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COUPLED-CHANNEL ANALYSIS OF LOW ENERGY NUCLEON-ANTINUCLEON INTERACTIONS

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1 INTRODUCTION

During past few years the experiments performed at the Low Energy Antiproton Ring (LEAR) at CERN and also at BNL (USA) and KEK (Japan) provided rich information on $N\bar{N}$ interactions at low energies.

There are two main features of proton-antiproton reactions at low energies which make them very distinct from pp interactions. The first is the large annihilation of a $p\bar{p}$ system (at antiproton beam momentum in the laboratory system p_L between 180 and 600 MeV/c the annihilation cross-section is about 2-3 times larger than the elastic one) whereas for pp interaction this process is absent. The second is the high anisotropy of elastic $p\bar{p}$ - scattering and $p\bar{p} \to n\bar{n}$ charge-exchange reaction which is produced by the interference of s - and higher waves. The analyses of data [1-3] lead to a significant p -wave contribution even at very small momenta to the elastic scattering cross section and to the annihilation which is quite different from pp scattering where the s - wave interaction dominates and the p -wave contribution is small.

One of the interesting features of the data is also the unusual behavior of the real-to-imaginary ratio of the $p\bar{p}$ forward elastic scattering amplitude, $\rho = Re\ f_{p\bar{p}}\ /\ Im\ f_{p\bar{p}}\ |_{\theta=0}$ which is large and negative at zero energy and changes rapidly when p_L grows.

The characteristic distance in low energy elastic and annihilation $p\bar{p}$ interaction is about 1 fm and for such distances the perturbation QCD theory is not applied. In this situation phenomenological analyses of data were made within the various potential models [4-8], but these analyses are in fact model dependent.

Other analyses fulfilled in the framework of the scattering-length expansion [9,10,15], on the contrary, are practically model-independent. These analyses incorporate analyticity and unitarity in the M -matrix formalism [11,12]. The $n\bar{n}$ channel was taken into account whose threshold is distant from $p\bar{p}$ ones at 98 MeV/c and affect the behavior of ρ and elastic $p\bar{p}$ -amplitude. The annihilation channels were considered by taking the elements of M -matrix in a complex form. But the arguments, given in paper [10], that the annihilation can be taken into account in a similar way are not consistent; particularly, in papers [9,10], mentioned above, there are discrepancies concerning the effective radii being complex or not. Below we discuss this problem, and our analysis involves the annihilation channels directly.

One point of interest is the question of resonances (or baryonium) and bound states in the $p\bar{p}$ system. In recent years, considerable experimental efforts have been made in search for this problem. Although several candidate states have been reported in the $p\bar{p}$ system, they have failed to be confirmed in $p\bar{p}$ cross section experiments. But if these resonances sit on a large nonresonant background, they might be visible more sensitively in the phase analysis rather than in cross section. Below we discuss the experimental data within the framework of phenomenological consideration and try to perform an analysis of that type.

The contents of the paper is the following. In section 2 we discuss briefly the M -matrix method and the problem of complex scattering lengths and radii. In section 3 the M -matrix effective range expansion is formulated for the $N\bar{N}$ system. In section 4 numerical fits to the data are presented. Section 5 is devoted to discussions of the fitted parameters, positions of the resonance and bound states and their connection with the behavior of the ρ ratio. In section 6 brief conclusions and the problems are presented.

2 MULTICHANNEL M - MATRIX METHOD

The multichannel effective range theory is a part of M - matrix method and it is based on the well-known effective range method in the case of elastic scattering.

The partial wave amplitude T_l satisfying the elastic unitary condition

$$Im\, T_l = 2i \mid T_l \mid^2 \tag{1}$$

may be written in the physical region in the form

$$T_l = \left(e^{2i\delta_l} - 1\right)/2i \;. \tag{2}$$

For the amplitude f_l determined by $T_l = k^{2l+1} f_l$ the unitary condition is

$$Im\, f_l^{-1} = -k^{2l+1} \;, \tag{3}$$

where k is the center-of-mass momentum,

$$k = \sqrt{[s-(m_1+m_2)^2][s-(m_1-m_2)^2]/4s} \;, \tag{4}$$

m_1 and m_2 are the masses of colliding particles. From equation (1) it follows that f_l may be represented in the form

$$f_l = \frac{1}{M_l(k) - ik^{2l+1}} \tag{5}$$

where $M_l(k)$ is the real function: $Im\, M_l(k) = 0$. Note also that

$$T_l^{-1} = -i + M_l/k^{2l+1} \quad and \quad M_l = k^{2l+1} cot\, \delta_l \;.$$

Function $M_l(k)$ is analytic in the complex k plane in the vicinity of $k=0$ and may be expanded in a series in powers of k^2

$$M_l(k) = \frac{1}{a_l} + \frac{1}{2} r_l\, k^2 + ... \tag{6}$$

At the threshold $f_l = a_l$. The parameters a_l and r_l are called "scattering lengths" and "effective radii", respectively, and are real because otherwise $Im\, M_l(k) \neq 0$.

Above the threshold the positions of possible resonances are determined by the relation $M_l(k) = 0$. Below the threshold $k \rightarrow i \mid k \mid$ and the zero of the denominator of formula (3) in the complex k -plane corresponds to a bound state, when the pole lies in the upper k half-plane (the first/physical sheet in the s -plane) and to a virtual state, when the pole lies in the lower k half-plane (the second/unphysical sheet in the s -plane).

The simplest way to include the inelasticity into this scheme is to change real a_l to complex $a_l' + ia_l''$. This leads to violation of elastic unitary condition (1) and elastic and total cross section are proportional to $a_l'^2$ and $a_l''^2$, respectively. But this simplest way is not sufficient if it is necessary to describe the transition amplitudes into concrete final states.

A suitable effective range method for reactions of a system of several two-body channels has been developed by M.Ross and G.Shaw [11,12]. For partial amplitudes $T_{ij} \equiv \langle j \mid T_l \mid i \rangle$ the unitary relations are

$$T_{ij} - T_{ij}^{+} = 2i \sum_n T_{in}^{+} T_{nj} \;, \tag{7}$$

where i, j, n - are numbers of channels. For amplitudes f_{ij} which are determined by the relation

$$\hat{T} = \hat{k}^{l+1/2} \hat{f} \hat{k}^{l+1/2} ,$$

where $\hat{k}$ is the diagonal matrix momentum $k_{ij} = k_i \delta_{ij}$ and k_i is given by eq.(4) the unitary condition has the form

$$Im \hat{f}^{-1} = -\hat{k}^{2l+1} . \tag{8}$$

Equation (8) is satisfied automatically if $\hat{f}$ is written in the form (the partial-wave index l for f_{ij} and M_{ij} is implied everywhere below)

$$\hat{f} = (\hat{M} - i\hat{k}^{2l+1})^{-1} , \tag{9}$$

where matrix $\hat{M}(s)$ is real and symmetrical and its elements are analytical (outside of dynamical cuts) functions free of unitarity singularities. Thus, they can be expanded in a series in powers of k^2 near any energy E_0 in the neighborhood of the threshold of one of the channels:

$$M_{ij} = M_{ij}(E_0) + \frac{1}{2} R_{ij} (k_i^2 - k_i^2(E_0)) + \dots , \tag{10}$$

where $M_{ij}(E_0)$, R_{ij} are the real constants.

The partial wave analysis based on expansion (10) is often used and we refer, for example, to the $\pi\pi$ analysis of Protopopescu et.al. [13] where the $K\bar{K}$ channel was taken into account and three first terms of expansion of $M_{ij}(s)$ were used. Note that Ross and Shaw demonstrated [11] that in a simple model a good approximation is $R_{ij} \simeq 0$ if $i \neq j$ even when the diagonal elements R_{ii} are large and numerically quite different.

Before we shall formulate our problem consistently, discuss briefly a discrepancy which often occurs in a phenomenological analysis of multichannel reactions using the effective range formalism with the so-called "complex scattering lengths". There are several ways to introduce that length.

i) Shaw and Ross [12] considered the case where just one initial channel is accessible experimentally. The complex energy-dependent scattering length $a(k)$ is introduced in this channel n by the formula

$$T_{nn} = -k_n^{2l+1} [1/a(k) + ik_n^{2l+1}]^{-1} \tag{11}$$

and $1/a(k)$ depends on the M -matrix elements:

$$-1/a(k) = M_{nn} + \mathcal{M}_{nn}^{-1} \sum_{i \neq n} M_{ni} \mathcal{M}_{in} , \tag{12}$$

where $\mathcal{M}_{ij}$ is the i, j minor of $(M - ik^{2l+1})$. The role of $a(k)$ in $T_{i,j}$ $(i, j \neq n)$ is complicated. It is seen from (11) that the energy dependence of $1/a(k)$ is not simple. In fact, the "zero-range" approximation, i.e. taking the matrix $R_{i,j}$ equal to zero is not equivalent to a constant scattering length. The complex scattering length can be used to describe some of observations but as it seen from (12) the knowledge of all $R_{i,j}$ is needed for the full effective range analysis.

ii) Dalitz et.al.(see [14]) used the complex scattering lengths in the K -matrix formalism. The hermitian K -matrix is related to the scattering amplitude T and M -matrix by

$$T = K (1 - iK)^{-1} , \quad M = \hat{k}^{2l+1} K^{-1} \hat{k}^{2l+1} .$$

For a two-channel problem (for simplicity)

$$K = \begin{pmatrix} \alpha & \beta \\ \beta & \gamma \end{pmatrix} \quad and \quad T_{11} = \frac{1}{A^{-1} - i} ,$$

where

$$A = \alpha + i\beta^2/(1 - i\gamma) \equiv a + ib$$

is interpretable as a complex (energy-dependent) length for the first channel since in the elastic case $(\beta = 0)$ $T_{11} = e^{i\delta_l} sin\delta_l$ and $A = \alpha = tg\delta_l$.

Though the amplitude

$$T_{12} = \frac{\beta}{1 - i\gamma} \frac{1}{1 - iA}$$

does not depend only on a and b, both quantities $| T_{11} |^2$ and $| T_{12} |^2$

$$| T_{11} |^2 = \frac{a^2 + b^2}{a^2 + (1 + b^2)} , \quad | T_{12} |^2 = \frac{b}{a^2 + (1 + b^2)} \tag{13}$$

do depend on them and it is possible to fit the cross sections σ_{11}, σ_{12} if $A = a + ib$ is assumed to be complex constant (zero effective range).

But, besides the energy-dependence of A, other amplitudes, for example T_{22}, cannot be evaluated only from a and b.

iii) In the framework of the M -matrix method complex scattering lengths in all channels were used in papers [9,10]. Note that matrix elements M_{ij} are not evaluated via the combination $A = a + ib$ and the complex lengths in the discussed papers are not the same as in the previous part. The reason for suggesting the lengths in M -matrix to be complex is the following. The amplitude f_{11} for the case of two coupled channels (for example) may be written in the form

$$f_{11} \;=\; 1/\,[(M_{11} - ik_1^{2l+1}) - M_{12}^2\,/\,(M_{22} - ik_2^{2l+1})]\,. \qquad (14)$$

If the second term in the square brackets is approximately constant for the concrete physical problem, it may be treated as a complex addition to $M_{11}(E_0)$ and the opening of the inelastic channel can be practically treated by introducing the complex scattering length (but the effective range is still real in contradiction with the assumption in paper [9]; see also [15]).

In paper [10] these arguments were presented for nucleon-antinucleon scattering for the purpose to take into account the annihilation channels for the two channel $p\bar{p}, n\bar{n}$ -problem. The results of our investigation demonstrate that these arguments are not correct enough.

3 EFFECTIVE RANGE EXPANSION FOR THE $N\bar{N}$ SYSTEM

The effective range analysis of $N\bar{N}$ interaction is rather complicated since one should take into account the $p\bar{p}$ and $n\bar{n}$ channels, annihilation channels and Coulomb attraction. As has been discussed above, we shall not use so ill-defined phenomena as the "complex scattering length", and shall try to include the annihilation channels into our scheme directly. For technical reasons we suppose that annihilation channels are effectively reduced to one channel that consists of two particles with equal masses m_0. The ratio m_0/m_p is, in principle, the parameter of the method, but the results are not rather sensitive to its variation in the physically reasonable region between about 0.6 and 0.8. Below we fix it equal to 0.7.

The matrix f_{ij} has the form

$$\hat{f}(s) = \begin{pmatrix} M_{11} - ik_1^{2l+1} & M_{12} & M_{13} \\ M_{21} & M_{22} - ik_2^{2l+1} & M_{23} \\ M_{31} & M_{32} & M_{33} - ik_3^{2l+1} \end{pmatrix} \qquad (15)$$

where indices 1, 2, 3 stand for channels of $p\bar{p}, n\bar{n}$ and annihilation. The momenta k_i are:

$$k_1 = \left(\frac{s - 4m_p^2}{4}\right)^{\frac{1}{2}}, \quad k_2 = \left(\frac{s - 4m_n^2}{4}\right)^{\frac{1}{2}}, \quad k_3 = \left(\frac{s - 4m_0^2}{4}\right)^{\frac{1}{2}}.$$

The real matrix $M_{ij}(s)$ is symmetrical because of the time-reversal invariance.

We obtain for the amplitudes discussed below:

$$\begin{aligned} f_{11} &= (M_{22} - ik_2^{2l+1} - x_l)/D_l\,, \\ f_{22} &= (M_{11} - ik_1^{2l+1} - y_l)/D_l\,, \\ f_{12} &= -(M_{12} - \sqrt{x_l y_l})/D_l\,, \end{aligned} \qquad (16)$$

where

$$\begin{aligned} x_l = M_{23}^2/(M_{33} - ik_3^{2l+1}),\;\; y_l &= M_{13}^2/(M_{33} - ik_3^{2l+1})\,, \\ D_l = (M_{11} - ik_1^{2l+1} - y_l)(M_{22} - ik_2^{2l+1} - x_l) &- (M_{12} - \sqrt{x_l y_l})^2 \\ &= det(\hat{f}^{-1})/(M_{33} - ik_3^{2l+1}). \end{aligned} \qquad (17)$$

It is seen from eq. (16),(17) that if momenta k_3 and all matrix elements M_{i3} are not constants, it is impossible to take the annihilation channels into account effectively by renorming matrix elements M_{ij} with indices $i, j = 1, 2$. Just the same situation occurs in our problem: i) the variations of k_3^2 and k_1^2 are equal when the momentum of an incident antiproton p_L grows from zero; ii) all our fits lead to at least one of the $M_{i3}(s)$ which depends substantially on energy.

Another problem is connected with isotopic invariance and its role for the matrix elements M_{ij}. Usually, this phenomenon is treated as a kinematical one, i.e., the M_{ij} are assumed to be unaffected by the mass splitting. In particular, the constant terms $M_{11}(E_0)$ and $M_{22}(E_0)$ which are named in papers [9,10] the scattering lengths of $p\bar{p}$ and $n\bar{n}$

coincide and can be expressed in terms of the isospin scattering lengths $a_0^{(l)}$ and $a_1^{(l)}$ with $I=0$ and $I=1$:

$$M_{11}(E_0)=M_{22}(E_0)=\frac{a_0^{(l)}+a_1^{(l)}}{2a_0^{(l)}a_1^{(l)}}\quad,\quad M_{12}(E_0)=\frac{a_1^{(l)}-a_0^{(l)}}{2a_0^{(l)}a_1^{(l)}}\,. \tag{18}$$

The effective radii of $p\bar{p}$ and $n\bar{n}$ interactions are supposed to be equal to each other too, but for $p\bar{p}\to n\bar{n}$ the analogous term is supposed to equal zero.

In distinction with this approach, we do not make such a not well-founded assumption that in the case when the cross section $p\bar{p}\to n\bar{n}$ is of the same order of magnitude as the elastic one, i.e. large isotopic-invariance violation, the matrix elements M_{ij} are unaffected by this violation. We want to stress that if one takes into account the isospin violation not only by proton and neutron mass difference but with inclusion of the charge-exchange channel, the violations become dynamical and the difference between amplitudes of strong interactions f_{11} and f_{22} may be large by definition. Note, that we use equal initial values of adjustible parameters for the matrix elements M_{11} and M_{22}, but this equality breaks down in the processes of fitting procedure due to the effect of charge-exchange channel.

Therefore, we do not suppose any artificial restriction on coefficients of M_{ij} and physical values of the scattering lengths and effective radii will only be results of the fit. Thus, we use the actual scattering amplitudes but $a_0^{(l)}$ and $a_1^{(l)}$ are calculated via actual physical length, i.e. via the values of amplitudes f_{ij} at the corresponding thresholds. To end this discussion, let us write down these scattering lengths for the two channel $p\bar{p}$ -, $n\bar{n}$ -problem without annihilation and Coulomb attraction (for simplicity). Using eq.(10) with taking the $n\bar{n}$ threshold for E_0 we obtain:

$$\begin{aligned}
f_{11}\,|_{k_1=0} &= \left[a_{11}^{-1}-\frac{1}{2}r_{11}\Delta-\frac{(a_{12}^{-1}-\frac{1}{2}r_{12}\Delta)^2}{a_{22}^{-1}-\frac{1}{2}r_{22}\Delta+\Delta^{1/2}}\right]^{-1},\\
f_{22}\,|_{k_2=0} &= \left[a_{22}^{-1}-\frac{a_{12}^{-1}}{a_{11}^{-1}-i\Delta}\right]^{-1}, \qquad\qquad (19)\\
f_{12}\,|_{k_1=0} &= -(a_{12}^{-1}-\frac{1}{2}r_{12}\Delta)[(a_{11}^{-1}-\frac{1}{2}r_{11}\Delta)(a_{22}^{-1}-\frac{1}{2}r_{22}\Delta+\Delta^{\frac{1}{2}})\\
&- (a_{12}^{-1}-\frac{1}{2}r_{12}\Delta)^2]^{-1},
\end{aligned}$$

where

$$\Delta \;=\; m_n^2-m_p^2\,,\quad a_{ij}^{-1}\equiv M_{ij}(E_0);$$

f_{22} is complex indeed. Only in the case when channels $p\bar{p}$ and $n\bar{n}$ are not coupled ($M_{12}=0$ and $\sigma(p\bar{p}\to n\bar{n})=0$) and $\Delta=0$, the channels $p\bar{p}$ and $n\bar{n}$ coincide and the equality $a_{11}=a_{22}$ must hold. It is clear that it is not correct to demand this equality *a priory* especially when $\sigma(p\bar{p}\to n\bar{n})$ is compatible with the cross section of an elastic process.

We take into account partial amplitudes with $l=0$ and $l=1$ and use the effective range expansion of elements M_{ij} in the standard form [11]

$$M_{ij}^{l=0}=\frac{1}{a_{ij}}+\frac{1}{2}r_{ij}k_2^2\,,\quad M_{ij}^{l=1}=\frac{1}{b_{ij}}-\frac{3}{2}\frac{1}{R_{ij}}k_2^2\,, \tag{20}$$

where the $n\bar{n}$ threshold was taken for E_0 and only first two terms in the row (10) were kept.

The differential cross section of the elastic scattering amplitude and charge-exchange reaction are expressed through the spin average amplitudes:

$$\begin{aligned}
d\sigma_{11}/d\Omega &= \mid F_C(\theta)+c^2[f_{11}^{(0)}+3k_1^2e^{2i(\delta_1-\delta_0)}(1+\eta^2)f_{11}^{(1)}cos\theta]\mid^2,\\
d\sigma_{12}/d\Omega &= \frac{c^2k_2}{k_1}\mid f_{12}^{(0)}+3k_1k_2e^{i(\delta_1-\delta_0)}(1+\eta^2)^{1/2}f_{12}^{(1)}cos\theta\mid^2, \qquad (21)
\end{aligned}$$

where the Coulomb amplitude [10,12,18]

$$F_C(\theta) \;=\; -(2k_1 sin^2\frac{\theta}{2})^{-1}\,\eta\,exp\,[-i\,\eta\,ln\,(sin^2\frac{\theta}{2})]\,, \tag{22}$$

$\eta=-1/k_1a_B$, $a_B=2/\alpha m_p\simeq$ 57.6 fm is the Bohr radius of the $p\bar{p}$ atom,

$$c^2=-2\pi\eta/[1-exp\,(2\pi\eta)]\,,\;\;\delta_l=arg\,\Gamma(l+1+i\eta).$$

According to papers [10,12], besides the Coulomb factors explicitly contained in eqs.(21) and (22), the amplitudes f_{ij} have still additional Coulomb corrections produced by changing $(M_{11}-ik_1^{2l+1})$ to $(M_{11}-ik_1^{2l}\,\bar{k}_1\prod_l+\Delta_C^{(l)})$, where

$$\bar{k}_1=k_1\,[c^2-2i\,\eta\,h(k_1a_B)]\,,\;\;h(x)\simeq-\gamma+ln\,x+1.2/x^2,\;if\;x\gg1\,,$$

$\gamma = 0.577...$ is the Euler constant, $\prod_l = 1$, if $l = 0$ and $\prod_l = 1 + \eta^2$, if $l = 1$, Δ_C is the Coulomb correction to the scattering lengths: $\Delta_C^{(l=0)} \simeq -0.08 fm^{-1}$, $\Delta_C^{(l=1)} \simeq 0$.

4 NUMERICAL ANALYSIS OF THE EXPERIMENTAL DATA

Our data set covers differential elastic and charge exchange $p\bar{p}$ cross sections, angle-integrated cross sections $\sigma_{el}(p\bar{p})$, $\sigma_{tot}(p\bar{p})$, $\sigma_{ex}(p\bar{p})$ and the real-to-imaginary ratio of the $p\bar{p}$ forward elastic scattering amplitude ρ . At our disposal there are differential cross section data for elastic $p\bar{p}$ scattering at three values of incident momenta: $p_L =$ 181, 287 and 505 MeV/c in the full angular region [1]. The differential cross section data for charge-exchange $p\bar{p} \rightarrow n\bar{n}$ scattering exist for a large set of momenta p_L [3,24], and we use few of them which are close to available momenta for elastic scattering. We use the data of refs. [1,3,16,17,19,20] for angle-integrated cross sections and data of refs. [16,21,22] for ρ values. The information for ρ at zero energy comes from the energy shift and width, $\Delta E_{1s} - i\Gamma_{1s}/2$ of the 1s shift of a $p\bar{p}$ atom (antiprotonic hydrogen) [25,26]. The experimental data are $\Delta E_{1s} = (0.8 + 0.2)\ KeV$, $\Gamma_{1s} = (1.0 + 0.2)\ KeV$. Note also that the present experimental data for the cross sections are averaged over spins and therefore our theoretical investigation does not include the spin dependence.

We analyze the whole set of the above-mentioned data simultaneously, i.e. we make the energy-independent fit. The curves describing the differential cross section for elastic scattering are shown in Fig.1-3. Two typical results for the differential cross section for charge-exchange scattering at two values of momenta are shown in Fig.4 and 5. One sees that the agreement of theory with experiment in a wide energy interval is not bad. Of course, when we made the energy-dependent analysis, we got the excellent χ^2 quality at each energy for both elastic and charge-exchange differential cross sections.

Fig.6 shows the ratio ρ and Fig.7 shows the angle integrated cross sections $\sigma_{el}(p\bar{p})$, $\sigma_{tot}(p\bar{p})$ and $\sigma_{ex}(p\bar{p})$ as a function of antiproton beam momentum.

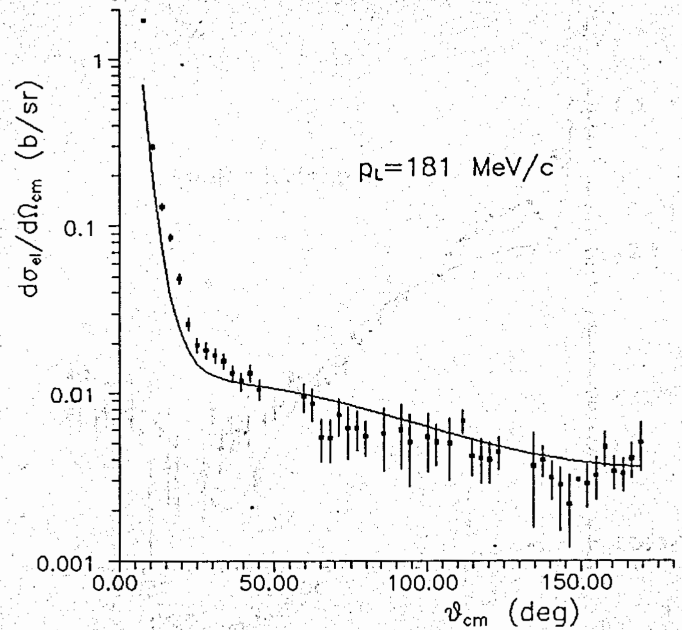

FIG.1. Differential cross section for $p\bar{p}$ elastic scattering at $p_L = 181 MeV/c$. Experimental data from ref.[1].

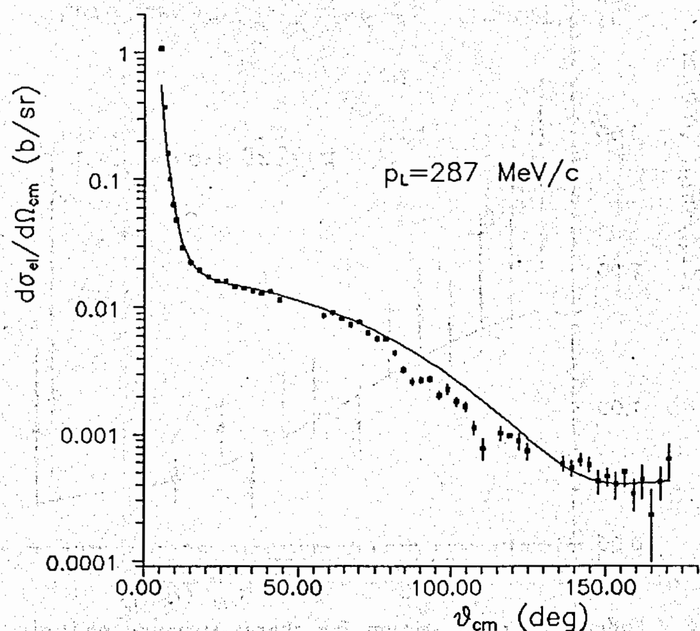

FIG.2. The same as in Fig.1., but $p_L = 287 MeV/c$.

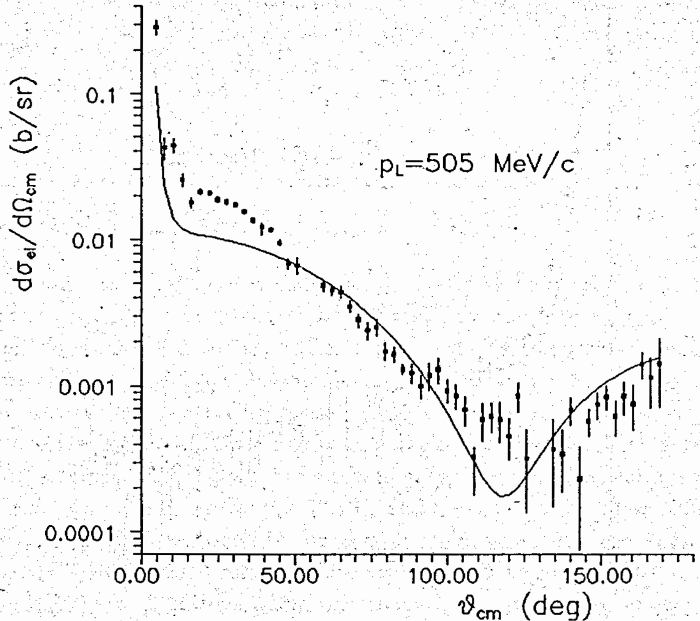

FIG.3. The same as in Fig.1., but $p_L = 505 MeV/c$.

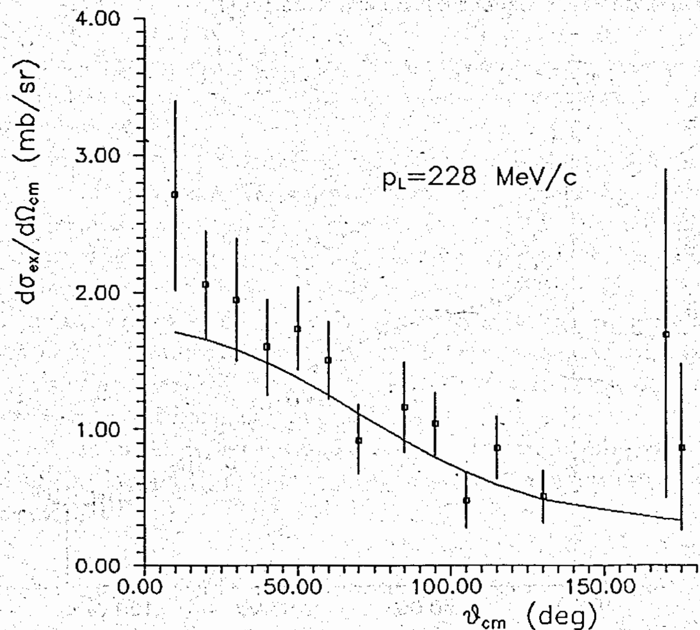

FIG.4. Differential cross section for charge-exchange scattering at $p_L = 228 MeV/c$. Experimental data from ref.[24].

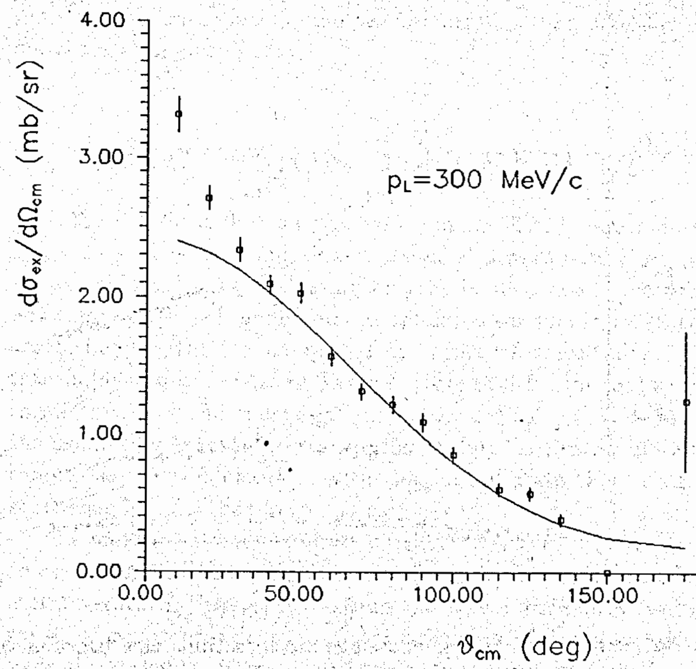

FIG.5. The same as in Fig.4., but $p_L = 300 MeV/c$.

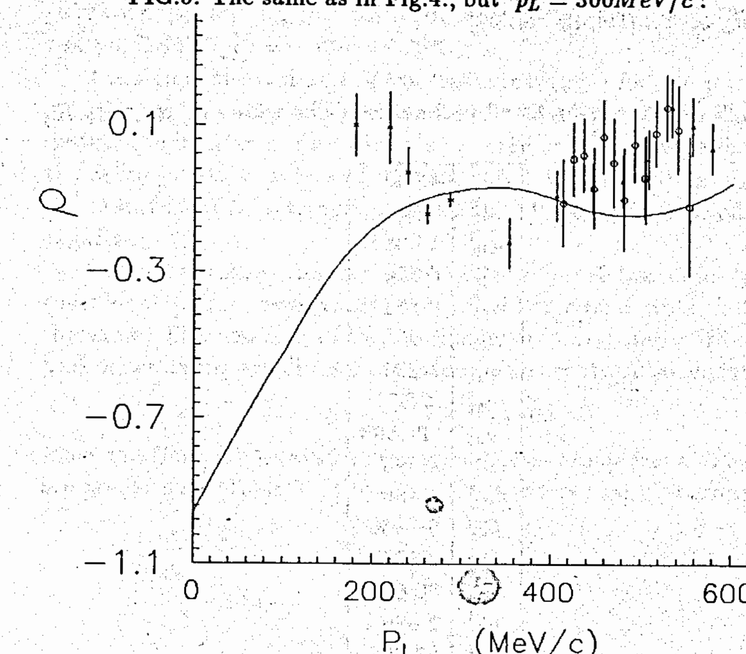

FIG.6. Real-to-imaginary ratio of forward $p\bar{p}$ scattering as a function of beam momentum. Experimental data: $\ast$ - ref.[16], ϕ - ref.[21], ▲ - ref.[22].

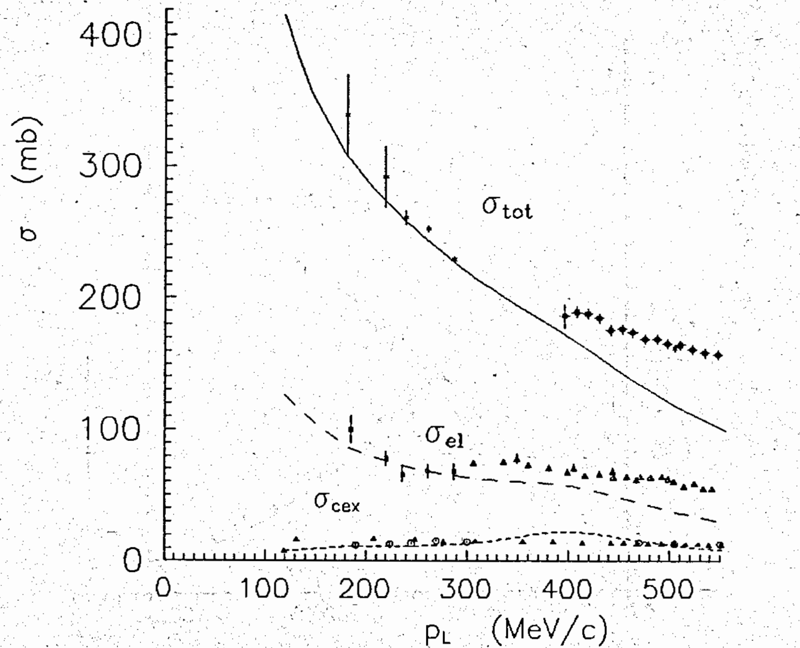

FIG.7. Total, elastic and charge-exchange cross sections as a function of beam momentum. Experimental data: ■ - ref.[1], ϕ - ref.[3], ✶ - ref.[16], ● - ref.[17], ▲ - ref.[19], △ - ref.[20].

TABLE 1. Values of the fitted parameters. The values of a_{ij}, b_{ij}, R_{ij} in fm, b_{ij} in fm^3.

a_{11}^{-1}	5.025
a_{22}^{-1}	9.484
a_{33}^{-1}	14.664
a_{12}^{-1}	6.311
a_{13}^{-1}	8.874
a_{23}^{-1}	7.920
r_{11}	3.291
r_{22}	-7.577
r_{33}	-11.808
b_{11}^{-1}	4.420
b_{22}^{-1}	1.450
b_{33}^{-1}	95.560
b_{12}^{-1}	0.804
b_{13}^{-1}	20.616
b_{23}^{-1}	2.080
R_{33}^{-1}	22.892

The values of the fitted parameters are listed in Table 1. The 16 real parameters were adjusted (the parameters C_{11} and C_{22} were fixed and equal zero, for simplicity).

5 DISCUSSION AND RESULTS

We comment first on the values of the fitted parameters listed in Table 1. As has been mentioned above, real parameters a_{ij}^{-1} and b_{ij}^{-1} do not coincide with scattering lengths in respective channels and are not compatible with analogous complex parameters in ref.[9,10,15]. The scattering lengths are computed as values of amplitudes f_{ij}^{l} at the thresholds and presented in Table 2. Note that for the aim to investigate the influence of the charge-exchange channel to $p\bar{p}$ and $n\bar{n}$ channels the scattering lengths are computed without inclusion of Coulomb interactions, i.e. for channels with $p\bar{p}$ initial state they are the 'strong scattering lengths' but not physical ones.

One sees from Table 2 that threshold values of f_{11}^{l} and f_{22}^{l} differ from each other and isospin invariance is broken rather sufficiently. It is not surprising, as it follows from the above discussion, and is not the result of our approach only. For the purpose of demonstrating this, we have calculated the above-mentioned values using the formulas and coefficients of paper [9]: $f_{11}^{(0)} = -0.131+i0.818$, $f_{22}^{(0)} = -0.044+i0.730$, $f_{11}^{(1)} = 1.695+i0.174$, $f_{22}^{(1)} = 1.385+i0.672$.

The sign of the real part of the scattering length for $p\bar{p}$ scattering is negative for s -wave and positive for p -wave, and it may lead to the destructive interference between the s -and p -waves contributions to the ratio ρ. As to values of r_{ij} and R_{ij}, they do not correspond to the radius of $N\bar{N}$ interaction and in the M -matrix method may be negative.

A way to understand the unusual behavior of the ratio ρ is to consider the $N\bar{N}$ resonances (baryonium) or bound states below $p\bar{p}$ threshold. The positions of the resonances or bound states of the strong part of scattering amplitudes are given by the roots of the equations:

$$Re\ Det\,(\hat{f}_{ij}^{-1}) \;=\; 0. \tag{23}$$

when the Coulomb interactions are not taken into account. The results are presented in Table 3. It was surprising for us that all performed fits

TABLE 2. The scattering lengths computed as the values of strong parts of scattering amplitudes $f_{ij}^{(l)}$ at the corresponding thresholds at $k_i = 0$. The values of $f_{ij}^{(l=0)}$ in fm, $f_{ij}^{(l=1)}$ in fm^3.

$f_{11}^{(0)}$	$-0.867 + i0.902$
$f_{22}^{(0)}$	$0.201 + i0.170$
$f_{12}^{(0)}$	$0.079 - i0.368$
$f_{11}^{(1)}$	$0.242 + i0.609$
$f_{22}^{(1)}$	$0.737 + i0.038$
$f_{12}^{(1)}$	$-0.123 - i0.151$

TABLE 3. $N\bar{N}$ bound states and resonance. Masses and total and partial widths of the resonance are in MeV.

l	m	Γ	$\Gamma_{p\bar{p}}$	$\Gamma_{n\bar{n}}$	Γ_{annih}
1	1809				
1	1843				
1	1875.8				
0	1942.5	46	17	14.6	14.4

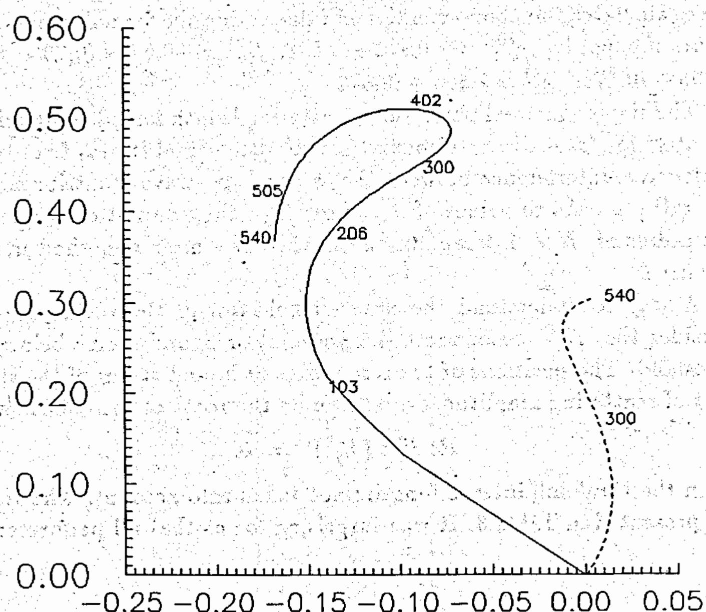

FIG.8. Argand diagrams for $p\bar{p}$ scattering for s- wave (left curve) and p -wave (right curve).

lead to resonance in s -wave at an incident antiproton momentum of about 500 MeV/c or 1940 MeV energy in the *c.m.s.* It is very interesting because among the reported baryonium candidates, the $S(1935)$ resonance (list of relevant information see in [23]) has a long history. It was observed as a peak in both total and annihilation cross sections of the $p\bar{p}$ interactions [19,27-31] and the existence of a rather narrow S -resonance seemed to be established. However, other experiments did not confirm the existence of enhancement in the S -region in total cross sections. In paper [17] reanalysis of the data of few groups was made and no evidence was found for narrow structure (less than 10 MeV) in the $p\bar{p}$ total cross section but it was pointed out that the broad S resonance (width ≥ 20 MeV.) is statistically inconsistent with the data.

The Argand diagrams, plotted in Fig.8 show the elastic $p\bar{p}$ amplitude T_{11} for s - and p -waves. The Argand curve for s -wave has a circle due to the resonance behavior of the amplitude. The total and partial widths of the resonance are presented in Table 3 and they have been calculated following paper [32]. As our fit for total cross section is not good enough at beam momentum above $400 MeV/c$, we are not sure that the resonance parameters are rather correct, but it is interesting that though the resonance is clearly seen in the partial wave, it is not visible in the cross sections where it is hidden by the background. Parameters of the background and its influence on the parameters of the resonance will be studied in a subsequent paper.

One topic of interest is to establish the connection between the behavior of the $\rho(p_L)$ and existence of the resonance and bound states. For this purpose we have experimented with artificially changing value of $\rho(0)$ in a wide interval. All fits describing the cross sections well enough lead to the s -wave resonance with a mass from about 1930 to 1945 MeV. But the positions of bound states are very sensitive to the value of $\rho(0)$ and to the fit of the cross sections. Therefore, the behavior of the ρ ratio is strongly correlated with the existence and position of bound states in $N\bar{N}$ amplitudes.

Note that the fits with energy-independent $M_{33}^{(l)}$ are sufficiently worse than those described above. As was mentioned, this means that annihilation channels cannot be taken into account by assuming that matrix elements $M_{ij}^{(l)}$ are complex.

6 CONCLUSION

We have made the coupled channel M -matrix investigation for low energy $N\bar{N}$ interaction, which takes into account $p\bar{p}$, $n\bar{n}$ and annihilation channels. Several features of $N\bar{N}$ interactions were the topics of our interest: high anisotropy, rapid rise of the ρ ratio from a large and negative value at a zero energy, possible resonances and bound states, the values of low-energy parameters, such as scattering lengths.

We have used the effective range expansion of matrix elements $M_{ij}^{(l)}$ and kept two first terms in this expansion. The coefficients of these series are real in accordance with the M -matrix theory whereas in similar investigations on this topic the complex coefficients were used. Considering the annihilation channels by taking into account the M -matrix elements in a complex form is formally suitable but not consistent because of the following reasons: i) though it leads to additional inelasticity, any information about position of the inelastic channels is lost; ii) being complex, only few elements of the M -matrix are necessary for taking effectively into account the inelastic channels and it is not clear which matrix elements are to be considered as complex and why.

Breaking of the isotopic invariance due the important role of the charge exchange channel affects very significantly the parameters of strong interactions. It is not a result of our concrete investigation but is a result of inelastic unitarity for partial amplitudes. Particularly, the radii of interactions in $p\bar{p}$ and $n\bar{n}$ channels in potential models are not equal to each other *a priory* .

Our analysis leads to a rather wide ($\Gamma \simeq 46\ MeV$) resonance in s -wave at invariant mass 1942 MeV (which produces the resonance behavior of s -partial wave) in spite of it being not visible in the total cross section. The existence of this resonance and the position of the bound states are strongly connected with the unusual behavior of $\rho(p_L)$. The existence of bound states in p -wave near the threshold leads to an important role of this wave in $N\bar{N}$ interactions. These two results made our work different from papers [9,10]. They are the consequences of an adequate account of unphysical cut in the $p\bar{p}$ elastic scattering. Our conclusion agrees with that of paper [33] where the similar procedure was made.

The important question is the range of applicability of our approach. One of the problems arises from the dynamical cuts in the partial wave amplitudes due to the t -channel meson exchange. The second is the use of the effective range approximation, whereas in NN scattering the term proportional to k^4 is important at a rather small energy. The third is to take account of d -wave. All these problems are now under investigations.

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Хеннер В.К., Мещеряков В.А. E2-91-360
Многоканальный анализ низкоэнергетических нуклон-антинуклонных взаимодействий

Многоканальный M-матричный метод применен к системе $N\bar{N}$; учтены упругое $p\bar{p}$-рассеяние, реакция перезарядки $p\bar{p} \to n\bar{n}$ и канал $p\bar{p}$-аннигиляции. Показано, что за счет большого влияния канала перезарядки, параметры сильных взаимодействий не удовлетворяют стандартным предположениям гипотезы изотопической инвариантности. В отличие от ранних подходов матричные элементы M-матрицы действительны. Мы описываем дифференциальные и полные сечения процессов $p\bar{p} \to p\bar{p}$ и $p\bar{p} \to n\bar{n}$, полные сечения $\sigma_{tot}(p\bar{p})$ и отношение действительной и мнимой части амплитуды упругого $p\bar{p}$ рассеяния до значений импульсов 600 МэВ/с. Процедура минимизации приводит к резонансу в s-волне при импульсе 500 МэВ/с и связанному состоянию в p-волне вблизи порога.

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Henner V.K., Meshcheryakov V.A. E2-91-360
Coupled-Channel Analysis of Low Energy Nucleon-Antinucleon Interactions

Multichannel M-matrix method is applied to an $N\bar{N}$-system with taking into account elastic $p\bar{p}$-scattering, charge-exchange $p\bar{p} \to n\bar{n}$ reaction and $p\bar{p}$ annihilation. It is shown that contrary to standard assumptions the isotopic violation due the large influence of the charge exchange channel affects very significantly the parameters of strong interactions. Contrary to previous descriptions, M-matrix elements are treated as real functions. We describe the differential and total cross-sections for $p\bar{p} \to p\bar{p}$ and $p\bar{p} \to n\bar{n}$ processes, cross-section $\sigma_{tot}(p\bar{p})$ and the real-to-imaginary ratio of the forward amplitude of elastic $p\bar{p}$ scattering up to beam momenta p_{lab}=600 MeV/c. The fitting leads to a resonance in the s-wave at about p_{lab} = 500 MeV/c and to bound states in p-wave near the $p\bar{p}$ threshold.

The investigation has been performed at the Laboratory of Theoretical Physics, JINR.

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