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ON THE SUBTLETIES OF THE SPECTRUM OF LIGHT MESON RESONANCES
I. THE MODEL

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1. Introduction

A search for unusual meson resonances - glueballs and hybrids - comes across unsatisfactory knowledge of the background of "standard" excitations of a $q\bar{q}$-system obeying the same quantum number J^{PC}. Of course, the investigation of "standard" excitations has a great interest itself.

In this paper the spectrum of radial and orbital excitations (with LS=0) of light mesons (consisting of u, d, and s quarks) is calculated within a relativized potential model elaborated by Gerasimov [1] on the basis of the quasi-potential approach given by Todorov [2]. A similar approach was developed by Godfrey and Isgur [3] to describe the spectrum of all mesons and their excitations from the pion to the upsilon.

We do not pursue such global purposes and we concentrate our efforts on the best description of light mesons only. A proximity of our results to the results [3] justifies our approach. Also we do not consider radial and orbital excitations with $LS\neq 0$ in order to avoid ambiguities carried by uncertainties of the spin-orbit interaction, and η-η'-mesons because for these mesons the annihilation channel, which is not enough studied, is essential.

In Section 2 we briefly remind general features of the employed model [1,2] (In Appendix A we give the application of this model to our case in more detail). At the same time we compare the calculated spectrum of radial and orbital excitations (with LS=0) with experimentally observed resonances of light mesons.

The data analysis on the basis of predicted resonances will be presented in Part II of this work.

2. The predicted spectrum of radial and orbital excitations of light mesons

The recipe of the relativization of the potential $q\bar{q}$ – model given by Gerasimov [1] consists in the following. First, we have to solve a nonrelativistic problem on the eigenstates of relative motion of quark and antiquark coupling by elected (nonrelativistic) potential*). Secondly, we substitute instead of quark and antiquark masses their effective energies

$$\varepsilon_{q,\bar{q}} = [W^2 \pm (m_q^{\,2} - m_{\bar{q}}^{\,2})]/(2W), \tag{1}$$

where W is the total energy in the c. m. s. Thirdly, we have to resolve the equation for W

$$W = (\varepsilon_q^{\,2} + m_q^{\,2})/(2\varepsilon_q) + (\varepsilon_{\bar{q}}^{\,2} + m_{\bar{q}}^{\,2})/(2\varepsilon_q) + E(\varepsilon_q ,\varepsilon_{\bar{q}}), \tag{2}$$

where $E(\varepsilon_{\bar{q}} ,\varepsilon_q)$ is just a manifest (not numerical!) solution of the nonrelativistic problem for the binding energy eigenvalue with the above substitution $m_q \Rightarrow \varepsilon_q$, $m_{\bar{q}} \Rightarrow \varepsilon_{\bar{q}}$.

In our problem, we consider a combination of the Coulomb-like potential with the linear potential

$$V(r) = - \kappa/r + r/a^2. \tag{3}$$

We impose an additional requirement for the binding energy of the ground state to vanish,

$$E_{1S} = 0, \tag{4}$$

and thus $\varepsilon_q = m_q$, $\varepsilon_{\bar{q}} = m_{\bar{q}}$ in this state. Under this ansatz we have a supplementary dimensionless equality

$$(2\mu a)^{2/3}\kappa = \lambda_0 \approx 2.144..., \ (1/\mu = 1/m_q + 1/m_{\bar{q}}), \tag{5}$$

which gives a connection between the parameters κ and a.

*) Remark, all this procedure is justifiable if the potential behaves as a Lorentz-scalar that is usually implied for the confinement potential.

We propose that the parameter λ has a fixed value (5) for all states. So, all calculations will be performed at this universal value λ_0.

For the manifest solution of the nonrelativistic problem with the potential (3) we employ the interpolating formulae presented in Appendix A being somewhat improved of the corresponding formulae from the work [4].

Then, we are in need of introducing a spin-spin interaction, and we put it into the linear mass operator in the form of a contact term

$$\hat{M} = W + 4\hat{C}\, \mathbf{s}_q \circ \mathbf{s}_{\bar{q}}\, \delta(\mathbf{r}). \tag{6}$$

For matrix elements of the operator $\hat{C}$ between i and j states of the $q\bar{q}$-system we propose

$$C_{ij} = C/(\varepsilon_q^{\,i}\varepsilon_q^{\,j}\varepsilon_{\bar{q}}^{\,i}\varepsilon_{\bar{q}}^{\,j})^{1/2} \tag{7}$$

with C as a universal constant. Really, we will consider two different variants: first, when the spin-spin coupling is determined by this formula (7) and which we call the "weak" spin-spin coupling, and second, when we substitute in formula (7) masses of a quark and an antiquark instead of their effective energies and which we call the "strong" (nonrelativistic) spin-spin coupling. In the latter case we have matrix elements independent of states

$$C_{ij} = C/(m_q m_{\bar{q}}). \tag{8}$$

Now, the unknown parameters of the problem are: masses of nonstrange (m_0) and strange (m_s) quarks, the strength of the spin-spin interaction (C), and the parameter of, for instance, linear potential (a). These four parameters are fixed by the reproduction of experimental masses of $\pi(140)$–, $\rho(770)$–, $K(496)$–, and $b_1(1235)$-mesons. The first three masses are calculated as total energies of the ground states of our problem with taking into account the mixing of radial excitations with the ground state stipulated by the spin-spin interaction. The mass of b_1-meson is calculated as the total energy of the P-wave orbital excitation of the $q\bar{q}$-system with

S=0 (The mixing of orbital excitations is ignored). Then, the parameters take the values: for the case of "strong" spin-spin coupling

$$m_0=336 \text{ MeV}, \; m_s=491 \text{ MeV}, \; C=115 \text{ MeV}, \; a=3.74 \text{ GeV}^{-1},$$

and for the case of "weak" spin-spin coupling

$$m_0=311 \text{ MeV}, \; m_s=492 \text{ MeV}, \; C=152 \text{ MeV}, \; a=3.58 \text{ GeV}^{-1}.$$

The masses of other meson states (with LS=0) are calculated within these two schemes at these fixed values of parameters.

The results of our calculations of masses of radial (L=0) excitations of pseudoscalar (0^{-+}) and vector (1^{--}) mesons are presented in Table 1. In particular, the calculation of masses $K^*(892)$ and $\phi(1020)$ serves as a control of our approximation. The bigger value of calculated mass of the ϕ-meson comparing to its experimental value can be partially stipulated by the neglect of a mixture of strange and non-strange quarks for the ω- and ϕ-mesons.

The results of our calculations for the orbital (with S=0) excitations together with their subsequent radial excitations are presented in Table 2.

In the same Table 1 and 2, we present for the comparision the results of paper /3/. As we can see, for the *first* radial excitations of both S- and P-states our results are in good agreement with the results /3/. For the second and higher radial excitatitions as well as for higher (D and F) orbital excitations our preditions are 100-150 MeV lower than the corresponding levels in paper /3/. It can be explained by a smaller inclination of the linear potential in our case than an analogous value in work /3/.

Now, we can compare the results of our calculations (and calculations of paper /3/ as well) with the available experimental data /5/ shown in Tables 1 and 2. One can be convinced of a coincidence of our predictions of π-, K-, ρ-, ω-, and ϕ-mesons (Table 1, second column) with the corresponding observed resonances. It is wonderful that the latter settle down

Table 1

Mass spectrum (in MeV) of radial excitations of pseudoscalar and vector light mesons

Pseudoscalar mesons, $J^{PC}=0^{-+}(L=0,\ S=0)$						
π-resonances						
Our calculations [a]	<u>137.5</u>±0.5	1295±65	1790±65	2165±65	2490±70	2770±70
Theory /3/	150	1300	1800			
Data /5/	137.5	1300±100	1770±30			
K-resonances						
Our calculation [a]	<u>496</u>	1435±55	1900±60	2255±65	2575±70	2840±70
Theory /3/	470	1450	2020			
Data /5/	496	~1460	~1830			
Vector mesons, $J^{PC}=1^{--}(L=0,\ S=1)$						
ρ/ω-resonances						
Our calcucation [a]	<u>768</u>	1420∓15	1870±5	2230±15	2545±20	2820±25
Theory /3/	770	1450	2000			
Data /5/	768 /782	1450±8 /1391±18	1700±20 /1594±12	~2150		
K^*-resonances						
Our calculation [a]	896	1530±2	1960±15	2305±25	2615±30	2880±35
Theory /3/	900	1580	2110			
Data /5/	892	1412±12	1714±20			
ϕ-resonances						
Our calculation [a]	1037±6	1640±15	2050±25	2385±35	2685±40	2945±45
Theory /3/	1020	1690				
Data /5/	1019.4	1680±50				

[a] In our calculations up and down deviations correspond to "weak" and "strong" spin-spin coupling schemes respectively. Underlined values were taken as input ones.

Table 2

Mass spectrum (in MeV) of orbital excitations of light mesons

1^{+-}-mesons (L=1, S=0)

b_1-resonances

Our calculation[a]	<u>1235</u>	1715±20	2095±30	2415±40	2695±45	2945±50
Theory /3/	1220	1780				
Data /5/	1233±10					

K_1-resonances

Our calculation[a]	1365±8	1820±25	2180±35	2490±40	2765±50	3010±55
Theory /3/	1340	1900				
Data /5/	1270±10 1402±7	~1800				

h_1-resonances($s\bar{s}$)

Our calculation[a]	1490±15	1920±30	2270±40	2570±45	2835±50	3075±60
Theory /3/	1470	2010				
Data /5/	1380±20					

2^{-+}-mesons (L=2, S=0)

π_2-resonances

Our calculation[a]	1560±10	1945±25	2295±35	2580±45	2845±50	3105±55
Theory /3/	1680	2130				
Data /5/	1665±20	2100±150				

K_2-resonances

Our calculation[a]	1670±20	2040±30	2375±40	2655±45	2915±50	3165±60
Theory /3/	1780	2230				
Data /5/	1580 1768±14	2247±17				

ϕ_2-resonances

Our calculation[a]	1780±25	2135±35	2455±45	2730±50	2980±55	3230±60
Theory /3/	1890					
Data	?					

(continuation of Table 2)

3^{+-}-mesons(L=3, S=0)

π_3-resonances

Our calculation[a]	1830±20	2170±30
Theory /3/	2030	
Data	?	

K_3-resonances

Our calculation[a]	1920±25	2255±35
Theory /3/	2120	–
Data /5/	?	2324±24

ϕ_3-resonances

Our calculation[a]	2015±30	2340±40
Theory /3/	2220	
Data	?	

[a] The notations are the same as in Table 1.

between our preditions for the "strong" and "weak" spin-spin coupling. So, we can rely upon a plausibility of our approach for other cases.

The only exclusion takes place for the radial excitation of K^*(892)-meson: our prediction for the mass of this excitation is 1530±2 MeV (the prediction /3/ is even higher: 1580 MeV/) in comparision with the observed mass of K^*(1410): 1412±12 /5/ (or even smaller 1367±54 MeV as it was observed in the latest experiment cited in /5/). Moreover, there is an enigmatic degeneration of this strange resonance with the corresponding nonstrange $\rho(1450)/\omega(1390)$-resonances found recently /5/. So, K^*(1410)-meson looks as a superfluous resonance with respect to the "standard" excitations of the $q\bar{q}$-system.

Further, the positions of second radial excitations (Table 1, third column) of π- and K-pseudoscalar mesons are in agreement with the observed resonances [5]. A comparision for second radial excitations of vector mesons with experimental data is more difficult in consequence of the existence of the D-wave orbital excitations possessing the same quantum number 1^{--} in the same mass region. In the Part II of our work we shall try to elucidate this situation and discuss some evidence for the existence of the $\rho/\omega(1200)$-resonances corresponding to the strange $K^*(1410)$-resonance.

Finally, the approximate conformity of evaluated orbital (and their radial) excitations of light mesons (with S=0) with the crresponding experimental resonances is obtained as we can see from Table 2. Remark only that experimentally observed resonances K_1 and K_2 are mixtures of states with different magnitudes of total quark-antiquark spin S=0 and S=1 due to the mass difference between strange and nonstrange quarks involved in the spin-orbit coupling. So the physical resonances $K_1(1270)$ and $K_1(1400)$ are mixtures of $^1P_1$ and $^3P_1$ states and $K_2(1580)$ and $K_2(1770)$ are mixtures of $^1D_2$ and $^3D_2$ states (thus, they have not a definite C-parity). However, we consider only $^1P_1$ -states and $^1D_2$ -states, and therefore, we cannot directly compare the positions of these states with experimentally observed resonances.

I would like to thank S. B. Gerasimov for numerous discussions and especially for the explanation of his method of relativization of the potential model.

Appendix A. The Relativized Potential Model

The Schrödinger equation for the radial wave function with the potential (3) can be rewritten in the form

$$d^2u(\rho)/d\rho^2 +[\zeta-\rho+\lambda/\rho-L(L+1)/\rho^2]u(\rho) = 0 \qquad (A.1)$$

with the dimensionless variable ρ and the only dimensionless parameter λ are

$$\rho = (2\mu^2/a^2)^{1/3}r \text{ , } \lambda = (2\mu a)^{2/3}\kappa \qquad (A.2)$$

where $\mu=m_q m_{\bar{q}}/(m_q+m_{\bar{q}})$. The radial function and the binding energy are defined by

$$R(r) = [(2\mu)^{1/2}/a]\ u(\rho)/\rho \text{ ,} \qquad (A.3)$$

$$E_{nL} = \zeta_{nL}/(2\mu a^4)^{1/3} \text{ .} \qquad (A.4)$$

The eigenvalues ζ_{nL} are characterized by the principal quantum number $n=n_r+L+1$ where n_r is the radial quantum number equal to the number of nodes of radial wave function (n_r=0,1,2,...),and L is orbital quantum number (L=0,1,2,...).

For the S-wave functions we have the Fermi-Schwinger sum rule

$$R^2(0) = 2\mu \int_0^\infty R^2(r)[dV(r)/dr]r^2dr$$
$$= (2\mu/a^2)[1+\lambda\langle\rho^{-2}\rangle] \qquad (A.5)$$

where

$$\langle\rho^{-2}\rangle = \int_0^\infty d\rho\ u^2(\rho)/\rho^2 \quad \text{with} \quad \int_0^\infty d\rho\ u^2(\rho) = 1.$$

For the manifest form of a solution we imploy somewhat improving the interpolating formulae [4], so that they turn into Coulomb-like dependences in the limit $\lambda\Rightarrow\infty$

$$\zeta_{nL} = [\zeta_{nL}(0)+b_{nL}\lambda+c_{nL}\lambda^2]/(1+d_{nL}\lambda^3)-\lambda^2/(4n^2), \qquad (A.6)$$

$$\langle\rho^{-2}\rangle_n = [\langle\rho^{-2}\rangle_n(0)+\beta_n\lambda+\gamma_n\lambda^2]/(1+\delta_n\lambda^3)+\lambda^2/(2n^3). \quad (A.7)$$

The values of the coefficients in these formulae are given by Tables A1 and A2.

The condition (4) means

$$\zeta_{1S}(0) = 0. \qquad (A.8)$$

Using (A.6) we find

$$\lambda_0 = 2.144... \qquad (A.9)$$

Table A1

Coefficients in Eq. (A6)

The state	$\zeta_{nL}(0)$	b_{nL}	c_{nL}	d_{nL}
1S	2.338	−0.848	0.201	0.0262
2S	4.088	−0.582	0.032	-3.4×10^{-5}
3S	5.521	−0.474	0.014	3.9×10^{-5}
4S	6.787	−0.408	0.008	1.4×10^{-5}
5S	7.903	−0.325	0.007	2.3×10^{-5}
6S	8.973	−0.288	0.004	-1.0×10^{-5}
1P	3.361	−0.510	0.037	-4.1×10^{-5}
2P	4.885	−0.411	0.016	-2.2×10^{-5}
3P	6.215	−0.360	0.010	1.5×10^{-5}
4P	7.406	−0.314	0.007	1.2×10^{-5}
5P	8.508	−0.281	0.005	2.1×10^{-5}
6P	9.543	−0.255	0.004	0.9×10^{-5}
1D	4.248	−0.395	0.036	0.0028
2D	5.630	−0.330	0.008	-1.4×10^{-5}
3D	6.892	−0.303	0.007	2.6×10^{-5}
4D	8.030	−0.273	0.005	2.2×10^{-5}
5D	9.092	−0.249	0.003	~ 0
6D	10.095	−0.229	0.003	~ 0
1F	5.056	−0.325	0.011	3.9×10^{-5}
2F	6.358	−0.291	0.007	0

Table A2

Coefficients in Eq. (A7)

The radial state	$<\rho^{-2}>_n(0)$	β_n	γ_n	δ_n
1S	1.122	0.620	−0.359	−0.0461
2S	0.821	0.287	−0.032	~ 0
3S	0.695	0.196	−0.010	-5.9×10^{-5}
4S	0.622	0.151	−0.005	-6.9×10^{-5}
5S	0.570	0.124	−0.004	~ 0
6S	0.532	0.105	−0.003	~ 0

The eigenvalues of ζ_{nL} at this value of λ are presented in Table A3. In the same table we present the ratio of magnitudes of radial wave functions of n-radial state and of the ground state "at zero"

$$\chi_n \equiv R_n(0)/R_1(0) = [(1+\lambda_0<\rho^{-2}>_n)/(1+\lambda_0<\rho^{-2}>_1)]^{1/2}. \quad (A.10)$$

Table A3

The eigenvalues ζ_{nL} and values of χ_n at λ_0=2.144

n	ζ_{nS}	ζ_{nP}	ζ_{nD}	ζ_{nF}	χ_n
1	0	2.15	3.35	4.34	1
2	2.70	3.95	4.84	5.72	−0.695
3	4.44	5.42	6.23	–	0.619
4	5.87	6.72	7.43	–	−0.580
5	7.19	7.90	8.55	–	0.558
6	8.34	8.99	9.60	–	−0.548

In accordance with the usual convention signs of consequent wave function "at zero" choose alternate.

Now the Eq. (2) is rewritten in the form

$$W^4-2(m_q^2+m_{\bar q}^2)W^2+(m_q^2-m_{\bar q}^2)^2 =$$
$$= (2/a)^{4/3}[W^4-(m_q^2-m_{\bar q}^2)^2]^{2/3}\zeta_{nL}(\lambda). \quad (A.11)$$

Solutions of this equation gives eigenvalues of the total energy W in the c. m. s. without a consideration of spin-spin interaction. We take into account the latter under propositions (6) and (7) or (8). Now, we have for the matrix elements of the mass matrix: in the case of "weak" spin-spin coupling

$$<i|\hat{M}|j> = W_i\delta_{ij} +$$
$$+16m_0^2C\chi_i\chi_j<s_q\cdot s_{\bar q}>W_iW_j/[(W_i^4-\Delta^4)(W_j^4-\Delta^4)]^{1/2}, \quad (A.12)$$

where $\Delta^4 \equiv (m_q^2-m_{\bar q}^2)^2$, and in the case of "strong" spin-spin coupling

$$<i|\hat{H}|j> = W_i\delta_{ij} + 4m_0^{\,2}C\chi_i\chi_j<s_q\cdot s_{\bar{q}}>/(m_q m_{\bar{q}}). \qquad (A.13)$$

In Eqs. (A.12) and (A.13) $<s_q\cdot s_{\bar{q}}> = 1/4, -3/4$ for vector and pseudoscalar mesons respectively. Now, the task consists in the diagonalization of matrix (A.12) or (A.13). We take into account 6 radial states.

References

1. Gerasimov S.B. In: Proc. 16 Int. Symp. Ahrenshoop, Berlin-Zeuthen, 1982, p. 31.
2. Todorov I.T. In: Proc. JINR-CERN Scool on High Energy Physics, JINR E2-5813, Dubna, 1971, p. 7.
3. Godfrey G., Isgur N. - Phys. Rev., 1985, v. D32, p. 189.
4. Eichten E. et al. - Phys. Rev., 1978, v. D17, p. 3090.
5. Particle Data Group - Phys. Lett., 1990, v. B239, p. 1.

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Говорков А.Б. E2-91-358
Об особенностях спектра резонансов
легких мезонов
I. Модель

Спектр реальных и орбительных возбуждений (с LS = 0) легких мезонов, вычисленный на основе релятивизованной потенциальной кварковой модели, хорошо согласуется с экспериментально наблюдавшимися резонансами. Исключение составляет радиальное возбуждение $K^*(892)$-мезона.

Работа выполнена в Лаборатории теоретической физики ОИЯИ.

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Govorkov A.B. E2-91-358
On the Subtleties of the Spectrum
of Light Meson Resonances
I. The Model

It is found that the spectrum of radial and orbital excitations (with LS = 0) of light mesons evaluated on the basis of a relativized potential quark model is consistent with experimentally observed resonances. The only exception is the radial excitation of K*(892).

The investigation has been performed at the Laboratory of Theoretical Physics, JINR.

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