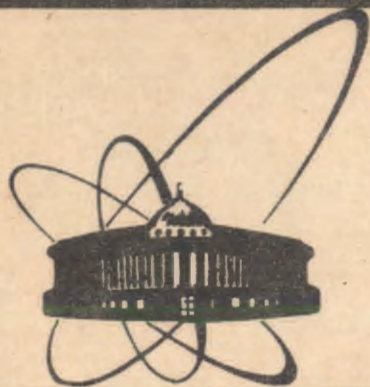


91-464



СООБЩЕНИЯ
ОБЪЕДИНЕННОГО
ИНСТИТУТА
ЯДЕРНЫХ
ИССЛЕДОВАНИЙ
ДУБНА

E2-91-464

G.V.Efimov, S.N.Nedelko*

PHASE STRUCTURE OF THE TWO-DIMENSIONAL ϕ^4
FIELD THEORY WITHIN THERMO FIELD DYNAMICS

*Tomsk State University, USSR

1991

INTRODUCTION

In this paper we will consider the phase structure of the models

$$L(x, t) = \frac{1}{2} \phi(x, t) (\square - m^2) \phi(x, t) - \frac{1}{4} g \phi^4(x, t) \quad (1)$$

$$L(x, t) = \frac{1}{2} \phi(x, t) (\square + \frac{1}{2} m^2) \phi(x, t) - \frac{1}{4} g \phi^4(x, t) \quad (2)$$

in the two-dimensional space-time at finite temperature.

These Lagrangians describe a one-component scalar field $\phi(x, t)$. The parameters m and g are positive. The Lagrangians are invariant under the transformation $\phi \rightarrow -\phi$.

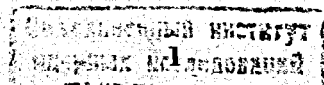
The models are superrenormalizable. The renormalization can be performed by normal ordering in the Hamiltonians.

If the dimensionless coupling constant G and the temperature θ

$$G = \frac{g}{2\pi m^2}, \quad \theta = \frac{T}{m}$$

is small enough, the Lagrangian (1) describes the interaction symmetric under transformation $\phi \rightarrow -\phi$, but the Lagrangian (2) describes the situation of spontaneous symmetry breaking.

The models under consideration are popular objects for investigation of dynamical symmetry reconstruction. This phenomenon is present in many profound four-dimensional



field theories (see, for example, [1]). The investigation of two-dimensional models (1),(2) plays a testing role for different nonperturbative methods, such as gaussian effective potential approximation (GEP) [2-4] or constructive quantum field theory [5,6]. At the same time there is an analogy between the model (2) and real systems in condensed state physics [7].

There are three groups of basic problems, methods and corresponding results concerning the phase structure of the models (1),(2).

The exact theorems have been proved within the constructive quantum field theory [5,6]. These theorems have strictly established the existence of the phase transition in the model (1) for the zero temperature case and given the arguments that the transition should be of the second order [6,8]. But these arguments do not exclude completely the possibility of the first order transition as it has been noted in papers [6]. At the same time, these theorems do not give any information about the value of critical coupling constant.

The fruitful attempts to construct phases in the explicit form and to calculate the approximate critical value of G was made by several authors within the variational GEP [2-4] and beyond the GEP approximation [9].

The restoration of symmetry in model (2) at high temperature was investigated by many authors within the one-loop or two-loop effective potential calculations (see for example [7,10]).

Unfortunately, there is a general feature of the variational approaches such as GEP. They do not give a direct opportunity to control the exactness of approximation [11]. At the same time the results of [7,10] are valid only for high temperature limit. Thus it seems to be interesting to construct an explicit form of the phases of the models (1),(2) for arbitrary θ , G and to attract for this purpose a method permitting a simple accuracy check.

We will solve that problem by the method of canonical transformations. This approach has been used for investigation of the model (1) at zero temperature [12]. Essentially, it is a field-theoretical version of the technique being used in the theory of superconductivity and so on for transition to the quasiparticle picture.

The apparatus of thermo field dynamics (TFD) [13,14] provides a natural way to take into account the thermal effects within the canonical quantization approach. Since the usual operator formalism of quantum field theory can be straightforwardly extended to TFD, the method of canonical transformations keeps essentially the same structure as in the zero-temperature formulation [12].

We consider canonically quantized within TFD theories

(1), (2) and formulate the problem as follows

what representation of canonical commutation relations (c.c.r.) is suitable for different values of the parameters G , θ and what physical picture corresponds to this representation?

Within this formulation the notions "a phase" and "a representation of c.c.r." have the same sense. According to this we define the phase transition as a transition from one representation to another.

Our approach consists in the following steps.

1) We construct canonically quantized theory in representation having a suitable physical interpretation for $G \ll 1$, $\theta \ll 1$.

2) We perform canonical transformations of field variables going to field with new mass and vacuum condensation and obtain a new representation of c.c.r.

The canonical transformation should be introduced in such a way that the total Hamiltonian in any representation (r) has the "correct" form. It means that

$$H = H_0^{(r)} + H_I^{(r)} + R^{(r)} + VE^{(r)}$$

Here $H_0^{(r)}$ is the standard free Hamiltonian. The interaction Hamiltonian $H_I^{(r)}$ contains the field operators in a degree more than two. The counter-terms operator $R^{(r)}$ is defined by $H_0^{(r)}$ and $H_I^{(r)}$, and it leads to cancellation of all

divergencies in perturbation theory. The constant $E^{(r)}$ has a sense of a vacuum energy density (here V is a large finite volume).

We should stress that the renormalization scheme as a general prescription has to be the same in all possible representations.

3) We perform classification of c.c.r. and then we choose the phase being suitable for a given values of G, θ . There are two mutually additional principles for this choice. A representation (r) is suitable, if

(α) the vacuum energy density $E^{(r)}$ is smaller in this representation, than in other ones;

(β) a coupling (we may call it "an effective coupling") is weaker in this representation than in other possible representations.

For two-dimensional models (1), (2) an effective coupling in any representation is given by the dimensionless coupling constant

$$G_{\text{eff}}(G) = \frac{g}{2\pi M(G)}, \quad (3)$$

where $M(G)$ is the mass of the field in the free Hamiltonian $H_0^{(r)}$.

One can see that the principle (β) is more preferable because of the following reasons.

First, from a physical point of view a value of vacuum

energy has no any significance because it does not contribute to the S-matrix elements. On the contrary, an effective coupling strength can be measured in experiments and has a direct physical importance.

Second, we are unable to get an exact vacuum energy, hence we are forced to compare the lowest contribution to this energy for different representations. Obviously, it is not a very good way.

Third, it is natural to suppose that the large coupling constant in the Hamiltonian means that the representation of c.c.r. connected with $H_0^{(r)}$ does not describe the real physical states and cannot be considered as a suitable representation for the total Hamiltonian.

Our results concerning the phase structure of the models (1), (2) are the following.

For the model (1) we have found that for any value of θ there is a phase transition of the first order at $G=G_c(\theta)$ accompanied by symmetry breaking.

There are two phase boundaries at (G,θ) -plane in correspondence to two phase transitions of the first order in the system (2). The symmetry is restored in the region between the boundaries and is broken for other values of G,θ . One of the boundaries lies in the domain of applicability of high temperature expansion ($\theta \gg G$) and it is in good quantitative

agreement with existing results [7,10,15]. If the temperature is high enough, the symmetry is restored in both systems (1) and (2) for any fixed value of G .

For arbitrary temperature the regime of strong coupling is absent in both models (1),(2) in the sense that the constant $G_{eff}(G,\theta)$ is small for any G, θ excluding the domain of phase transitions where it is of order $o(1)$. Also the coupling $G_{eff}(G,\theta)$ decreases if G increases.

In other words it means that our approximation is good enough and hence we have an accurate quantum field-theoretical description of the systems, unless the values of G,θ are outside the critical regions. Particularly our result that all phase transitions in our consideration turn out to be of the first order may be qualitatively true but may be not. We have no any reasonable answer.

The next important point consists in that there is difference in low-temperature phase diagrams obtained by the above-mentioned alternative principles (α) and (β).

According to (α) we find the phase boundaries as the points of equality of energy density estimations for the S- and BS-phase. The critical values of G at zero temperature, obtained in this way, coincide with the numerical results of the GEP-approximation [2,4]. It is not surprising, since at zero temperature our approach is equivalent to the GEP-approximation from calculational point of view. It turns out that within this way the phase diagrams at low temperature

correspond to a qualitatively wrong conclusion: a symmetry should be broken when temperature increases.

On the contrary, (β)-principle, i.e. a comparison of the effective coupling constants, leads to a qualitatively right result: a symmetry is restored if the temperature increases. Moreover, at zero temperature a phase transition accompanied by symmetry rearrangement is absent at all in the model (2).

Reliability of the results based on the comparison of the effective coupling constants is corroborated by calculation of perturbative corrections to the vacuum energy estimations.

We want to say in conclusion that the following picture seems to be reliable for QFT. As soon as a coupling becomes strong enough or temperature is high enough, a system cannot exist any more in the state being a ground state in a weak coupling regime. As a result, the system transits to a new phase where an interaction between the new particles which are excitations under the new vacuum turns out to be weak again. Thus, a description of strong coupling regime in QFT is reduced to a construction of such a c.c.r. representation that an interaction is weak in this representation.

1. THERMO FIELD DYNAMICS

It is convenient to deal with a general form of the Lagrangian density

$$L(x,t) = \frac{1}{2} \dot{\phi}(x,t) (\square - m^2) \phi(x,t) - g' \phi^3(x,t) - \frac{1}{4} g \phi^4(x,t) \quad (1.1)$$

Putting $g' = 0$ we get the Lagrangian (1), but if $g' = m\sqrt{g/2}$ then we get the Lagrangian (2), shifted to the minimum of the classical potential (a constant term we will omit).

The detailed description of TFD can be found for example in [13]. We confine ourselves to the brief formulation.

The total Hamiltonian for the system (1.1) has the following form in the TFD-formalism: $H = H_0 + H_I$ where

$$H = H_0 + H_I \quad (1.2)$$

$$H_0 = \frac{1}{2} \int dx: \left[\pi^2(x) + (\nabla\phi(x))^2 + m^2\phi^2(x) \right]: \quad ,$$

$$H_I = \int dx: \left[\frac{g}{4} \phi^4(x) + g' \phi^3(x) \right]: \quad ,$$

$$\phi(x) = \int \frac{dk}{\sqrt{2\pi}} \frac{1}{\sqrt{2\omega(k)}} \left[a(k) e^{ikx} + a^*(k) e^{-ikx} \right] \quad , \quad (1.3)$$

$$\pi(x) = i \int \frac{dk}{\sqrt{4\pi}} \sqrt{\omega(k)} \left[a(k) e^{ikx} - a^*(k) e^{-ikx} \right] \quad , \quad (1.4)$$

$\tilde{H} = H^*[\tilde{\phi}, \tilde{\pi}]$, where

$$\tilde{\phi}(x) = \int \frac{dk}{\sqrt{2\pi}} \frac{1}{\sqrt{2\omega(k)}} \left[\tilde{a}(k)e^{ikx} + \tilde{a}^+(k)e^{-ikx} \right]$$

$$\tilde{\pi}(x) = \frac{1}{i} \int \frac{dk}{\sqrt{4\pi}} \sqrt{\omega(k)} \left[\tilde{a}(k)e^{ikx} - \tilde{a}^+(k)e^{-ikx} \right]$$

$$\omega(k) = \sqrt{k^2 + m^2}, \quad [a(k), a^+(k')] = \delta(k-k') \quad (1.6)$$

$$[\tilde{a}(k), \tilde{a}^+(k')] = \delta(k-k') \quad (1.7)$$

The temperature-dependent operators $\alpha(k, \beta)$, $\tilde{\alpha}(k, \beta)$ are introduced by canonical transformation:

$$\alpha(k, \beta) = a(k) \text{Cosh}(\chi(k, \beta)) - \tilde{a}^+(k) \text{Sinh}(\chi(k, \beta))$$

$$\tilde{\alpha}(k, \beta) = \tilde{a}(k) \text{Cosh}(\chi(k, \beta)) - a^+(k) \text{Sinh}(\chi(k, \beta))$$

with the inverse transformation given by

$$a(k) = \alpha(k, \beta) \text{Cosh}(\chi(k, \beta)) + \tilde{\alpha}^+(k, \beta) \text{Sinh}(\chi(k, \beta)) \quad (1.8)$$

$$\tilde{a}(k) = \tilde{\alpha}(k, \beta) \text{Cosh}(\chi(k, \beta)) + \alpha^+(k, \beta) \text{Sinh}(\chi(k, \beta)) \quad (1.9)$$

This is the Bogolyubov transformation and it can be performed in the operator form

$$\alpha(k, \beta) = U^{-1}(\beta) a(k) U(\beta), \quad \tilde{\alpha}(k, \beta) = U^{-1}(\beta) \tilde{a}(k) U(\beta),$$

where

$$U(\beta) = \exp \left\{ \int dk \chi(k, \beta) \left[\tilde{a}(k) a(k) - a^+(k) \tilde{a}^+(k) \right] \right\} \quad (1.10)$$

A ground state of the field system at temperature $T=1/\beta$ is defined by the relations

$$\alpha(k, \beta) |o(\beta)\rangle = 0,$$

$$\tilde{\alpha}(k, \beta) |o(\beta)\rangle = 0.$$

The parameter $\chi(k, \beta)$ is defined by

$$\text{Sinh}(\chi(k, \beta)) = [\exp(\beta\omega) - 1]^{-1/2} \quad (1.11)$$

The connection between TFD and zero-temperature field theory is given essentially by the relations

$$|o(\beta)\rangle = U^{-1}(\beta) |o\rangle \otimes |o\rangle,$$

$$a(k) = \bar{a}(k) \otimes 1, \quad \tilde{a}(k) = 1 \otimes \bar{a}(k),$$

$$H = \bar{H} \otimes 1, \quad \tilde{H} = 1 \otimes \bar{H}.$$

They reflect the necessity to use the quasiparticle picture for the description of quantum field system at finite temperature. The state $|o\rangle$ and the operators $\bar{a}(k)$, \bar{H} , are the ground state, the annihilation operator and the Hamiltonian at zero temperature.

Due to (1.11) the vacuum expectation value of any observable coincides with its statistical average. So the energy density of a system in the thermodynamic equilibrium is given by

$$E = V^{-1} \langle o(\beta) | H | o(\beta) \rangle.$$

2. CANONICAL TRANSFORMATION

Here we proceed in the following way.

- 1) We construct a new unitary unequivalent representation of the relations (1.6), (1.7) by a canonical transformation of the operators $a(k)$ and $\tilde{a}(k)$.
- 2) We fix the parameters of the canonical transformation by a requirement providing the correct form of a new Hamiltonian.
- 3) We choose a representation, which is suitable for given G, θ according to our criteria $(\alpha), (\beta)$.
- 4) We calculate perturbation corrections to energy density estimations using the Hamiltonians in different representations and reform the phase picture.

As the first step we perform the canonical transformation

$$a(k) \rightarrow a_f(k) - \sqrt{2\pi m} B \delta(k) = U_2^{-1}(f) U_1^{-1}(B) a(k) U_1(B) U_2(f)$$

where

$$U_1(B) = \exp \left\{ -\sqrt{2\pi m} B \int dk \delta(k) [a(k) - a^+(k)] \right\}$$

$$U_2(f) = \exp \left\{ \frac{1}{2} \int dk \lambda(k, f) [a(-k) a(k) - a^+(k) a^+(-k)] \right\}$$

the quantity B is a constant.

The operator U_2 has the same structure as $U(\beta)$ ((1.10)).

It is easy to get the following relations:

$$a(k) = a_f(k) \text{Cosh}(\lambda(k, f)) + a_f^+(-k) \text{Sinh}(\lambda(k, f)) \quad (2.1)$$

$$a^+(k) = a_f^+(k) \text{Cosh}(\lambda(k, f)) + a_f(-k) \text{Sinh}(\lambda(k, f))$$

which are analogous to (1.8), (1.9).

If the parameter λ has the form

$$\lambda(k, f) = \frac{1}{2} \ln \left[\frac{\omega(k)}{\omega_f(k)} \right]$$

$$\omega_f(k) = \sqrt{k^2 + M^2}, \quad M^2 = m^2(1+f)$$

then using the relations (2.1) one can get a new representation for the fields $\phi(x)$ and $\pi(x)$ ((1.3-4))

$$\phi(x) = \int \frac{dk}{\sqrt{2\pi}} \frac{1}{\sqrt{2\omega_f}} \left[a_f(k) e^{ikx} + a_f^+(k) e^{-ikx} \right] \equiv \phi_f(x) \quad (2.2)$$

$$\pi(x) = i \int \frac{dk}{\sqrt{4\pi}} \sqrt{\omega_f} \left[a_f(k) e^{ikx} - a_f^+(k) e^{-ikx} \right] \equiv \pi_f(x)$$

The U_1 -transformation leads to the shifting of the field ϕ at the constant B , thus

$$\phi = \phi_f + B \quad (2.3)$$

Analogous transformation should be performed for $\tilde{a}(k)$ in

order to obtain $\tilde{a}_f(k)$.

The operators a_f, \tilde{a}_f annihilate the state

$$|o\rangle\rangle = U_2^{-1}(f)\tilde{U}_2^{-1}(f)U_1^{-1}(B)\tilde{U}_1^{-1}(B)|o\rangle\rangle|o\rangle.$$

where \tilde{U} -transformations coincide with $U_{1(2)}$ up to the substitution of \tilde{a}, \tilde{a}^* for a, a^* . This equation is a formal relation between the ground states of two unitary nonequivalent representations.

In order to consider the thermal effects we follow the above mentioned TFD-prescriptions and get the relation

$$a_f(k) = \alpha_f(k, \beta) \text{Cosh}(\chi_f(k, \beta)) + \tilde{\alpha}_f^*(k, \beta) \text{Sinh}(\chi_f(k, \beta)) \quad (2.4)$$

where $\text{Sinh}(\chi_f(k, \beta)) = [\exp(\beta\omega_f) - 1]^{-1/2}$.

The operators $\alpha_f, \tilde{\alpha}_f$ annihilate the state

$$|o(\beta)\rangle\rangle = U_f^{-1}(\beta)|o\rangle\rangle,$$

where

$$U_f(\beta) = \exp \left\{ \int dk \chi_f(k, \beta) [\tilde{a}_f(k) a_f(k) - a_f^*(k) \tilde{a}_f^*(k)] \right\}.$$

Using equations (2.2-3) we now express the Hamiltonian (1.2) in terms of ϕ_f, π_f and then taking into account (2.4) go to the normal ordering of the operators $\alpha_f, \tilde{\alpha}_f$. As a result we obtain the expression

$$H = VE + H'_0 + H'_1 + H_1,$$

where

$$H'_0 = \frac{1}{2} \int dx: [\pi_f^2(x) + (\nabla\phi_f(x))^2 + M^2\phi_f^2(x)]:,$$

$$H'_1 = \int dx: \left[\frac{g}{4} \phi_f^4(x) + (gB+g')\phi_f^3(x) \right]:, \quad (2.5)$$

$$H_1 = \int dx: \left[\frac{1}{2} \phi_f^2(x)R(f, B) + \phi_f(x)P(f, B) \right]:,$$

$$R(f, B) = -m^2f + 3gB^2 - 3gD_\theta(f) + 3g'B, \quad (2.6)$$

$$P(f, B) = Bm^2 + gB[B^2 - 3D_\theta(f)] + 3g'[B^2 - D_\theta(f)],$$

$$E = \frac{1}{2} m^2 B^2 + L_\theta(f) + \frac{g}{4} [B^4 - 6B^2 D_\theta(f) + 3D_\theta^2(f)] + g'B[B^2 - 3D_\theta(f)], \quad (2.7)$$

$$L_\theta(f) = \frac{m^2}{8\pi} \left\{ f - 4\pi D_\theta(f) + 4(1+f) \left[2s(\theta/\sqrt{1+f}) + d(\theta/\sqrt{1+f}) \right] \right\},$$

$$D_\theta(f) = \frac{1}{4\pi} \ln(1+f) - \frac{1}{\pi} d(\theta/\sqrt{1+f}),$$

$$d(z) = \int_0^\infty \frac{du}{\sqrt{1+u^2}} \left(\exp\left\{ \frac{1}{z} \sqrt{1+u^2} \right\} - 1 \right)^{-1},$$

$$s(z) = \int_0^{\infty} \frac{duu^2}{\sqrt{1+u^2}} \left(\exp\left\{ \frac{1}{z} \sqrt{1+u^2} \right\} - 1 \right)^{-1},$$

$$d(0) = 0, \quad s(0) = 0.$$

Since the symbol of normal ordering in (2.5) relates to the operators α_f , $\tilde{\alpha}_f$, it is obvious that the quantity E given by (2.7) is an estimation from above on the energy density of the state $|\alpha(\beta)\rangle\rangle$.

Let us put the coefficients R, P equal to zero

$$R(f, B) = 0 \quad (2.8)$$

$$P(f, B) = 0$$

This requirement leads to $H_1=0$ and, hence, provides a correct form of the Hamiltonian, i. e. the free Hamiltonian H'_0 has a standard form, the interaction Hamiltonian H'_1 contains the fields in degrees more than two and renormalization is realized by the normal ordering in H'_0 and H'_1 .

It is easy to check the equivalence of (2.13), (2.14) to equations

$$\frac{\partial E(f, B)}{\partial B} = 0$$

$$\frac{\partial^2 E(f, B)}{\partial B^2} = M^2 = m^2(1+f)$$

which are analogous to the minimum and stability conditions for the effective potential [1].

We will stress that equations (2.8) define the minimum of energy density as a function of two variables f, B only for $\theta=0$. Hence, only at $\theta=0$ our and GEP numerical results should coincide. At finite temperature our technique differs from the variational approach.

3. THE SYMMETRIC MODEL (1)

Putting the constant g' equal to zero and using the equations (2.8) and (2.6) one can get the following equations for f and B:

$$B[gB^2 - 3gD_{\theta}(f) + m^2] = 0 \quad (3.1)$$

$$3gB^2 - 3gD_{\theta}(f) - m^2 f = 0$$

There are two phases in correspondence to two solutions of the system (3.1).

1) The symmetric under $\phi_f \rightarrow -\phi_f$ phase (S).

Putting $B=0$ in the second equation of (3.1) we get

$$\frac{2}{3G} f = - \ln(1+f) + 4d(\theta/\sqrt{1+f}) \quad (3.2)$$

This equation defines the mass M of the field and has a unique solution for any G, θ . Using the relations (2.5), (2.7) and (3.2) we obtain the Hamiltonian and energy density for

this phase

$$H'_0 = \frac{1}{2} \int dx: \left[\pi_f^2(x) + (\nabla \phi_f(x))^2 + M^2 \phi_f^2(x) \right]:$$

$$H'_I = \int dx: \left[\frac{g}{4} \phi_f^4(x) \right]: \quad (3.3)$$

$$E_s = \frac{m^2}{8\pi} \left\{ \left[\frac{2}{3G} + 1 \right] f + \frac{f^2}{3G} + 4(1+f) \left[2s(\theta/\sqrt{1+f}) + d(\theta/\sqrt{1+f}) \right] \right\}$$

2) The phase with broken symmetry.

Using the nonzero solution for B we can rewrite (3.1) in the form

$$B^2 = \frac{1+f}{4\pi G} \quad (3.4)$$

$$\frac{1}{3G} f + \frac{1}{G} = \ln(1+f) - 4d(\theta/\sqrt{1+f})$$

The second equation in (3.4) defines the mass of the field. A numerical analysis shows that this equation has solution only for such θ, G that $G \geq G_c(\theta)$. The function $G_c(\theta)$ is given in Fig.1 by the solid line. Using the equations (2.5), (2.7) and (3.4) we get the Hamiltonian and the vacuum energy in the BS-phase

$$H'_I = \int dx: \left[\frac{g}{4} \phi_f^4(x) \pm gB\phi_f^3 \right]: \quad (3.5)$$

$$E_s = \frac{m^2}{8\pi} \left\{ -\frac{1}{2G} + \left[1 - \frac{1}{3G} \right] f - \frac{f^2}{6G} + 4(1+f) \left[2s(\theta/\sqrt{1+f}) + d(\theta/\sqrt{1+f}) \right] \right\}$$

The free Hamiltonian has a standard form of (3.3). The value of condensate B plays the role of a parameter of order.

At the last step of our consideration we choose a phase, which is suitable for a given θ, G .

Comparing the effective coupling constants $G_{eff}(\theta)$ (3) in the S- and BS-phase we find that the phase-stable boundary is given by the same function $G_c(\theta)$ (solid line in Fig.1). As soon as a solution for the BS-phase exists, the coupling in this phase is weaker than in the symmetric phase (Fig.2).

A comparison of the energy densities E_s and E_b leads to the boundary given by the dashed line AB in Fig.1 at low temperature and coinciding with $G_c(\theta)$ at higher temperature. The section AB represents the points of equality of E_s, E_b . For higher temperature $\theta > \theta_B$ (see fig.1) we are unable to find such points like these since $E_b(G, \theta) < E_s(G, \theta)$ for any $G \geq G_c(\theta)$ (remember that the solutions for the BS-phase are absent for $G < G_c(\theta)$). The value $G_A = 1.625\dots$, corresponding to the point A coincides with the critical coupling constant in the GEP-approximation [1,2] up to the notation since at $\theta=0$, and only in this case, equations (3.1), (3.2) coincide with the equations minimizing the gaussian effective potential, and the energy densities E_s and E_b are equal to GEP in the local minima at the origin and out of it.

Thus, there is a qualitative difference in low temperature diagrams obtained by two alternative principles. Moreover, the section AB in Fig.1 corresponds to a wrong result: a symmetry is broken if the temperature increases. This can be explained by the roughness of our approximation of the energy densities, given by E_s and E_b , in the region of $G \approx 0(1)$. The effective coupling constant is large enough in both phases in this region (see Fig.2), hence, perturbation corrections have to be large and can change the boundary.

Let us calculate these corrections to energy density estimations E_s and E_b at zero temperature, for simplicity. For this purpose we use the Hamiltonians given in (3.3), (3.5). We take into account the corrections up to the order $o(G^3)$ for the phase S and up to $o(G_{eff}^2)$ for the phase BS. The necessary Feynman diagrams are placed in Fig.3,4. The result turns out to be equal

$$\Delta E_s = \frac{m^2}{8\pi} \left[-1.671 \cdot G^2 + 4.0388 \cdot G^3 + o(G^4) \right] \quad (3.6)$$

$$\Delta E_b = \frac{M^2}{8\pi} \left[-1.758 \cdot G_{eff} - 4.316 \cdot G_{eff}^2 - o(G_{eff}^3) \right] \quad (3.7)$$

for S and BS respectively.

One can see that (3.6) represents the alternating asymptotic series. The series for (3.7) is nonalternating.

This is the usual situation for a system with degenerate vacuum (see for example [15]). So we can make a Borel summation of (3.6). It is not so for (3.7) because of singularity on the integration contour. Nevertheless, we make the summation defining the Cauchy mean value of the integral. The result is

$$\Delta E_s = \frac{m^2}{8\pi} \left[\int_0^\infty dt e^{-t} \frac{1 + 0.8057 \cdot Gt}{1 + 0.8057 \cdot Gt + 0.8355 \cdot (Gt)^2} - 1 \right]$$

$$\Delta E_b = - \frac{M^2}{8\pi} \left[\text{V.P.} \int_0^\infty dt e^{-t} \frac{1 + 0.5305 \cdot G_{eff} t}{1 - 1.2275 \cdot G_{eff} t} - 1 \right]$$

The solid and dashed lines in Fig.5 represent the estimations on the energy density without and with the perturbative corrections, respectively. One can see that the corrections "shift" the critical point from $G_A \approx 1.625$ to $G_c(0) \approx 1.44$.

Thus, we conclude that the phase boundary is given by the function $G_c(\theta)$ both at high so and low temperature (solid line in Fig.1).

The order parameter and the mass $M(G, \theta)$ (fig.6) are discontinues at the boundary; hence, the phase transition is of the first order.

Since $G_{eff}(G, \theta)$ is smaller than $G_c(0) = 1.44 \dots$ and decreases when G increases, we conclude that the strong coupled regime does not exist in the model (1) (see Fig. 2)

and our description of the system is good enough outside the critical region.

4. THE TWO-WELL POTENTIAL (2)

Using once more equations (2.6) but with $g' = m\sqrt{g/2}$ we obtain for (2.8) the following form:

$$gB^3 + 3m\sqrt{g/2} \cdot B^2 + B[m^2 - 3gD_\theta(f)] - 3m\sqrt{g/2} \cdot D_\theta(f) = 0 \quad (4.1)$$

$$3gB^2 + 3m\sqrt{2g} \cdot B - 3gD_\theta(f) - m^2f = 0$$

According to the solutions of this system there are two phases with broken symmetry and one being symmetric.

1) S-phase.

The first equation in (4.1) has a solution

$$B = - \frac{1}{\sqrt{4\pi G}} \quad (4.2)$$

Using (4.2) in the second equation of (4.1) we obtain a relation defining the mass of the field in the S-phase

$$\frac{2}{3G} f + \frac{1}{G} = - \ln(1+f) + 4d(\theta/\sqrt{1+f}) \quad (4.3)$$

The unique solution exists for any G, θ . Relations (2.5), (2.7) and (4.2), (4.3) lead to the Hamiltonian in the form

(3.3) and to the following energy density:

$$E_s = \frac{m^2}{8\pi} \left\{ \frac{1}{2G} + \left[\frac{2}{3G} + 1 \right] f + \frac{f^2}{3G} + 4(1+f) \left[2s(\theta/\sqrt{1+f}) + d(\theta/\sqrt{1+f}) \right] \right\}$$

2) BS-phases.

Using the rest solutions of the first equation of (4.1)

$$B = - \frac{1 \pm \sqrt{1+f}}{\sqrt{4\pi G}} \quad (4.4)$$

we get the equation for $M(G, \theta)$

$$\frac{1}{3G} f = \ln(1+f) - 4d(\theta/\sqrt{1+f}) \quad (4.5)$$

This equation has the solutions only for such θ, G that $G \leq G_c^{(1)}(\theta)$ or $G \geq G_c^{(2)}(\theta)$. The functions $G_c^{(1)}$ and $G_c^{(2)}$ are given in Fig. 7 by the solid lines. There are two solutions and they are equal to each other for $G = G_c^{(1)}(\theta)$ or $G_c^{(2)}(\theta)$. These solutions represent two different phases with broken symmetry. Let us demonstrate this for the case $\theta=0$. Since $d(0)=0$, one can see that (4.5) has a trivial solution $f=0$ for any G , leading to the initial representation with the mass m and with the vacuum condensate $\sigma = \pm 1/\sqrt{4\pi G}$. This is the first BS-phase. In order to exclude this trivial solution from (4.5) we rewrite it in the form ($\theta=0$)

$$\frac{1}{3G} = \frac{\ln(1+f)}{f} \quad (4.6)$$

This equation has a unique solution for any G and define the

second BS-phase with the mass and vacuum condensate given by

$$M^2 = m^2(1+f), \quad \sigma = \pm \sqrt{\frac{1+f}{4\pi G}}$$

One can see from (4.6) that

$$M^2 = m^2 \exp\left\{-\frac{1}{3G}\right\} \quad \text{as } G \rightarrow 0.$$

Nonanalyticity of M at the point $G=0$ means that the difference between m and M cannot be obtained within the perturbative calculations and that the first and second solutions of (4.5) represent the two really different phases.

One can see from Fig.7 that $G_c^{(1)}(0)=G_c^{(2)}(0)=G_c$. Using $f=0$ in (4.6) we get the value of $G_c=1/3$. The region at the phase plane (Fig.7) below the boundary $G_c^{(1)}(\theta)$ corresponds to the first BS-phase, but the region above the boundary $G_c^{(2)}(\theta)$ represents the second BS-phase.

The free Hamiltonian has a standard form. The interaction Hamiltonian and the vacuum energy density in the BS-phases take the following form

$$H'_I = \int dx: \left[\frac{g}{4} \phi_f^4(x) \pm g\sigma \phi_f^3 \right]:$$

$$E_B = \frac{m^2}{8\pi} \left\{ \left[1 - \frac{1}{3G} \right] f - \frac{f^2}{6G} + 4(1+f) \left[2s(\theta/\sqrt{1+f}) + d(\theta/\sqrt{1+f}) \right] \right\}.$$

At the last step we choose the phase being suitable for a given θ, G .

The phase-stable boundaries, obtained by comparing the energy densities (sections AB, CD in Fig.7) and by comparing the effective coupling constants (solid lines in Fig.7) differ from each other at low temperature. The reasons for such a difference are the same as in the case of the model (1). That is why we do not discuss them here. We will note only that the values $G_A=.19\dots$ and $G_D=.64\dots$ corresponding to the points A, D in Fig.7, coincide up to the notation with the critical coupling constants, obtained by the GEP-approximation [1] by the reason which has already been pointed out in the previous section.

Let us discuss an improved phase picture, given by the functions $G_c^{(1)}(\theta)$ and $G_c^{(2)}(\theta)$ (solid lines in Fig.7).

One can see that there are two phases with broken symmetry and one being symmetric. At zero temperature the symmetry is broken for any G , but a phase transition without symmetry rearrangement takes place at $G_c=1/3$.

The symmetry is restored for any fixed value of G if temperature is high enough.

The phase transitions are of the first order since the mass and the order parameter σ are discontinuous at the phase boundaries, as in Fig.8 (dashed line for $\theta=1$).

The effective coupling constant $G_{eff}(G, \theta)$ is small unless

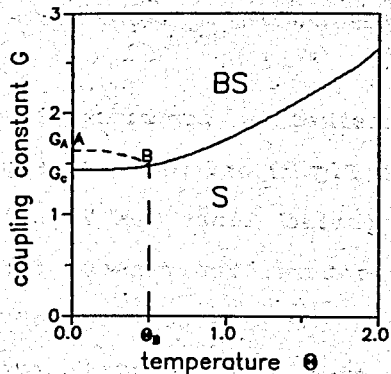


Fig.1
Phase plane for the model (1). Solid line denotes $G_c(\theta)$, dashed line is obtained by comparing the vacuum energies.

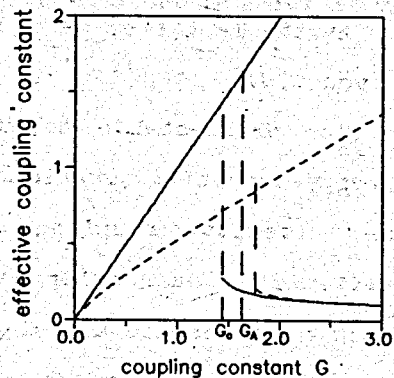


Fig.2
Effective coupling constants for the model (1). Upper lines relate to the S-phase. The case $\theta=1$ is given by dashed line.

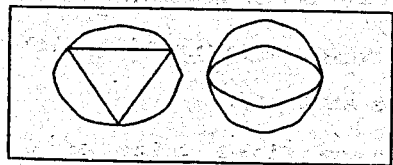


Fig.3

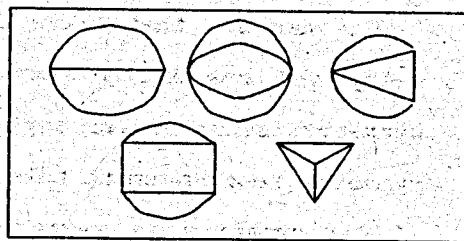


Fig.4

Vacuum diagrams $o(G^2)$, $o(G^3)$ for the S-phase. Vacuum diagrams $o(G_{eff})$, $o(G_{eff}^2)$ for the BS-phase.

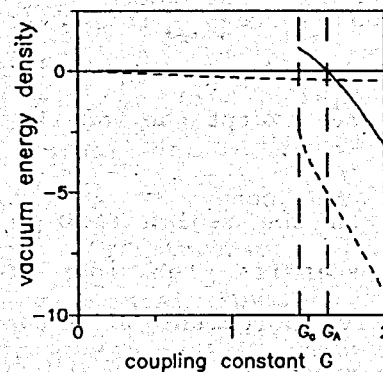


Fig.5
Vacuum energy densities for the model (1). Solid lines correspond to E_S , E_B , dashed lines denote $E_S + \Delta E_S$, $E_B + \Delta E_B$.

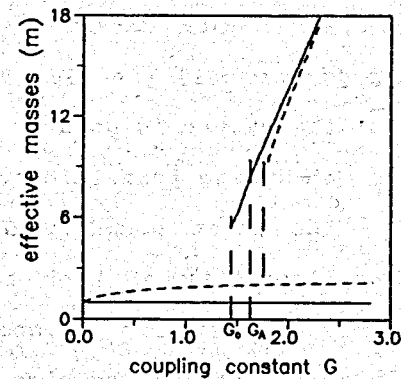


Fig.6
The masses for the model (1). Solid lines correspond to $\theta=0$, dashed lines - $\theta=1$. Upper lines relate to BS-phase.

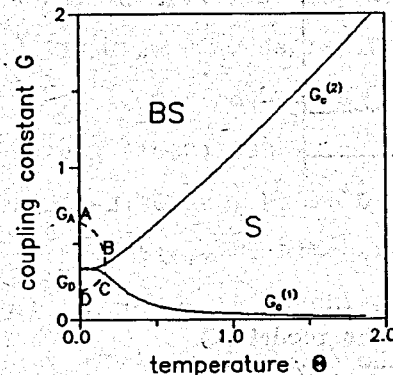


Fig.7
Phase plane for the model (2). The order parameter for the model(2). Dashed lines are obtained by comparing the vacuum energies. Solid line relates to $\theta=0$, dashed lines correspond to $\theta=1$.

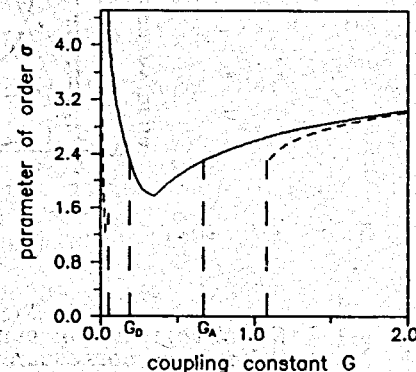


Fig.8

(G, θ) is outside the critical regions where G_{eff} is of an order of $o(1)$ (see Fig.9). This means that our description of the system is good enough everywhere, except the locality of the phase transitions.

The phase boundary $G_c^{(1)}(\theta)$ lies in the region $\theta \gg G$ of applicability of high temperature expansion. Behaviour of this boundary is in good quantitative agreement with the results obtained by high temperature expansion [7,10,16].

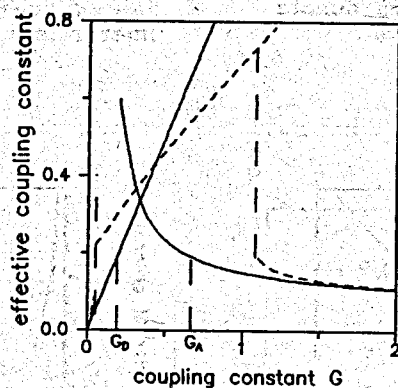


Fig.9
Effective coupling constants for the model (2). Solid lines correspond to $\theta = 0$, the case $\theta = 1$ is given by dashed lines. Upper dashed line relates to the S-phase.

REFERENCIES

1. A. D. Linde, Rep. Prog. in Phys. v42 (1979) 389; S. Coleman, E. Weinberg, Phys. Rev D7 (1973) 1888
2. P. M. Stevenson, Phys. Rev. D32 (1985) 1389
3. P. M. Stevenson, I. Roditi, Phys. Rev. D33 (1986) 2305; I. Roditi, Phys. Lett. B169 (1986) 264; B. Alles, R. Tarrach, Phys. Rev. D33 (1986) 1718
4. S.-J. Chang, Phys. Rev. D12 (1975) 1071
5. B. Simon, R. Griffiths, Comm. in Math. Phys., v33 (1973) 145; J. Glimm, A. Jaffe, Quantum Physics. A Functional Integral Point Of View (Springer-Verlag, New York, Heidelberg, Berlin, 1981).
6. O. Mc Bryan, J. Rosen, Comm. in Math. Phys. v51 (1976) 97; J. Glimm, A. Jaffe, Phys. Rev. D10 (1975) 536
7. V. A. Ossipov, V. K. Fedyanin, sov. j. Theor. Math. Phys. v73 (1987) 393
8. S.-J. Chang, Phys. Rev. D13 (1976) 2778
9. L. Polley, U. Ritchel, Phys. Lett. B221 (1989) 44
10. L. Dolan, R. Jackiv, Phys. Rev. D9 (1974) 3320
11. Proceedings of the International Workshop "Variational Calculations in Quantum Field Theory", eds L. Polley and D. Pottinger, World Scientific, Singapore (1988); R. Minoz-Tapia, J. Taron, R. Tarrach, Int. J. Mod. Phys.

A3 (1988) 2143

12. G. V. Efimov, Int. J. Mod. Phys. A4 (1989) 4977
13. H. Umezawa, H. Matsumoto, M. Tachiki, Thermo field dynamics and condensed states (North-Holland, Amsterdam, 1982);
I. Ojima, Ann. Phys. (NY) 137 (1981) 1;
N. P. Landsman, Ch. G. van Weert, Phys. Rep. 147 (1987) 141;
H. Matsumoto et al, Phys. Rev. D29 (1984) 2838
14. I. Roditi, Phys. Lett. B177 (1986) 85
15. D. I. Kazakov, Preprint JINR, E2-80-62, Dubna, 1980
16. R. Su, P. Bi, G. Ni, J. Phys. A16 (1983) 2445

Received by Publishing Department
on October 25, 1991.