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PHASE STRUCTURE OF THREE- AND FOUR-DIMENSIONAL ϕ^4 FIELD THEORY

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1. INTRODUCTION

In this paper we will investigate vacuum structure of the field model with the following classical Lagrangian density

$$L(x) = \frac{1}{2}\,\phi(x)(\Box - m^2)\phi(x) - \frac{1}{4}\,g\,\phi^4(x) \qquad (1.1)$$

in space-time R^d for d=3 and 4. Here x=(**x**,t). The constants m^2 and g are positive. The dimensionless coupling constant

$$G = \frac{g}{(2\pi m)^{4-d}}$$

is a parameter of the theory.

In more than four dimensions there are rigorous proofs of triviality of ϕ^4 theory- either non-interacting or else inconsistent[1,2]. But in exactly four dimensions the proofs are incomplete[2]. More physical although aproximate approach to the problem is provided by variational estimations of an effective potential[3-8]. Unfortunately, usefullness of a variational approach in QFT is restricted by some problems (see Feunman's paper in Ref.3). The Hamiltonian of the model (1.1) is not a well-defined operator in the Hilbert space for d>2 because of the highest order ultraviolet divergences. Hence, a procedure of variational estimation with the help of the trial wave functionals is not defined either[9]. Another problem arises from impossibility of controlling the approximation accuracy directly within the variational calculations[10].

We follow the way which consists in appreciation of perturbative QFT, particularly, its renormalization (R) structure, as a kind of initial condition for strong coupling problem[11-13]. Here we continue the investigations, undertaken in Refs.12,13, where the phase structure of various scalar field models in R^2 and R^3 has been considered by the canonical transformation method.

The essence of our approach consists in combination of two powerful methods of QFT - canonical transformations and renormalization group (RG). The idea of such a combination originates from the fundamental properties of the local QFT: existence of nonequivalent representations of canonical commutation relations (c.c.r.) and UV-divergences (e.g., see Refs.14,15 and references therein). From a physical viewpoint, existence of nonequivalent representations means that the ground state of the QFT-system is not unique. At the same time, vacuum instability originates from the radiative corrections to the physical parameters of the system. Renormalization means actually an appreciation of leading radiative corrections. Hence, the R-structure of the theory should contain the main (at least qualitative) information about its vacuum structure (see also Ref.16).

According to this intuitive motivation, our starting points are

* the phases appear in QFT as nonequivalent c.c.r. representations;

* the renormalization structure of the theory contains basic information about its phase structure.

It is well-known how to construct an appropriate QFT if renormalized coupling constant G is small enough. The standard canonical quantization in representation, given by the Fock space of scalar particles with renormalized mass m, should be performed. This is an "initial" phase of the model (1.1). Having this in our mind, we want to know what is our system in strong coupling regime, i.e. for large G? We formulate the problem as follows

What representation of c.c.r. is suitable for different values of G and what physical picture corresponds to this representation ?

Our approach consists in the following steps.

(1) We construct a canonically quantized theory in representation having a suitable physical interpretation for $G \ll 1$. Renormalization scheme should be fixed. It means that

* one-parameter class $R_{(\mu)}$ of R-schemes is chosen;

* renormalization scale μ is fixed by the relation

$$\frac{m(\mu)}{\mu} = C \quad ,$$

where $m(\mu)$ is renormalized mass and C is a constant.

(2) We perform canonical transformation of the field variables and get the representation with the new mass of the field and nonzero vacuum condensate. The canonical transformation should be introduced in such a way that the total Hamiltonian has a

"correct" form in any representation (r)

$$H = H_0^r + H_I^r + H_{ct}^r + VE^r \quad . \qquad (1.2)$$

Here H_0^r is the standard free Hamiltonian. The interaction Hamiltonian H_I^r contains the field operators in degree more than two. The constant E^r has a sense of vacuum energy density. The counter-term operator H_{ct}^r is defined by H_0^r and H_I^r and corresponds to the same R-scheme in all phases.

We consider R-scheme being the same in two representations with different masses m and M if

* the same $R_{(\mu)}$-class is used in both representations;
* renormalization scales μ and ν in the first and second representations obey the relation

$$\frac{m}{\mu} = \frac{M}{\nu} \ . \qquad (1.3)$$

(3) We perform classification of the phases and choose the phase suitable for a given value of G. There are two mutually additional principles for this choice. A representation (r) is suitable, if

(α) the vacuum energy density E^r is smaller in this representation than in others;

(β) an effective coupling is weaker in this representation than in other possible representations.

Usually energy criterion (α) is used in phase transition theory. But one can see that in QFT the principle (β) is preferable due to several reasons. Vacuum energy has no any significance from a physical viewpoint since it does not contribute to the S-matrix elements. We are unable to get an exact vacuum energy; hence we are forced to compare the lowest contribution to this energy for different representations. At the same time, it is natural to suppose that the large coupling constant in the Hamiltonian (1.2) means that the representation of c.c.r. connected with H_0^r does not describe the real physical states and can not be considered as a suitable representation for the total Hamiltonian. Incidentally, our calculations in Refs.12,13 show that the principles (α), (β) do not contradict each other. We will use the criterion (β) in this paper and omit all vacuum energy counter-terms.

Now we discuss briefly the contents of various sections of the paper.

In sect.2 we consider three-dimensional model and show stability of our method under the choice of R-scheme in the initial representation at least in superrenormalizable case.

In sect.3 we investigate the vacuum structure of four-dimensional ϕ^4 theory. The simplest consideration is provided by mass-independent R-schemes. Although the exact form of RG-functions γ_m and β is unknown, we can consider all logical possibilities and make some conclusions. Independently of the form of γ_m and β the phase with broken symmetry is absent in four-dimensional theory (at least for mass-independent R-schemes). This result is completely different from the situation in R^2 and R^3 where the phase with broken symmetry exists[12,13]. We shall discuss the reasons for such difference.

Simple assumption about the form of exact γ_m-function provides the existence of two symmetric phases with different masses m and M=t(g)m. Interaction in phase with mass M is defined by the effective coupling constant G(g) which is small for large g. One can conclude that $(\phi^4)_4$ field system exists in symmetric phases with masses m and M in weak and strong coupling regimes, respectively. Our consideration of four-dimensional model is valid for any mass-independent R-scheme. The question about its validity in a more general case is open.

In sect.4 we consider some speculations concerning an asymptotically free case.

2. THREE-DIMENSIONAL ϕ^4 THEORY

2.1 HAMILTONIAN $(\phi^4)_3$

Here we investigate the R-scheme dependence of the results obtained in our previous paper[13].

First of all we represent our problem in a form convinient for this investigation.

The model is superrenormalizable. A finite number of divergent diagrams contribute to the mass renormalization

(Fig.1). The renormalized Lagrangian looks like

$$L_R = \frac{1}{2}\,\phi(\,\square - m_B^2\,)\,\phi - \frac{1}{4}\,g\,\phi^4 \;.$$

The bare mass has the form

$$m_B^2 = m^2(\mu) + \delta m^2(\mu) \;.$$

The dimensionless perturbation coupling constant

$$G(\mu) = \frac{g}{2\pi m(\mu)}$$

is a parameter of the model.

The Hamiltonian density in representation with mass $m(\mu)$ looks like

$$H = H_0 + H_I + H_{ct} \qquad (2.1)$$

$$H_0 = \frac{1}{2}\left(\pi^2 + (\nabla\phi)^2 + m^2(\mu)\phi^2\right), \quad H_I = \frac{1}{4}\,g\,\phi^4$$

$$H_{ct} = \frac{1}{2}\,\delta m^2(\mu)\phi^2 \;.$$

The operators ϕ and π obey the standard canonical commutation relations. It has been stressed in the introduction that R-scheme should be fixed in the initial representation (2.1). This representation is suitable for $G(\mu)\ll 1$.

2.2 CANONICAL TRANSFORMATION

To investigate strong coupling regime $G(\mu)\gg 1$, we perform canonical transformation to the variables Φ, Π with the new mass $M=m(\mu)\cdot t$ and vacuum condensate B=const

$$(\,\phi\,,\,\pi\,) \longrightarrow (\,\Phi + B,\,\Pi\,)\,, \qquad (2.2)$$

accompanied by the scale transformation

$$\mu \longrightarrow \nu = \mu \cdot t$$

to provide condition (1.3)

$$\frac{m(\mu)}{\mu} = \frac{M}{\nu} .$$

Transformation (2.2) can be written in an explicit form in terms of the creation and annihilation operators (Bogoliubov canonical transformation, see Refs.14,17).

As a result, we get the following representation for Hamiltonian density:

$$H = H'_0 + H'_I + H'_{ct} + H_1$$

$$H'_0 = \frac{1}{2}\left(\Pi^2 + (\nabla\Phi)^2 + M^2\Phi^2 \right) ,$$

$$H'_I = \frac{1}{4} g \left(\Phi^4 + 4B\Phi^3\right)$$

$$H'_{ct} = \frac{1}{2} \delta m^2(\mu\cdot t)\Phi^2 + \delta m^2(\mu\cdot t)B\Phi ,$$

$$H_1 = \frac{1}{2}\left(m^2(\mu\cdot t) +3gB^2 - M^2\right)\Phi^2 + \left(m^2(\mu\cdot t) + gB^2 \right)B\Phi .$$

To provide the correct form (1.2) of the total Hamiltonian we put $H_1=0$ and get the following equations for parameters B, t

$$\begin{aligned} & m^2(\mu\cdot t) + 3gB^2 - m^2(\mu)t^2 = 0 \\ & B\left[m^2(\mu\cdot t) + gB^2 \right] = 0 \end{aligned} \qquad (2.3)$$

Eqs.(2.3) describe the phase structure of the model. The solution $B\neq0$ corresponds to the phase with broken symmetry under $\Phi\longrightarrow-\Phi$. Symmetric phase is described by the solution B=0. Details can be found in Ref.13. Here we restrict ourselves to the symmetric case and consider the R-scheme dependence of

Eqs.(2.3). They are reduced to the following equation for t in the symmetric case B=0

$$\frac{m^2(\mu\cdot t)}{m^2(\mu)} = t^2 . \qquad (2.4)$$

The model at hand is superrenormalizable and we are able to calculate the running mass $m(\mu\cdot t)$ exactly using different R-schemes.

3.1. ZERO-MOMENTUM R-SCHEME WITH ARBITRARY "MASS" μ

In this scheme mass counter-terms are given by diagrams (Fig.1) with zero external momentum and arbitrary "mass" μ in propagators. This is one of the possible in $(\phi^4)_3$ ways to introduce mass scale μ. Let us denote

$$m_B^2 = \bar{m}^2(\mu) + \delta\bar{m}_a^2(\mu) + \delta\bar{m}_b^2(\mu) , \qquad (2.5)$$

where counter-terms correspond to diagrams (a) and (b) in Fig.1. They can be easily calculated

$$\delta\bar{m}_a^2(\mu) = -3g\Delta_{reg}(\mu) , \quad \delta\bar{m}_b^2(\mu) = 3!g^2\Sigma_{reg}(\mu) ,$$

with

$$\Delta_{reg} = \frac{1}{(2\pi)^2}\, \mathrm{reg}\int_0^\infty \frac{du\,u^2}{u^2+\mu^2}$$

$$\Sigma_{reg} = \frac{1}{(4\pi)^2}\, \mathrm{reg}\int_0^\infty \frac{dt}{t}\, e^{-3\mu t} . \qquad (2.6)$$

An appropriate regularization should be introduced here.

Let the initial representation (2.1) be constructed within the usual zero momentum scheme. It corresponds to a particular choice of μ in Eq.(2.5) equal to renormalized mass m, i.e. the condition

$$\overline{m}(m) = m \qquad (2.7)$$

fixes the standard zero-momentum scheme within R-class with arbitrary mass μ.

Eq.(2.4) takes the form

$$\frac{\overline{m}^2(m\cdot t)}{m^2} = t^2 \qquad . \qquad (2.8)$$

Using the R-invariance of bare mass and Eq.(2.7) as the initial condition, we obtain

$$m^2 - 3g\Delta_{reg}(m^2) + 6g^2\Sigma_{reg}(m^2) = m_B^2 \qquad (2.9)$$
$$= \overline{m}^2(\mu) - g\Delta_{reg}(\mu) + 6g^2\Sigma_{reg}(\mu).$$

Eq.(2.9) can be rewritten as follows

$$\overline{m}^2(\mu) = m^2 + 3g\left(\Delta_{reg}(\mu)-\Delta_{reg}(m)\right) - 6g^2\left(\Sigma_{reg}(\mu)-\Sigma_{reg}(m)\right) .$$

Now we can remove regularization and get the expression for running mass

$$\overline{m}^{\,2}(\mu) = m^2\left[1 + \frac{3}{2} G \left(1-\frac{\mu}{m}\right) + \frac{3}{2} G^2 \ln\left(\frac{\mu}{m}\right)\right] \quad (2.10)$$

here $G=g/2\pi m$. The function $\overline{m}^2(\mu)$ is given in Fig.2.

Using Eq.(2.10) with $\mu=m\cdot t$ in Eq.(2.8) we get the following equation for t

$$t^2 - 1 + \frac{3}{2} G(t-1) - \frac{3}{2} G^2\ln(t) = 0 . \qquad (2.11)$$

Eq.(2.11) has two solutions. The first $t\equiv1$ corresponds to the initial representation. Another solution $t=t(G)$ describes the second symmetric representation. In particular, one can find the asymptotics

$$t(G) \xrightarrow[G\gg1]{} \left(\frac{3}{2} G^2\ln(G)\right)^{1/2} ,$$

$$G_{eff} = \frac{G}{t(G)} \xrightarrow[G\gg1]{} \left(\frac{3}{2} \ln(G)\right)^{-1/2} \ll 1 ,$$

here $G_{eff}(G)$ is the effective perturbation coupling constant in the phase with mass $M=t(G)m$.

2.2. DIMENSIONAL REGULARIZATION, MS-SCHEME

Here we use the following notation

$$\varepsilon = 3 - d , \quad \alpha = g/2\pi ,$$
$$m_B^2 = m^2(\mu) + \delta m_a^2 + \delta m_b^2 , \qquad (2.12)$$

$m(\mu)$ is a running mass in the MS-scheme. Standard calculations give the following result for the diagram (a) in Fig.1

$$\delta m_a^2 = -3g\ m(\mu) \frac{\pi^{d/2}}{(2\pi)^d} \left(\frac{2\pi\mu}{m(\mu)}\right)^{\varepsilon} \Gamma(1-d/2) \ .$$

Putting d=3 we get finite result

$$\delta m_a^2 = \frac{3}{2}\ \alpha\ m(\mu) \qquad . \qquad (2.13)$$

Such finiteness is the usual artefact of dimensional regularization if physical dimension is odd. That is why we include finite contribution (2.13) into the mass renormalization.

Calculation of the diagram (b) in Fig.1 gives the following result for zero external momentum

$$\Sigma_{reg} = \frac{3}{4}\alpha^2 \left[\frac{1}{\varepsilon} + \ln\left(\frac{4\pi\mu^2}{m^2(\mu)}\right) - \gamma_E + o(\varepsilon) \right] . \quad (2.14)$$

In the MS-scheme we should introduce into the counter-term only the divergent part of this expression

$$\delta m_b^2 = \frac{3}{4}\alpha^2 \frac{1}{\varepsilon} \quad (2.15)$$

Using Eqs.(2.13) and (2.15) in Eq.(2.12) we get the following expression for bare mass

$$m_B^2 = m^2(\mu) + \frac{3}{2}\alpha\, m(\mu) + \frac{3}{4}\alpha^2 \frac{1}{\varepsilon} . \quad (2.16)$$

Let us go to a new scale ν in Eq.(2.16). The following change should be done (e. g. Ref.18)

$$g \longrightarrow g_{new} \left(\frac{\mu}{\nu}\right)^{\varepsilon} \qquad \left(g_{new} \xrightarrow[\varepsilon\to 0]{} g \right) .$$

Using this substitution we get

$$m_B^2 = m^2(\nu) + \frac{3}{2}\alpha_{new} m(\nu) + \frac{3}{4}\alpha^2_{new}\frac{1}{\varepsilon} + \frac{3}{2}\alpha^2_{new} \ln\left(\frac{\mu}{\nu}\right) \quad (2.17)$$

with the obvious condition

$$m(\nu)\Big|_{\nu=\mu} = m(\mu) . \quad (2.18)$$

From Eqs.(2.16) and (2.17)) we get the following equation

$$m^2(\mu) + \frac{3}{2}\alpha\, m(\mu) + \frac{3}{4}\alpha^2 \frac{1}{\varepsilon} =$$

$$= m^2(\nu) + \frac{3}{2}\alpha_{new}\, m(\nu) + \frac{3}{4}\alpha^2_{new}\frac{1}{\varepsilon} + \frac{3}{2}\alpha^2_{new}\ln\left(\frac{\mu}{\nu}\right) .$$

After removing regularization ($\varepsilon\to 0$) we get the equation

$$m^2(\nu) + \frac{3}{2}\alpha\, m(\nu) - \frac{3}{2}\alpha^2 \ln\left(\frac{\nu}{\mu}\right) - m^2(\mu) - \frac{3}{2}\alpha\, m(\mu) = 0. \quad (2.19)$$

Solution of Eq.(2.19), obeying the condition (2.18) looks like

$$m(\nu) = - m(\mu) - \frac{3}{2}\alpha + \sqrt{\left[2m(\mu) + \frac{3}{2}\alpha\right]^2 + 6\alpha^2 \ln\left(\frac{\nu}{\mu}\right)} . \quad (2.20)$$

The function $m(\nu)$ is shown in Fig.3. The point ν_0 $(m(\nu_0)=0)$ is defined by Eq.(2.20) as follows

$$\nu_0 = \mu \exp\left\{ - \frac{m(\mu)\left[\, m(\mu) + \alpha\,\right]}{2\alpha^2} \right\} < \mu .$$

Putting $\nu=\mu\cdot t$ and $m(\nu)=m(\mu)\cdot t$ in Eq.(2.19) we obtain the equation

$$t^2 - 1 + \frac{3}{2}G(t-1) - \frac{3}{2}G^2\ln(t) = 0 . \quad (2.21)$$

Here $G=\alpha/m(\mu)$. One can see that Eq.(2.21) coincides with Eq.(2.11). Thus, the function $t(\cdot)$ turns out to be the same in both R-schemes, although the running masses $\overline{m}(\nu)$ and $m(\nu)$ are completely different functions (compare Figs.2,3).

3.3 DIMENSIONAL REGULARIZATION, ZERO-MOMENTUM R-SCHEME

Let us change substraction prescription for the diagram (b) in Fig.1. We introduce into the counter-term δm_b^2 not only the pole part of expression (2.14) but also its finite terms. It is a kind of zero-momentum scheme. Bare mass looks like

$$m_B^2=\hat{m}^2(\mu) + \frac{3}{2}\alpha\hat{m}(\mu)+$$

$$+ \frac{3}{4}\alpha^2 \left[\frac{1}{\varepsilon} - \gamma_E + \ln\left(\frac{4\pi\mu^2}{\hat{m}^2(\mu)}\right) + o(\varepsilon) \right]. \qquad (2.22)$$

Going in Eq.(2.22) to a new scale ν in a standard way, we get the equation

$$\hat{m}^2(\nu) - \hat{m}^2(\mu) + \frac{3}{2}\alpha\left(\hat{m}(\nu) - \hat{m}(\mu)\right) - \frac{3}{2}\alpha^2 \ln\left(\frac{\nu}{\mu}\right) + \qquad (2.23)$$

$$+ \frac{3}{4}\alpha^2 \ln\left(\frac{\hat{m}^2(\mu)\ \nu^2}{\mu^2\ \hat{m}^2(\nu)}\right) = 0 \, .$$

The last term in Eq.(2.23) originates from the last term in Eq.(2.14). One can see that the running mass $\hat{m}(\nu)$ defined by Eq.(2.23) differs from the masses $\overline{m}(\nu)$ and $m(\nu)$ (see Eqs.(2.10),(2.19)). Nevertheless, substitution $\nu=\mu\cdot t$, $\hat{m}(\nu)=\hat{m}(\mu)\cdot t$ in Eq.(2.23) leads to an equation coinciding with Eqs.(2.11),(2.21)

$$t^2 - 1 + \frac{3}{2}\hat{G}(t-1) - \frac{3}{2}\hat{G}^2\ln(t) = 0 \quad ,$$

$$\hat{G} = g/2\pi\hat{m}(\mu) \quad .$$

The difference in the dimensionless coupling constant is not important since this constant is a free parameter of the equation.

Calculations undertaken in this section do not prove the scheme invariance of Eq.(2.4) but they give us enough experience to conclude that Eq.(2.4) is stable under the choice of R-scheme although running mass $m(\mu\cdot t)$ in (2.4) depends

on R-scheme very strongly. The same conclusion is valid for the case B≠0. We can say that the results of Ref.13 should hardly depend on R-scheme and, hence, they should reflect the real physical properties of ϕ^4 field system in R^3.

3. FOUR-DIMENSIONAL ϕ^4-THEORY

3.1 HAMILTONIAN $(\phi^4)_4$

The renormalized Lagrangian looks like

$$L_R(x) = \frac{1}{2}\phi(x)(\ \Box - m^2(\mu)\ )\phi(x) - \frac{1}{4}g(\mu)\ \phi^4(x) + \qquad (3.1)$$

$$+\frac{1}{2}\left(Z_2-1\right)\phi(x)\Box\phi(x) - \frac{1}{2}\ \delta m^2(\mu)\ \phi^2(x) - \frac{1}{4}\left(Z_1-1\right)g(\mu)\ \phi^4(x).$$

We omit vacuum energy counter-terms in (3.1). The R-scheme in (3.1) should be fixed. In other words, $R_{(\mu)}$-class is chosen and ratio $m(\mu)/\mu$ is fixed.

The Hamiltonian density has the form

$$H = H_0 + H_I + H_{ct}$$

where

$$H_0= \frac{1}{2}\left(\pi^2 + (\nabla\phi)^2 + m^2(\mu)\phi^2\right), \quad H_I= \frac{1}{4}g(\mu)\phi^4 \qquad (3.2)$$

$$H_{ct} = \frac{1}{2}\left[\left(Z_2^{-1}- 1\right)\pi^2 + \left(Z_2- 1\right)(\nabla\phi)^2 + \phi^2\ \delta m^2(\mu)\ \right] +$$

$$\frac{1}{4}\left(Z_1 - 1\right) g(\mu)\ \phi^4 \, .$$

Here $\phi=\phi(\mathbf{x})$, $\pi=\pi(\mathbf{x})$.

Canonical quantization means that the fields ϕ,π are operators and obey the standard equal-time commutation relations.

Representation (3.2) is suitable for weak coupling regime $g(\mu)<<1$. Having this in our mind we will investigate the strong coupling limit $g(\mu)>>1$.

3.2. CANONICAL TRANSFORMATION

Let us perform the following canonical transformation

$$\left\{\phi,\pi\right\} \longrightarrow \left\{z_2^{-1/2}\,\Phi + z_2^{-1/2}\,B\ ,\ z_2^{1/2}\,\Pi\right\}. \qquad (3.3)$$

Here (ϕ,π) are the field operators with mass $m(\mu)$, (Φ,Π) are the field operators with mass $M=t\cdot m(\mu)$, B is a constant having a sense of vacuum condensate. According to the equivalence condition (1.3) this canonical transformation should be accompanied by the scale transformation $\mu\longrightarrow\nu=\mu\cdot t$. This is the origin of the presence of finite renormalization constant z_2 in (3.3).

The total Hamiltonian density takes the following form in the new representation:

$$H = H_0' + H_I' + H_{ct}' + H_1$$

$$H_0' = \frac{1}{2}\Big[\, \Pi^2 + (\nabla\Phi)^2 + M^2\Phi^2 \,\Big] \quad ,$$

$$H_I' = \frac{1}{4}\, g(\nu) \Big[\, \Phi^4 + 4B\Phi^3 \,\Big] \quad ,$$

$$\begin{aligned} H_{ct}' = & \frac{1}{2}\Big[(Z_2'^{-1} - 1)\,\Pi^2 + (Z_2' - 1)(\nabla\Phi)^2 \Big] + \\ & \frac{1}{2}\,\Phi^2 \Big[\, \delta m^2(\nu) + 3(Z_1' - 1)g(\nu)B^2 \,\Big] + \\ & \frac{1}{4}\,(Z_1' - 1)g(\nu)\,(\, \Phi^4 + 4B\Phi^3 \,) + \qquad (3.4) \\ & \Big[\, \delta m^2(\nu) + (Z_1' - 1)g(\nu)B^2 \,\Big]B\Phi \quad , \end{aligned}$$

$$\begin{aligned} H_1 = & \frac{1}{2}\Big[\, m^2(\nu) + 3g(\nu)B^2 - M^2 \,\Big]\Phi^2 + \\ & \Big[m^2(\nu) + g(\nu)B^2\Big]\, B\Phi \end{aligned}$$

here $\nu=\mu\cdot t$, $M=m(\mu)\cdot t$.

To provide the correct form (1.2) of the total Hamiltonian we put $H_1=0$, or equivalently

$$\begin{aligned} & m^2(\mu\cdot t) + 3g(\mu\cdot t)B^2 - m^2(\mu)t^2 = 0 \\ & B\Big[\, m^2(\mu\cdot t) + g(\mu\cdot t)B^2 \,\Big] = 0 \end{aligned} \qquad (3.5)$$

The quantities $m(\mu\cdot t)$, $g(\mu\cdot t)$ and $m(\mu)$, $g(\mu)$ are connected by the scale RG-transformation and $m(\mu\cdot t)$ and $g(\mu\cdot t)$ can be obtained from the RG-equations

$$\begin{aligned} & t\,\frac{dg(\mu\cdot t)}{dt} = \beta\Big(g(\mu\cdot t), \frac{m(\mu\cdot t)}{\mu\cdot t}\Big) \\ & \frac{t}{m^2(\mu\cdot t)}\,\frac{dm^2(\mu\cdot t)}{dt} = -\gamma_m\Big(\, g(\mu\cdot t), \frac{m(\mu\cdot t)}{\mu\cdot t}\Big) \end{aligned} \qquad (3.6)$$

with the initial conditions

$$m(\mu\cdot t) = m(\mu)\ , \quad \text{for}\ \ t = 1$$

$$g(\mu\cdot t) = g(\mu)\ , \quad \text{for}\ \ t = 1\ .$$

Two possibilities follow from the second Eq.(3.5): B=0 (symmetric phase) and B≠0 (broken symmetry phase).

3.2. SYMMETRIC PHASE (B=0)

Putting B=0 we get an equation for t in the symmetric phase

$$\frac{m^2(\mu\cdot t)}{m^2(\mu)} = t^2 \qquad (3.7)$$

We should note that Eq.(3.7) has the same form as Eq.(2.4) for $(\phi^4)_3$. The renormalization group shows itself in a superrenormalizable three-dimensional case in the simplest form and we are able to solve (2.4) exactly for any R-scheme. The situation is quite different for four-dimensional model and our further consideration is concentrated around RG-equations (3.6). We must choose certain R-scheme. We will stress that we have not any general proof of the scheme invariance of Eqs.(3.5). At the same time our calculations with $(\phi^4)_3$ in sect.2 allow to assume that Eqs.(3.5) should hardly depend on the choice of the R-scheme.

Eqs.(3.6) take a simple form in any mass-independent R-scheme and can be easily solved as follows

$$m^2(\mu\cdot t) = m^2(\mu)\ \exp\left\{ -\int_{g(\mu)}^{g(\mu\cdot t)} dx\ \frac{\gamma_m(x)}{\beta(x)} \right\}$$

$$\int_{g(\mu)}^{g(\mu\cdot t)} dx\ \frac{1}{\beta(x)} = \ln(\ t\ ) \qquad . \qquad (3.8)$$

Appreciation of the first Eq.(3.8) in Eq.(3.7) leads to the equation

$$\int_{g(\mu)}^{g(\mu\cdot t)} dx\ \frac{\gamma_m(x)}{\beta(x)} = -\ \ln(\ t^2\ ) \qquad . \qquad (3.9)$$

It is convenient to use the second Eq.(3.8) in Eq.(3.9). As a result, we obtain the following equations

$$\int_g^G dx\ \frac{2\ +\ \gamma_m(x)}{\beta(x)} = 0 \ ,$$

$$\ln(\ t\ ) = \int_g^G dx\ \frac{1}{\beta(x)} \ , \qquad (3.10)$$

where we denote $g=g(\mu)$, $G=g(\mu\cdot t)$.

Eqs.(3.10) define the symmetric representations of the model. Since exact functions γ_m and β are unknown, we have nothing to do as to consider all logical possibilities.

Obviously, a trivial solution t≡1, G≡g exists for any γ_m and β. It corresponds to the initial representation (3.2).

The behaviour of γ_m and β at small x can be calculated perturbatively. Integrand in the first Eq.(3.10) for x⟶0 behaves as follows

$$F(x) = \frac{2\ +\ \gamma_m(x)}{\beta(x)} \xrightarrow[x\longrightarrow 0]{} \frac{2}{\beta_1 \alpha^2} \ , \qquad (3.11)$$

here $\alpha=3!x/(4\pi)^2$, $\beta_1=3/2$. It is known that the β-function is positive for $x\in(0,g^*)$ where UV-fixed point g^* may be finite or infinite. If the function F(x) ((3.11)) does not change sign in the interval $(0,g^*)$, then Eqs.(3.10) have only trivial solution $G \equiv g$, $t \equiv 1$.

Another possibility is illustrated in Fig.4. The second solution of Eqs.(3.10) exists if the function $F(x)$ changes sign at any point $g_c \in (0,g^*)$. Let us assume that such situation takes place. For example, let $\gamma_m = -ax$, $\beta = bx^2$ $(g^* = \infty)$ with $a>0$, $b>0$. Eqs.(3.10) take the following form after integration

$$-\frac{1}{G} + \frac{1}{g} - \frac{a}{2}\cdot \ln\left(\frac{G}{g}\right) = 0$$

$$b\cdot \ln(t) = \frac{1}{g} - \frac{1}{G} \quad .$$

Asymptotics of $G(g)$ and $t(g)$ in a strong coupling limit look like

$$G(g) \xrightarrow[g\to\infty]{} \frac{2}{a\cdot \ln(g)} \ll 1$$

$$t(g) \xrightarrow[g\to\infty]{} g^{-a/2b} \ll 1 \quad .$$

This example illustrates the following general picture. The qualitative dependences $G(g)$ and $t(g)$ are given in Figs.5 and 6, respectively. The effective coupling constant G depends

on g only, moreover

$$\begin{aligned} G &\longrightarrow g^* \ , && \text{if } g \longrightarrow 0 \\ G &= g \ , && \text{if } g = g_c \\ G &\longrightarrow 0 \ , && \text{if } g \longrightarrow g^* \end{aligned} \qquad (3.12)$$

and, since $\beta(x)>0$,

$$\begin{aligned} t(g) &\longrightarrow \infty \ , && \text{if } g \longrightarrow 0 \\ t(g_c) &= 1 && \\ t(g) &\longrightarrow 0 \ , && \text{if } g \longrightarrow g^* \ . \end{aligned} \qquad (3.13)$$

From comparison of the coupling constants in (3.12) (see Introduction) we conclude that our system exists in the symmetric phase with mass $m(\mu)$ in the weak coupling limit $(g\to 0)$, but another symmetric phase with mass $M \ll m(\mu)$ and effective coupling constant $G(g) \ll 1$ turns out to be suitable in the strong coupling limit $g\to g^*$. A kind of a phase transition takes place at the point $g=g_c$.

3.3. DYNAMICAL SYMMETRY BREAKING ($B\neq 0$)

The Hamiltonian H'_{ct} in (3.4) reflects the well-known fact that the counter-terms for ϕ^4-model with spontaneous symmetry breaking are defined completely by the counter-terms of the symmetric model (e.g. Ref.18). Hence, the running mass $m(\mu\cdot t)$ and coupling constant $g(\mu\cdot t)$ in Eqs.(3.5) are given by the same Eqs.(3.6) both for $B=0$ and $B\neq 0$.

Eqs.(3.5) for $B\neq 0$ can be easily rewritten as follows

$$B^2 = -\frac{m^2(\mu \cdot t)}{g(\mu \cdot t)} \qquad (3.14)$$

$$t^2 = -2\,\frac{m^2(\mu \cdot t)}{m^2(\mu)} \quad .$$

Eqs.(3.14) have not a real solution since $m^2(\mu\cdot t)>0\ \forall g(\mu),t$ (at least within mass-independent R-schemes). We conclude that our system (3.1) have not a representation with $B\neq 0$; and hence, dynamical symmetry breaking is absent in $(\phi^4)_4$.

This result differs from the situation in $(\phi^4)_{2,3}$ where the c.c.r. representation with $B\neq 0$ exists, although phase transition accompanied by symmetry breaking takes place only in $(\phi^4)_2$ /12,13/. We will stress that this difference originates from an essentially different behaviour of the running mass $m(\nu)$ in $(\phi^4)_{2,3}$ and $(\phi^4)_4$. One can see from Figs.2,3 that there are such values of scale ν that the running mass turns out to be negative for all considered R-schemes. Situation in R^2 is analogous. At the same time, the running mass is positive in the four-dimensional case.

Absence of symmetry breaking in $(\phi^4)_4$ is not in contradiction with the following heuristic consideration. As B.Simon has noted in Ref.16, intuitively clear reason for symmetry breaking in $(\phi^4)_2$ comes from normal operator ordering, in other words from the bubble diagram in Fig.1,a. Its contribution changes a sign of the bare mass in the strong coupling limit. A contrary picture takes place in $(\phi^4)_3$ since two

diagrams contribute to m_B with different signs (Fig.1). The bare mass turns out to be positive for large g and symmetry breaking is absent. The situation in $(\phi^4)_4$ is completely different since the bare mass is represented by alternating series. This series may be positive for any value of g and the reason for appearance of the phase with broken symmetry may be absent at all.

We can now compare the behaviour of the ϕ^4-systems in space-time R^d for d=2,3 and 4. The situation is presented in Table 1 and reflects correlation between renormalization and phase structures.

4. SPECULATIONS

Let us consider a toy theory- model (1.1) with negative coupling constant $g\to -g$. Such model turns out to be asymptotically free /19/. One can see that the first Eq.(3.10) is not sensitive to the sign of the β-function. Hence, the relations (3.12) are not changed and Fig.5 keeps qualitative validity. At the same time, the function t(g) is changed cardinally, as it follows from the second Eq.(3.10). We have a picture contrary to (3.13) (compare Figs.6,7)

$$t(g)\longrightarrow 0\ ,\ G\longrightarrow g^*\ ,\quad \text{if } g\longrightarrow 0$$
$$t(g)\longrightarrow \infty\ ,\ G\longrightarrow 0\ ,\quad \text{if } g\longrightarrow g^*\ . \qquad (4.1)$$

Such a situation in the strong coupling regime $g\to g^*$ (here g^* is an infrared fixed point) has the following physical inter-

Table 1

	$G \ll 1$	$G \gg 1$
R^2	$\frac{m^2}{2}\phi^2 + \frac{g}{4}\phi^4$	$\frac{M^2}{2}\Phi^2 + \frac{g}{4}\Phi^4 + gB(g)\Phi^3$ SSB
R^3		$\frac{M^2}{2}\Phi^2 + \frac{g}{4}\Phi^4$
R^2	$g(\phi^2 - \phi_0^2)^2 = \frac{m^2}{2}\phi^2 + \frac{g}{4}\phi^4 + m\sqrt{\frac{g}{2}}\,\phi^3$	$\frac{M^2}{2}\Phi^2 + \frac{g}{4}\Phi^4 + gB(g)\Phi^3$ SSB
R^3		$\frac{M^2}{2}\Phi^2 + \frac{g}{4}\Phi^4$
R^2	$\frac{m^2}{2}\sum_1^N \phi_i^2 + \frac{g}{4}\left[\sum_1^N \phi_i^2\right]^2$	$\frac{M^2}{2}\Phi^2 + \frac{M_0^2}{2}\sum_1^{N-1}\Phi_i^2 + \frac{g}{4}\left[\sum_1^{N-1}\Phi_i^2 + \Phi^2\right]^2 + gB\Phi\left[\sum_1^{N-1}\Phi_i^2 + \Phi^2\right]$ SSB
R^3		$\frac{M^2}{2}\sum_1^N \Phi_i^2 + \frac{g}{4}\left[\sum_1^N \Phi_i^2\right]^2$
R^4	$\frac{m^2}{2}\phi^2 + \frac{1}{4}g\phi^4$ (mass-independent R-schemes)	$\frac{M^2}{2}\Phi^2 + \frac{1}{4}G_{eff}(g)\Phi^4$ if $\exists g_c \in (0, g^*)$: $2+\gamma_m = 0$; ? if $\forall g \in (0, g^*)$ $2+\gamma_m > 0$

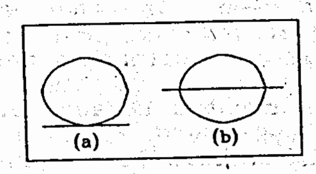

Fig.1
Mass renormalization diagrams for $(\varphi^4)_3$

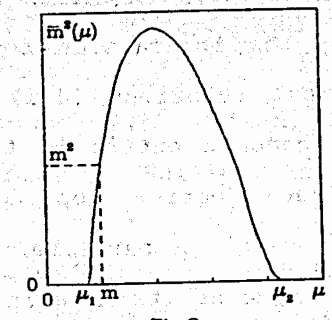

Fig.2
The running mass given by Eq.(2.10)

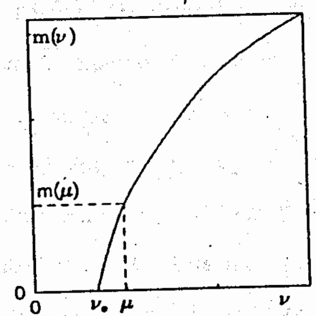

Fig.3
The running mass given by Eq.(2.20)

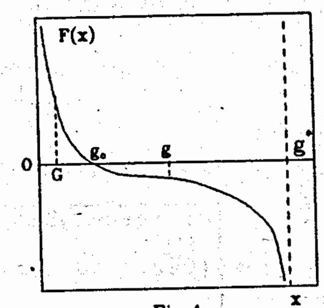

Fig.4
Possible behaviour of the integrand in the first Eq.(3.10)

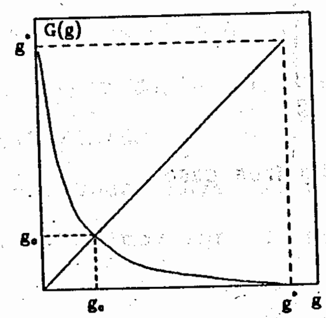

Fig.5
The effective coupling constant for $(\varphi^4)_4$

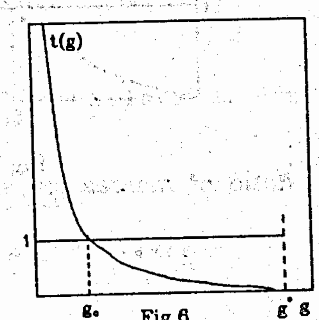

Fig.6
Ratio of masses in symmetric phases for $(\varphi^4)_4$

pretation. Our system exists in a phase with mass $M=t(g)m \gg m$; moreover, the interaction between particles is weak $G(g) \ll 1$. Asymptotical relations (4.1) suggest that two-point Green's function becomes entire in the limit $g \to g^*$. In other words, although an effective coupling is small and approaches zero in the limit $g \to g^*$, a particle cannot be created because of the infinite value of its mass in this limit. This is a situation of the so-called analytical confinement.

This example illustrates a scenario of confinement which may turn out to be reliable in physical asymptotically free models.

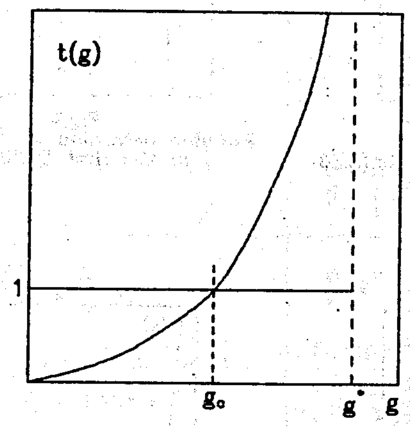

Fig.7
Ratio of masses for asymptotically free case

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