should not answer the usual question which asks at which point the force is applied. We restrict ourselves solely to the interpretation of the force $\vec{F}(t)$ as a vector-valued measure of interaction of the system with its environment which violates the conservation law of the total momentum.

Under Galilean transformations the force $\vec{F}(t)$ transforms in the usual way

$$\vec{F}(t) \to \vec{F}'(t') = \mathcal{R}\vec{F}(t). \tag{9}$$

Similarly, the transformation rule for the momentum $\vec{\mathcal{P}}(t)$ must be of the form

$$\vec{\mathcal{P}}(t) \to \vec{\mathcal{P}}'(t') = \mathcal{R}\vec{\mathcal{P}}(t) + \mathcal{M}\vec{u} \tag{10}$$

and introduces the notion of the Galilean mass $\mathcal{M}$ of the system of N particles as a whole. The notion of its inertial mass will be introduced below. We do not introduce the notion of individual masses of the particles because for them we have neither individual momenta nor individual forces. Therefore, for the considered particles we can write neither equations (5) nor equations (6). The particles of the considered systems have only individual kinematics but are deprived from individual dynamics. In this sense the particles are "confined in a system" because they cannot realize themselves as individual mechanical entities. They may exist only as constituents of collections and there is no possibility to extract from these collections any individual constituent or a cluster of constituents.

We are fully aware that our point of view on the nature of confined particles is very radical and it may be rejected by the argument that in the domain of classical mechanics all particles should be treated in a classical way. Confined particles observed in Nature are objects quite different from the usual particles just due to their being absent in a free state. They cannot therefore have all the attributes of usual particles and any theory of confined particles has to take this fact into account from the very begining. Otherwise, irrespective of any eventual partial success, a theory of that type is physically wrong. Realistic confined particles are quantum objects and any realistic consistent theory of such particles must be of a quantum type. Unfortunately, all present quantum theories are based

on classical prototypes in which the confinement of particles is excluded. To have physically well motivated theory of confined particles, we should construct it from an assumption different than the usual theories have incorporated in. It is therefore highly necessary to study alternative classical **prototypes for new quantum theories** and this was the main motivation of writing the present paper.

In addition, it should be taken into account that all detection methods in particle physics are based on the principles of classical physics because the interaction of quantum particles with measurement apparata is always classical. Therefore, independently from any quantum theory which may explain the confinement any complete theory of confined particles has to look for alternative approaches already on the classical level.

One crucial novelty of our approach to Galilean covariant mechanics /2/ consists in rejecting all known force laws and in replacing them by a system of differential equations from which the influences and forces may be determined. In Newton's mechanics the influences $\vec{I}_j(t)$ and forces $\vec{F}_j(t)$ always are related by simple relations

$$\dot{\vec{F}}_j(t) = M_j \vec{I}_j(t). \tag{11}$$

In more sophisticated forms of mechanics the differential equations which relate $\vec{I}_j$ and $\vec{F}_j$ are of the form

$$\phi_\ell(\vec{F}, \dot{\vec{F}}, \ddot{\vec{F}}; \vec{I}, \dot{\vec{I}}, \ddot{\vec{I}}) = 0 \tag{12}$$

where ϕ_ℓ is a function of its arguments collectively denoted here by letters without the label j and the label ℓ runs over a set which is sufficient to produce a complete system of differential equations for specification of all $\vec{I}_j(t)$ and $\vec{F}_j(t)$. In any mechanics of confined particles we have only one force $\vec{F}(t)$ and N influences $\vec{I}_j(t)$, and consequently we should modify the standard way of writing the differential equations (11) and (12). To do this, let us observe that in the standard mechanics the force $\vec{F}(t)$ may be represented in the form

$$\vec{F}(t) = \sum_{j=1}^{N} \vec{F}_j(t) \tag{13}$$

where $\vec{F}_j(t)$ are the forces acting on the jth particle. From this representation and (11) we find the equation

$$\dot{\vec{F}}(t) = \sum_{j=1}^{N} M_j \vec{I}_j(t) \qquad (14)$$

which apart from the undetermined inertial masses contains quantities used also in the mechanics of confined particles. We cannot however take this equation as one of the equations of the environment because confined particles have no inertial masses and the meaning of the coefficients in (14) is not clear. We have argued in Ref. /2/ that all constants that appear in the equations for influences and forces may be interpreted as coupling constants of the model of the particle environment, and following this line of reasoning we could take instead of (14) the equation

$$\dot{\vec{F}}(t) = \sum_{j=1}^{N} \alpha_j \vec{I}_j(t) \qquad (15)$$

where α_j ($j = 1,2,\ldots N$) are fixed coupling constants. However, such an approach leaves too much arbitrariness. The situation is much better for identical particles because for such particles all relations should be symmetric under all permutations of quantities ascribed to individual particles. This means that for identical particles instead of (15) we will have

$$\dot{\vec{F}}(t) = \alpha \sum_{j=1}^{N} \vec{I}_j(t) \qquad (16)$$

and we have to do only with one coupling constant the meaning of which is easily established. To find the interpretation of α , let us observe that the case of identical particles is the only case for which the usual notion of the center of mass has a purely geometrical meaning independent of the values of masses of particles. In fact, for a system of identical particles the usual definition of the center of mass leads to the following identification of its position vector

$$\vec{X}(t) = \frac{1}{N} \sum_{j=1}^{N} \vec{x}_j(t) \qquad (17)$$

its velocity

$$\vec{V}(t) = \frac{1}{N} \sum_{j=1}^{N} \vec{v}_j(t) \qquad (18)$$

and its acceleration

$$\vec{A}(t) = \frac{1}{N} \sum_{j=1}^{N} \vec{a}_j(t). \qquad (19)$$

Clearly, all these concepts have a pure geometrical and kinematical meaning as the average position, velocity and acceleration, respectively. We may therefore use these notions also for systems of confined particles for which the notion of mass of the individual particles is not defined. The equation of motion for any system of confines particles as whole will then be of the form

$$\vec{F}(t) = \mathbb{M}\, \vec{A}(t) \qquad (20)$$

where $\mathbb{M}$ is the inertial mass of the system as a whole. Differentiating now this equation we get

$$\dot{\vec{F}}(t) = \frac{\mathbb{M}}{N} \sum_{j=1}^{N} \dot{\vec{a}}_j(t) = \frac{\mathbb{M}}{N} \sum_{j=1}^{N} \vec{I}_j(t) \qquad (21)$$

and comparing this result with equation (16) we come to the identification

$$\alpha = \frac{\mathbb{M}}{N} \qquad (22)$$

and this completes the interpretation of the coupling constant α .

Clearly, equation (16) is only one of equations which should determine the influences. We still need more equations to determine the individual influences of each particle. Whatever their form may be, they should not contain the total force $\vec{F}(t)$ because the behaviour of an individual confined particle should be independent of the interaction of the system as a whole with its environment. Consequently, the environment should never be able to disjoin the particles into separate clusters or constituents. This may be achieved if we postulate that the only equation in which participate $\vec{F}(t)$ is equation (16). The eventual equations for determining $\vec{F}(t)$ should arise from treating at least two many-particle systems because the force $\vec{F}(t)$ is connected with the interaction

of the system as a whole and not with the interaction of its individual constituents.

Since the remaining equations which contain individual influences should determine only relative motion of the confined particles we must, instead of (12), postulate equations of the type

$$\phi_j\left(\vec{I}_{j\kappa}, \dot{\vec{I}}_{j\kappa}, \ddot{\vec{I}}_{j\kappa}\right) = 0 \qquad (23)$$

where

$$\vec{I}_{j\kappa}(t) \equiv \vec{I}_j(t) - \vec{I}_\kappa(t) \qquad (24)$$

are the relative influences.

Solving equations (23) together with (16) we will find all individual influences $\vec{I}_j(t)$ and we may integrate all kinematical equations (1)–(3). Each equation needs however some initial conditions in order to specify the physical quantity determined by this equation, and the overall picture of the motion of particles will crucially depend on the choice of initial conditions. To keep the confined particles together in some finite regions of space, we must additionally assume the condition

$$\sup_{-\infty<t<\infty} \left|\vec{x}_j(t) - \vec{x}_\kappa(t)\right| < \infty . \qquad (25)$$

This is a condition on the initial preparation of the system and may not be satisfied. Note, however, that (25) is not a defining property of confined particles but only one of their particular feature.

To illustrate the proposed approach, we complete the paper with a simple example of the system of two confined particles for which equations (23) are of the oscillator type

$$\ddot{\vec{I}}_{12} + \omega^2 \vec{I}_{12} = 0 \qquad (26)$$

and for which the external force $\vec{F}(t)$ is constant in time. Clearly, equation (26) has the solution

$$\vec{I}_{12}(t) = \vec{I}\cos\omega t + \vec{J}\sin\omega t \qquad (27)$$

where $\vec{I}$ and $\vec{J}$ are two integration constants which describe the internal structure of the system and which have to be determined from some kind of initial conditions. From (16) and (27) we get then the individual influences in the form

$$\vec{I}_1(t) = \tfrac{1}{2}\vec{I}\cos\omega t + \tfrac{1}{2}\vec{J}\sin\omega t \qquad (28)$$

$$\vec{I}_2(t) = -\tfrac{1}{2}\vec{I}\cos\omega t - \tfrac{1}{2}\vec{J}\sin\omega t$$

and integrating equations (1)–(3) we get the trajectories of particles in the form

$$\vec{x}_1(t) = \vec{\alpha}_1 + \vec{\beta}_1 t + \vec{\gamma}_1 t^2 - \frac{1}{2\omega^3}\vec{I}\sin\omega t + \frac{1}{2\omega^3}\vec{J}\cos\omega t \qquad (29)$$

$$\vec{x}_2(t) = \vec{\alpha}_2 + \vec{\beta}_2 t + \vec{\gamma}_2 t^2 + \frac{1}{2\omega^3}\vec{I}\sin\omega t - \frac{1}{2\omega^3}\vec{J}\cos\omega t$$

where $\vec{\alpha}_j, \vec{\beta}_j$ and $\vec{\gamma}_j$ for $j=1,2$ are integration constants to be determined from initial conditions. Condition (25) requires that

$$\vec{\beta}_1 = \vec{\beta}_2$$

and (30)

$$\vec{\gamma}_1 = \vec{\gamma}_2 .$$

Imposing now the initial conditions

$$\vec{x}_j(t_0) = \vec{x}_{j0} \qquad (31)$$

$$\vec{v}_j(t_0) = \vec{v}_{j0}$$

$$\vec{a}_1(t_0) - \vec{a}_2(t_0) = \vec{a}_0$$

and using equation (20) we get the solution (29) in the form

$$\vec{x}_1(t) = \vec{x}_{10} + \frac{\vec{v}_{10}+\vec{v}_{20}}{2}(t-t_0) + \frac{\vec{P}}{2\mathcal{M}}(t-t_0)^2 +$$

$$+ \frac{1}{2\omega^2}\vec{a}_0\left[1 - \cos\omega(t-t_0)\right] + \frac{\vec{v}_{10}-\vec{v}_{20}}{2\omega}\sin\omega(t-t_0), \qquad (32)$$

$$\vec{X}_2(t) = \vec{X}_{20} + \frac{\vec{v}_{10}+\vec{v}_{20}}{2}(t-t_0) + \frac{\vec{F}}{2M}(t-t_0)^2 -$$

$$- \frac{1}{2\omega^2}\vec{a}_0\left[1-\cos\omega(t-t_0)\right] - \frac{\vec{v}_{10}-\vec{v}_{20}}{2\omega}\sin\omega(t-t_0).$$

It is easy to see that the relative motion of the particles is independent of the force $\vec{F}$ and therefore no external force can disjoin the particles.

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