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QCD-MOTIVATED NAMBU-JONA-LASINIO MODEL WITH QUARK AND GLUON CONDENSATES

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1. Introduction

The Nambu-Jona-Lasinio (NJL) model at quark level was first studied in 1976 in refs. [1]. In the last decade, interest in this model was continuously increasing [2]- [6] thus expanding the range of its application in recent years [7]- [13]. More thorough studies were made of chiral anomalies and Skyrme-like terms with fourth-order derivatives as well as of intrinsic properties of the model connected with nondiagonal transitions of mesons [3,5,8], mixing of flavours [7]- [9], dependence of the model parameters on temperature and density [10], phase transitions [11], relation to QCD [12]- [14] and the description of diquarks [15], baryons and light nuclei [12]- [14]. This list being far from complete shows a sufficiently wide spectrum of possible applications of the NJL model.

In all the above-mentioned papers, the NJL model with the effective chiral four-quark Lagrangian was usually used. The basic independent quantities of the model are the masses of constituent quarks connected with the quark condensate, the cut-off parameter Λ determining the boundary of the region of spontaneous chiral symmetry breaking and the four-quark interaction constant κ. The purpose of the present paper is to investigate a QCD-motivated NJL model containing a nonperturbative gluon condensate. We will then show how the basic parameters and model equations of the resulting chiral σ-model will change with this quantity taken into account.

Let us start with QCD and decompose the gluon field $\mathbf{G}^a_\mu$ into a condensate field G^a_μ and the quantum fluctuations g^a_μ around it

$$\mathbf{G}^a_\mu(x) = G^a_\mu(x) + g^a_\mu(x) \ . \tag{1}$$

By assumption the first part of the field yields a nonvanishing gluon condensate

$$< \mathrm{vac}|\frac{g^2}{4\pi} : \mathbf{G}^a_{\mu\nu}(0)\mathbf{G}^{\mu\nu}_a(0) : |\mathrm{vac}> = < \mathrm{vac}|\frac{g^2}{4\pi} G^a_{\mu\nu}(0)G^{\mu\nu}_a(0)|\mathrm{vac}> , \tag{2}$$

where $G^a_{\mu\nu}$ is the field strength tensor. Integrating in the generating functional of QCD over the quantum field $g^a_\mu(x)$ and approximating the (unknown) nonperturbative gluon propagator by a δ-function we get an effective chiral four-quark interaction of the NJL type. In this case, the condensate field $G^a_\mu(x)$ enters into the standard Lagrangian of the NJL model through the covariant derivative of the quark field

$$D_\mu q = (\partial_\mu + ig\frac{\lambda_a}{2}G^a_\mu)q \ , \tag{3}$$

where g is the QCD coupling constant and $\lambda_a/2$ are the generators of the color group $SU(N_c)$.

2. QCD-motivated NJL model for scalar and pseudoscalar mesons

The effective chiral quark Lagrangian describing interactions of composite scalar and pseudoscalar mesons in the presence of condensate gluon fields can be written as [2]- [5] [2]

$$\mathcal{L}(q,G) = \bar{q}(i\hat{D} - m^0)q + \frac{\kappa}{2}[(\bar{q}\tau^\alpha q)^2 + (\bar{q}i\gamma_5\tau^\alpha q)^2] \ , \tag{4}$$

where $\hat{D} = \gamma^\mu D_\mu$, D_μ is the covariant derivative (3), τ^α are the Pauli matrices of the flavor group $SU(2)_F$ ($\tau^0 \equiv 1$; summation over α is understood), and q are fields of current quarks with mass m^0. Upon introducing meson fields, the Lagrangian (4) turns into the equivalent form

$$\mathcal{L}'(q,G,\tilde{\sigma},\varphi) = -\frac{(\tilde{\sigma}_\alpha + \varphi_\alpha)}{2\kappa} + \bar{q}(i\hat{D} - m^0 + \tilde{\sigma} + i\gamma_5\varphi)q \tag{5}$$

with $\tilde{\sigma} = \tilde{\sigma}_\alpha\tau^\alpha$, $\varphi = \varphi_\alpha\tau^\alpha$. The vacuum expectation value of the isoscalar-scalar field $\tilde{\sigma}_o$ turns out to be nonzero ($<\tilde{\sigma}_0> \neq 0$). To pass to a physical field σ_0 with $<\sigma_0> = 0$ one usually performs a field shift leading to a new quark mass m to be identified with the mass of constituent quarks,

$$-m^0 + \tilde{\sigma}_0 = -m + \sigma_0; \quad \tilde{\sigma}_\alpha = \sigma_\alpha \quad (\alpha = 1,2,3), \tag{6}$$

where m is determined from the gap equation (see below).

Let us for a moment neglect the gluon condensate in (5) ($\hat{D} \to \hat{\partial}$). Integrating in the generating functional associated with the Lagrangian (5) over the quark fields, evaluating the resulting quark determinant by a loop expansion and including thereby only second-order field derivatives gives then an expression corresponding to the linear σ-model [2,3]

$$\mathcal{L}'' = -\frac{\tilde{\sigma}_\alpha^2 + \varphi_\alpha^2}{2\kappa} + \mathrm{Tr}\{[p^2 I_2 + 2(I_1 + m^2 I_2)][(\sigma - m)^2 + \varphi^2] - I_2[(\sigma - m)^2 + \varphi^2]^2\} \ . \tag{7}$$

Here I_1 and I_2 are divergent integrals regularized with a cut-off parameter Λ characterizing the scale of chiral-symmetry breaking,

$$\begin{aligned} I_1 &= -i\frac{N_c}{(2\pi)^4}\int\frac{d^4k\,\theta(\Lambda^2 + k^2)}{m^2 - k^2} = \frac{3}{(4\pi)^2}[\Lambda^2 - m^2\ln(\frac{\Lambda^2}{m^2} + 1)] \ , \\ I_2 &= -i\frac{N_c}{(2\pi)^4}\int\frac{d^4k\,\theta(\Lambda^2 + k^2)}{(m^2 - k^2)^2} = \frac{3}{(4\pi)^2}[\ln(\frac{\Lambda^2}{m^2} + 1) - (1 + \frac{m^2}{\Lambda^2})^{-1}] \ , \end{aligned} \tag{8}$$

and N_c is the number of colors. In configuration space we have $p^2\varphi^2 \to \varphi(-\Box)\varphi$ etc.

Now let us see how the Lagrangian (7) changes when condensate gluon fields are taken into account. Here the effect of gluon condensate corrections will be calculated with an accuracy up to squared terms in the field strength $G^a_{\mu\nu}(x)$. For evaluating the quark determinant with external fields we shall use the "heat kernel" technique which has been employed in papers [5,16]. Then, instead of (7) we get [3]

$$\begin{aligned} \mathcal{L}^G = \mathrm{Tr}\{ &-\frac{\varphi^2 + (\sigma - m + m^0)^2}{4\kappa} + [p^2(I_2 + \frac{1}{96}\frac{G^2}{m^4}) + \\ &+ 2[I_1 + m^2 I_2 + \frac{1}{48}\frac{G^2}{m^2}(1 + \frac{1}{2})]][(\sigma - m)^2 + \varphi^2] - (I_2 + \frac{1}{96}\frac{G^2}{m^4})[(\sigma - m)^2 + \varphi^2]^2\}, \end{aligned} \tag{9}$$

where

$$G^2 = \frac{\alpha}{\pi} < (G^a_{\mu\nu})^2 >, \qquad \alpha = \frac{g^2}{4\pi} \ .$$

Now let us determine physical quantities and model parameters.

3. Basic quantities and model equations.

In order that the Lagrangian (9) takes its usual form (with the standard coefficient of the kinetic term) one should perform the following field renormalizations:

$$\sigma_\alpha = g_\sigma\sigma_\alpha^R, \quad \varphi_\alpha = g_\varphi\varphi_\alpha^R, \quad g_\sigma = g_\varphi = \frac{1}{2}(I_2 + \frac{G^2}{96m^4})^{-1/2} . \tag{10}$$

Then, from the requirement for terms linear in σ to vanish we get a modified "gap" equation

$$m = m^o + 8\kappa m I_1 + \kappa\frac{G^2}{6m} \ . \tag{11}$$

After renormalization of the meson fields we get for the square part of the Lagrangian (9) determining the mass terms (omitting the field index R),

$$\begin{aligned} \mathcal{L}^G_m &= -\frac{g_\sigma^2}{4}\mathrm{Tr}\{(\frac{1}{\kappa} - 8I_1 - \frac{G^2}{6m^2})(\sigma^2 + \varphi^2) + (4m)^2(I_2 + \frac{G^2}{96m^4})\sigma^2\} = \\ &= -\frac{g_\sigma^2 m^0}{4\kappa m}\mathrm{Tr}(\sigma^2 + \varphi^2) - m^2\mathrm{Tr}\sigma^2 \ , \end{aligned} \tag{12}$$

where in the last line of eq.(12) use of the gap equation (11) has been made. We then get for the meson masses the known equations

$$\begin{aligned} m_\pi^2 &= \frac{g_\varphi^2 m^0}{\kappa m} = \frac{m^0 m}{\kappa F_\pi^2} \approx -\frac{2m^0 < \bar{q}q >}{F_\pi^2} \quad \text{(Gell-Mann-Oakes-Renner formula)}, \\ m_\sigma^2 &= m_\pi^2 + 4m^2. \end{aligned} \tag{13}$$

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[2] This type of interaction results from the current $\times$ current interaction of quarks due to gluon exchange after applying a Fierz transformation to color and Dirac indices. For simplicity, we shall omit here vector and axial-vector channels and consider an unbroken flavour group $SU(2)_F$ with equal quark masses $m_u^0 = m_d^0$.

[3] Higher order terms of the form $g^3 f_{abc}\frac{<G^a_{\mu\nu}G^b_{\nu\sigma}G^c_{\sigma\mu}>}{m^6} + \ldots$ give less essential contributions of about several percent [18], and will therefore, be neglected. Note that proper-time regularized integrals are replaced here by momentum cut-off regularized integrals I_1, I_2.

Here we have used the Goldberger-Treiman identity $g_\varphi = m/F_\pi$ and the expression for the total quark condensate $<\bar{q}q>$ which will be derived below (see eq. (25)). The Goldberger -Treiman identity leads to the following expression for the pion decay constant F_π

$$F_\pi^2 = \frac{m^2}{g_\varphi^2} = m^2(4I_2 + \frac{G^2}{24m^2}) . \quad (14)$$

Finally, the effective coupling constants of meson interactions can be read off from the interaction terms

$$\mathcal{L}_{\sigma\varphi^2} = m\mathrm{Tr}(\sigma\varphi^2) ,$$
$$\mathcal{L}_{(\sigma^2+\varphi^2)^2} = -\frac{g_\sigma^2}{4}\mathrm{Tr}(\sigma^2+\varphi^2)^2 . \quad (15)$$

Thus, the form of meson interactions remains unchanged after the inclusion of gluon condensate fields [2,3]. Only the expression of the coupling constant $g_\sigma(g_\varphi)$ changes.

4. Determination of model parameters

Until now we have not considered vector and axial-vector mesons. However, one should bare in mind that since axial-vector mesons do exist, nondiagonal transitions of the type $\pi \to a_1$ play an essential role in the NJL model. The effect of these nondiagonal terms on different physical quantities in the NJL model is well studied (see refs. [3,5,8]). Specifically, if they are taken into account, there arises an additional renormalization of pseudoscalar fields

$$g'_\varphi = g_\sigma Z^{1/2}, \quad (16)$$

where

$$Z = (1-\frac{6m^2}{m_{a_1}^2})^{-1} = 2[1+\sqrt{1-(\frac{2g_\rho F_\pi}{m_{a_1}})^2}] \quad (17)$$

with m_{a_1} being the mass of the a_1 meson.

The analysis of vector meson fields shows that in the NJL model the constant g_ρ from ρ-meson decay $\rho \to 2\pi(\frac{g_\rho^2}{4\pi} \approx 3)$ and the $\sigma q\bar{q}$-coupling constant g_σ are related by [1]- [5]

$$g_\rho = \sqrt{6}g_\sigma . \quad (18)$$

Then, from eqs. (14), (16) and (18) we get for the constituent quark mass the expression

$$m_u^2 = \frac{m_{a_1}^2}{12}[1-\sqrt{1-(\frac{2g_\rho F_\pi}{m_{a_1}})^2}] . \quad (19)$$

It is seen from (17) and (19) that $1-(\frac{2g_\rho F_\pi}{m_{a_1}})^2 \geq 0$ and hence the minimal mass of the a_1 meson equals $m_{a_1} = 2g_\rho F_\pi = \sqrt{2}m_\rho = 1.1GeV$, where in the last step the KSFR relation has been used. This is just Weinberg's relation. From this the estimates $m_u = 330MeV$ and $Z = 2$ follow.

The value of the gluon condensate is taken from the data on the hadron process $e^+e^- \to$ hadrons (see ref.[17])

$$G^2 = \frac{\alpha}{\pi} < G^a_{\mu\nu}G^{\mu\nu}_a > = [(410\pm 80)MeV]^4. \quad (20)$$

Taking into account the additional renormalization constant Z due to $\pi - a_1$ mixing, eq. (14) together with eq. (8) gives for F_π

$$F_\pi^2 = \frac{N_c m^2}{(2\pi)^2 Z}[\ln(\frac{\Lambda^2}{m^2}+1)-(1+\frac{m^2}{\Lambda^2})^{-1}+\frac{\pi^2}{6N_c m^4}G^2] . \quad (21)$$

Hence, using the values $F_\pi = 93MeV$, $N_c = 3$ and eq. (20) we find for the parameter Λ the estimate

$$\Lambda = 700 \ MeV . \quad (22)$$

Let us first calculate that part of the quark condensate $<\bar{q}q>'$ which does not explicitly contain the gluon condensate G^2 [4]

$$<\bar{q}q>' = \mathrm{Tr}(\frac{1}{i\hat{\partial}-m}) = -4mI_1 = -(200 \ MeV)^3. \quad (23)$$

The gap equation (11) can then be rewritten in terms of quark and gluon condensates as

$$m = m^0 - 2\kappa <\bar{q}q>' + \frac{\kappa}{6m}\frac{\alpha}{\pi} < G^a_{\mu\nu}G^{\mu\nu}_a > \equiv m^0 - 2\kappa <\bar{q}q>, \quad (24)$$

where we have introduced the notion of the total quark condensate $<\bar{q}q>$ which includes also gluon condensate corrections,

$$<\bar{q}q> = <\bar{q}q>' - \frac{1}{12}\frac{\alpha}{\pi}\frac{< G^a_{\mu\nu}G^{\mu\nu}_a >}{m} = -(245 \ MeV)^3. \quad (25)$$

We see that this number is close to the standard value of the quark condensate.

With eqs. (24) and (13) we find the constant κ

$$\kappa^{-1} = (\frac{m_\pi F_\pi}{m})^2 - \frac{2<\bar{q}q>}{m} = 0.091 \ GeV^2, \quad \kappa = 11 \ GeV^{-2}. \quad (26)$$

Finally, we can determine the current quark mass m^0,

$$m^0 = \frac{m_\pi^2 F_\pi^2 \kappa}{m} = m + 2\kappa <\bar{q}q> \approx 5 \ MeV. \quad (27)$$

[4]Note, however, that also in this case there exists an indirect dependence on G^2 via the constituent quark mass m (eq. (24)) and the cut-off Λ. To compare with quark condensates used in the QCD sum rule approach one must subtract a perturbative part $\mathrm{Tr}(\frac{1}{i\hat{\partial}-m_0})$ which is however, negligible for $SU(2)_F$ [4].

This is also a standard value. Thus, our model gives a reasonable self-consistent description of the most important model parameters and physical quantities.

5. Summary and conclusions.

The above investigation shows that corrections due to the gluon condensate which naturally arise in our QCD-motivated NJL model provide quite reasonable results. In particular, the gluon condensate $\frac{\alpha}{\pi} < G^a_{\mu\nu} G^{\mu\nu}_a >$ turns out to contribute to various quantities, for instance g_φ, F_π, m and $< \bar{q}q >$, without changing thereby the form of meson mass formulae and interaction terms in the effective meson Lagrangian. Thus, in the considered approximation the main effects of the gluon condensate are the decrease in the value of the low-energy cut-off scale Λ from 1.25 GeV [2,3] to 700 MeV and the increase in the coupling constant of the effective four-quark interaction κ from 5 GeV^{-2} to 11 GeV^{-2}.

It has been argued in the literature that in models with a gluon condensate a dynamical gluon mass could appear. For example, the authors of ref. [19] presented a description of the gluon condensate based upon an analogy to the Landau-Ginzburg theory of superconductivity. As a result of that analysis they predicted a gluon mass of the order of 600 MeV. In our approach, in the case of a massive gluon, we would have the relation $\kappa = \frac{g^2}{4\pi} \cdot \frac{4\pi}{2N_c M_G^2}$. At low energies one has $\frac{g^2}{4\pi} \approx 1$, which leads to an estimate $M_G \approx \sqrt{\frac{2\pi}{N_c \kappa}} \approx 440 MeV$ which is not too different from the above value.

The effect of the gluon condensate in nonlinear chiral Lagrangians was studied in ref. [20] as well, particularly for estimating its influence on the low-energy coefficients. The coefficients of the G^2 terms obtained in their expressions for F_π and in the quark condensate coincide with the coefficients of the present paper. However, a definite advantage of the present approach is the existence of an inherent mechanism for spontaneous breaking of chiral symmetry and the appearance of constituent quarks and meson masses on the basis of a simple effective four-quark interaction arising from gluon exchange. In the above-mentioned paper the mechanism of spontaneous breaking of chiral symmetry is brought in from outside by using additional assumptions. Another essential difference between both approaches is that we cannot neglect the quark condensate $< \bar{q}q >'$ since it is an important contribution of the four-quark interaction caused by quantum fluctuations of the gluon field. Finally, let us mention that the bosonization approach is in some sense complementarly to the more phenomenological approach of QCD sum rules [18]. Indeed, in our case composite hadrons arise as a result of two combined nonperturbative effects: i) by the ladder summation of (soft) gluon-mediated four-quark interactions and ii) the nonperturbative contributions of the quark and gluon condensates.

In the nearest future we are planning to extend these studies to nonlinear effective Lagrangians with fourth order derivatives in order to describe the coefficients of the low-energy expansion in a self-consistent way.

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Эберт Д., Волков М.К. E2-91-362

КХД мотивированная модель Намбу-Иона-Лазинио с кварковым и глюонным конденсатами

Проведено систематическое изучение роли непертурбативного глюонного конденсата, возникающего в КХД мотивированной НИЛ модели. Исследовано действие глюонного конденсата на индуцированные мезонные связи, пионную константу распада, кварковый конденсат и массовые формулы. Интересный результат связан с изменением Λ-шкалы нарушения киральной симметрии и универсальной четырехкварковой константы связи κ.

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QCD-Motivated Nambu-Jona-Lasinio Model with Quark and Gluon Condensates

We present a systematic study of the role of the nonperturbative gluon condensate arising in a QCD-motivated NJL model. The effects of the gluon condensate on induced meson couplings, the pion decay constant, quark condensate and mass formulae are investigated. An interesting result is the change of the scale Λ of chiral symmetry breaking and of the universal four-quark coupling κ.

The investigation has been performed at the Laboratory of Theoretical Physics, JINR.

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