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QUANTUM FIELD RENORMALIZATION GROUP IN GYROTROPIC MAGNETIC HYDRODYNAMICS

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1 Introduction

The application of the quantum-field theory (QFT) method, including a renormalization group technique (RG), to hydrodynamics has shown the ability of QFT to describe general physical laws of the isotropic turbulence. This approach has been successfully used for the theoretical explanation of the phenomenological Kolmogorov power law [1, 2]. Furthermore, the theoretical estimation [3, 4, 5] of Kolmogorov constant was done. The RG technique has also been applied to find the critical indexes of composite operators presented in energy-momentum conservation laws for the turbulent liquid [6], in a passive admixture problem [7] and in stochastic magnetic hydrodynamics (MHD) [8]. The influence of the anisotropy on the turbulent current in above mentioned framework was recently studied [9, 10].

In this paper the RG technique in the quantum field model of gyrotropic MHD is used. The results obtained for ordinary MHD [8] are generalized to the case of the gyrotropic medium. The well known stochastic model of the stationary isotropic turbulence for the incompressible medium - Wyld model (WM) [11] is considered as a starting point. It is expressed in terms of the Navier-Stokes equation for the velocity field and the equation for magnetic field driven by Gaussian random forces with given 2×2 matrix D of the hydrodynamic, magnetic and mixed noise correlators. The forces simulate the stochasticity of the medium. The concrete form of D is to be chose on the basis of certain phenomenological conceptions of the internal turbulence mechanism.

In ref. [8] the multiplicative renormalizability of the quantum field MHD has been proved for the general form of this matrix. It provided the possibility to apply the whole standard RG technique and to study an asymptotic behaviour of the theory. The existence of two infrared-stable fixed points was established. These points induce the existence of two critical regimes: the trivial magnetic regime and the kinetic one (the later is of the Kolomogorov type).

The main problem of stochastic turbulence is an infrared one. It consists in the existence of infrared singularities in series of a perturbative theory for the correlation functions in the case of the physical energy pumping. The direct summation of this singularities is not a trivial problem. This situation has its counterpart in the critical phenomena theory. It could be solved by roundabout and well known way using RG technique in both cases.

There is an additional problem in the gyrotropic MHD: the instability of the theory, induced by the exponential increasing of the magnetic fluctuations in the large scales range(see e.g. [12]). The elimination of this instability leads to the formation of large-scale magnetic field. This type of the hydrodynamic energy transfer to the magnetic energy, by the instability mechanism, is called a turbulent dynamo. The removal of the instability in quantum field gyrotropic MHD can be achieved by means of a nice and very well known spontaneous symmetry breaking mechanism with the following creation of homogeneous stationary magnetic field. The special case, when only the hydrodynamic noise does not vanish, was shown in [13] and the results of the RG analysis for the ordinary MHD [8] were used. In this particular case, only the Kolmogorov critical regime is presented in ordinary MHD, and the gyrotropic MHD is not different from the ordinary one in meaning of critical behaviour. Thus, the spontaneous creation of magnetic field is a property of critical Kolmogorov regime.

The critical properties of the gyrotropic MHD are not known in the case of the arbitrary noise matrix D. To provide the multiplicative renormalizability and consequent application of RG to be necessary to extend the theory by means of the extra dissipative terms with a new gyrotropic Prandtl numbers. Therefore, also a critical behaviour of this gyrotropic MHD is more complicated. Apriori, the existence of the former stable regime of the Kolmogorov type is not clear. If it is existed, the answer to the question about the attractive region of such regime is necessary.

In one-loop approximation, the existence of the critical regimes mentioned above for ordinary MHD are also demonstrated in gyrotropic MHD. The large range of the physically allowed quantities of the gyrotropic Prandtl numbers is from the attractive region of the Kolmogorov regime. It was established solving a Gell-Mann-Low equations for the invariant charges.

2 The formulation of the problem

The interaction of electrically neutral conductive turbulent medium (with the unitary magnetic permeability) with the magnetic field is described by the MHD equations driven by random forces. These equations for the incompressible medium have the following form (see, i.e. [8]):

$$\nabla_t \varphi = \nu \Delta \varphi - (\theta\partial)\theta - \partial p + F^{\varphi} \tag{1}$$
$$\nabla_t \theta = \nu' \Delta \theta - (\theta\partial)\varphi + F^{\theta} \tag{2}$$

where $\nabla_t = \partial_t + (\varphi\partial)$ is a covariant derivative. The first equation is the known Navier-Stokes equation for transverse velocity field $\varphi(x) = \varphi_i(\mathbf{x}, t)$ with the additional nonlinear contribution of the Lorentz force (the longitudinal contribution is ascribed to pressure p).The second equation for magnetic field $\theta(x) = \theta_i(\mathbf{x}, t)$ (it is connected with magnetic induction $\mathbf{B}$ by the relation $\theta_i = B_i/\sqrt{4\pi\varrho}$, where ϱ is a medium density) follows from the Maxwell equations for continuous medium. The magnetic diffuse coefficient ν' coincides with the coefficient of molecular viscosity in the dimensionality. Further, the relation $\nu' = u\nu$ is used with dimensionless inverse magnetic Prandtl number (PN) u.

The random forces are assumed to have a Gaussian distribution with $< F >= 0$ and given 2×2 matrix of the noise correlators $D =< FF >$. The matrix elements are: the hydrodynamic $D^{\varphi\varphi}$ noise, the magnetic $D^{\theta\theta}$ one and the mixed $D^{\varphi\theta}$ one. They simulate the specific form of the energy pumping into system, which compensates the dissipative losses. On the one hand, the symmetries of the system restrict the form of noises, and, on the other hand, these noises make the possibility of statistical simulation of some symmetry breaking, for example, the reflex symmetry. Therefore, the form of the equations for the ordinary and gyrotropic MHD is the same. The difference is only in the form of the noises (this form will be set later).

The problem (2) is equivalent to quantum theory with a double number of the fields $\Phi = \varphi, \theta, \varphi', \theta'$ in accordance to the general theorem of stochastic quantization [2]. The corresponding action take the following form:

$$S(\Phi) = \frac{\varphi' D^{\varphi\varphi}\varphi'}{2} + \frac{\theta' D^{\theta\theta}\theta'}{2} + \frac{\varphi' D^{\varphi\theta}\theta'}{2} + \frac{\theta' D^{\theta\varphi}\varphi'}{2} + \varphi'[-\partial_t\varphi + \nu_0\Delta\varphi$$
$$-(\varphi\partial)\varphi + (\theta\partial)\theta] + \theta'[-\partial_t\theta + u_0\nu_0\Delta\theta - (\varphi\partial)\theta + (\theta\partial)\varphi] \quad (3)$$

Hereafter in the similarly expressions, the integration over **x**, t and the traces over the vector indexes are implied. The auxiliary fields φ', θ' have the same tensor structure as the fields φ, θ, i.e. they are vectorial and transversal. As it is usually in QFT, the action (3) is considered to be unrenormalized with the bare constants marked by the subscript "0" . The basic objects of the study are the Green functions of the fields Φ or the correlation functions and response functions in the terminology of the original problem (2). They can be determined by means of the generating functional G:

$$G(A) = \int D\Phi exp[S(\Phi) + A\Phi] \quad (4)$$

Here, $D\Phi$ denotes the functional measure of the integration over the fields Φ with all normalization coefficients. The Green functions are the functional derivatives with respect to an external sources $A = A^{\varphi}, A^{\theta}, A^{\varphi'}, A^{\theta'}$, i.e. they are the functional averaged values of the corresponding number of the fields ϕ with a weight $exp[S(\phi)]$. The equivalence of (2) and QFT (3) means, that Green functions determined by (4), are equaled to correlation ones, obtained immediately by the averaging of the solution of the equations (2) with a weight $exp[-\frac{1}{2}FD^{-1}F]$ over the external random forces.

The Feynman diagrammatic expansion of the Green functions is constructed with the aid of the action (3).The matrix of propagators (the lines in the diagrams) $\Delta = \mathbf{K}^{-1}$ can be obtained from the squared part of the action $\frac{1}{2}\phi\mathbf{K}\phi$. The explicit form of the matrix Δ is given in appendix I for the extended theory. The non-squared terms in (3) generate the vertexes in the diagrams. After the symmetrization, we have:

$$\varphi'(\varphi\partial)\varphi = \frac{1}{2}\varphi_i' v_{isl}\varphi_s\varphi_l \qquad \varphi'(\theta\partial)\theta = \frac{1}{2}\varphi_i' v_{isl}\theta_s\theta_l$$

$$\theta'(\varphi\partial)\theta - \theta'(\theta\partial)\varphi = \theta_i'\bar{v}_{isl}\theta_s\varphi_l$$

$$v_{isl} = i(k_l\delta_{is} + k_s\delta il) \qquad \bar{v}_{isl} = i(k_s\delta il - k_l\delta is) \quad (5)$$

Now, we choice the concrete form of the noise matrix D in the momentum-frequency $(\mathbf{k}, \omega)$ representation. These noises are transversal for the incompressible liquid. The action (3) must be a scalar in the ordinary (non-gyrotropic) MHD. The field φ is a vector, θ - a pseudovector, therefore the noises $D^{\varphi\varphi}$ and $D^{\theta\theta}$ are the tensors, i.e. they are proportional to transverse projector $P_{is} = \delta_{is} - k_ik_s/k$ ($k \equiv \mid \mathbf{k} \mid$), and the noise $D^{\varphi\theta}$ is a pseudotensor. There is only one transverse pseudotensor of second rank - the trace $\varepsilon_{isl}k_l/k$ (ε_{isl} - full antisymmetric tensor of third rank)

The action for gyrotropic MHD can possess a scalar terms as well as a pseudoscalar ones. Hence, the tensor structure of all noises is a linear combination of both tensor and pseudotensor. The correlators have a following form:

$$D^{\varphi\varphi}_{is} = g_0\nu_0^3k^{4-2\mu-2\epsilon}\mathbf{P}^1_{is} \qquad D^{\theta\theta}_{is} = g_0'\nu_0^3k^{4-2\mu-2a\epsilon}\mathbf{P}^2_{is}$$
$$D^{\varphi\theta}_{is} = g_0''\nu_0^3k^{4-2\mu-(1+a)\epsilon}\mathbf{P}^3_{is} \quad (6)$$

Here, $\mathbf{P}_{is} = P_{is} + i\rho\varepsilon_{isl}k_l/k$ with some new dimensionless real parameters $\rho \equiv \rho_1, \rho_2, \rho_3$, satisfying the condition $\mid \rho \mid \leq 1, \rho_3^2 \leq \mid \rho_1\rho_2 \mid$. Simultaneously, the standard scalar parts [8] of the noises are explicitly written in (6). The constants g_0, g_0', g_0'' play a role of the bare coupling ones, 2μ is the dimension of the space **k** (we shall be interested by the case $2\mu = 3$, at last), a, ϵ are free parameters of the theory. The value $\epsilon = 2$ corresponds to the Kolmogorov energy pumping from infra-red region of the small momentums **k**. Notice, that the parameter ϵ is independent on the dimension of the space. In dimensional regularization, which we use, it plays the same role as the analogical one in known $(4-\epsilon)$ Wilson scheme [14].

3 Renormalization

As usually, we solve the initial infrared problem for the physical value $\epsilon = 2$ by transfer to the region of such values ϵ, where the ultraviolet divergences (UD) appear. For the value $\epsilon = 0$ (the limited case) the theory is becoming logarithmic one (the bare coupling constans are becoming dimensionless). In this case the power of the UD is independent on the order of the diagrammatic expansion and these UD can be eliminated by some of the procedures of the ultraviolet renormalization. After that, the RG technique can be applied and the return to the former physical value of the parameter ϵ is possible. In dimensional regularization the UD manifest themselves like the poles of ϵ. They can be eliminated by the addition of the appropriated counterterms to the "intermediate" action, which could be obtained by the replacement of the bare parameters e_0 in (3) to renormalized ones e: $e_0 \to eM^{d_{e_0}}$, where d_{e_0} is the scaling dimension e_0 (see later), M is a scale setting parameter. The counterterms are formed of the superficial UD, which are presented to one-particle irreducible (1-PI) Green functions [15]. If these counterterms have the same form as the terms of action (3), the UD can be eliminated by redefinition the parameters of the original QFT. The theory is becoming multiplicatively renormalizable.

The classification of the UD by power counting is possible. MHD is a double scaling theory. All parameters and fields have a momentum d^p, frequency d^ω and total $d = d^p + 2d^\omega$ scaling dimensions [8]. They are shown in table.

Table

	φ, θ	φ', θ'	ν_0, ν	M	g_0	g_0'	g_0''	$g, g', g'', u, u_0, \rho_0$
d^p	-1	$2\mu+1$	-2	1	2ϵ	$2a\epsilon$	$(1+a)\epsilon$	0
d^ω	1	-1	1	0	0	0	0	0
d	1	$2\mu-1$	0	1	2ϵ	$2a\epsilon$	$(1+a)\epsilon$	0

The scaling dimensionality of N_Φ-particles 1-PI Green function Γ is

$$d_\Gamma = 2\mu + 2 - N_\Phi d_\Phi, \qquad N_\Phi d_\Phi = N_\varphi d_\varphi + N_{\varphi'} d_{\varphi'} + N_\theta d_\theta + N_{\theta'} d_{\theta'}$$

The superficial UD are simple polynoms of the momentum and the frequency. The power of these polynoms is determined by the formal ultraviolet index of the divergence δ (UI). In the logaritmic theory ($d_{g_0} = 0$ for all g_0) it takes place $\delta = d_\Gamma$. If $\delta \geq 0$ the diagramm possesses the UD. In this theory the "real" UI $\delta' = \delta - N_{\varphi'} - N_{\theta'}$ is used, too [2]. It shows, that the real power of the divergence of the diagrams with external fields φ', θ' is always less then the formal one. It should be noted, that 1-PI

Green functions with $N_\varphi = N_\theta = 0$ are equal to zero [2].

The following 1-PI Green functions can posses the superficial UD:

$$< \varphi'\varphi > \quad < \theta'\theta > \quad < \varphi'\theta > \quad < \theta'\varphi > \quad (\delta = 2, \delta' = 1)$$

$$< \varphi'\varphi\varphi > \quad < \varphi'\theta\theta > \quad < \varphi'\varphi\theta > \quad < \theta'\varphi\theta > \quad < \theta'\theta\theta >$$
$$(\delta = 1, \delta' = 0)$$

In the ordinary MHD only the 1-PI Green functions $< \varphi'\varphi >$ $< \theta'\theta >$ and the vertex $< \varphi'\theta\theta >$ possess the UD [8]. The corresponding counterterms are: $\sim \nu\varphi'\Delta\varphi$, $\nu\theta'\Delta\varphi$ and $\varphi'(\theta\partial\theta)$. Other Green functions have not the UD due to Galilelian invariance (GI), reflex symmetry and the property of the transverse of antisymetric vertex (5) (TV): $k_i \bar{v}_{i,s,l} = 0$.

In gyrotropic MHD the reflex symmetry is broken, therefore, the UD can be presented in 1-PI Green functions with odd sum of the external fields θ, θ'. The vertexes $< \varphi'\varphi\theta >$, $< \theta'\theta\theta >$ do not possess the UD due to the same reasons - GI and PTV. Thus, UD remain only in Green functions $< \varphi'\theta >$, $< \theta'\varphi >$ and generate the counterterms $\sim \nu\varphi'\Delta\theta$, $\nu\theta'\Delta\varphi$. The terms of such form aren't presented in the former action (3). For this reason, it is necessary to consider the extended theory with the additional cross dissipative terms $v\nu\varphi'\Delta\theta$, $w\nu\theta'\Delta\varphi$. We will call a new totally dimensionless parameters v, w as the inverse gyrotropic magnetic Prandtl numbers (GPN). To determine the physical region of the value of both GPN and PN to solve the linearized equations MHD (2) without external forces. These equations with cross dissipative terms have the following vector form:

$$\partial_t \phi = \tilde{\nu}\Delta\phi, \quad \text{where} \quad \phi = \begin{pmatrix} \varphi \\ \theta \end{pmatrix} \quad \tilde{\nu} \equiv \nu \begin{pmatrix} 1 & v \\ w & u \end{pmatrix}$$

The solution is (in the momentum-time representation):

$$\phi(\mathrm{k}, t) = exp(-\tilde{\nu}p^2 t\phi(\mathrm{k}))$$

. The physical solutions must be attenuated, thus the real parts of the eigenvalues (EV) of the matrix $\tilde{\nu}$ must be positive. The physical region of PN and GPN can be determined from this condition. The EV are equal to $(1+u)(1 \pm \sqrt{1 - 4(u - vw)/(1+u)^2}/2$. In the general case they are complex. If simultaneously, $u > -1, u \geq vw, vw \geq -(1-u)^2/4$ then the EV are real and positive. The corresponding solutions are clean-attenuated. The oscillated and attenuated solutions are obtained for the case $u \geq -1$ and $vw \leq -(1-u)^2/4$.

It is necessary to emphasize a following comment. The orders of using dimensional regularization are only a set of some formal rules, allowing to simplify strongly the analysis of critical behaviour of the theory. There aren't the counterterms with dimensional parameters because the parameters of ultraviolet cutoff's type Λ are absent. However, these infrared-significant couterterms can appear in ordinary scheme with cut-off (for examples, the counterterm $\Lambda^2\phi^2$ in the ϕ^4-theory). The initial action must possess the corresponding "mass" terms. The renormalized coefficients at these terms must turn into zero in the critical regime. The dimensional scheme ignores both the terms of the initial action and corresponding counter terms. This procedure is selfconsistent and provides the correct results for the anomalous dimensions. In the gyrotropic MHD the Green functions $< \theta'\theta >, < \theta'\varphi >$ (in case $2\mu = 3$) also possess the divergences of such type. The corresponding counterterms have the form $\chi\Lambda\theta' rot\theta$, $\bar{\chi}\Lambda\theta' rot\varphi$. In one-loop approximation $\chi = \chi_1 g + \chi_2 g' + \chi_3 g''$, $\bar{\chi} = \bar{\chi}_1 g + \bar{\chi}_2 g' + \bar{\chi}_3 g''$. The numerical coefficients depend on the parameter u, v, w. The first rotor term generates the instability of the theory. Therefore, its direct insertion into the action (3) is not allowed. This term can be eliminated in another way using the mechanism of the spontaneous symmetry breaking [13]. On the other hand, it can not be done for the arbitrary constants g, g', g'', but only in case if $g' = g'' = v = w = 0$ and $g, u \neq 0$. Later we will see, that it corresponds to Kolmogorov regime. That way we are interested in this regime. It can be easily see, that $\bar{\chi} = 0$ in this case. It means, that the second rotor term is also absented. Note, that the last term doesn't generate the instability.

4 RG analysis

In this section the extended gyrotropic MHD with the cross dissipative terms is considered. The corresponding action S^G has the following form:

$$S^G(\Phi) = S(\Phi) + v_0\nu_0\varphi'\Delta\theta + w_0\nu_0\theta'\Delta\varphi \qquad (7)$$

Here, $S(\Phi)$ is the initial action (3). All UD can be eliminated using five independent renormalization constants $Z_i, i = 1\dots5$. We obtain the renormalized action:

$$\begin{aligned} S^G_R(\Phi) = {} & \frac{\varphi' D_R^{\varphi\varphi}\varphi'}{2} + \frac{\theta' D_R^{\theta\theta}\theta'}{2} + \frac{\varphi' D_R^{\varphi\theta}\theta'}{2} + \frac{\theta' D_R^{\theta\varphi}\varphi'}{2} + \\ & \varphi'[-\partial_t\varphi + Z_1\nu\Delta\varphi + Z_4 v\nu\varphi'\Delta\theta - (\varphi\partial)\varphi + Z_3(\theta\partial)\theta] + \\ & \theta'[-\partial_t\theta + Z_2 u\nu\Delta\theta + Z_5 w\nu\theta'\Delta\varphi - (\varphi\partial)\theta + (\theta\partial)\varphi] \qquad (8) \end{aligned}$$

Here D_R denotes the renormalized noises. The action (8) is connected with unrenormalized one (7) by the formulae of the multiplicative renomalization: $S^G_R(\Phi, e) = S^G(Z_\Phi, e_0)$, where Z_ϕ are the renormalization constants of the fields ϕ. The renormalized parameters e are related to the bare ones:

$$\begin{gathered} g_0 = gM^{2\epsilon}Z_g \quad g'_0 = g'M^{2a\epsilon}Z_{g'} \quad g''_0 = g''M^{(1+a)\epsilon}Z_{g''} \\ \rho_0 = \rho \quad \text{for all} \quad \rho \qquad \nu_0 = \nu Z_\nu \quad u_0 = uZ_u \quad v_0 = vZ_v \quad w_0 = wZ_w \qquad (9) \end{gathered}$$

$$\begin{gathered} Z_g = Z_1^{-3} \quad Z_{g'} = Z_1^{-3}Z_3 \quad Z_{g''} = Z_1^{-3}Z_3^{1/2} \\ Z_\nu = Z_1 \quad Z_u = Z_1^{-1}Z_2 \quad Z_v = Z_1^{-1}Z_4 \quad Z_w = Z_1^{-1}Z_5 \\ Z_\varphi = Z_{\varphi'} = 1 \quad Z_\theta = Z_{\theta'}^{-1} = Z_3^{1/2} \qquad (10) \end{gathered}$$

The RG-functions - β-functions and the anomalous dimensions of the fields γ_Φ and the parameters γ_e can be expressed by the renormalization constants Z:

$$\gamma_{e(\Phi)} \equiv \tilde{\mathcal{D}}_M lnZ_{e(\Phi)} \quad \beta_g \equiv \tilde{\mathcal{D}}_M g \quad g \equiv g, g', g'', u, v, w \qquad (11)$$

Here $\tilde{\mathcal{D}}_M \equiv M\frac{\partial}{\partial M}|_{e_0}$ denotes the derivative with respect to the parameter M at the fixed values of the bare parameters e_0. Later one will be used the similar operation $\mathcal{D}_M \equiv M\frac{\partial}{\partial M}|_{e_0}$ at the fixed values of the renormalized parameters e. Using (9), (10), (11) we obtain the following expression for the β-functions:

$$\begin{gathered} \beta_g = g(-2\epsilon + 3\gamma_1) \quad \beta_{g'} = g'(-2a\epsilon + 3\gamma_1 - \gamma_3) \\ \beta_{g''} = g''(-\epsilon - a\epsilon + 3\gamma_1 - 1/2\gamma_3) \quad \beta_u = u(\gamma_1 - \gamma_2) \\ \beta_v = v(\gamma_1 - \gamma_4 + 1/2\gamma_3) \quad \beta_w = w(\gamma_1 - \gamma_5 + \gamma) \qquad (12) \end{gathered}$$

Replacing $g'' = \alpha(gg')^{1/2}$ one can see, that $\beta_\alpha = 0$. It means that α isn't a charge but only an arbitrary parameter. Its value isn't fixed. It follows that the theory is five-charged.

5 One-loop approximation

In this section the constants Z and RG-functions are calculated in one-loop approximation. We use the minimal subtraction scheme, where the constants Z possess only the poles over the parameter ϵ.

The corresponding Feynman graphs of the 1-PI Green functions $<\varphi'\varphi>$, $<\varphi'\theta>$, $<\theta'\theta>$, $<\theta'\varphi>$, are shown in fig.1. Twenty four graphs are corresponded to the 1-PI vertex $<\varphi'\theta\theta>$. Six typical Feynman graphs are shown in fig.2. The remaining graphs can be obtained by all permutations of the fields in the internal lines.

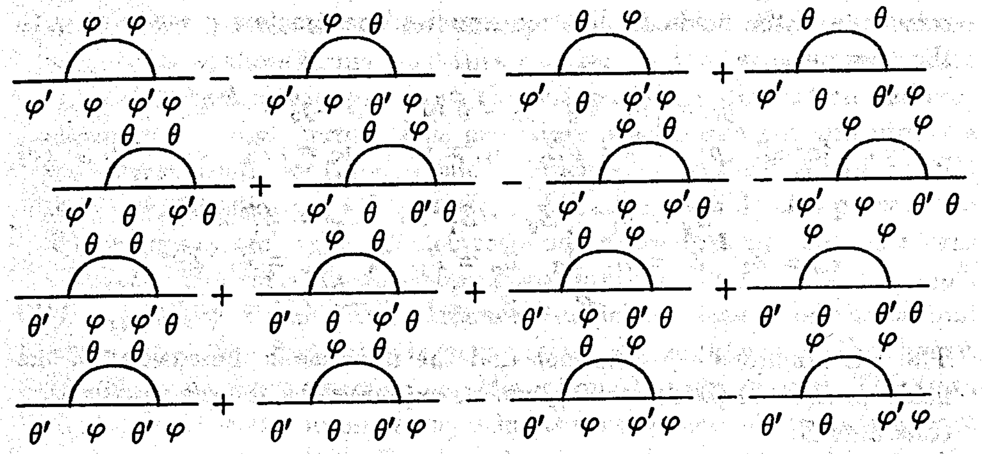

Figure 1:

The calculations of the singular parts of the graphs give the following expressions for constants Z:

$$Z_i = 1 - A_{i1}\frac{g_1}{\epsilon} - A_{i2}\frac{g_2}{a\epsilon} - 2A_{i3}\frac{\alpha(g_1g_2)^{1/2}}{(1+a)\epsilon} \quad i = 1\dots 5 \tag{13}$$

where $g_1 \equiv g/B$, $g_2 \equiv g''/B$, $B = 2\mu(2\mu+2)(4\pi)^{\mu}\Gamma(\mu)$ (Γ - gamma-function). The coefficients A depend on the space dimension, PN u, v, w and are written in Appendix II. RG-functions $\gamma_i \equiv \tilde{\mathcal{D}}_M lnZ_i$ $(i = 1\dots 5)$ can be determined from the relation:

$$\gamma_i = [\beta_{g_1}\partial_{g_1} + \beta_{g_2}\partial_{g_2} + \beta_u\partial_u + \beta_v\partial_v + \beta_w\partial_w]lnZ_i \tag{14}$$

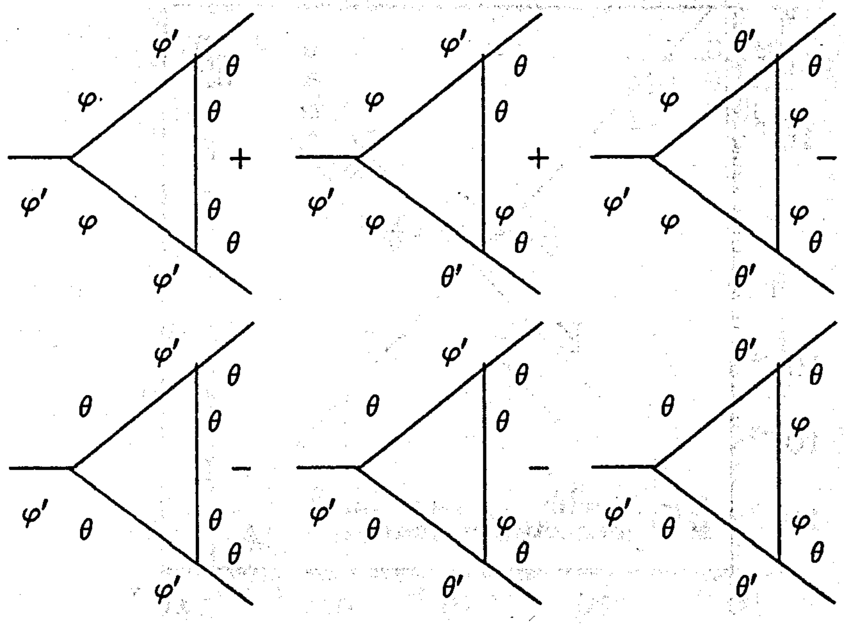

Figure 2:

For one-loop calculations of these functions it is necessary to set $\beta_{g_1} \simeq -2\epsilon g_1$, $\beta_{g_2} \simeq -2a\epsilon g_2$, $\beta_u \simeq \beta_v \simeq \beta_w \simeq 0$ in (14). We obtain

$$\gamma_i = 2[A_{i1}g_1 + A_{i2}g_2 + A_{i3}\alpha(g_1g_2)^{1/2}] \tag{15}$$

The substitution (15) into (12) gives β-functions :

$$\begin{aligned}
\beta_{g_1} &= g_1[-2\epsilon + 6A_{11}g_1 + 6A_{12}g_2 + 6A_{13}\alpha(g_1g_2)^{1/2}] \\
\beta_{g_2} &= g_2[-2a\epsilon + 2(3A_{11} - A_{31})g_1 + 2(3A_{12} - A_{32})g_2 \\
&\quad +2(3A_{13} - A_{33})\alpha(g_1g_2)^{1/2}] \\
\beta_u &= u[2(A_{11} - A_{21})g_1 + 2(A_{12} - A_{22})g_2 + 2(A_{13} - A_{23})\alpha(g_1g_2)^{1/2}] \\
\beta_v &= v[(2A_{11} - 2A_{41} + A_{31})g_1 + (2A_{12} - 2A_{42} + A_{32})g_2 \\
&\quad +(2A_{13} - 2A_{43} + A_{33})\alpha(g_1g_2)^{1/2}] \\
\beta_w &= w[(2A_{11} - 2A_{51} - A_{31})g_1 + (2A_{12} - 2A_{52} - A_{32})g_2 \\
&\quad +(2A_{13} - 2A_{53} - A_{33})\alpha(g_1g_2)^{1/2}]
\end{aligned} \tag{16}$$

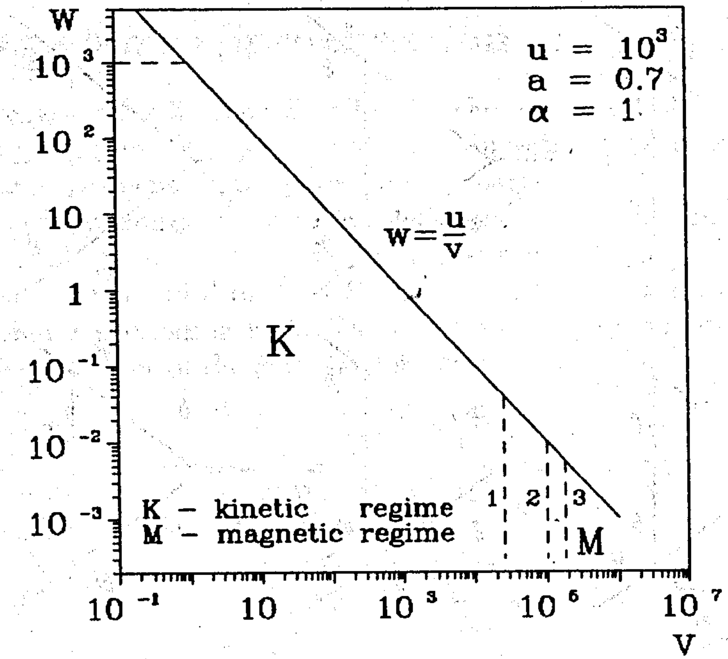

Figure 3:

The final goal of RG analysis is in the establishment of the asymptotic behavior of the theory. The renormalized Green functions satisfy the basic RG equation:

$$[\mathcal{D}_M + \sum_{i=1}^{5} \beta_{g_i} \partial_{g_i} - \nu\gamma_\nu \partial_\nu + \gamma_{\varphi'} N_{\varphi'} + \gamma_{\theta'} N_{\theta'}] W_N^R = 0 \qquad (17)$$

where $g_i \equiv g_1, g_2, u, v, w$ and W_N^R are the connected Green functions . The general solution of (17) is the arbitrary function of the first integrals - invariant charges (IC) $\bar{g}_i$ and invariant variable $\bar{z}$, which corresponds to the variable $z = \omega/\nu M^2$. They satisfy the following equations:

$$s\frac{d\bar{g}_i(s)}{ds} = \beta_{g_i}(\bar{g}(s)) \qquad \bar{g}_i(s)\,|_{s=1} = g_i \qquad (18)$$

$$s\frac{d\bar{z}(s)}{ds} = -[2 - \gamma_\nu(\bar{g}(s))]\bar{z} \qquad \bar{z}(s)\,|_{s=1} = z \qquad (19)$$

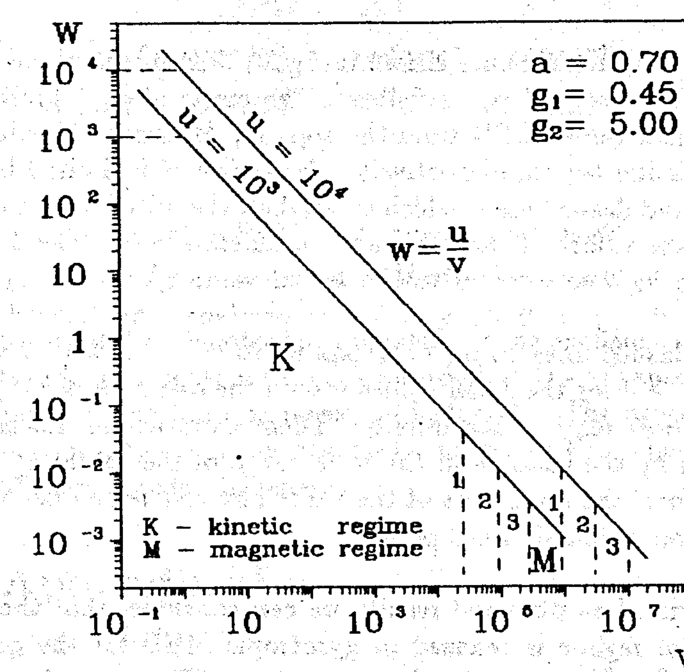

Figure 4:

where $s = p/M$. Formally, the asymptotic behaviour of the Green functions may be inferred finding the fixed point g_i^*, which is the solution of the equations $\beta_{g_i}(g^*) = 0$. From (16) we obtain five complicated algebraic equations with two arbitrary parameters a, α. The fixed points g_i^* are infrared-stable, if the matrix $\partial\beta_{g_i}/\partial g_k\,|_{g=g^*}$ is positively determined. There are two infrared-stable fixed points: the Gaussian $g_i^* = 0$ (it is stable if $a \leq 1.16$) and nontrivial $g_1 = 2\epsilon/9$, $u = 1.393$, $g_2 = u = v = w = 0$ (it is stable if $a \geq 0.25$). Note, that they coincide with fixed points (the magnetic and kinetic) earlier found in ordinary MHD. The kinetic fixed point provides the existence of asymptotic critical regime of the Kolmogorov type.

We numerically solved Gell-Mann-Low equations (18) for the various initial values of the invariant charges g_i. It provides the possibility to analyse the attracting regions of infrared fixed points. The initial values of g_1, g_2 are unknown, but initial values of $u \gg 1$ for the realistic medium, and the values of v, w are limited by the values u. In particular case, the attracting regimes are shown in fig.3 , fig.4 in the plane GPN w, v for several values g_1, g_2, u and α. The curve $w = \frac{u}{v}$ limits the region of physical values GPN from the top, K, M denote kinetic and magnetic attracting region, respectively. The region K is limited by the curve $wv = u$ and dashed lines, which depend on the initial values of the g_1, g_2, u and the values of the arbitrary parameters α, a. The 1-,2-,3-dashed lines in fig.3 correspond to the initial value $g_1 = 0.45$, $g_2 = 5$; $g_1 = 0.1$, $g_2 = 0.1$; $g_1 = 0.45$, $g_2 = 0.1$, respectively. Analogously, the same marked dashed lines on fig.2 correspond to $\alpha = 1$, $\alpha = 0$, $\alpha = -1$. If $wv \geq u$ or $w \geq u$ for the small values v then the solutions of (18) (the phase trajectories) tend to the infinity. These pictures are not significantly affected by the changes in the wide range of the values g_1, g_2, u and α. Therefore, the large part of the GPN physical region lies in the attracting region of kinetic fixed point.

On the basic of all obtained results we can conclude, that the Kolmogorov critical regime is realized in gyrotropic MHD for the general case of the arbitrary matrix of noise correlators. That way, the explanation of the generation of homogeneous magnetic field by means of the spontaneous symmetry breaking mechanism earlier accomplished in paper [13] is suitable to this case as well.

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6 Appendix I

The propagators of the extended theory have the following form:

$$\Delta^{\varphi\varphi'} = MR \quad \Delta^{\theta\theta'} = LR \quad \Delta^{\theta\varphi'} = -SR \quad \Delta^{\varphi\theta'} = -VR \qquad (20)$$

$$\Delta^{\varphi'\varphi} = R^T M^T \quad \Delta^{\theta'\theta} = R^T L^T \quad \Delta^{\theta'\varphi} = -R^T S^T \quad \Delta^{\varphi'\theta} = -R^T V^T \qquad (21)$$

$$\begin{aligned}
\Delta^{\varphi\varphi} &= RR^T(D^{\varphi\varphi} MM^T - D^{\varphi\theta} MV^T - D^{\theta\varphi} VM^T + D^{\theta\theta} VV^T) \\
\Delta^{\theta\theta} &= RR^T(D^{\varphi\varphi} SS^T - D^{\varphi\theta} L^T S - D^{\theta\varphi} LS^T + D^{\theta\theta} LL^T)
\end{aligned}$$

$$\begin{aligned}
\Delta^{\varphi\theta} &= RR^T(-D^{\varphi\varphi} MS^T + D^{\varphi\theta} ML^T + D^{\theta\varphi} VS^T - D^{\theta\theta} VL^T) \\
\Delta^{\theta\varphi} &= RR^T(-D^{\varphi\varphi} SM^T + D^{\varphi\theta} SV^T + D^{\theta\varphi} LM^T + D^{\theta\theta} LV^T), \qquad (22)
\end{aligned}$$

where

$$R = (LM - SV)^{-1} \quad V = v\nu k^2 \quad S = w\nu k^2$$
$$L = -\,\mathrm{i}\,\omega + \nu k^2 \quad M = -\,\mathrm{i}\,\omega + u\nu k^2$$

The superscript "T" denotes the operation of the exchange $\omega \to -\omega$ and $\mathbf{k} \to -\mathbf{k}$. The propagators (20) are retarded, (21) - advanced, and both are proportional to the transverse projector P_{is}. The propagators (22) are proportional to the mixed transverse projector $\mathbf{P}_{is}$. The $\Delta^{\varphi'\varphi'}$, $\Delta^{\varphi'\theta'}$, $\Delta^{\theta'\varphi'}$, $\Delta^{\theta'\theta'}$ are equaled to zero.

7 Appendix II

$$\begin{aligned}
A_{11} &= \frac{4\mu^2 - 2}{4st^2}(u^3 + u^2 - 2uvw - uw^2 + w^2) + \frac{2 - 2\mu}{4s^2t^2}(u^4 + \\
&\quad 2u^3 - u^2w^2 + u^2 - 2uvw - 4uw^2 + 2v^2w^2 + 2vw^3 - w^2)
\end{aligned}$$

$$\begin{aligned}
A_{12} &= \frac{4\mu^2 - 2}{4st^2}(uv^2 + u - v^2 - 2vw + 1) + \frac{2 - 2\mu}{4s^2t^2}(u^2v^2 - \\
&\quad u^2 + 4uv^2 + 2uvw - 2u - 2v^3w - 2v^2w^2 + v^2 - 1)
\end{aligned}$$

$$\begin{aligned}
A_{13} &= \frac{4\mu^2 - 2}{2st^2}(-u^2v + v^2w + vw^2 - w) + \frac{2 - 2\mu}{2s^2t^2}(-u^3v - \\
&\quad 3u^2v + uv^2w + uvw^2 + 3uw - wv^2 - vw^2 + w)
\end{aligned}$$

$$A_{21} = \frac{2\mu + 2}{2s^2tu}[(2\mu - 2)(-u^2 - u + vw - w^2) + u^2 + u - vw - w^2]$$

$$A_{22} = \frac{2\mu + 2}{2s^2tu}[(2\mu - 2)(-u - v^2 + vw - 1) - u + v^2 + vw - 1]$$

$$A_{23} = \frac{2\mu+2}{s^2tu}[(2\mu-2)(uv+w)-uv+w]$$

$$\begin{aligned} A_{31} = {} & \frac{1}{s^3t^2}(u^4+3u^3+3u^2+u-u^2v^2-3u^2vw-2u^2w^2- \\ & uv^2-4uvw-3uw^2+v^3w+3v^2w^2+3vw^3-vw+w^4-w^2) \end{aligned}$$

$$\begin{aligned} A_{32} = {} & \frac{1}{s^3t^2}(-u^3+u^2v^2-3u^2-3u+u^2vw+3uv^2+4uvw+uw^2- \\ & v^4-3v^3w-3v^2w^2+2v^2-vw^3+3vw+w^2-1) \end{aligned}$$

$$\begin{aligned} A_{33} = {} & \frac{2}{s^3t^2}(-u^3v-2u^2v+u^2w+uv^3+ \\ & 2uv^2w+uvw^2-uv+2uw-v^2w-2vw^2-w^3+w) \end{aligned}$$

$$\begin{aligned} A_{41} = {} & \frac{1}{4st^2v}[(4\mu^2-2)(-u^2v+u^2w+2uw-vw^2-w^3)- \\ & 2\mu(u^2v+u^2w-2uw+vw^2-w^3)]+\frac{1}{2s^3t^2v}(u^4v+u^4w- \\ & 2u^3w+2u^2v^2w+3u^2vw^2-u^2v-u^2w^3-5u^2w+4uv^2w+ \\ & 8uvw^2-2uw-2v^3w^2-4v^2w^3-2vw^4+vw^2-w^3) \end{aligned}$$

$$\begin{aligned} A_{42} = {} & \frac{1}{4st^2v}[(4\mu^2-2)(2uv-v^3-v^2w+v-w)- \\ & 2\mu(2uv+v^3-v^2w-v-w)]+\frac{1}{2s^3t^2v}(2u^3v+u^2v^3- \\ & u^2v^2w+5u^2v+u^2w-8uv^2w-4uvw^2+2uv+2v^4w+ \\ & 4v^3w^2+v^3+2v^2w^3-3v^2w-2vw^2-v-w) \end{aligned}$$

$$\begin{aligned} A_{43} = {} & \frac{1}{2st^2v}[(4\mu^2-2)(-u^2+uv^2-u+w^2)- \\ & 2\mu(-u^2-uv^2+u+w^2)]+\frac{1}{s^3t^2v}(-u^4-u^3v^2- \\ & u^3+2u^2vw+u^2w^2+u^2-2uv^3w-2uv^2w^2-uv^2- \\ & 2uvw+u+2v^2w^2+2vw^3+w^2) \end{aligned}$$

$$A_{51} = -\frac{(2-2\mu)(2\mu+2)u}{s^2t}$$

$$A_{52} = -\frac{(2-2\mu)(2\mu+2)v}{s^2tw}$$

$$A_{53} = \frac{2(2-2\mu)(2\mu+2)u}{s^2tw}$$

Here, $s=1+u$, $t=u-vw$.

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