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ON THE VECTOR MESON PRODUCTION IN NAMBU-JONA-LASINIO MODEL

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1. Introduction

Vector meson production in the nucleons and the nuclei collisions can give some new information about dynamics of the quark interaction at short distances but theoretical description of the hadron dynamics at low and intermediate energies on the quark level requires an effective theory of strong interaction. In recent years the Nambu-Jona-Lasinio (NJL) type models [1] have been considered as a realistic candidate for it. The models provide spontaneous breakdown of the chiral symmetry and with a minimal number of parameters yield quite reasonable results for the meson [2]-[5] and baryon [6]-[16] sectors and describe different hadron properties at low energies [17],[18]. So it seems to be very interesting apply this model for the particle production at low energies.

The chiral invariant four-quark interaction of NJL type predicts also existence of "bounded" quark pairs, diquarks, which transform like a color antitriplet. The diquark properties in the frame work of the NJL-model have been studied in papers [17],[19]-[22]. However as compared with the meson the properties situation in the diquark sector is not so clear. Really, in the papers [21] and [22] the same mass relations were obtained, but final conclusions on the diquark masses are different. It was claimed in [21] that the scalar diquark mass is approximately equal to the constituent quark mass ($\sim 300 MeV/c^2$), but in ref.[22] it is about twice the constant quark mass. The difference can be explained by different values of the input parameters including the coupling constants in the (qq)-channels. Obviously these parameters can be fixed from the calculation of an "observable" where the diquark degrees of freedom give the main contribution. One of such possible observables may be the vector meson production in the interaction of two constituent quarks in a nucleon-nucleon collision.

The simplest subprocess is $qq \rightarrow VD$ (where D is a diquark) with the subsequent diquark decay. On the second stage its quarks join the quarks of the primary nucleons to form the measured hadrons. Some other subprocesses are also possible.

To describe interactions between quarks, mesons, and diquarks, one should formulate the Feynman rules for NJL-model. For this aim we use the results of ref.[23] where the Feynman rules for quantum corrections were deduced. As usual, our consideration is valid only when the relevant momentum are less than the cutoff momentum to be introduced because of unrenormalizability of the NJL-model.

Recall that the NJL-model does not contain the confinement mechanism. Therefore the model predictions should be considered as only approximate even for the colorless meson sector. When one considers the dynamics of color diquarks, the model has to be completed by the corresponding confinement mechanism.

The paper is organized as follows. In sect.2 we discuss the mass relations and Feynman rules for NJL-model. In sect.3 we fix the model parameters. In sect.4 we analyze main diagrams which contribute to the elementary $qq \to Vqq$ subprosess, perform numerical calculation of the vector meson production in proton- proton collisions. A summary is given in sect.5.

2. Mesons and diquarks in the NJL-model, Feynman rules

Our starting point is the following chiral invariant Lagrangian of the NJL-type which has been already investigated in ref.[22]:

$$\begin{aligned}\mathcal{L} \;=\;& \bar{q}(i\hat{\partial} - m_0)q + \frac{G_1}{2}[(\bar{q}\lambda^a q)^2 + (\bar{q}i\gamma_5\lambda^a q)^2] \\ & -\frac{G_2}{2}[(\bar{q}\lambda^a\gamma_\mu q)^2 + (\bar{q}\lambda^a\gamma_\mu\gamma_5 q)^2] \\ & +\frac{\tilde{G}_1}{4}[(\bar{q}_a\gamma_5 C\bar{q}_b^T\epsilon_{abc})(q_{a'}^T C^{-1}\gamma_5 q_{b'}\epsilon_{a'b'c}) \\ & +(\bar{q}_a iC\bar{q}_b^T\epsilon_{abc})(q_{a'}^T iC^{-1}q_{b'}\epsilon_{a'b'c})] \\ & -\frac{\tilde{G}_2}{4}[(\bar{q}_a\gamma_\mu\gamma_5 C\bar{q}_b^T\epsilon_{abc})(q_{a'}^T C^{-1}\gamma_\mu\gamma_5 C^{-1}q_{b'}\epsilon_{a'b'c}) \\ & +(\bar{q}_a\gamma_\mu C\bar{q}_b^T\epsilon_{abc})(q_{a'}^T C^{-1}q_{b'}\epsilon_{a'b'c})],\end{aligned} \qquad (1)$$

where q symbolizes the Dirac spinors with the three flavor components, m_0 is the current mass matrix, $\lambda^i (i = 0, ..., 8)$ are the $SU(3)$ Gell-Mann matrices, $C = i\gamma^0\gamma^2$ is the charge conjugation matrix, ϵ_{abc} is the antisymmetric tensor defined in the color space and a, b, c are the color indices. We assume that the coupling constants for the $(\bar{q}q)$ pairs - G_α differ from the corresponding coupling constants for (qq) pairs - $\tilde{G}_\alpha$ where $\alpha = 1, 2$ corresponds to scalar/pseudoscalar or vector/axial vector channels. In the following we restrict our consideration to the $SU(2)$ flavor subsector with u and d quarks.

One of the way to obtain the Feynman rules for the NJL-model is to rewrite of the generating functional

$$Z_{NJL} = \int \mathcal{D}\bar{q}\mathcal{D}q \, \exp \, i \int \mathrm{d}x \, (\mathcal{L}(x) + \eta^T Q + Q^T \eta) \qquad (2)$$

in terms of the "renormalized" meson and diquark fields after integrating over the quark field and eliminating the linear terms in the following form:

$$Z'_{NJL} = \int \mathcal{D}M \, \mathcal{D}D^* \, \mathcal{D}D \, \exp \, i \int \mathrm{d}x \, (\mathcal{L}_2(x) + \mathcal{L}'(x) + \eta^T \frac{1}{K_R}\eta), \qquad (3)$$

where $Q = \binom{\bar{q}^T}{q}$, $Q^T = (\bar{q}, q^T)$ are the quark double spinors, η^T and η are the quark sources, symbol $\mathcal{D}M, \mathcal{D}D, \mathcal{D}D^+$ mean the integration over the meson and diquark scalar M_S, D_S, pseudoscalar M_P, D_P , vector M_V^μ, D_V^μ and axial vector M_A^μ, D_A^μ field, the matrix operator K_R is determined by renormalized fields

$$K_R = \frac{1}{2}\begin{pmatrix} -D_R C & i\hat{\partial} - M - M_R \\ i\hat{\partial}^T + M - M_R & -CD_R^+ \end{pmatrix}, \qquad (4)$$

where M is the quark constituent mass and M^R, D^R are equal:

$$M_R = g_1^S(M_{S_R} + i\gamma_5 M_{P_R}) - g_2^V(\gamma_\mu M_R^\mu + \gamma_\mu\gamma_5 M_A^\mu), \qquad (5)$$

$$D_R = 2[\tilde{g}_1(\gamma_5 D_{S_R} + iD_{P_R}) - \tilde{g}_2(\gamma_5 D_R^\mu + \gamma_\mu D_A^\mu)] \qquad (6)$$

with the renormalized coupling constants determined as

$$\begin{gathered} g_1^S = (12I_2)^{-1/2}, \; \tilde{g}_1^P = (1+a)^{1/2} g_1^S, \; g_2^{V,A} = (8I_2)^{-1/2}; \\ 2\tilde{g}_1^P = (4I_2)^{-1/2}, \; \tilde{g}_1^S = (1+\tilde{a})^{1/2}\tilde{g}_1^P, \; 2\tilde{g}_2^{V,A} = (32/3I_2)^{-1/2} \end{gathered} \qquad (7)$$

and I_2 is a logarithmical by divergent integral

$$I_2(M, \Lambda) = -\frac{i}{(2\pi)^4}\int^{\Lambda} \frac{\mathrm{d}^4 k}{(k^2 - M^2)^2}. \qquad (8)$$

The constants $a, \tilde{a} \neq 0$ take into account $M(P \to A)$ and $D(S \to V)$ mixing [2]: $a = (1 - 6M^2/m_A^2)^{-1} - 1 \simeq 6M^2/m_\rho^2$, $\tilde{a} \simeq 6M^2/m_{D_V}{}^2$.

The Lagrangian $\mathcal{L}_2$ contains "divergent" terms of the quark determinant including the quadratic terms of the fields and their derivatives:

$$\mathcal{L}_2 = -\sum_{a=S,PS}\frac{1}{2}[(M_R^a\partial^2 M_R^a + m_{D_a}^2) + D_R^{a\,+}\partial^2 D_R^a + m_{D_a}^2 D_R^{a\,+} D_R^a$$
$$- \sum_{a=V,A}\frac{1}{4}(\partial_\mu M^a_{R\nu} - \partial_\nu M^a_{R\mu})(\partial^\mu M_R^{\nu a} - \partial^\nu M_R^{\mu a}) - \frac{1}{2} M^a_{R\mu} M_R^{a\mu} \quad (9)$$
$$+\frac{1}{2}(\partial_\mu D^{a\,+}_{R\nu} - \partial_\nu D^{a\,+}_{R\mu})(\partial^\mu D_R^{\nu a} - \partial^\nu D_R^{\mu a}) - m^2_{D_a} D^{a\,+}_{R\mu} D^{a\mu}]$$
$$+[M_R - M_R,\ D_R - D_R,\ M_R - D_R \quad \textit{interaction terms}]$$

and $\mathcal{L}'$ in (3) stands for the sum of the convergent part of the quark determinant. The masses in (9) are given by the mass relations:

$$m_\pi^2 = \frac{m_0}{12G_1I_2M}(1+a);\quad m_\sigma^2 = m_\pi^2(1+a)^{-1} + 4M^2; \quad (10)$$
$$m_\rho^2 = (8G_2I_2)^{-1};\quad m_A^2 = m_\rho^2 + 6M^2.$$
$$m_{D_S}{}^2 = [m_\pi^2(1+a)^{-1} + (3\kappa-1)(12G_1I_2)^{-1}](1+\tilde{a}),\quad \kappa = \frac{\tilde{G}_1}{G_1};$$
$$m_{D_{PS}}{}^2 = m_{D_S}{}^2 + 4M^2; \quad (11)$$
$$m_{D_A}{}^2 = (\frac{8}{3}\tilde{G}_2I_2)^{-1},\quad m_{D_V}{}^2 = 6M^2 + m_{D_A}{}^2,$$

where κ is the ratio of the coupling constants in diquark to meson sectors in (1).

The generating functional (3) is the basic point for calculating for the Feynman diagram in which the $\bar{q}qM$ and $q^TqD^+(\bar{q}.\bar{q}^TD)$ Yukawa types vertices with coupling constants (7) are implied by the last terms in the exponent. Since

$$\frac{g_i^2}{4\pi} \sim \frac{1}{ln(\Lambda^2/M^2)} \ll 1, \quad (12)$$

it can be used as the power expansion parameters.

3. Fixing the parameters

By now the theory formally contains the current quark mass m_0, the cutoff Λ, the coupling constants G as parameters. For their fixing we use

the following conditions:
The width of $\rho \to \pi\pi$ decay is determined by the coupling constant $g_\rho = 2g_2$. Experiment shows that $(2g_2^V)^2/4\pi \simeq 3$ thus yielding

$$I_2(M,\Lambda) \simeq (24\pi)^{-1}. \quad (13)$$

The constituent quark mass and cutoff Λ can be obtain from the Goldberger-Treiman relation

$$M = f_\pi g_1^\pi = f_\pi\ [(1+a)/12I_2]^{1/2};\quad f_\pi = 93.3MeV$$

and eq.(13). If the $(P-A)$ - mixing is ignored, we obtain $M = 234MeV/c^2$. If we use the experimental value for A_1 - meson ($\sim 1.27GeV$ [24]), then $M \simeq 276MeV/c^2$, if we use the theoretical formula $m_{A_1}^2 = m_\rho^2 + 6M^2$, we obtain $M \simeq 350MeV/c^2$. On the other hand, the absence of the confinement mechanism may also distort the estimation. So, in further analysis we will use the mass of a constituent quark in the range of $234 \div 350MeV/c^2$, remembering that eqs.(13),(14) give $M \simeq 350MeV/c^2$, which qualitatively agrees with the constituent quark model prediction.

The coupling constant G_2 is fixed by the ρ - meson mass: $G_2 = (8I_2m_\rho^2)^{-1}$. The coupling constant G_1 and m_0 may be obtained from the gap equation and the π - meson mass.

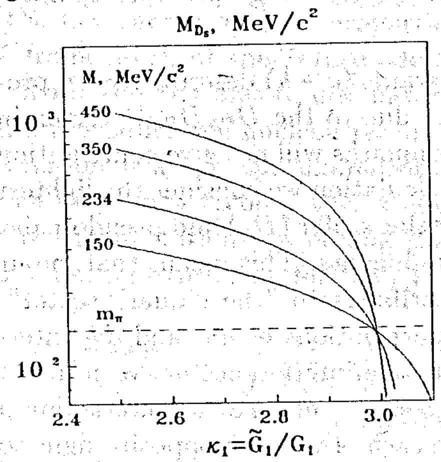


Fig.1. Scalar diquark mass for different constituent quark masses.

For the diquark sector we also need the coupling constants $\tilde{G}_{1,2}$ or $\kappa_{1,2} = \tilde{G}_{1,2}/G_{1,2}$ which have influence on the diquark masses. Figure 1 shows the dependence of the isoscalar diquark mass on κ_1. One can see that as $\kappa_1 \to 3$, the isoscalar diquark mass is closer (or smaller) than the pion mass. In the case of complete identify of the coupling constants in the $(\bar{q}q)$ and (qq) - channels ($\kappa_1 = \kappa_2 = 1$) diquark mass comes out several times larger than expected from the naive constituent model $m_D \sim 2M \sim 2/3m_N$ where m_N is the nucleon mass. In further calculation we choose

m_D (or κ) as a free parameter and suppose $m_D = 2M(1+\delta)$, with $|\delta| \ll 1$, which is in agreement with the argumentation of ref. [22].

4. Vector meson production in *pp*-collision

The main Feynman diagrams of the vector meson production in qq - interaction in the lowest order of the $(g_i^2/4\pi)$ expansion are depicted in Fig.2. The first four graphs correspond to the one-boson-exchange contribution and the remaining graphs describe the interactions with di-

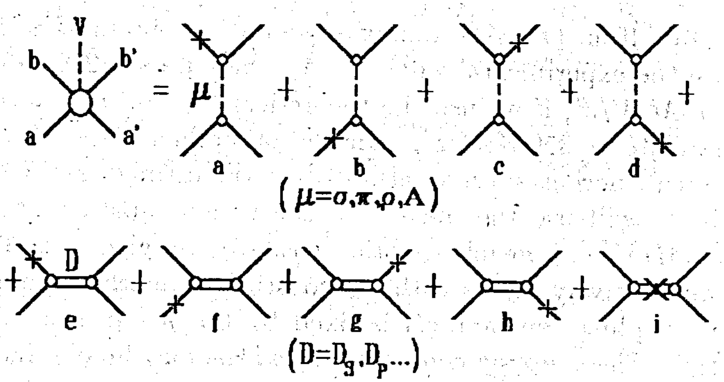
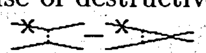
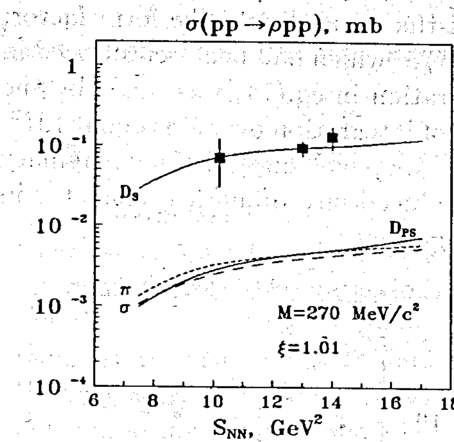


Fig.2. Subprocesses $qq \to Vqq$. The crosses on graphs correspond to the $\bar{q}qV$ and DD^*V - vertices.

quarks in intermediate states. The diagrams $(a-h)$ describe meson production due to the qqV-interaction and due to the $D_{PS}D_{PS}V$-vertex in the last graph. In the case of isoscalar diquarks will not give contribution to the ω/ρ meson production. In our calculation we consider the lightest $(\pi, \sigma, \rho, \omega)$ mesons and the lightest isoscalar scalar (D_S) and pseudoscalar (D_{PS}) diquarks which consist of ud-components. This means that in our approximation only ud-interaction contributes to "the diquark sector". In "the meson exchange sector" the contributions of uu- and dd- interactions are negligible because of destructive interference between direct and exchange diagrams:  ets. For ud-interaction ρ and ω exchange amplitudes are equal to each other with opposite signs so $\rho+\omega$ exchange may be omitted. The amplitude T_i^{ud} of the subprocesses shown in diagrams $(a-i)$ in Fig.2 is calculated in a straight forward way by using the Feynman rules with parameters given by eqs. (7), (10), (11), (13).

Calculation of the cross section of $pp \to Vpp$ process needs not only



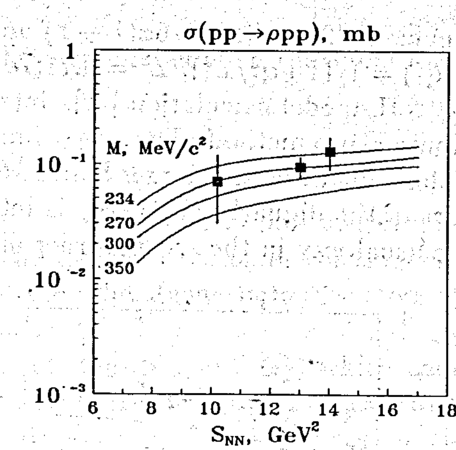


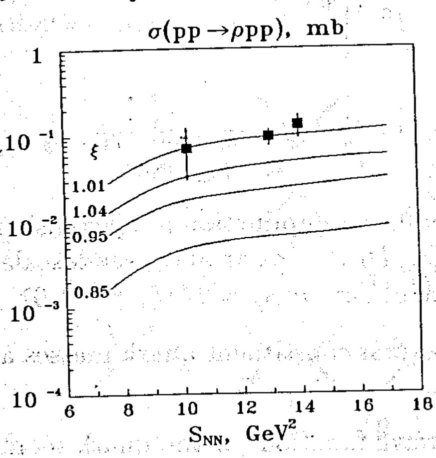
Fig.3. Different mechanisms of ρ-meson production in pp-collision. "π, σ": one boson exchanges; "D_S, D_{PS}": scalar and pseudoscalar diquark contributions; $M = 270 MeV/c^2$, $m_{D_S} = 2M\xi$, $\xi = 1.01$.

Fig.4. ρ meson production for several constituent quark masses at fixed $\xi = m_{D_S}/2M = 1.01$.

amplitudes T_i^{ud} but also the quark wave function (of the quark momentum distribution) of the nucleons in initial and final states. Description of the low energy nucleon properties and the nucleon wave function in the frame work of the NJL-model is an independent and complicated problem. But the first results have shown [17], [18] that the NJL-model could reproduce the nucleon radius and form factor. For our qualitative estimation as a first approximation we use a simple and objective model which takes into account this result and the assumption of the quark - string fragmentation model [25], [26] on the quark distribution. We use the following expression for the $pp \to Vpp$ cross section:

$$\sigma^{pp\to Vpp} = \int \frac{(1-4M^2/s')^{1/2}}{4(4\pi)^4 s(s-4M^2)} \times \sum_i |4\, T_i^{ud\to Vud}(s,s',\tau,x,y)\, F(\mathbf{q}_1^2)\, F(\mathbf{q}_2^2)|^2 \times f_1(x)\, f_2(y)\, \mathrm{d}\tau\, \mathrm{d}s'\, \mathrm{d}\Omega_{b'}\, \mathrm{d}x\, \mathrm{d}y, \qquad (14)$$

where $s = (P_a + P_b)^2 = ((x^2\mathbf{P}_n^2 + M^2)^{1/2} + (y^2\mathbf{P}_n^2 + M^2)^{1/2})^2 - \mathbf{P}_n^2(x-y)^2$, $\mathbf{P}_n^2 = s/2 - m_n^2$, $\tau = (P_b - P_V)$, $q_{1,2} = P_{a,b} - P_{a',b'}$, $s' = (P_{a'} + P_{b'})^2$. We choose the simplest space-volume form of the quark distribution in a nucleon [27]: $f(x) = 6x(1-x)$ and the usual dipole-like form factor: $F(\mathbf{q}^2) = 1/(1+\mathbf{q}^2/L^2)$, $L^2 = 0.71 GeV^2/c^2$ which had been reproduced in the NJL-model calculation [17]. Integration in eq.(14) was made by the Monte-Carlo method. The lower limit of integration over s' is equal $4M^2$. If the diquark mass is larger than $2M$ ("unstable" diquark) the imaginary part of the diquark self-energy is introduced into diquark propagator in the usual way in the second order of $g^2_{Dqq}/4\pi$.



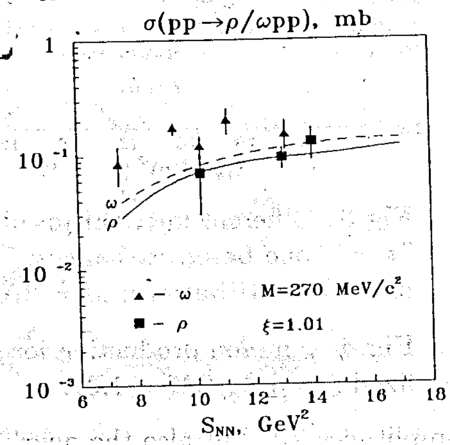


Fig.5. ρ meson production for several diquark masses at fixed $M = 270 MeV/c^2$.

Fig.6. ρ and ω meson production. $M = 270 MeV/c^2$, $\xi = 1.01$.

The results of calculation of the vector meson production as a function of the nucleon-nucleon center of mass-energy squared are shown in Figs.3 - 5. Experimental data are taken from ref.[28]. Figure 3 shows the contribution from different mechanisms with $m_{D_S} = 2M\xi$, $M = 270 MeV/c^2$, $\xi = 1.01$. One can see that the main contribution comes from the diquark mechanism diagrams $(e-h)$ with a scalar diquark and diagram (i) with a pseudoscalar diquark. Boson exchanges give a small contribution mainly because of an additional τ-dependence. The cross section decreases with M, m_D increasing at fixed ξ ($\xi \sim 1.01$) - Fig.4 and $|1-\xi|$ increasing at fixed M ($M = 270 MeV/c^2$) - Fig.5. Figure 6 shows that the cross sections of ρ and ω production are close to each other, and choosing $m_{D_S} \sim 2M$ with $M = 270 MeV/c^2$ in principal one can qualitatively describe the experiment. In the case of $m_D \sim M$ the diquark contribution gets compared with the meson exchange and the total theoretical prediction becomes several times smaller than the experimental data.

5. Summary and conclusion

In the present paper we study the vector meson production within the Nambu-Jona-Lasinio model. The model Lagrangian contains chiral invariant four-quark interactions in the meson and diquark sectors with different values of the coupling constants. This means that the model includes at least two additional parameters ($\tilde{G}_1$ and $\tilde{G}_2$) which cannot be fixed only from analysis of the meson properties. The parameters determine the diquark masses and, as a consequence, the cross sections of the processes which include the diquark degrees of freedom. We have analyzed the diagrams which give the main contribution to the vector meson production in a collision of two constituent quarks. The extension of the model to the nucleon-nucleon collision needs some assumption about the properties of the quark-diquark vertex of a nucleon. We have made some simplest assumption on the vertices (form factors) and shown that in principle one can choose the parameters of the NJL-model so as describe the chiral phenomenology and the production processes if one chooses the scalar diquark mass approximately equal to two constituent quark masses with the quark mass in the vicinity of $300 MeV/c^2$. These values of diquark masses are close to those discussed in ref.[22]. A more complete and consistent analysis of this problem will be done in our further investigations.

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