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TWO FORMULATIONS OF RELATIVITY THEORY

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A weekly newspaper "Science" published in 1889 a short note [1], a letter to the editor, concerning the result which became later a famous experiment Michelson-Morley [2]. The hypothesis was suggested in it by G.F.Fitzgerald for explaining a negative result of the indicated experiment: "... the length of material bodies changes, according as they are moving... by an amount depending on the square of the ratio of their velocity to that of light". This article remained quite unnoticed and in 1892 H.Lorentz, a famous Dutch physicist, made a suggestion [3] that all moving objects undergo contraction in the direction of motion. Further on this effect was referred to as Lorentz-Fitzgerald contraction. It is expressed by the following simple formula

$$\ell_L = \ell_0 (1 - v^2/c^2)^{1/2} = \ell_0 \gamma^{-1} . \quad (1)$$

Here ℓ_0 and ℓ_L are the longitudinal sizes on an object at rest and in motion, respectively; v is the velocity of object motion, c is the light velocity, γ is the Lorentz-factor. The phenomenon of changing the longitudinal sizes of moving objects is one of the striking effects of relativism. As a matter of fact, it gave impetus to revising the space-time picture — the base of physical world description. The notion "relativism" itself embraces a wide range of questions and phenomena. Its foundation is relativistic theory which deals with the most common laws. This theory brought in revolutionary representation concerning space and time and criticized traditional conceptions formulated by Newton. So, it discovered new ways of comprehending natural phenomena. Just the theory of relativity served as the basis for relativization of many fields of physics beginning with electrodynamics mechanics, thermodynamics and so on. The establishment finitness of the velocity of light propagation by O.Römer (1676) should be considered as the initial point of relativism. The generally accepted present-day interpretation (its space-time structure is meant) of relativity theory is based on Einstein's definition of space sizes related to the determination of simultaneous positions of measurable object elements [4]. In other words, "instantaneous (or synchronous) distances" serve as the basis for this approach. Just the indicated condition (t = const. for the coordinates in a moving reference system) permits one to obtain the contraction effect of Lorentz-Fitzgerald as a necessary consequence of Lorentz transformations for coordinates, this basic "instrument" of relativity theory.

In 1949, i.e. 60 years after Fitzgerald's paper, B.Kwal's paper [5], having a kindred fate, appeared. It has remained unnoticed for a long time too. The article considered the problems of the electromagnetic field energy and momentum of a moving electron ("problem 4/3"). Apparently, exactly in this paper another formula was first proposed for the transformation of a space volume element instead of Lorentz habitual contraction, which can be expressed in terms of longitudinal sizes as follows:

$$\ell_r = \ell_0 (1 - v^2/c^2)^{-1/2} = \ell_0 \gamma. \qquad (2)$$

As is seen, according to eq.(2), the longitudinal sizes of moving objects should increase by a factor of γ. Perhaps, it should be noted here that 10 years later, in 1959, J.Terrell[6] had doubts that fast-moving bodies had to undergo Lorentz contraction. However, it should be stressed for justice that as long ago E.Fermi paid attention to this problem consisting in contradiction between the Abraham-Lorentz theory of electromagnetic mass and relativity theory. We should remind that the essence of this contradiction lies in the appearance of additional coefficient 4/3 in the expression for electromagnetic field momentum. Fermi's solution[7] was based on the covariant formulation of Hamilton's principle. This was provided by variation connected with the normal section of the world tube* of charge field whereas within the framework of the usual approach a variation is caused by the condition t = const. But just the latter condition leads to the generally accepted definition of the notion of length of a moving rod. Although in Fermi's approach the transformation formula for space volume did not figure, the introduction of the normal (nonsimultaneous) section of the world tube as a matter of fact was unevidently the mathematical base for another definition of the length of a moving rod. Moreover Fermi indicated that the usual approach (based on the condition t = const) evidently contradicted the relativity principle. However, nobody paid attention to this remark of him. Besides, if we restrict ourselves only to the condition itself, a similar statement can be found in Einstein's papers[8]. For example, we read in his article "Discourses concerning the foundations of theoretical physics": "... simultaneity of two space-distant events is not an invariant notion..."**.

In the sixties the relativistic formulation of statics (in particular, the famous paradox of Lewis-Tolman's rectangular lever) "the problem 4/3", the relativistic description of thermodynamics and so on were widely enough discussed. All this could be considered as the consequence of dissatisfaction of the existing solutions of these questions and finally the generally accepted approach itself. The introduction of the so-colled asynchronous formulation[9] should be considered as a result of that discussion. In essence, as stated above E.Fermi already discussed it. For example, in its framework the length of a moving rod is given by the distance between nonsimultaneous (asynchronous) position of its ends that reflects the name of the formulation. It is based on the mathematical eq.(2), i.e. within its scope the longitudinal sizes should increase

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**The continuous curve describing the behaviour of a point object in space-time, is called the world line. The set of such lines forms the world tube.*

***On the other hand, just such events-notches of a simultaneous position of the ends are used to determine the length of a moving rod.*

as a result of motion. However, at present this approach has not been generally recognized. In our opinion, this fact mainly due to the asynchronous interpretation was not based on a specific measurable procedure in contrast to the traditional (Einstein) approach. Moreover, this interpretation itself cannot give in principle such a procedure without reference to another (proper) reference system what is quite intolerable from the point of view of relativity principle. Perhaps, we have here a characteristic example that in physics theory the introduction of a new mathematical formula, allowing definite difficulties of this theory to be solved, is not yet enough for its acknowledgement. It is necessary that the quantities figuring in the formula should be based on physical measurements, i.e. on such a relativistic procedure in this case which does not depend on reference system.

The conception of relativistic length introduced at one time[10], which is based on the radar method of distance measurement, satisfies this condition undoubtedly. Reminding that in its framework the relativistic length (the length of a fast-moving rod) is called the half-sum of distances covered by a light signal forward and backward along the rod. Let for simplicity the rod be oriented and move to the direction of the x-axis from left to right . A signal is sent at the instant of flight of the left end. The light achieves the right end, is reflected there and goes back, to the left end. In the first case, when the signal achieves the right end, it covers the distance $\ell_f = (1 + v/c)\ell_0\gamma$. After refrection it travels the distance $\ell_b = (1 - v/c)\ell_0\gamma$. From here we in fact have the "elongation formula" (eq.(2)) for relativistic length $\ell_r = (\ell_f + \ell_b)/2$. It is evident that when $v \to c$ $\ell_r \cong \ell_f/2$. It should be emphasized that the quantities ℓ_f and ℓ_b define initially the distances between points which are taken at different instants of time. And they correspond exactly to two of the most characteristic modifications of retarded distances* when the electromagnetic field (described by the Lienar-Wiechert potential) propagates in the direction of source (charge) motion and in an opposite direction. Thus, one can say that the conception of relativistic length is a natural consequence of electrodynamics. Though the other thing cannot exist as this conception is based on the radar method of distance measerement.

As far as we know, space distances are compound elements of the space-time picture (structure) — the basis of bases of relativity theory. Therefore the transition from "instant distances" to light (or radar) ones means another (different from Einstein's) formulation of relativity theory. It should be noted that from the point of view of pure mathematics, the results obtained in terms of light distances can be always expressed through "instant" distances what is often done in electrodynamics (see, for example[12]). However, from the viewpoint of physics, this leads to a variety of paradoxes

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**As a matter of fact, the field at the given point is defined by the retarded (earlier) position of charge through which an electromagnetic interaction propagates with velocity c.*

and difficulties such as the Lewis-Tolman paradox, "the problem 4/3", the appearance of charge in a moving (neutral) conductor with current and so on*.

At first sight it seems that the considered alternative approach brings in little new since the radar procedure of measurement is in fact used in relativity theory from the very outset for sinchronization of distant clocks, in particular, if we take into account that at one time a new method of statement, the basis of which became observers having identical clocks and radars, has been suggested [13]. Rigid scales were thereby excluded from the theory. However, this approach was as a matter of fact of a more formal character as all previous conclusions corresponding to Einstein's approach remained valid due to the following transition to instantaneous distances. The essence of the other "radar formulation" consists in that it dealsexactly with distances between nonsimultaneous points directly observed in experiment (measured by the radar method). As is known, such distances are called retarded in electrodynamics. Light (radar) distances are their synonyms. As was noted, the main peculiarity of the alternative formulation is the increase of longitudinal sizes of relativistic objects with increasing their velocity according to eq.(2). Maybe, one of the most remarkable results is the cardinal change (in terms of retarded distances) of a behaviour character of an electromagnetic field. The equipotential surfaces of an electric field of a relativistic charge have the form of rotation ellipsoids stretched in the direction of motion (see fig.1). As is seen, the longitudinal sizes of a field as well as the spatial sizes of moving objects increase. Fields in front of the moving charge act on larger distances with increasing

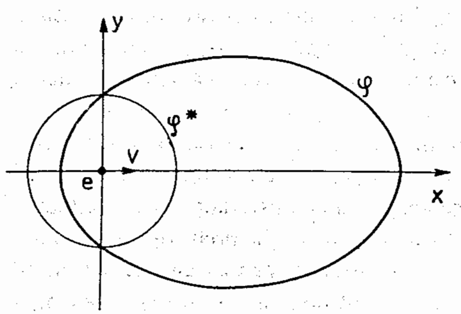

Fig.1. Lienard-Wiehert's equipotential ellipsoid, v = 0.75 c. The circle is the Coulomb equipotential.

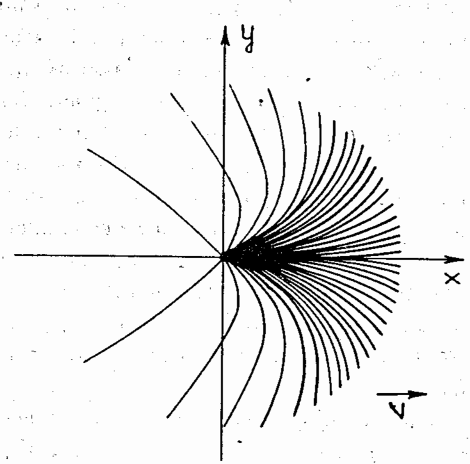

Fig.2. Force lines of the electrical field of a moving charge, v = 0.75 c.

As a result, one has to introduce fictitious quantities such as von Laue's energy flow, Poincare stresses and so on.

velocity. Therefore one can say that it is a kind of "relativistic long-range". The field of such a flying charge is presented by force lines in fig.2.

The new approach has not been generally recognized to date though some papers on this subject are going on to appear in the press. Laying aside the historical aspect of this problem, in our opinion, the main cause consists in that we as a matter of fact conceive space distances subconsciously only as distances between simultaneous point (events). When, say, the solution of the wave equation proves to be dependent on retarded distances, i.e. distances between nonsimultaneous points, we try to turn to a more habitual "instantaneous distance" whereas just retarded distances or quantities produced on their basis are observable in experiment. It is appropriate to remind here of the observability principle. According to this principle, notions and affirmations unaccessible to a direct empirical test should not take place in physics theory. Otherwise, unobservable quantities (and, such are in fact, "instantaneous distances" after establishing finiteness of the velocity of light propagation) should be eliminated from theory. The known Einstein (macroscopic) procedure of notch of simultaneous positions of the ends of a moving rod with the help of a set of preliminary synchronized clocks arranged in space seems quite realizable at first sight. However, the main field of application of relativity theory is the phenomenon of the microworld which Einstein's "macroscopic" procedure is simply unacceptable to. On the other hand, the features of "the radar procedure"* are seen at an attentive analysis of the relativistic effects. It is noteworthy that the modern quantum theory of fundamental interactions is based on quantum exchange, i.e. these interactions are of the type of sending and receiving quanta or, in other words, they (in particular electromagnetic) are of the "radar" type. Elementary particle physics, in particular in the region of high energies, has been found to deal in fact with relativistic length (and formation length related to it) [14].

However, it is appropriate here to dwell upon more known and habitual phenomena. As the specialists know, the diagram of the angular distribution of charge radiation stretches more forward in the direction of charge motion ("the projector effect") with increasing velocity. This is in full agreement with the behaviour of a relativistic charge field as described in figs.1 and 2. This result becomes particularly intelligible if we take into account that radiation can be treated as turning of virtual field photons to real one. Cherenkov radiation can serve as the most striking example [15]. Counters using Cherenkov radiation have found wide application in high energy physics. The velocity of fast particles is determined with their help on the basis of Cherenkov angle measurement. It is essential here that this angle is "light", i.e. is given exactly by the light distance. But the phenomenon of light aberration, discovered by Bradley as early as 1727 [16], is undoubtedly the very first example of manifestation of light distance.

Thus, on the other hand, light aberration and Cherenkov radiation can be considered as vey clear experimental evidence in favour of the radar formulation of relativity theory different from the traditional one.

Einstein's procedure of synchronization is based on the radar method by itself.

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