объединенный
институт
ядерных
исследований
дубна

E2-91-393

D.I.Kazakov, I.N.Kondrashuk*

LOW-ENERGY PREDICTIONS OF SUSY GUTS:
Minimal Versus Finite Model

Submitted to "International Journal of Modern Physics A"

*Physics Department, Leningrad State University

1991

1 Introduction

High accuracy measurements of the weak mixing angle $\sin^2\theta_W$ and the strong QCD coupling α_s at LEP made it possible to check the low-energy predictions of Grand Unified Theories and to abandon some of them. For this reason, e.g. the minimal non-supersymmetric SU(5) model is already excluded [2] (5% below the measured value of $\sin^2\theta_W$). The supersymmetrization of the SU(5) model has led to almost ideal agreement with experimental data [3], however, according to the recent LEP results its predictions are within one standard deviation from the measured values of $\sin^2\theta_W$ and α_s [1].

In a recent paper, ref.[1], this problem has been analyzed for the so-called $SU(5)\times U(1) flipped$ SUSY model [4],[5], where the prediction for $\sin^2\theta_W$ becomes an upper bound and there is much more freedom in the spectrum of the model.

We analyze here another possibility. Namely, along the lines of ref.[1] we compare the minimal SUSY SU(5) model with the non-minimal one which is distinguished among the others by the property of ultraviolet finiteness. This model is very rigid due to restriction on the particle content coming from the finiteness requirement. Besides the usual minimal set of supermultiplets it contains additional six higgs multiplets $3(\bar{5}+5)$ needed to achieve all loop finiteness [6]. We consider the influence of these new particles on the low-energy predictions such as $\sin^2\theta_W$ and proton decay and discuss the bounds on the spectra of extra higgses. We find that with a reasonable assumption about SU(5) symmetry breaking and the mass splitting for additional multiplets one can describe all experimental data.

2 The basic formulae and the mass spectrum

In view of constantly increasing accuracy of the measurement of low-energy characteristics [7], [8] an attempt to compare the predictions of any GUT with experimental data requires two-loop calculations. However, as has been mentioned in ref.[1] one-loop calculations of $\sin^2\theta_W$ are only 1% below the two-loop one while the value of the unification point M_x is about 20% above the two-loop result. Therefore, by fitting a general one-loop calculation to a two-loop one for central values of inputs, one can quickly reach the desired accuracy. Further on we shall use the one-loop analytical formulae with the two-loop numerical correction.

We assume below the following spectrum of particles:

1. three light generations of quarks and leptons with masses, except for t-quark, less than Z-boson mass m_Z;
2. the higgs boson with mass of order m_Z;
3. t-quark with mass m_t;
4. the second higgs boson with mass m_a;
5. the superpartner of t-quark with mass $m_{\tilde{t}}$;
6. the superpartners of the higgs bosons and of four vector bosons of electroweak theory - higgsino, wino and zino - with masses $m_{\tilde{W}}$;
7. five squarks with masses $m_{\tilde{q}}$;
8. three sleptons with masses $m_{\tilde{l}}$;
9. eight gluinos with masses $m_{\tilde{g}}$;
10. additional particles Q, U, D, L, E with masses laying in the "Great desert" between m_Z and M_x. The quantum numbers of these particles coincide with those of quarks and leptons of the Standard model. They appear in finite SUSY models, considered below.

Some comments are in order:

- The mass of t-quark is supposed now to be inside the "Great desert" [9]. We take it to be below 1 Tev.
- We neglect the mass difference between higgsino and wino and zino, except for some cases, for the reasons to be clear below.

- We consider the masses of superpartners of the t-quark and the second higgs doublet as free parameters in a wide energy range and discuss the consequences of this assumption.

Assuming universal supersymmetry breaking on the unification scale, we parametrize the light scales as follows [11]:

$$\begin{aligned} m_{\tilde{q}} &= \sqrt{7m_{\frac{1}{2}}^2 + m_0^2} \\ m_{\tilde{l}_l} &= \sqrt{0.5m_{\frac{1}{2}}^2 + m_0^2} \\ m_{\tilde{l}_r} &= \sqrt{0.15m_{\frac{1}{2}}^2 + m_0^2} \\ m_{\tilde{W}} &= m_{\frac{1}{2}} \\ m_{\tilde{g}} &= 3m_{\frac{1}{2}} \end{aligned} \qquad (1)$$

Bounding $m_{\frac{1}{2}}$ and m_0 from above by requiring that no supersymmetric masses exceed 1 Tev [12], and bounding from below by requiring no supersymmetric masses are below 40 Gev, one can achieve further restrictions on low-energy predictions.

Defining the spectrum consider now its influence on the evolution of gauge couplings in the Standard model. The renormalization group equations for them written to the one-loop order have the form

$$\frac{d\alpha_i}{dt} = \frac{b_i}{2\pi}\alpha_i^2, \qquad i = 1, 2, 3. \qquad (2)$$

where $\alpha_i = \frac{g_i^2}{4\pi}$, g_i being the corresponding gauge couplings of $U(1)$, $SU(2)$ and $SU(3)$. Note that we use the $\overline{MS}$ renormalization scheme.

Eqs.(2) should be supplemented with the boundary conditions determined on the scale m_Z:

$$\frac{1}{\alpha_{em}} = 127.3 \pm 0.3 \text{ [13]}, \ \sin^2\theta_W = 0.2329 \pm 0.013 \text{ [3]}, \ \alpha_3 = 0.111 \pm 0.003 \text{ [7], [8]}$$

On the unification scale M_x we assume the simplest equality condition

$$\alpha_1 = \alpha_2 = \alpha_3. \qquad (3)$$

The constants in the renormalization group equations (2), b_i, for the adopted field content are:

$$\begin{aligned}
b_1 &= \frac{109}{30}+\frac{17}{30}\delta_t+\frac{49}{60}\delta_{\tilde q}+\frac{4}{15}\delta_{\tilde t_r}+\frac{1}{60}\delta_{\tilde t_l}+\frac{3}{10}\delta_{\tilde l_l}+\frac{3}{5}\delta_{\tilde l_r}+\frac{2}{5}\delta_{\tilde W}+\frac{1}{10}\delta_a \\
&\quad +\frac{4}{5}n_U+\frac{1}{5}n_D+\frac{1}{10}n_Q+\frac{3}{10}n_L+\frac{3}{5}n_E \\
b_2 &= -\frac{7}{2}+\frac{1}{2}\delta_t+\frac{5}{4}\delta_{\tilde q}+\frac{1}{2}\delta_{\tilde l_l}+2\delta_{\tilde W}+\frac{1}{4}\delta_{\tilde t_l}+\frac{1}{6}\delta_a+\frac{3}{2}n_Q+\frac{1}{2}n_L \qquad (4) \\
b_3 &= -\frac{23}{3}+\frac{2}{3}\delta_t+\frac{5}{3}\delta_{\tilde q}+\frac{1}{6}\delta_{\tilde t_l}+\frac{1}{6}\delta_{\tilde t_r}+2\delta_{\tilde g}+\frac{1}{2}n_U+\frac{1}{2}n_D+n_Q
\end{aligned}$$

These general formulae for b_1, b_2 and b_3 are valid for any GUT. The particular values of the parameters δ and n depend on the spectrum of the model. Index $\tilde W$ when not specified refers to the superpartners of gauge and higgs bosons which are assumed to be degenerated in mass.

Solving RG equations with the boundary conditions (3) we get the following expression for $\sin^2\theta_W$ at low energies:

$$\sin^2\theta_W = \alpha_{em}\left[\frac{1}{\alpha_3}+\frac{2a_c}{\pi f_c(s)}+\frac{1}{2\pi}\sum f_s(h)\ln\frac{M_x}{M_h}-\frac{1}{2\pi}\sum f_s(l)\ln\frac{M_l}{m_Z}\right] \qquad (5)$$

where

$$\begin{aligned}
a_c &= 2\pi\left(\frac{3}{5\alpha_{em}}-\frac{8}{5\alpha_3}\right) && (6)\\
f_c(b) &= b_1+\frac{3}{5}b_2-\frac{8}{5}b_3 && (7)\\
f_s(b) &= b_2-b_3-\frac{4f_c(b)}{f_c(s)}. && (8)
\end{aligned}$$

Summation in eq.(5) runs over heavy (h) and light (l) masses, respectively. The argument b in eqs.(5) - (8) refers to the contribution of a particular particle and argument s to that of three standard generations, two higgs doublets and their light superpartners. The multiplier $f_c(s)$, which essentially defines the value of $\sin^2\theta_W$, does not take into account the heavy particles. Hence, moving the superpartner of any particle from one end of the "Great desert" to the other causes a noticeable change in $\sin^2\theta_W$. We consider this influence of the choice of the spectrum on the low-energy predictions of GUTs in the next section.

The unification scale in eq.(5) is found from [1]:

$$\ln\frac{M_x}{m_Z} = \frac{a_c}{f_c(s)}-\sum\frac{f_c(h)}{f_c(s)}\ln\frac{M_x}{M_h}+\sum\frac{f_c(l)}{f_c(s)}\ln\frac{M_l}{m_Z}. \qquad (9)$$

Summation here is performed like in eq.(5).

Knowing the unification scale one can find also proton life-time in the minimal SUSY SU(5) model [1,4]

$$\tau(p\to e^+\pi^0) = 0.66\cdot 10^{31}\left(\frac{M_x}{10^{15}Gev}\right)^4\left(\frac{0.042}{\alpha_{M_x}}\right)^2\ years. \qquad (10)$$

Substituting the value of M_x we get from eq.(9) the following value

$$\tau(p\to e^+\pi^0) = 2.4\cdot 10^{35}\ years. \qquad (11)$$

This does not contradict the experiment [14], which gives

$$\tau(p\to e^+\pi^0) > 5.5\cdot 10^{32}\ years. \qquad (12)$$

In the next section using the aforementioned formulae we discuss in more detail the influence of the choice of the particle spectrum on the low-energy predictions and compare two particular models: the minimal and finite N=1 SUSY SU(5) GUTs.

3 Low-energy predictions as functions of the mass spectrum

Consider first the simplest minimal non-supersymmetric $SU(5)$ model. Besides the standard three fermion generations it contains two higgs multiplets: $\underline{24}$ which breaks $SU(5)$ down to $SU(3)\times SU(2)\times U(1)$ and $\underline{\bar 5}$, breaking $SU(2)\times U(1)$ down to $U(1)_{em}$. The $\underline{24}$, being heavy, does not contribute to the renormalization group equations for the running couplings eq.(4). One has according to eqs.(7,8)

$$b_1=\frac{21}{5},\ b_2=-3,\ b_3=-7,\ f_c(s)=\frac{68}{5},\ f_s(t)=-\frac{11}{102}$$

This leads to (see eq.(5))

$$\sin^2\theta_W = \frac{3}{17} + \frac{9}{17}\frac{\alpha_{em}}{\alpha_3} + \frac{\alpha_{em}}{2\pi}\frac{11}{102}\ln\frac{M_t}{m_Z} + \sigma_2, \tag{13}$$

where $\sigma_2 = 0.003$ is the two-loop correction. Calculating $\sin^2\theta_W$ at $\alpha_3 = 0.110$ we find

$$\sin^2\theta_W = 0.217. \tag{14}$$

This value is much lower than the experimental result $\sin^2\theta_W = 0.233$.

A) *Minimal SUSY SU(5) Model*

We now consider the SUSY generalization of the minimal model. Assuming that all superpartners are light (with masses $\leq 1\ Tev$), one has from eqs.(4-9): $f_c(s) = 12$, that gives according to eq.(5)

$$\begin{aligned}\sin^2\theta_W &= \frac{1}{5} + \frac{7}{15}\frac{\alpha_{em}}{\alpha_3} + \frac{\alpha_{em}}{20\pi}[\ln\frac{M_t}{m_Z} - \ln\frac{M_a}{m_Z} \\ &+ \frac{5}{3}\ln\frac{M_{\tilde{e}_r}}{m_Z} - \frac{7}{6}\ln\frac{M_{\tilde{t}_l}}{m_Z} + \frac{1}{2}\ln\frac{M_{\tilde{q}}}{m_Z} + 2\ln\frac{M_{\tilde{l}_r}}{m_Z} \\ &- 3\ln\frac{M_{\tilde{l}_l}}{m_Z} + \frac{28}{3}\ln\frac{M_{\tilde{g}}}{m_Z} - \frac{44}{3}\ln\frac{M_{\tilde{W}}}{m_Z}] + \sigma_2,\end{aligned} \tag{15}$$

Taking the values of masses in eq.(1) to be $m_{\frac{1}{2}} = 40$ Gev, $m_0 = 1$ Tev, and assuming m_a and $m_{\tilde{t}}$ to be equal to 1 Tev, and M_t of the order of m_Z we find for $\alpha_3 = 0.110$

$$\sin^2\theta_W = 0.236. \tag{16}$$

This value of $\sin^2\theta_W$ is already very close to the experimental one being larger by one standard deviation. Assuming large masses (of the order of $10^9 - 10^{12}\ Gev$) of superpartners, one can slightly change the situation.

To be precise, we consider SUSY SU(5) GUT with a heavy t-squark and/or a heavy second higgs doublet. In the first case one has $f_c(s) = \frac{182}{15}$ Then $\sin^2\theta_W$ according to eq.(5) is

$$\sin^2\theta_W = \frac{18}{91} + \frac{43}{91}\frac{\alpha_{em}}{\alpha_3} + \Delta_{\tilde{t}} + \Delta_l + \sigma_2, \tag{17}$$

where

$$\Delta_{\tilde{t}} = \frac{109}{546}\frac{\alpha_{em}}{2\pi}\ln\frac{M_x}{M'_{\tilde{t}}}$$

is the contribution of a heavy t-squark (hereafter we denote by prime the masses of heavy superpartners) and

$$\begin{aligned}\Delta_l &= \frac{\alpha_{em}}{1092\pi}[-127\ln\frac{M_t}{m_Z} - 55\ln\frac{M_a}{m_Z} + 516\ln\frac{M_{\tilde{g}}}{m_Z} \\ &+ \frac{99}{2}\ln\frac{M_{\tilde{q}}}{m_Z} - 165\ln\frac{M_{\tilde{l}_l}}{m_Z} + 108\ln\frac{M_{\tilde{l}_r}}{m_Z} - 258\ln\frac{M_{\tilde{W}}}{m_Z}]\end{aligned}$$

is the total contribution of light particles. Assuming the aforementioned values of their masses we get

$$\Delta_l = 0.0004.$$

To use eq.(5) we have to estimate $\ln\frac{M_x}{M'_{\tilde{t}}}$, which can be done with the help of eq.(9):

$$\begin{aligned}\ln\frac{M_x}{m_Z} &= \frac{15\pi}{91}(\frac{3}{5\alpha_{em}} - \frac{8}{5\alpha_3}) + \frac{15}{182}[-\frac{1}{5}\ln\frac{M_t}{m_Z} + \frac{1}{5}\ln\frac{M_a}{m_Z} \\ &- \frac{11}{10}\ln\frac{M_{\tilde{q}}}{m_Z} + \frac{3}{5}\ln\frac{M_{\tilde{l}_r}}{m_Z} + \frac{3}{5}\ln\frac{M_{\tilde{l}_l}}{m_Z} - \frac{16}{5}\ln\frac{M_{\tilde{g}}}{m_Z} \\ &+ \frac{8}{5}\ln\frac{M_{\tilde{W}}}{m_Z}] + \frac{3}{364}\ln\frac{M_x}{M'_{\tilde{t}}}.\end{aligned} \tag{18}$$

Substituting here $\alpha_3 = 0.110$ we get

$$\ln\frac{M_x}{M'_{\tilde{t}}} = \frac{364}{361}\ln\frac{m_Z}{M'_{\tilde{t}}} + 32.64. \tag{19}$$

Thus, $\Delta_{\tilde{t}}$ is decreasing with the growth of $M'_{\tilde{t}}$. For $M'_{\tilde{t}} = 10^9 - 10^{12}$ Gev

$$\begin{aligned}\Delta_{\tilde{t}} &= 0.003 - 0.002 \\ \sin^2\theta_W &= 0.238 - 0.237,\end{aligned} \tag{20}$$

respectively. We see that the value of $\sin^2\theta_W$ became even higher, though it is still very close to the experimental one.

We have to check now whether the assumption of the heavy t-squark leads to any contradiction with the other experimental data, namely, with the limit on the proton life-time. One can get from eq.(10) that

$$\frac{\tau'}{\tau} = (\frac{m_Z}{M_{\tilde{t}}})^{4f_{23}(\tilde{t})} (\frac{M'_{\tilde{t}}}{M_0})^{\frac{4f_{23}(\tilde{t})}{f_{23}(\tilde{t})+1}}, \tag{21}$$

where τ' is the upper bound of proton life-time for the minimal SUSY SU(5) model with the heavy t-squark and τ is the same for light superpartners (11). The parameter M_0 is defined from the equation

$$\ln\frac{M_0}{m_Z} = \frac{\pi}{2}(\frac{\sin^2\theta_W}{\alpha_{em}} - \frac{1}{\alpha_3})$$

for $\alpha_3 = 0.110$ and $\sin^2\theta_W = 0.233$. As far as $f_{23}(\tilde{t}) = -\frac{1}{48}$, then

$$\frac{\tau'}{\tau} = (\frac{m_Z}{M_{\tilde{t}}})^{-\frac{1}{12}} (\frac{M'_{\tilde{t}}}{M_0})^{-\frac{4}{47}}. \tag{22}$$

Analyzing this equation we see that τ' is inversely proportional to $M'_{\tilde{t}}$. For $M'_{\tilde{t}} = 10^9 \; - \; 10^{12}$ Gev

$$\tau' = 1.2 \; - \; 6.6\cdot 10^{36}\ years; \tag{23}$$

Both values are obviously above the experimental limit (12).

In the same way one can analyze the case of a heavy superpartner of the second higgs doublet. Now we distinguish the contribution of the higgsino $\tilde{a}$ and wino and photino $\tilde{W}$. One has: $f_c(s) = \frac{56}{5}$ and $\sin^2\theta_W$ according to eq.(5) is

$$\sin^2\theta_W = \frac{3}{14} + \frac{3}{7}\frac{\alpha_{em}}{\alpha_3} + \Delta'_{\tilde{a}} + \Delta'_l + \sigma_2, \tag{24}$$

where

$$\Delta'_{\tilde{a}} = \frac{5}{7}\frac{\alpha_{em}}{2\pi}\ln\frac{M_x}{M'_{\tilde{a}}}$$

is the contribution of a heavy superpartner of the second higgs doublet and

$$\begin{aligned}\Delta'_l \;=\; & -\frac{\alpha_{em}}{2\pi}[-\frac{2}{21}\ln\frac{M_t}{m_Z} + \frac{5}{42}\ln\frac{M_{\tilde{t}_l}}{m_Z} - \frac{1}{6}\ln\frac{M_{\tilde{t}_r}}{m_Z}\\ + & -\frac{1}{42}\ln\frac{M_{\tilde{q}}}{m_Z} + \frac{2}{7}\ln\frac{M_{\tilde{l}_l}}{m_Z} - \frac{3}{14}\ln\frac{M_{\tilde{l}_r}}{m_Z} + \frac{11}{14}\ln\frac{M_{\tilde{W}}}{m_Z} - \frac{6}{7}\ln\frac{M_{\tilde{g}}}{m_Z}]\end{aligned}$$

is the total contribution of light particles. Assuming the same masses as before we have

$$\Delta_l = 0.0001.$$

To estimate the $\ln\frac{M_x}{M'_{\tilde{a}}}$, we use again eq.(9):

$$\begin{aligned}\ln\frac{M_x}{m_Z} \;=\; & \frac{5\pi}{28}(\frac{3}{5\alpha_{em}} - \frac{8}{5\alpha_3}) + \frac{5}{56}[-\frac{1}{5}\ln\frac{M_t}{m_Z} + \frac{1}{5}\ln\frac{M_a}{m_Z}\\ - & \frac{11}{10}\ln\frac{M_{\tilde{q}}}{m_Z} + \frac{3}{5}\ln\frac{M_{\tilde{l}_r}}{m_Z} + \frac{3}{5}\ln\frac{M_{\tilde{l}_l}}{m_Z} - \frac{16}{5}\ln\frac{M_{\tilde{g}}}{m_Z}\\ + & \frac{3}{5}\ln\frac{M_{\tilde{W}}}{m_Z} + \frac{1}{10}\ln\frac{M_{\tilde{t}_l}}{m_Z}] - \frac{1}{14}\ln\frac{M_x}{M'_{\tilde{a}}}.\end{aligned} \tag{25}$$

Substituting $\alpha_3 = 0.110$, we get

$$\ln\frac{M_x}{M'_{\tilde{a}}} = \frac{14}{15}[\ln\frac{m_Z}{M'_{\tilde{a}}} + 34.84]. \tag{26}$$

Again with increase of $M'_{\tilde{a}}$, $\Delta_{\tilde{a}}$ is decreasing. For $M'_{\tilde{a}} = 10^9 \; - \; 10^{12}$ Gev

$$\begin{aligned}\Delta'_{\tilde{a}} &= 0.012 \; - \; 0.008,\\ \sin^2\theta_W &= 0.260 \; - \; 0.256,\end{aligned} \tag{27}$$

respectively. We see that the value of $\sin^2\theta_W$ became essentially larger and is far from the experimental one, 0.233.

We also check the proton life-time. Substituting $f_{23}(\tilde{a}) = \frac{1}{4}$ instead of $f_{23}(\tilde{t})$ into eq.(21) we get:

$$\frac{\tau'}{\tau} = (\frac{m_Z}{M_{\tilde{a}}})(\frac{M'_{\tilde{a}}}{M_0})^{\frac{4}{5}}. \tag{28}$$

where τ' is as before the upper bound of proton life-time in a given model and τ is that in minimal $SUSY\,SU(5)$ with light particles. Here, contrary to the previous case, τ' is increasing with $M'_{\tilde{a}}$. For $M'_{\tilde{a}} = 10^9 \; - \; 10^{12}$ Gev

$$\tau' = 4.6\cdot 10^{28} \; - \; 1.1\cdot 10^{31}\ years. \tag{29}$$

For both scales τ' is smaller than the experimental bound (12), which means that the model at hand can not be considered as realistic.

The Model	The varying parameter of the mass spectrum, Gev	The value of $\sin^2\theta_W$ for $\alpha_s = 0,110$ without L and D particles	Required ratio $\frac{M_L}{M_D}$	The bound on $\frac{M_L}{M_D}$	The value of $\sin^2\theta_W$ for $\alpha_s = 0,110$ with L and D particles	Upper bound on $\tau(p \to e^+\pi^0)$ years	Comments
Minimal $SUSY\,SU(5)$		0.236			0.236	$2.4 \cdot 10^{35}$	1. $\sin^2\theta_W$ is beyond the observed area 2. τ_p is realistic
Minimal $SUSY\,SU(5)$ with heavy $\tilde{t}$-squark	$10^9 \leq M_{\tilde t} \leq 10^{12}$	0.238- 0.237			0.238- 0.237	$1.2 \cdot 10^{36}$ – $6.6 \cdot 10^{35}$	1. $\sin^2\theta_W$ is beyond the observed area 2. τ_p is realistic
Minimal $SUSY\,SU(5)$ with heavy higgsino $\tilde{a}$	$10^9 \leq M_{\tilde a} \leq 10^{12}$	0.260 - 0.256			0.260 - 0.256	$4.6 \cdot 10^{28}$ – $1.1 \cdot 10^{31}$	1. $\sin^2\theta_W$ is far beyond the observed area 2. τ_p below the experimental limit
Finite $SUSY\,SU(5)$ with light superpartners		0.236	5.3	$\frac{M_L}{M_D} > 0.13$	0.233	$3.6 \cdot 10^{37}$	1. $\sin^2\theta_W$ is in the observed area 2. τ_p is realistic
Finite $SUSY\,SU(5)$ with heavy $\tilde{t}$-squark	$10^9 \leq M_{\tilde t} \leq 10^{12}$	0.238 - 0.237	17.3 - 9.8	$\frac{M_L}{M_D} > 0.10$	0.233	$7.4 \cdot 10^{39}$ – $7.2 \cdot 10^{38}$	1. $\sin^2\theta_W$ is in the observed area 2. τ_p is realistic
Finite $SUSY\,SU(5)$ with heavy higgsino $\tilde{a}$	$10^9 \leq M_{\tilde a} \leq 10^{12}$	0.260 - 0.256	$2.0 \cdot 10^8$ – $1.2 \cdot 10^7$	$\frac{M_L}{M_D} > 50.0$	0.233	$3.8 \cdot 10^{48}$ – $1.1 \cdot 10^{48}$	1. $\sin^2\theta_W$ is in the observed area 2. τ_p is realistic 3. unrealistic $\frac{M_L}{M_D}$

B) *Finite SUSY SU(5) Model*

As far as the low-energy predictions of the minimal SUSY model considered above seem to be already in contradiction with precise data, one can try to improve the situation allowing for non-minimal models. To choose among the variety of non-minimal models, we consider an interesting particular example of UV finite $N = 1$ supersymmetric GUT based on SU(5) gauge group. Non-minimality is needed here to cancel all the ultraviolet divergences. The method of construction of such models is described in ref.[6] where it is shown that one-loop finiteness is crucial and allows one to construct all-loop finite theories. A complete classification of $N = 1$ supersymmetric theories satisfying the one-loop finiteness criteria are given in ref.[15]. Additional multiplets with respect to the minimal SUSY SU(5) model are: three quintets and three antiquintets of higgs superfields [15], [6]. As far as 5 and $\bar{5}$ have the following decomposition with respect to $SU(3) \times SU(2) \times U(1)$:

$$\begin{aligned} 5 &= (3, 1, -\frac{2}{3}) + (1, 2, 1) \\ \bar{5} &= (3, 1, \frac{2}{3}) + (1, 2, -1), \end{aligned}$$

the addition of extra higgs multiplets will after spontaneous symmetry breaking $SU(5) \rightarrow SU(3) \times SU(2) \times U(1)$ lead to new fermions and bosons with quantum numbers of D-antiquarks and L-leptons of the minimal SU(5) model . The quantum numbers of 5 differ from those of $\bar{5}$ by the hypercharge sign, but this does not influence the coefficient b_1 because it contains the square of the hypercharge of each particle. These new particles will change eq.(15) for $\sin^2\theta_W$ adding new terms according to the general formula eq.(5). However, because of these particles being heavy, their contribution will not be the main one.

We calculate $\sin^2\theta_W$ in this model assuming that all superpartners of the particles of the Standard model are light (less than 1 Tev). One has according to eq.(7)

$$f_s(D) = -\frac{3}{10}; \quad f_s(L) = \frac{3}{10}.$$

This gives

$$\begin{aligned} \sin^2\theta_W &= \frac{1}{5} + \frac{7}{15}\frac{\alpha_{em}}{\alpha_3} + \frac{\alpha_{em}}{20\pi}[\ln\frac{M_t}{m_Z} - \ln\frac{M_a}{m_Z} + \frac{5}{3}\ln\frac{M_{\tilde{t}_r}}{m_Z} \\ &- \frac{17}{6}\ln\frac{M_{\tilde{t}_l}}{m_Z} + \frac{1}{2}\ln\frac{M_{\tilde{q}}}{m_Z} + 2\ln\frac{M_{\tilde{l}_r}}{m_Z} - 3\ln\frac{M_{\tilde{l}_l}}{m_Z} \\ &+ \frac{28}{3}\ln\frac{M_{\tilde{g}}}{m_Z} - \frac{44}{3}\ln\frac{M_{\tilde{W}}}{m_Z}] - \frac{3n\alpha_{em}}{10\pi}\ln\frac{M_L}{M_D} + \sigma_2. \end{aligned} \tag{30}$$

The contribution of additional multiplets does not depend on M_x. This can be seen from eq.(7) as far as

$$f_s(L) = \frac{1}{2} - \frac{12}{5}\frac{1}{f_c(s)} = -f_s(D).$$

Hence, independently of the spectra of light particles which determine $f_c(s)$, the contribution of extra multiplets $n(5+\bar{5})$ to $\sin^2\theta_W$ is given by

$$-f_s(L)\ln\frac{M_L}{M_D}, \tag{31}$$

where we assumed the same doublet-triplet mass splitting in each multiplet. Otherwise one has to sum over all multiplets.

Because of a positive value of $f_s(L)$ the desired decrease of $\sin^2\theta_W$ due to the additional terms can be achieved if

$$\frac{M_L}{M_D} > 1.$$

This requirement can be easily realized in a concrete mechanism of SU(5) symmetry breaking. To decrease the value of $\sin^2\theta_W$ by 0.002 -0.003 one should require

$$3.0 \leq \frac{M_L}{M_D} \leq 5.3, \tag{32}$$

which can be realized on the scales 10^9 *Gev* $< M_D < M_L < 10^{12}$ *Gev*. Hence, properly choosing the mass splitting of additional multiplets one can obtain the value of $\sin^2\theta_W$ in the finite SUSY SU(5) model in accordance with the experimental data.

In full analogy with the minimal models we have to check that eq.(32) does not contradict the bound on the proton life-time (12). We get from eq.(10) that

$$\frac{\tau'}{\tau} = (\frac{M_L}{M_D})^3, \tag{33}$$

where as usual τ' is the upper bound of proton life-time in a given model and τ is the same in a minimal SUSY SU(5). Using eq.(32) we find that τ' is within the limits

$$6.7 \cdot 10^{36}\ years < \tau' < 3.6 \cdot 10^{37}\ years, \tag{34}$$

which does not confront the experimental data (12).

For completeness like in the minimal model we consider the case when some superpartners are heavy ($m \sim 10^9 - 10^{12}\ Gev$). For the heavy t-squark using eqs.(4), (6) - (8) we get $f_c(s) = \frac{182}{15}$. Equations (17) - (18) will obtain some additional contributions due to the new particles. $\sin^2\theta_W$ now according to eq.(5) is

$$\sin^2\theta_W = (\sin^2\theta_W)_{minimal} + \Delta_h, \tag{35}$$

where $(\sin^2\theta_W)_{minimal}$ is defined by eq.(17) and the contribution of heavy particles is contained in

$$\Delta_h = -\frac{165}{91}\frac{\alpha_{em}}{2\pi}\ln\frac{M_L}{M_D} \tag{36}$$

and also in $\ln\frac{M_x}{M_{\tilde{t}}'}$ in eq.(17) for $(\sin^2\theta_W)_{minimal}$. The latter contribution to eq.(18) is

$$\frac{27}{91}\ln\frac{M_L}{M_D},$$

and eq.(19) takes the form

$$\ln\frac{M_x}{M_{\tilde{t}}'} = \frac{364}{361}\ln\frac{m_Z}{M_{\tilde{t}}'} + 32.64 + \frac{108}{361}\ln\frac{M_L}{M_D}. \tag{37}$$

Together with eq.(36) the total contribution of extra heavy particles is

$$-\frac{57603}{32851}\frac{\alpha_{em}}{2\pi}\ln\frac{M_L}{M_D}.$$

To decrease the value of $\sin^2\theta_W$ (20) in the finite model with a heavy t-squark up to the experimental value, 0.233, we need the following doublet-triplet splitting for additional multiplets: for $M_{\tilde{t}}' = 10^9 - 10^{12}$ Gev

$$\frac{M_L}{M_D} = 17.3 - 9.8. \tag{38}$$

We have to check also the proton life-time prediction. Eq.(21) now is

$$\frac{\tau'}{\tau} = (\frac{m_Z}{M_{\tilde{t}}})^{4f_{23}(\tilde{t})}\ (\frac{M_{\tilde{t}}'}{M_0})^{\frac{4f_{23}(\tilde{t})}{f_{23}(\tilde{t})+1}}\ (\frac{M_L}{M_D})^{\frac{3}{f_{23}(\tilde{t})+1}}. \tag{39}$$

Substituting

$$f_{23}(\tilde{t}) = -\frac{1}{48},$$

we get

$$\frac{\tau'}{\tau} = (\frac{m_Z}{M_{\tilde{t}}})^{-\frac{1}{12}}\ (\frac{M_{\tilde{t}}'}{M_0})^{-\frac{4}{47}}\ (\frac{M_L}{M_D})^{\frac{144}{47}}. \tag{40}$$

Increasing $M_{\tilde{t}}'$ the life-time τ' is decreased like in the minimal model. For $M_{\tilde{t}}' = 10^9 - 10^{12}$ Gev

$$\tau' = 7.4 \cdot 10^{39} - 7.2 \cdot 10^{38}\ years. \tag{41}$$

Thus, we have no contradiction with experimental data within the whole energy scale of t-squark masses.

In the same way we analyze the situation when except for additional particles the superpartner of the second higgs doublet is heavy ($10^9 - 10^{12}$ Gev). In this case from eqs.(4), (6) - (8) it follows that $f_c(s) = \frac{56}{5}$. Eqs.(24) - (26) now have the form (35)

$$\sin^2\theta_W = (\sin^2\theta_W)_{minimal} + \Delta_h,$$

where $(\sin^2\theta_W)_{minimal}$ is defined by (24) and the contribution of extra heavy particles is contained in

$$\Delta_h = -\frac{2}{7}\frac{\alpha_{em}}{2\pi}\ln\frac{M_L}{M_D} \tag{42}$$

and in $ln\frac{M_x}{M'_{\tilde{a}}}$ of eq.(24) for $(\sin^2\theta_W)_{minimal}$. The latter can be found from eqs.(9), (25) and (26)

$$\ln\frac{M_x}{M'_{\tilde{a}}} = \frac{14}{15}\ln\frac{m_Z}{M'_{\tilde{a}}} + 32.51 + \frac{1}{20}\ln\frac{M_L}{M_D}. \tag{43}$$

The resulting contribution of heavy particles into $\sin^2\theta_W$ is then equal to

$$-\frac{99}{70}\frac{\alpha_{em}}{2\pi}\ln\frac{M_L}{M_D}.$$

To get a correct value of $\sin^2\theta_W$ (cf eq.(27), one needs the following mass splitting in additional multiplets: for $M'_{\tilde{a}} = 10^9 - 10^{12}$ Gev

$$\frac{M_L}{M_D} = 2.0\cdot 10^8 - 1.2\cdot 10^7. \tag{44}$$

which is obviously unreliable.

The influence of heavy particles on the proton life-time as in the previous case is given by eq.(39) with $f_{23}(\tilde{a})$ instead of $f_{23}(\tilde{t})$

$$\frac{\tau'}{\tau} = \left(\frac{m_Z}{M_{\tilde{a}}}\right)^{4f_{23}(\tilde{a})} \left(\frac{M'_{\tilde{a}}}{M_0}\right)^{\frac{4f_{23}(\tilde{a})}{f_{23}(\tilde{a})+1}} \left(\frac{M_L}{M_D}\right)^{\frac{3}{f_{23}(\tilde{a})+1}}. \tag{45}$$

Taking into account that

$$f_{23}(\tilde{a}) = \frac{1}{4},$$

we get

$$\frac{\tau'}{\tau} = \left(\frac{m_Z}{M_{\tilde{a}}}\right) \left(\frac{M'_{\tilde{a}}}{M_0}\right)^{\frac{4}{5}} \left(\frac{M_L}{M_D}\right)^{\frac{12}{5}}. \tag{46}$$

The restriction on the mass ratio coming from eq.(46) and requirement $\tau' > \tau_{exp}$, where τ_{exp} is an experimental lower bound on the proton life-time (12), give

$$\frac{M_L}{M_D} > \left(\frac{1}{436}\right)^{\frac{5}{12}} \left(\frac{m_Z}{M_{\tilde{a}}}\right)^{-\frac{5}{12}} \left(\frac{M'_{\tilde{a}}}{M_0}\right)^{-\frac{1}{3}}, \tag{47}$$

which means that

$$\frac{M_L}{M_D} > 50.0. \tag{48}$$

This inequality is obviously satisfied in the interval (44). However, the obvious pathology of (44) makes the model with that spectrum unsatisfactory.

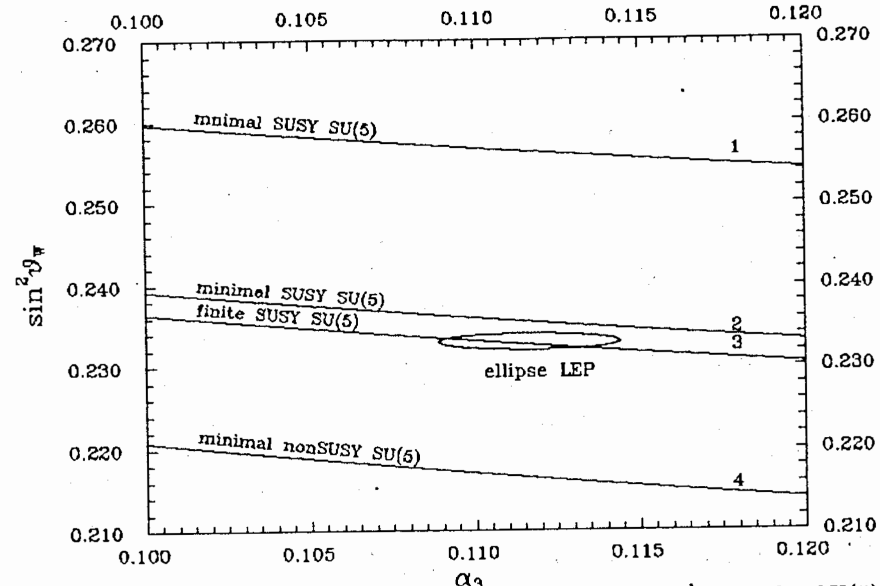
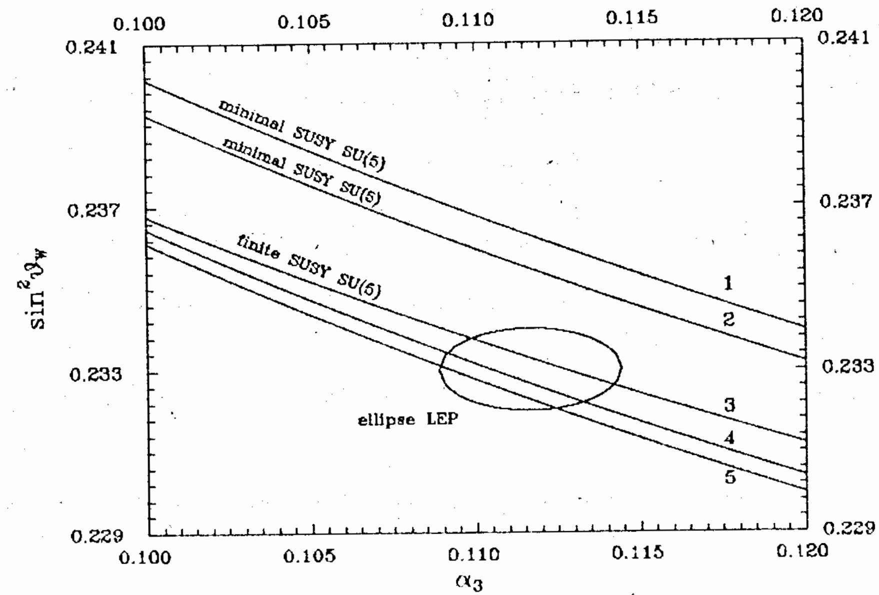


Fig. 1: 1) Minimal SUSY SU(5) with heavy higgsino; 2) Minimal SUSY SU(5) with light superpartners; 3) Finite SUSY SU(5) with light superpartners; 4) Minimal non-SUSY SU(5).

Fig. 2: 1) Minimal SUSY SU(5) with heavy $\tilde{t}$-squark; 2) Minimal SUSY SU(5) with light superpartners; 3) Finite SUSY SU(5) with heavy higgsino; 4) Finite SUSY SU(5) with light superpartners; 5) Finite SUSY SU(5) with heavy $\tilde{t}$-squark.



4 Conclusion

The results obtained above are summarized in table 1.

One can see from the table that among the models considered here the most reliable results are obtained in finite SUSY SU(5) models with light particles and with a heavy t-quark superpartner. Properly ajusting SU(5) mass splitting for additional multiplets one can always obtained an agreement with the experimental data. In the other cases the value of $\sin^2\theta_W$ is too big as compared to the modern data within the reasonable assumptions on the spectrum. The situation is illustrated in Figs. 1 & 2. The curves related to the finite SUSY SU(5) models correspond to the properly ajusted mass splittings as explained in the text.

References

[1] J. Ellis, S. Kelley, and D. V. Nanopoulos, *Phys. Lett.* **B 260** (1991) 131

[2] H. Georgi, H. Quinn and S. Weinberg, *Phys. Rev. Lett.* **33** (1974) 451

[3] J. Ellis, S. Kelley, and D. V. Nanopoulos, *Phys. Lett.* **B 249** (1990) 441

[4] S. M. Barr, *Phys. Lett.* **B 112** (1982) 219; J. P. Derendinger, Jihn E. Kim and D. V. Nanopoulos, *Phys. Lett.* **B 139** (1984) 170; I. Antoniadis, J. Ellis, J. Hagelin and D. V. Nanopoulos, *Phys. Lett.* **B 194** (1987) 231

[5] I. Antoniadis, J. Ellis, J. Hagelin and D. V. Nanopoulos, *Phys. Lett.* **B 231** (1989) 65

[6] A. V. Ermushev, D. I. Kazakov, O. V. Tarasov, *Nucl. Phys.* **B 281** (1987) 72

[7] A. D. Martin, R. G. Roberts and W. J. Stirling, *Durham University preprint*, DPT-90-76 (1990)

[8] U. Amaldi, *Review Talk at Texas/ESO-CERN Symposium on Relativistic Astrophysics, Particle Physics and Cosmology*, Brighton, Dec. 1990

[9] J. Ellis and G. L. Fogli, *Phys. Lett.* **B 249** (1990) 543

[10] J. Ellis, G. Ridolfi and F. Zwirner *Phys. Lett.* **B 237** (1990) 423

[11] A. B. Lahanas and D. V. Nanopoulos, *Phys. Rep.* **145** (1987) 1

[12] J. Ellis, K. Enqvist, D. V. Nanopoulos and F. Zwirner, *Mod. Phys. Lett.* **A 1** (1986) 57; R. Barbieri and G. F. Giudice, *Nucl. Phys.* **B 306** (1988) 63

[13] W. J. Marciano, *Brookhaven report* BLN-41498 (1988)

[14] IBM-3 Collaboration, R. Becker-Szendy et al., *Phys. Rev.* **D 42** (1990) 2974

[15] S. Hamidi, J. Patera and J. H. Schwarz *Phys. Lett.* **B 141** (1984) 349;
J. E. Bjorkman, D. R. T. Jones, S. Raby *Nucl. Phys.* **B 259** (1985) 503

**Received by Publishing Department
on August 22, 1991.**