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SEMILEPTONIC DECAYS OF D- AND B-MESONS
IN THE RELATIVISTIC QUARK MODEL

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1. Introduction

The semileptonic decays of mesons are a source for the determination of the Kobayashi-Maskawa (KM) matrix elements from the comparison of theoretical description and the experimental data. The main theoretical uncertainties in understanding the semileptonic decays arise from the matrix element of the weak current between meson states. Many theoretical models have been used to calculate this matrix element and the semileptonic decay widths [1-9]. Recently the new detailed experimental data on the width and form factors of the decay $D \rightarrow K^* \ell \nu_\ell$ was presented by the E691 collaboration [10]. This data contradicts almost all the predictions of the theoretical models [1-9]. One of the origins of the discrepancy may be in the neglect of the relativistic effects.

In this paper we calculate the semileptonic decay widths and form factors in the framework of the relativistic quark model based on the quasipotential method. This model has been used for the calculations of the quarkonium mass spectra [11], radiative decay widths [12] and pseudoscalar decay constants [13].

2. Semileptonic decays of mesons

The transition amplitude for the exclusive semileptonic decay $B \rightarrow A \ell \nu_\ell$ is

$$A(B \rightarrow A \ell \nu_\ell) = \langle A \ell \nu_\ell | H_{\text{eff}} | B \rangle = (G_F / \sqrt{2}) V_{ab} L_\mu^\mu H^\mu; \quad (1)$$

$$\text{where } H_{\text{eff}} = (G_F / \sqrt{2}) J_{\text{hadron}}^\mu J_{\text{lepton}, \mu}^\mu; \quad (2)$$

$$L^\mu = \ell \gamma^\mu (1 - \gamma_5) \nu_\ell \quad (3)$$

$$H^\mu = \langle A | \bar{a} \gamma^\mu (1 - \gamma_5) b | B \rangle ; \quad (4)$$

V_{ab} is the corresponding KM-matrix element, B is the initial pseudoscalar meson, A is the final pseudoscalar (vector) meson.

The hadronic matrix element (4) is usually decomposed in invariant form factors [3,4,5] defined by :

a) for the decay to the pseudoscalar meson $B \rightarrow A \ell \nu_\ell$

$$\langle A(p_A) | J_\mu^V | B(p_B) \rangle = f_+(q^2) (p_A + p_B)_\mu + f_-(q^2) (p_B - p_A)_\mu ; \quad (5)$$

b) for the decays to the vector meson $B \rightarrow A^* \ell \nu_\ell$

$$\langle A^*(p_A, e) | J_\mu^V | B(p_B) \rangle = i g(q^2) \epsilon_{\mu\nu\rho\sigma} e^{*\nu} (p_B + p_A)^\rho (p_B - p_A)^\sigma ; \quad (6)$$

$$\langle A^*(p_A, e) | J_\mu^A | B(p_B) \rangle = f(q^2) e_\mu^* + a_+(q^2) (e^* p_B) (p_A + p_B)_\mu + a_-(q^2) (e^* p_B) (p_B - p_A)_\mu ; \quad (7)$$

where $q = p_B - p_A$; $J_\mu^V = (\bar{a} \gamma^\mu b)$ and $J_\mu^A = (\bar{a} \gamma^\mu \gamma_5 b)$ are the vector and axial parts of the weak quark current, e_μ is the polarization vector of the vector meson A^* .

Since $q = p_\ell + p_{\nu_\ell}$, the terms proportional to q_μ , i.e. f_- and a_- give contributions proportional to the lepton masses and do not influence significantly the transition amplitude, except in the case of the heavy τ lepton.

The decay rate and differential decay rate can be calculated in terms of the invariant form factors. The formulas are [3]:

$$\frac{d\Gamma}{dy} = \frac{G_F^2 |V_{ab}|^2 K M_B^2 y}{96\pi^3} \left(|H_+|^2 + |H_-|^2 + |H_0|^2 \right) ; \quad (8)$$

where $y = q^2/M_B^2$; $K = \frac{M_B}{2} \left[\left(1 - \frac{M_A}{M_B} - y \right)^2 - 4 \frac{M_A}{M_B} y \right]^{1/2}$

For the decay to the pseudoscalar meson $B \rightarrow A \ell \nu_\ell$

$$H_+ = 0; \quad H_0 = -2Ky^{-1/2} f_+(y)$$

and for the decay to the vector meson $B \rightarrow A^* \ell \nu_\ell$

$$H_0 = \frac{M_B}{2M_A \sqrt{y}} \left[\left(1 - \frac{M_A}{M_B} - y \right) f(y) + 4K^2 a_+(y) \right] \quad (10)$$

$$H_+ = f(y) + 2M_B K g(y).$$

The ratio of the longitudinal and transverse decay widths is

$$\Gamma_L/\Gamma_T = \frac{\int dy |H_0(y)|^2 Ky}{\int dy (|H_+(y)|^2 + |H_-(y)|^2) Ky} \quad (11)$$

3. Relativistic corrections to the form factors in semileptonic decays

In the quasipotential method the bound system is described by the wave function $\Psi_M(p)$ satisfying the quasipotential equation [14], which can be written in the local Schrödinger-like form [15]:

$$\left(\frac{b^2(M)}{2\mu_R} - \frac{p^2}{2\mu_R} \right) \Psi_M(p) = \int \frac{d^3q}{(2\pi)^3} V(p, q; M) \Psi_M(q) ; \quad (12)$$

where the relativistic reduced mass is

$$\mu_R = \frac{E_1 E_2}{E_1 + E_2} = (1/4M^3) [M^4 - (m_1^2 - m_2^2)^2] ; \quad (13)$$

$$E_1 = \frac{M^2 - m_2^2 + m_1^2}{2M} ; \quad E_2 = \frac{M^2 - m_1^2 + m_2^2}{2M} ; \quad E_1 + E_2 = M ;$$

the square of the relative momentum p on the mass-shell is

equal to

$$b^2(M) = [M^2 - (m_1 + m_2)^2] [M^2 - (m_1 - m_2)^2] / 4M^2 \quad (14)$$

m_1, m_2 are the quark masses; M is the meson mass.

To construct the kernel of this equation $V(p, q, M)$ the quasipotential we assume that effective quark-antiquark interaction is the mixture of the single gluon exchange with the long-range vector and scalar linear confining potentials. We

also assume that quarks have an anomalous chromomagnetic moments κ .

Then the quasipotential is

$$V(p, q; M) = \bar{u}_1(p) \bar{u}_2(-p) \left\{ \frac{4}{3} \alpha_s D_{\mu\nu}(k) \gamma_1^\mu \gamma_2^\nu + v_{\text{conf}}^V(k) \Gamma_{12; \mu}^V + v_{\text{conf}}^S(k) \right\} u_1(q) u_2(-q), \quad (15)$$

where $k=p-q$; $D_{\mu\nu}(k)$ is the gluon propagator, $u_{1,2}(p)$ are the Dirac spinors; the effective long-range vector vertex is

$$\Gamma_\mu(k) = \gamma_\mu + \frac{i\kappa}{2m} \sigma_{\mu\nu} k^\nu$$

The vector and scalar confining potentials in configuration space are

$$\begin{aligned} v_{\text{conf}}^V(r) &= (1-\epsilon)(Ar+B) \\ v_{\text{conf}}^S(r) &= \epsilon(Ar+B) \end{aligned} \quad (16)$$

The explicit expression for the quasipotential with the account of the relativistic corrections of order v^2/c^2 can be found in [11]. The method of the numerical solution of (12) was described in [16].

The matrix element of the local current J between bound states [17,18] has the form

$$\langle A | J_\mu(0) | B \rangle = \int \frac{dp dq}{(2\pi)^6} \bar{\Psi}_A(p) \Gamma_\mu(p, q) \Psi_B(q), \quad (17)$$

where $\Gamma_\mu(p, q)$ is the two-particle vertex function.

In our case $J = J^A + J^V$ is the weak quark current and to calculate its matrix element between meson states it is necessary to consider the contributions to Γ from the diagrams on figs. 1, 2. The vertex functions obtained from these diagrams are

$$\Gamma_\mu^{(1)}(p, q) = \bar{u}_a(p_1) \gamma^\mu (1-\gamma_5) u_b(q_1) (2\pi)^3 \delta(p_2 - q_2) \quad (18)$$

and

$$\begin{aligned} \Gamma_\mu^{(2)}(p, q) &= \bar{u}_a(p_1) \bar{u}_q(p_2) \left\{ \gamma_{1\mu} (1-\gamma_5) \frac{\Lambda_b^{(-)}(k)}{\epsilon_b(k) + \epsilon_b(p_1)} \gamma_1^0 V(p_2 - q_2) + \right. \\ &+ \left. V(p_2 - q_2) \frac{\Lambda_a^{(-)}(k')}{\epsilon_a(k) + \epsilon_a(q_1)} \gamma_1^0 \gamma_{1\mu} (1-\gamma_5) \right\} u_b(q_1) u_q(q_2) \end{aligned} \quad (19)$$

where $k = p_1 - \Delta$; $k' = q_1 + \Delta$; $\Delta = p_B - p_A$; $\epsilon(p) = (m^2 + p^2)^{1/2}$;

$$\Lambda^{(-)}(p) = \frac{\epsilon(p) - (m\gamma^0 + \vec{\gamma}\vec{p})}{2\epsilon(p)}$$

The form of the relativistic corrections resulting from the vertex functions (19) is explicitly dependent on the Lorentz-structure of $q\bar{q}$ -interaction. Our previous analysis of the radiative decays [12] and mass spectrum [11] of quarkonia showed that in the $q\bar{q}$ -potential (16) the mixing parameter $\epsilon = -0.9$.

After substituting (18) and (19) in the matrix element (17) we make the expansion up to order v^2/c^2 with respect to the velocity of heavy quarks (b, a) and do not expand with respect to the velocity of a light quark (q). The final formulas for the form factors look like

1) for the decay $B \rightarrow A l \nu_l$

$$f_+(Y) = f_+(Y_{\text{max}}) I(Y) \mathcal{F}_1(Y) \quad (20)$$

$$\begin{aligned} f_+(Y_{\text{max}})^{(1)} &= \sqrt{M_A/M_B} \int \frac{d^3p}{(2\pi)^3} \bar{\Psi}_A(p) \left\{ 1 + \frac{M_B - M_A}{2m_a} \frac{p^2}{8} \left(\left(\frac{m_b - m_a}{m_b m_a} \right)^2 + \right. \right. \\ &+ \left. \frac{M_B - M_A}{2m_a} \left(5/m_a^2 + 1/m_b^2 + 2/3m_a m_b \right) + \frac{2}{3} \frac{M_B - M_A}{M_A} \left(\frac{m_b - m_a}{m_b m_a} \right) \left(\frac{\epsilon_q(p) - m_a}{\epsilon_q(p) m_a} \right) + \right. \\ &+ \left. \frac{M_B - M_A}{6} \left(\frac{1}{\mu} - \frac{p^2}{8} \left(1/m_a^2 \left(3/m_a + 1/m_b \right) + 1/m_b^2 \left(3/m_b + 1/m_a \right) \right) \right) \right\} * \\ &* \frac{\epsilon_q(p)}{M_A} (\not{p} \partial / \partial p) \left. \right\} \Psi_B(p) \end{aligned} \quad (21)$$

$$\begin{aligned}
(2) \quad f_{+S}(y_{\max}) = & \sqrt{M_A/M_B} \int \frac{d^3 p}{(2\pi)^3} \bar{\Psi}_A(p) \left\{ -\frac{M_B - M_A}{2m_a} \left(\frac{M_B - \epsilon_b(p) - \epsilon_q(p)}{m_a} \right) + \right. \\
& + \frac{M_B - M_A}{12m_a} p^2 \left(1/m_a^2 + 1/m_b^2 \right) - \frac{M_B - M_A}{12} \left(1/m_a^2 + 1/m_b^2 \right) \left(M_B + M_A - \epsilon_b(p) - \epsilon_a(p) - \right. \\
& \left. \left. - 2\epsilon_q(p) \right) \frac{\epsilon_q(p)}{M_A} (p\partial/\partial p) \right\} \Psi_B(p) \quad (22)
\end{aligned}$$

$$\begin{aligned}
(2) \quad f_{+V}(y_{\max}) = & \sqrt{M_A/M_B} \int \frac{d^3 p}{(2\pi)^3} \bar{\Psi}_A(p) \left\{ \frac{M_B - M_A}{m_a} \frac{p^2}{12\mu} \left((1+\kappa)(1/m_a - 1/m_b) - \right. \right. \\
& \left. \left. - \frac{2}{\epsilon_q(p) + m_q} \right) - \frac{M_B - M_A}{12} (1+\kappa)(1/m_a^2 - 1/m_b^2)(M_B - M_A - \epsilon_b(p) + \right. \\
& \left. + \epsilon_a(p)) \frac{\epsilon_q(p)}{M_A} (p\partial/\partial p) + \frac{M_B - M_A}{6(\epsilon_q(p) + m_q)\mu} \left(M_B + M_A - \epsilon_b(p) - \epsilon_a(p) - \right. \right. \\
& \left. \left. - 2\epsilon_q(p) \right) \frac{\epsilon_q(p)}{M_A} (p\partial/\partial p) \right\} \Psi_B(p) \quad (23)
\end{aligned}$$

2) for the decay $B \rightarrow A^* \ell \nu_\ell$

$$g(y) = g(y_{\max}) I(y) \mathcal{F}_1(y) \quad (24)$$

$$\begin{aligned}
(1) \quad g(y_{\max}) = & \sqrt{M_A/M_B} \int \frac{d^3 p}{(2\pi)^3} \bar{\Psi}_A(p) \left\{ \frac{1}{2m_a} \left(1 - \frac{p^2}{8} \left(5/m_a^2 + 1/m_b^2 - 2/3m_a m_b \right) - \right. \right. \\
& \left. \left. - \frac{p^2(m_a + \epsilon_q(p))}{12\mu M_A \epsilon_q(p)} - \frac{p^2}{12M_A} \left(\frac{m_b - m_a}{m_b m_a} \right) + \frac{m_a}{3} \left(\frac{m_b - m_a}{m_b m_a} - \right. \right. \right. \\
& \left. \left. - \frac{p^2}{8} \left(1/m_a^2 \left(3/m_a - 1/m_b \right) - 1/m_b^2 \left(3/m_b - \right. \right. \right. \right. \\
& \left. \left. \left. - 1/m_a \right) \right) \right) \frac{\epsilon_q(p)}{M_A} (p\partial/\partial p) \right\} \Psi_B(p) \quad (25)
\end{aligned}$$

$$(2) \quad g_S(y_{\max}) = \sqrt{M_A/M_B} \int \frac{d^3 p}{(2\pi)^3} \bar{\Psi}_A(p) \left\{ \frac{1}{2m_a} \left(-\frac{M_B - \epsilon_b(p) - \epsilon_q(p)}{m_a} + \right. \right.$$

$$\begin{aligned}
& \left. + \frac{p^2}{6} \left(1/m_a^2 - 1/m_b^2 \right) - \frac{m_a}{6} \left(1/m_a^2 - 1/m_b^2 \right) \left(M_B + M_A - \epsilon_b(p) - \epsilon_a(p) - \right. \right. \\
& \left. \left. - 2\epsilon_q(p) \right) \frac{\epsilon_q(p)}{M_A} (p\partial/\partial p) \right\} \Psi_B(p) \quad (26)
\end{aligned}$$

$$\begin{aligned}
(2) \quad g_V(y_{\max}) = & \sqrt{M_A/M_B} \int \frac{d^3 p}{(2\pi)^3} \bar{\Psi}_A(p) \left\{ \frac{1}{2m_a} \left(\frac{p^2}{6} (1+\kappa) \left(1/m_a^2 + 1/m_b^2 - \right. \right. \right. \\
& \left. \left. - \frac{1}{\epsilon_q(p)\mu} \right) + \frac{m_a}{6} (M_B - M_A - \epsilon_b(p) + \epsilon_a(p)) (1+\kappa) \left(1/m_a^2 + 1/m_b^2 - \frac{1}{\epsilon_q(p)\mu} \right) * \right. \\
& \left. * \frac{\epsilon_q(p)}{M_A} (p\partial/\partial p) - \frac{p^2}{6\epsilon_q(p)} \left(\frac{m_b - m_a}{m_b} \right) + \frac{m_a}{6} \left(M_B + M_A - \epsilon_b(p) - \epsilon_a(p) - \right. \right. \\
& \left. \left. - 2\epsilon_q(p) \right) \frac{2\epsilon_q(p)}{(\epsilon_q(p) + m_q) M_A} \left(\frac{m_b - m_a}{m_b m_a} \right) (p\partial/\partial p) \right\} \Psi_B(p) \quad (27)
\end{aligned}$$

$$f(y) = f(y_{\max}) I(y) \mathcal{F}_2(y) \quad (28)$$

$$f(y_{\max}) = \sqrt{4M_A M_B} \int \frac{d^3 p}{(2\pi)^3} \bar{\Psi}_A(p) \left\{ 1 - \frac{p^2}{8\mu^2} + \frac{p^2}{6m_b m_a} \right\} \Psi_B(p) \quad (29)$$

$$a_+(y) = a_+(y_{\max}) I(y) \mathcal{F}_1(y) \quad (30)$$

$$\begin{aligned}
(1) \quad a_+(y_{\max}) = & \frac{1}{\sqrt{4M_A M_B}} \int \frac{d^3 p}{(2\pi)^3} \bar{\Psi}_A(p) \left\{ \left(1 + \frac{M_A}{M_B} \right) \left(1 - \frac{p^2}{2} \left(\frac{1}{4\mu^2} - \frac{1}{3m_b m_a} \right) \right) - \right. \\
& \left. - \frac{M_A}{m_a M_B} \left(1 - \frac{p^2}{8} \left(5/m_a^2 + 1/m_b^2 + 2/3m_a m_b \right) - \frac{p^2}{6M_A \mu} \right) - \frac{M_A \epsilon_q(p)}{\mu M_B} \left(1/3 \left(1 - \right. \right. \right. \\
& \left. \left. - \mu \frac{p^2}{8} \left(1/m_a^2 \left(3/m_a + 1/m_b \right) + 1/m_b^2 \left(3/m_b + 1/m_a \right) \right) \right) + \frac{M_B - M_A}{3(m_b + m_a)} \left(1 - \right. \right. \\
& \left. \left. - \frac{3p^2}{8} \left(1/m_a^2 + 1/m_b^2 \right) \right) \right) (p\partial/\partial p) \right\} \Psi_B(p) \quad (31)
\end{aligned}$$

$$(2) \quad a_{+S}(y_{\max}) = \frac{1}{\sqrt{4M_A M_B}} \int \frac{d^3 p}{(2\pi)^3} \bar{\Psi}_A(p) \left\{ \frac{M_A}{m_a M_B} \left(-\frac{M_B - \epsilon_b(p) - \epsilon_q(p)}{m_a} + \right. \right.$$

$$\begin{aligned}
& + \frac{p^2}{6} \left(1/m_a^2 + 1/m_b^2 \right) - \frac{m_a}{6} \left(M_B + M_A - \epsilon_b(p) - \epsilon_a(p) - 2\epsilon_q(p) \right) \left(1/m_a^2 + \right. \\
& \left. + 1/m_b^2 \right) \frac{\epsilon_q(p)}{M_A} \left(p\partial/\partial p \right) \left. \right\} \Psi_B(p) \quad (32)
\end{aligned}$$

$$\begin{aligned}
(2) \\
a_{+v}(y_{\max}) = & \frac{1}{\sqrt{4M_A M_B}} \int \frac{d^3 p}{(2\pi)^3} \bar{\Psi}_A(p) \left\{ \frac{M_A}{m_a M_B} \left(\frac{p^2}{6} (1+\kappa) \left(1/m_a^2 - 1/m_b^2 \right) + \right. \right. \\
& + \frac{m_a}{6} (M_B - M_A - \epsilon_b(p) + \epsilon_a(p)) (1+\kappa) \left(1/m_a^2 - 1/m_b^2 \right) \frac{\epsilon_q(p)}{M_A} \left(p\partial/\partial p \right) - \\
& - \frac{p^2}{3(\epsilon_q(p) + m_q)\mu} + \frac{m_a}{6\mu} \left(M_B + M_A - \epsilon_b(p) - \epsilon_a(p) - \right. \\
& \left. \left. - 2\epsilon_q(p) \right) \frac{-2\epsilon_q(p)}{(\epsilon_q(p) + m_q)M_A} \left(p\partial/\partial p \right) \right\} \Psi_B(p) \quad (33)
\end{aligned}$$

where indexes (1) and (2) correspond to the diagrams in figs. 1 and 2, s and v - to the scalar and vector potentials of $q\bar{q}$ -interaction; $\mu = m_b m_a / (m_b + m_a)$; $y_{\max} = ((M_B - M_A)/M_B)^2$; $(p\partial/\partial p)$ acts on the wave function $\bar{\Psi}_A(p)$. The dependence of the form factors on the momentum transfer was found to be

$$I(y) = \int \frac{d^3 p}{(2\pi)^3} \bar{\Psi}_A(p + \frac{m_q}{M_A} \Delta) \Psi_B(p) \quad (34)$$

where $\Delta^2 = (p_B - p_A)^2 = (M_B^2(1-y) + M_A^2)^2 / 4M_B^2 - M_A^2$
and

$$\mathfrak{F}_1(y) = \frac{2\sqrt{2} M_A}{M_B [(y - \tilde{y} - M_A/M_B)^2 - (M_A/M_B)^2]^{1/2}} \quad (35)$$

$$\mathfrak{F}_2(y) = 2^{-1/2} \left[1 + \frac{2M_A}{M_B(y - \tilde{y})} \right]^{1/2} \quad (36)$$

where $\tilde{y} = (M_B^2 + M_A^2)/M_B^2$.

The functions \mathfrak{F}_1 and \mathfrak{F}_2 emerge from the lower and upper.

Table 1. Comparison of experimental data on $D \rightarrow K^* e \nu_e$ semileptonic decay form factors with theoretical predictions.

Form factor	Experiment E691	Our results	IS [1]	BW [2]	GS [3]	KS [4]
$A_1(0)$	$0.46^{+0.05}_{-0.05}$	0.43	0.8	0.9	0.8	1.0
$A_2(0)$	$0.0^{+0.2}_{-0.1}$	0.29	0.8	1.2	0.6	1.0
$V(0)$	$0.9^{+0.3}_{-0.1}$	0.50	1.1	1.3	1.5	1.0

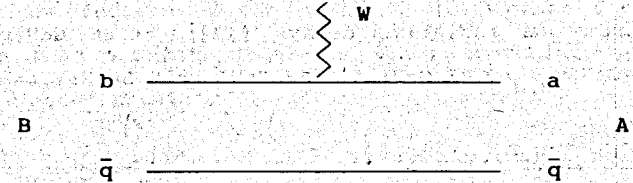


Fig.1 The lowest order vertex function.

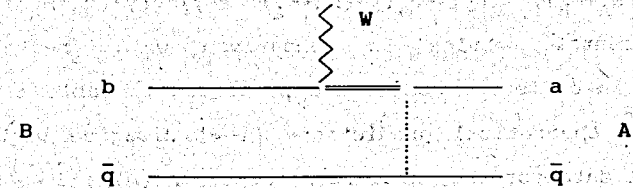


Fig.2 The vertex function with the account of the quark interaction. Dashed line corresponds to the effective potential (15). Bold line denotes the negative-energy part of quark propagator [18].

components of Dirac spinors u_a in eq.(18) respectively. We also replaced the heavy quark mass m_a by the mass of the corresponding meson M_A for simplicity.

In the limit $v^2/c^2 \rightarrow 0$ the form factors (20)-(33) reduce to the standard expressions, obtained in the nonrelativistic quarks models [3,5,6].

4. Results and discussion

In this section we present the results of the numerical calculations of the form factors and semileptonic decay widths of D- and B-mesons. The quark masses and parameters of the potential were determined earlier from the analysis of meson mass spectrum and radiative decays [12]: $m_D = 4.88$ GeV, $m_C = 1.55$ GeV, $m_S = 0.5$ GeV, $m_{u,d} = 0.33$ GeV; $A = 0.18$ GeV², $B = -0.30$ GeV, $c = -0.9$, $\kappa = -1$.

The most interesting is the decay $D \rightarrow K^* e \nu_e$ because recently the E691 collaboration [10] determined the decay form factors from the analysis of the angular correlation structure in this process. Their definition of form factors slightly differs from ours. The connection between them is the following:

$$A_1(y) = f(y)/(M_A + M_B); \quad A_2(y) = -a_+(y)(M_B + M_A); \quad V(y) = g(y)(M_B + M_A).$$

The experimental results in comparison with theoretical predictions in different models are presented in Table 1. While the previous theoretical predictions [1-4] disagree with the experimental data for the axial-form factors $A_1(0)$ and $A_2(0)$, we get the results in accord with these data.

For the decay width we obtain

$$\Gamma(D \rightarrow K^* e \nu_e) \approx 4.3 \cdot 10^{10} \text{ s}^{-1}$$

to be compared with the experimental data

$$\begin{aligned} \Gamma_{\text{exp}}(D \rightarrow K^* e \nu_e) &= (4.2 \pm 0.7 \pm 0.5) \cdot 10^{10} \text{ s}^{-1} \quad (\text{E691 [10]}) \\ &= (3.6 \pm 0.8) \cdot 10^{10} \text{ s}^{-1} \quad (\text{PDG [19]}). \end{aligned}$$

The predicted ratio of the longitudinal and transverse decay

$$\text{widths is } \Gamma_L / \Gamma_T \approx 1.5$$

while experimentally [10]

$$\Gamma_L / \Gamma_T = 1.8_{-0.4}^{+0.6} \pm 0.3 \quad (\text{E691}).$$

The decay width for $D \rightarrow K e \nu_e$ is predicted to be

$$\Gamma(D \rightarrow K e \nu_e) \approx 9.1 \cdot 10^{10} \text{ s}^{-1}$$

and the experimental data are

$$\begin{aligned} \Gamma_{\text{exp}}(D \rightarrow K e \nu_e) &= (8.8 \pm 1.2 \pm 1.4) \cdot 10^{10} \text{ s}^{-1} \quad (\text{E691 [20]}) \\ &= (7.8 \pm 1.2 \pm 0.9) \cdot 10^{10} \text{ s}^{-1} \quad (\text{MARK [21]}) \\ &= (8.1 \pm 1.2) \cdot 10^{10} \text{ s}^{-1} \quad (\text{PDG [19]}). \end{aligned}$$

The form factors of the semileptonic B-meson decays have not been measured yet. Only the decay branching ratios are known. Our model predicts

$$\Gamma(B \rightarrow D e \nu_e) \approx 9.0 \cdot 10^{12} |V_{bc}|^2 \text{ s}^{-1} \quad \text{and} \quad B(B \rightarrow D e \nu_e) \approx 10.6 |V_{bc}|^2.$$

It should be compared with the experimental data

$$\begin{aligned} B(B \rightarrow D e \nu_e) &= \left. \begin{aligned} (1.7 \pm 0.6 \pm 0.4)\% & \quad (\text{ARGUS [22]}) \\ (1.6 \pm 0.6 \pm 0.4)\% D^0 & \\ (1.8 \pm 0.6 \pm 0.3)\% D^+ & \end{aligned} \right\} \quad (\text{CLEO [23]}) \\ &= (1.8 \pm 0.8)\% \quad (\text{PDG [19]}) \end{aligned}$$

The prediction for the decay $B \rightarrow D^* e \nu_e$ is

$$\Gamma(B \rightarrow D^* e \nu_e) \approx 2.3 \cdot 10^{13} |V_{bc}|^2 \text{ s}^{-1} \quad \text{and} \quad B(B \rightarrow D^* e \nu_e) \approx 27.1 |V_{bc}|^2$$

and

$$\Gamma_L / \Gamma_T \approx 1.2$$

The experimental data are

$$\begin{aligned} B(B \rightarrow D^* e \nu_e) &= \left. \begin{aligned} (5.4 \pm 0.9 \pm 1.3)\% & \quad (\text{ARGUS [24]}) \\ (4.1 \pm 0.8 \pm 0.9)\% D^{*0} & \\ (4.6 \pm 0.5 \pm 0.7)\% D^{*+} & \\ (7.0 \pm 1.8 \pm 1.4)\% & \quad (\text{Crystal Ball [25]}) \end{aligned} \right\} \quad (\text{CLEO [23]}) \end{aligned}$$

$$\text{and } \Gamma_L / \Gamma_T = \begin{aligned} &0.83 \pm 0.33 \pm 0.13 \quad (\text{CLEO}) \\ &0.85 \pm 0.45 \quad (\text{ARGUS}) \end{aligned}$$

So we can extract the value of KM matrix element:

$$|V_{cb}| = 0.041 \pm 0.006.$$

Our analysis have shown that relativistic effects play a significant role in semileptonic decays of mesons. It was found that taking into account relativistic corrections to the form factors of semileptonic decays it is possible to get a good description of all available experimental data on $D \rightarrow K(K^*) \ell \nu_\ell$ and $B \rightarrow D(D^*) \ell \nu_\ell$ decays.

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