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ATOMS AND HADRONS IN GAUGE THEORIES

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CONTENTS.

Dedicated to the memory of D.I.Blokhintsev

1 LECTURE

INTRODUCTION AND STATEMENT OF PROBLEMS.

Bound state theories, the uniqueness of radiative gauge, contradictions between the theory and the practice in the description of an atom, gauge dependence and gauge invariance, relativistic theory of the bilocal field.

I would like to begin with recalling the decade of theoretical physics from 1964 to 1974, which could be called the "gold age". That time there were suggested the first quark models, the parton model and scaling, the current algebra and chiral Lagrangians, the unified theory of weak and electromagnetic interactions and quantum chromodynamics; there were formulated the basic notions and methods of quantum gauge field theory, ideas of the string theory, supersymmetry and conformal theory. It was the time of a fruitful unification of theory, phenomenology and experiment, the time of rapid invention of new ideas, conceptions, principles, notions and terms of the modern high energy physics.

Beginning in 1974 the forefront of theoretical physics moved to the problem of confinement and hadronization, and having not solved this problem in the eighties turned out to be at "ethereal distances" of a superstring theory of all interactions, which is still far from reality but at the same time is very beautiful to be thought of as a chimera.

The idea of strings themselves arose from the phenomenology of strong interactions, and one of the ways to overcome the difficulties in creating a unified theory consists the solution of hadronization and confinement at least at the level of correctness of the modern quantum electrodynamics.

My lectures represent a review of attempts in creating a self-consistent theory of bound states of quarks and gluons.

There attempts should take into account the experience of describing atoms as bound states in QED, all the more that hadrons made of heavy quarks are very much alike atoms.

Let me produced with a few words about the general relativistic theory of bound states which can be considered at three levels: relativistic classical theory [1], relativistic quantum mechanics [2], and quantum field theory (QFT).

At the first two levels the theory is in fact formulated only for a "direct interaction" which is called the "action at a distance" (for example, the Coulomb or the Newton forces).

The higher the level, the less are the contradictions with the corresponding theory of bound states. For example, there is no classical relativistic theory of two-particle interactions which does not contradict the notion "trajectory" [1]. Nevertheless, a consistent quantum mechanics of two particles can be constructed.

There are difficulties only in generalizing this theory to many- body systems.

We hope that the next level (QFT) will allow one to solve this task too. However, just in the case of QFT the status of well- known direct interactions becomes indefinite.

On the one hand, direct interactions in QFT present a unique example, for which there exist a physically consistent theory of bound states in their rest frame. Going beyond the scope of direct interactions additional hypotheses (of the type of "instantaneity" in the quasipotentional approach [5]) have to be introduced.

On the other hand in all gauge theories, including gravity, direct interactions appear only after a definite choice of gauge. For example, to describe atoms in QED, one choose the Coulomb (or radiative) gauge, where the total propagator for the interactions between two charged currents $J^{(1)}, J^{(2)}$ is given as a sum of propagator for Coulomb interaction **C** and the propagator for transversal photons **T**:

$$\mathbf{C}+\mathbf{T}=J_0^{(1)}\frac{1}{\mathbf{q}^2}J_0^{(2)}+J_i^{(1)}(\delta_{ij}-q_i\frac{1}{\mathbf{q}^2}q_j)J_j^{(2)}\frac{1}{q_0^2-\mathbf{q}^2}\ . \qquad (1.1)$$

In accordance with the practice of the description of bound states in QED the Coulomb interaction **C** is considered nonperturbatively and the second, transversal part **T**, as the radiation correction (by means of perturbation theory). In other words, an "atom" is formed by the static infrared singularity of the Coulomb field, $\mathbf{q}^2=0$, and this singularity belongs only to the Coulomb gauge and explicitly depends on an external vector $\eta^0_\mu=(1,0,0,0)$ (called the time-axis vector). The expressions **C**,**T** and sum (1.1) are relativistic-noncovariant.

The main questions are how to restore the relativistic covariance and how to prove the gauge independence of physical results.

There is a difference between the practical and theoretical answers to these questions.

The "theoretical proof" of relativistic covariance consists in a transition to another, relativistic, gauge that does not depends on the time axis and in the proof of the independence of physical results of gauge due to the gauge invariance (when a gauge change is made by the gauge transformation).

Really, multiplying the term **C** by the factor one $(q_0^2-\mathbf{q}^2)/(q_0^2-\mathbf{q}^2)$, we can decompose the sum (1.1), $\mathbf{C}+\mathbf{T}$. into the Feynman covariant propagator **F** and the longitudinal term **L**

$$\begin{aligned}\mathbf{C}+\mathbf{T} &= \mathbf{F}+\mathbf{L}=\\ &= -\frac{J^{(1)}\cdot J^{(2)}}{(q\cdot q)}+\frac{(J_0^{(1)}q_0)(J_0^{(2)}q_0)-(J_i^{(1)}q_i)(J_j^{(2)}q_j)}{\mathbf{q}^2(q\cdot q)}, \qquad (1.2)\\ (A\cdot B) &= A_0B_0-A_iB_i)\ .\end{aligned}$$

The whole "theoretical proof" of the relativistic covariance of eq. (1.2) (see ref. [6]) is the statement that the contribution of the longitudinal term **L** equal zero. This takes really place because of the current conservation law

$$J_0^{(1,2)}q_0=J_i^{(1,2)}q_i\ . \qquad (1.3)$$

So we get the equality

$$\mathbf{C}+\mathbf{T}=\mathbf{F}=-\frac{J^{(1)}\cdot J^{(2)}}{q\cdot q}\ . \qquad (1.4)$$

In this explanation of the relativistic covariance the transversal photon **T** is considered as a relativistic correction, and the Coulomb field **C** and the potential model are understood as a nonrelativistic approximation and a nonrelativistic model respectively.

However the given proof which is based on gauge independence of physical results is valid only for scattering and dissociation processes but not for the task of bound states.

If the omit the term **L** in eq. (1.2), it is easy to see that the propagator in the relativistic gauge (1.4) loses the static Coulomb pole ($\mathbf{q}^2=0$) which forms the "atom". The term **L** disappears only on mass-shell of charged elementary particles. But the latter are off mass-shell in the Bethe-Salpeter equation. Here the currents (J) turn into the vertices (Γ) which do not satisfy the conservation law (1.3).

For the description of atoms we cannot use any gauge and any time-axis η_μ. When the atom spectrum is calculated in a relativistic gauge, one adds the Coulomb field $\mathbf{C}(\eta)$ by hand [7]. Futhermore from one and the same expression (1.4) we can get the wave function for an atom at rest as well as for a moving atom in dependence on our choice of the time-axis in the term $\mathbf{C}(\eta)$. There is the series of work [7, 8] devoted to the proof of the gauge independence of an atom spectrum. In these treatments, the Coulomb interaction is used in the rest frame with the choice of the time-axis $\eta_\mu=(1,0,0,0)$. However, the authors of those papers have not taken into account that the vector η_μ (contained in the Coulomb part of the interaction) can be indeed arbitrary, and that a transition from one vector η_μ to another η'_μ ($\eta'_\mu\cdot\eta'_\mu=1$) is realized by means of a special gauge change.

It is easy to check that the usual Lorentz transformation, or a special gauge change ($\eta\to\eta'$) break the relativistic dispersion law (i.e. $P\cdot P=M^2_{B.S.}$, where P_μ is the total momentum and $M_{B.S.}$ is the mass of the bound state [9]). The dispersion law is invariant only under a combined realization of the usual Lorentz and the special gauge transformation ($P\to P', \eta\to\eta'$) in such a way that the time-axis is parallel to the total momentum ($\eta_\mu\sim P_\mu, \eta'_\mu\sim P'_\mu$). This parallelism is equivalent to the Markov-Yukawa principle concerning the choice of the bound state bilocal relative space and time with respect to the bound state total momentum operator [10, 11]. The need for a combined transformation has first been pointed out by Heisenberg and Pauli [12] in their pioneering papers on quantization of electrodynamics.

Thus we have seen that there not only exists a gauge dependence of bound state calculations but that this dependence is necessary to provide relativistic covariance.

Thus for the task of the description of bound states the following questions arise:

1. How looks the relativistic covariance for "atoms" like if the interaction (1.1) depends on the external time-axis η_μ?
2. Why is the Coulomb gauge favoured?
3. How can one describe "atoms" in other gauges?
4. What is the status of the potential model?
5. What is the time-axis for many-particle relativistic bound systems?

6. Does the gauge dependence of physical results mean gauge noninvariance?

In the present Lectures I try to give answers just to these questions and I will construct a relativistic gauge theory for atoms and hadrons.

Let me begin with answering the last question because the confusion of the notions "gauge independence" and "gauge invariance" is the main obstacle for solving the problems of the relativistic bound states. First let us recall such notions as "gauge invariance", "choice of gauge", and "change of gauge". The gauge invariance of the Lagrangian

$$L = -\frac{1}{4}F_{\mu\nu}^2 \; ; \; F_{\mu\nu} = \partial_\mu A_\nu - \partial_\nu A_\nu \tag{1.5}$$

means that it does not vary under gauge transformations of the fields

$$A_\mu^\lambda = A_\mu + \partial_\mu \lambda \; : \; L(A^\lambda) = L(A). \tag{1.6}$$

The "choice of the gauge" is a specific gauge transformation λ^f depending on the field A, so that the new field $A^f_\mu[A] = A_\mu + \partial_\mu \lambda^f[A]$ satisfies the additional condition ("gauge condition")

$$f(A^f[A]) = 0. \tag{1.7}$$

The quantization of the fields and the Feynman rules are always formulated in terms of a certain gauge: $f_1 = 0, f_2 = 0, ;$, for example,

$$A_3^{(3)} = 0 \; ; \; \partial_i A_i^T = 0. \tag{1.8}$$

We would like to draw your attention to some not well known consequences of these definitions.

i). The explicit solution of gauge condition (1.8) gives the physical variables A^f as a functional on the initial fields A_i. In QED this is the axial field

$$A_\mu^{(3)}[A] = (\delta_{\mu\nu} - \partial_\mu \frac{1}{\partial_3}\delta_{3\nu})A_\nu \; , \; (A_3^{(3)}[A] = 0),$$

or the transversal field

$$A_i^T[A] = (\delta_{ij} - \partial_i \frac{1}{\partial^2}\delta_j)A_j \; , \; (\partial_i A_i^T[A] = 0), \tag{1.9}$$

and so on. These functionals are invariant under gauge transformations of the initial fields in the sense of eq. (1.6). So, any gauge choice is a transition from the initial fields to the gauge invariant physical variables (i.e. "gauge" (1.8) is the choice of variables).

If the physical results depend on a "gauge choice" (1.8), this does not mean that the gauge invariance principle is broken, as any "choice of variables" (1.7) is gauge

invariant. In the last statement we cannot substitute the term "gauge" by the term "variables", otherwise, we get nonsense: "any choice of gauge" is "gauge invariant". To exclude that kind of nonsense, we should use the terms "choice of variables", "change of variables" instead of "choice of gauge", "change of gauge".

ii). Different "variables" are connected with each other by a transformation, which is called the "change of variables". For example, the variable A_i^T is expressed in terms of the variables $A^{(3)}$ by means of the transformation

$$A_i^T[A^{(3)}] = (v^T[A^{(3)}])(A_i^{(3)} - \frac{1}{ie}\partial_i)(v^T[A^{(3)}])^{-1} \; , \tag{1.10}$$

$$\psi^T = v^T[A^{(3)}]\psi^{(3)} \; ; (v^T[A^{(3)}] = \exp\{ie\frac{1}{\partial^2}\partial_i A_i\}) \; .$$

This transformation does not differ in sense from the same transformation of variables in any nongauge theory. Any physical results including the average (the Green function)

$$< \psi^T \ldots \bar{\psi}^T > = < v^T[A^{(3)}]\psi^{(3)} \ldots \bar{\psi}^{(3)}(v^T[A^{(3)}])^{-1} > \tag{1.11}$$

do not depend on the change of variables. The main problem consists in that the r.h.s. of (1.11) contains not only the modification of the Feynman rules (i.e. the "gauge change") but also the spurious diagrams induced by the artificial factor $v^T[A^{(3)}]$, which do not follow from the initial Lagrangian or Hamiltonian. An example of this spurious diagrams is the longitudinal part L in eq. (1.2).

In term "change of gauge" differs from the term "change of variables" as it means only the modification of the Lagrangian Feynman rules. Both terms coincide in the case, when the contribution from the spurious diagrams is equal to zero, for example, for the S- matrix of scattering with the asymptotical states on mass- shell.

But off mass- shell (i.e. for bound state physical values) the artificial spurious diagrams in general give a nonzero contribution, and the dependence on the "gauge" takes place. Nevertheless as we have seen above, this does not mean gauge noninvariance.

The problem is that, to reproduce the observed Lamb shift in atomic spectra, all "variables" (except the Coulomb ones) must consist of contributions from both usual Feynman rules diagrams and man- made spurious ones. The latter restore just the Feynman rules in the Coulomb (or radiative) variables. The radiative variables are the only ones which do not demand these spurious diagrams.

Before we discuss the problems of the favourity of the "radiative variables", we shall recall the main statements of the general relativistic theory of bilocal fields, as atoms and hadrons are described by bilocal fields:

$$\mathcal{M}(x_1, x_2) = \mathcal{M}(z|X) \; ; \; z_\mu = (x_1 - x_2)_\mu \; ; \; X_\mu = (\frac{x_1 - x_2}{2})_\mu \; . \tag{1.12}$$

We suppose that z and X are the relative and total bound state coordinates, respectively; x_1, x_2 are the coordinates of two particles which form a bound state.

For example, the bilocal field of a positronium (e^+e^-- atom) has in the rest frame the form:

$$\mathcal{M}(z|X) = e^{iM_A X_0}\psi(z)\delta(z_0) \tag{1.13}$$

The delta- function $\delta(z_0)$ means the instantaneity of two elementary particles forming an atom [1], $\psi(z)$ is the wave function satisfying the Schrödinger equation with the Coulomb potential. The instantaneity of the atom reflects the instantaneity of the Coulomb interaction:

$$W_{int} = -\frac{1}{2}\int dx_1 dx_2 \psi(x_2)\bar{\psi}(x_1)\mathcal{K}(x_1x_2)\psi(x_1)\bar{\psi}(x_2) \quad (1.14)$$

where $\mathcal{K}$ is the Coulomb kernel

$$\mathcal{K}(x_1x_2) = (\gamma_0)V_C(z)\delta(z_0)(\gamma_0) \quad (1.15)$$

In 1978 S.Love [7] had proved the theorem that the radiation corrections, breaking the instantaneity of the interaction (1.15), do not break the instantaneity of the atom bilocal field (1.13).

The main question in connection with the problems discussed here concerns the form of action (1.14) for a moving atom.

It is well known that the wave function of a relativistic atom (used for the description of the creation of atoms and of their destruction) is constructed by the usual boost operation

$$\mathcal{M}(x_1,x_2) \to \mathcal{M}'(x_1,x_2) = e^{i\mathcal{P}'\cdot X}\psi(z^{\perp})\delta(z\cdot\eta') , \quad (1.16)$$

where $\mathcal{P}'$ is total momentum

$$\mathcal{P} = (\sqrt{\vec{\mathcal{P}}'^2 + M_A^2}, \ \vec{\mathcal{P}} \neq 0) \equiv M_A \cdot \eta'_\mu , \quad (1.17)$$

and

$$z^{\perp}_\mu = z_\mu - \eta'_\mu(z\cdot\eta'_\mu) \quad (1.18)$$

is the transversal part of the relative coordinate z_μ with respect to the total momentum.

This relativistic atom bilocal field is described by the action (1.14) with the moving Coulomb kernel

$$\mathcal{K}(x_1,x_2) = \mathcal{K}(z|X) = \eta\!\!\!/'\, V(z^{\perp})\, \eta\!\!\!/'\, \delta(z\cdot\eta') \quad (1.19)$$

This means that we choose the new radiative gauge depending on the arbitrary unit time-like vector η'_μ (that one calls by the time-axis of quantization) and this vector has been chosen parallel to the total momentum of an atom ($\eta'_\mu \sim \mathcal{P}'_\mu$). According to the Love theorem the old structure of the bilocal field (1.13) cannot be restored by the radiation corrections. We cannot say here that the atom wave function does not depend on gauge.

So, the usual boost of the matrix elements with the atom wave function corresponds to the Lorentz transformation of field operators accompanied by the gauge change. Just

[1] "A proton yesterday and electron today do not make an atom [13].

this field transformation law has first been pointed out by Heisenberg and Pauli in 1930 [10].

Another question is "What is the relativistic atom for which we do not change gauge ?" The answer to this question is given by the general theory of bilocal fields [10],[11].

It is easy to see that the bilocal fields (1.13) and (1.16) satisfy the Markov- Yukawa condition [10].

$$z_\mu \frac{\partial}{\partial X_\mu}\mathcal{M}(z|X) = 0 , \quad (1.20)$$

which means that the bilocal field is an irreducible representation of the Lorentz group (i.e. it has the mass $\mathcal{P}^2 = M_A^2$ and a spin). Expression (1.20) is a generalized condition of irreducibility of vector, tenzor and other fields ($\partial_\mu A_\mu = 0; \partial_\mu T_{\mu\nu} = 0; \cdots$)[11].

If we shall not change gauge, then the irreducibility condition is not fulfilled and the relativistic dispersion law breaks down, $\mathcal{P}^2 \neq M_A^2$ (see for example ref. [9]). Thus, not only the wave function but also spectrum depend on gauge.

We should prove the favourity of the Coulomb gauge with the time-axis η =(1, 0, 0, 0) for the description of an atom at rest and find the internal mechanism of transition to another gauge with any time-axis η'_μ ($\eta'_\mu \cdot \eta'_\mu = 1$).

We will see below that the definition (1.12) of the total and relative coordinates $X = (x_1 + x_2)/2$ and $z = x_1 - x_2$ respectively is universal for quarks with an arbitrary mass, including also constituent masses depending on momenta.

For the description of baryons and many-quark systems in an arbitrary relativistic reference frame, one should generalize the Markov-Yukawa condition (1.20) for the bilocal field to the N -local field $\mathcal{M}(x_1, x_2, \cdots, x_N)$. By analogy with a bilocal system, we introduce for the N -local field the total and relative coordinates

$$X_\mu = \frac{1}{N}\sum_{i=1}^{N}(x_i)_\mu , \quad z_\mu^{(i)} = (x_i)_\mu - x_\mu , \quad (1.21)$$

which are connected by the identity

$$\sum_{i=1}^{N} z_\mu^{(i)} \equiv 0 . \quad (1.22)$$

Then, the natural generalization of the Markov-Yukawa condition takes the form

$$z_\mu^{(i)}\frac{\partial}{\partial X_\mu}\mathcal{M}(z_\mu^{(1)}, z_\mu^{(2)}, \cdots, z_\mu^{(N)}|X) , \ (i = 1, 2, \cdots, N) \quad (1.23)$$

Let $\mathcal{P}_\mu$ be an eigenvalue of the operator for the operator for the total 4-momentum, and η_μ be the unit vector in the direction $\mathcal{P}$ ($\eta_\mu \sim \mathcal{P}_\mu$). Owing to the condition (1.23) the N- local function $\underline{\mathcal{M}}(p_\mu^{(1)}, p_\mu^{(2)}, \cdots, p_\mu^{(N)}|\mathcal{P})$, being the Fourier transform of $\mathcal{M}(z_\mu^{(1)}, z_\mu^{(2)}, \cdots, z_\mu^{(N)}|X)$ with respect to all coordinates, depends only on the transversal relative momenta

$$p_\mu^{(i)\perp} = p_\mu^{(i)} - \eta_\mu(p^{(i)}\cdot\eta) , \sum_{i=1}^{N} p_\mu^{(i)\perp} = 0 , \quad (1.24)$$

$$\mathcal{M}(p_\mu^{(1)}, p_\mu^{(2)}, \cdots, p_\mu^{(N)}|\mathcal{P}) \equiv \mathcal{M}(p_\mu^{(1)\perp}, p_\mu^{(2)\perp}, \cdots, p_\mu^{(N)\perp}|\mathcal{P}) \qquad (1.25)$$

So, the main results of the first Lecture are, firstly the introduction of the term "choice of variables" instead of "choice of gauge" and secondly the necessary of the choice of the time- axis of quantization and boundary conditions in accordance with the Markov- Yukawa condition (1.20), if we want to describe bound states as irreducible representations of the Poincare group.

2 LECTURE

MINIMAL QUANTIZATION OF ELECTRODYNAMICS.

Constraints and the Heisenberg relation, quantization without gauge, covariance of a "instantaneous" atom, the time axis and the Belinfante tensor.

In the first Lecture we established the contradictions between the experience of the description of bound states QED and the conventional "theoretical proof" of relativistic covariance and gauge independence based mainly on the experience of the theory of scattering of free particles.

In the scattering theory the relativistic covariance is a consequence of the gauge independence of the physical results. On the other hand, in the theory of bound states a gauge dependence is manifestly present, moreover it is even necessary to provide relativistic covariance.

Futhermore the notions "gauge independence" and "gauge invariance" are often confused which obstructs realization and understanding of all these contradictions.

The first step to overcome this last obstacle was the introduction of the notion "choice of variables" instead of "choice of gauge". We have shown, that the choice of variables itself is a gauge-invariant procedure leading to the establishment of definite Feynman rules in perturbation theory. The dependence of the atomic-physics results on the choice of the Feynman rules (i.e. in the old language on the "choice of gauge") does not mean a breaking of gauge invariance.

In the context of the bound state problem one has to give evidence in favour of the Feynman rules in the "Coulomb gauge", or, in the new language, in the "transversal variables".

The present Lecture is devoted to the solution of just this task.

Let us consider the quantization of the two simplest systems in classical mechanics with the Lagrangians

$$\mathcal{L}_1 = \frac{\dot{q}^2}{2}; \quad \mathcal{L}_2 = \frac{q^2}{2}.$$

The Lagrangian $\mathcal{L}_1$ describes a nontrivial dynamics of a point with the momentum $p = \partial\mathcal{L}_1/\partial\dot{q} = \dot{q}$ and with the Euler equation $\ddot{q} = 0$. Quantization means the transition to operators for the momentum and coordinate with the commutation relation $i[\hat{p}, \hat{q}] = \hbar$

The Lagrangian $\mathcal{L}_2$ describes a physical point at rest with zero momentum $p = \partial\mathcal{L}_2/\partial\dot{q} = 0$ and zero coordinate $q = \partial\mathcal{L}_2/\partial q = 0$, as a solution of Euler equation.

This is the simplest example of a system with zero momentum and in this sense it is analogous to gauge theories, where the equality to zero of the momentum is treated as a constraint [14] imposed on dynamical variable intended for a quantization in the spirit of the first system $\mathcal{L}_1$. This constraint is not unique. We should also consider the secondary constraint, which is obtained by commuty the Hamiltonian $\mathcal{H}_2 = -\mathcal{L}_2$ with the first constraint: $-i[\hat{p},\mathcal{H}_2]=\dot{\hat{q}}=0$. It is easy to see that both constraints contradict the commutation relation $i[\hat{p},\hat{q}]=\hbar$ and the Heisenberg relation $<\hat{p}><\hat{q}>\geq\hbar$ which forbids the simultaneous fixation of momentum and coordinate eigenvalues. In principle the second system with the Lagrangian $\mathcal{L}_2$ cannot be considered as a quantum one.

An example of such a nonquantum static system represents the Lagrangian of electrodynamics, in which one neglects the space components of the gauge field ($A_i=0$):

$$\mathcal{L}=-\frac{1}{4}F_{\mu\nu}^2+A_\mu J_\mu|_{A_i=0}=\frac{1}{2}(\partial_i A_0)^2+A_0J_0\equiv\mathcal{L}_C\ . \tag{2.1}$$

Upon the solution of Euler equation

$$\partial_i^2 A=J_0\ ;\ A_0(\mathbf{x})=-\frac{1}{4\pi}\int dy\frac{1}{|\mathbf{x}-\mathbf{y}|}J_0(\mathbf{y})\equiv\frac{1}{\partial^2}J_0(\mathbf{x}) \tag{2.2}$$

the Lagrangian (2.1) exactly coincides with the Coulomb current interaction (1.14)

$$\mathcal{L}_C=\frac{1}{2}J_0\frac{1}{\partial^2}J_0\ , \tag{2.3}$$

which is the basis of the nonperturbative calculation of the atom spectrum and wave function in the lowest order in radiation.

Thus, the practice of the description of an atom is equivalent to the application of the method of constructing dynamical variables in gauge theory by means of an explicit solution of the Euler equation for the time component A_0 (this equation is called the Gauss equation).

In the first paper on QED Heisenberg and Pauli outlined just such a quantization method. Later on, to restore the manifest relativistic covariance, methods were suggested of equal- footing quantization of all gauge field components. These methods have been developed mostly by Dirac [14], and by Faddeev [15] (see, also ref. [16]).

So, all methods of gauge fields quantization can roughly be devided into two approaches. In the first approach one tries to quantize all components to conserve the manifest relativistic covariance (we shall call such an approach Dirac quantization), in the second one- one solves explicitly all constraints and quantizes only the remaining minimal set of physical degrees of freedom (we call such an approach "minimal quantization"). The "minimal quantization" of gauge theories has been formulated in ref. [17]- [20], and its main problem is the proof of relativistic covariance.

Let us consider the solution of this problem using as an example a free electromagnetic field (1.5)

$$\mathcal{L}(x)\ =\ -\frac{1}{4}F_{\mu\nu}^2=\frac{1}{2}F_{0i}^2-\frac{1}{2}B_i^2\ ; \tag{2.4}$$

$$F_{0i}\ =\ \partial_0A_i-\partial_iA_0\ ;\ B_i=\epsilon_{ijk}\partial_jA_k\ .$$

In accordance with the prescription of the minimal quantization method at the beginning, we should solve the classical equation for the "static" component A_0 having zero momentum:

$$\partial^2A_0=\partial_i\partial_0A_i\Longrightarrow A_0=\frac{1}{\partial^2}\partial_j\partial_0A_j\ . \tag{2.5}$$

It is easy to see that the constraint (2.5) and its solutions are invariant under the gauge transformation (1.6). The substitution of solution (2.5) into Lagrangian (2.4) leads to the expression

$$\mathcal{L}_G=\frac{1}{2}(\partial_0A_i^T)^2-\frac{1}{2}(\partial_jA_k^T)^2\ . \tag{2.6}$$

$\mathcal{L}_G$ depends only on the transverse field (1.9):

$$A_i^T=(\delta_{ij}-\partial_i\frac{1}{\partial_j^2}\partial_j)A_j\ ;\partial_iA_i^T\equiv0\ , \tag{2.7}$$

which is a gauge- invariant functional over the initial fields A_i.

The fact of the removal of two field components (the time and longitudinal ones) by solving one gauge- invariant equation (2.5) can be shown more clearly, by an example of the electric tension $F_{0i}=\partial_0A_i-\partial_iA_0$ which explicitly depends on four components (A_0,A_i). Inserting into the solution of (2.5)

$$F_{0i}=\partial_0A_i-\partial_iA_0|_{A_0=\frac{1}{\partial^2}\partial_j\partial_0A_j}=\partial_0\left[A_i-\partial_i\frac{1}{\partial_j^2}\partial_jA_j\right]=\partial_0A_i^T[A]$$

it can be found that this tensor depends only on two transversal fields A_j^T (2.7). The longitudinal component disappears due to gauge invariance of the tensor F_{0i}.

To present the dependence on two variables explicitly, we introduce two transverse unit vectors

$$e_i^a(\partial)\ ;\ (\partial_ie_i^a(\partial)=0)\ ;\ a\ =\ 1,2\ ,$$

which together with the vector of differentiation ∂_i form a complete set of three vectors. The completeness condition reads

$$\sum_{a=1,2}e_i^ae_j^a+\partial_i\frac{1}{\partial_i^2}\partial_j=\delta_{ij}\ ,$$

or

$$\sum_{a=1,2}e_i^ae_j^a=\left(\delta_{ij}-\partial_i\frac{1}{\partial_i^2}\partial_j\right)\ . \tag{2.8}$$

The last equality allows to introduce, instead of three dependent fields A_i^T, two independent ones, A^a:

$$A_i^T=\left(\delta_{ij}-\partial_i\frac{1}{\partial_i^2}\partial_j\right)A_j^T=\sum_{a=1,2}e_i^a(e_j^aA_j^T)=\sum_{a=1,2}e_i^aA^a\ . \tag{2.9}$$

In terms of these independent fields the Lagrangian (2.6) has the form

$$\mathcal{L}_G=\sum_{a=1,2}\frac{1}{2}\partial_\mu A^a\partial^\mu A^a\ . \tag{2.10}$$

Now, quantization means to establish commutation relations for the two independent components

$$i[\partial_0 A^a(\mathbf{x},t)\ ,\ A^b(\mathbf{y},t)] = \delta^{ab}\delta(\mathbf{x}-\mathbf{y})\ . \qquad (2.11)$$

Dynamical quantities can be constructed in the usual way from the canonical energy- momentum tensor of the system (2.10) [21],

$$T^G_{\mu\nu} = \sum_{a=1,2} \partial_\mu A^a \partial_\nu A^a - \delta_{\mu\nu}\mathcal{L}_G\ , \qquad (2.12)$$

which coincides upon the solution of the Gauss equation (2.5) with the gauge- invariant energy- momentum tensor of the initial theory (2.4) (which is called the Belinfante tensor)

$$T^B_{\mu\nu} = F_{\mu\nu}F^\alpha_\nu - \delta_{\mu\nu}\mathcal{L}\ . \qquad (2.13)$$

Thus, we quantized the gauge theory, without using the gauge constraint (1.7) as the initial assumption. In the "minimal method" the term "choice of gauge" is not needed at all, and the "choice of variables" is fulfilled uniquely by explicitly solving the Gauss equation and by the way of constructing quantities. There, for the minimal quantized gauge theory the term "construction of variables" is even more suited.

It should be noted that in terms of the Dirac quantization we got the "Feynman rules" in the radiative gauge. However, one cannot say that the expression (2.9) represents a definite "gauge" in the conventional classical meaning. In fact, after the Lorentz transformation of the initial fields upon the solutions of the Gauss equation (2.5),

$$\begin{aligned} \delta^0_L A_k(x) &= \epsilon_i(x'_i\partial_0 - x'_0\partial^{x'}_i)A_k(x') + A_0(x') \\ x'_k &= x_k + \epsilon_k x_0\ ;\ x'_0 = x_0 + \epsilon_k x_k\ ; \\ \delta^0_L\partial_k &= \epsilon_k\partial_0\ ;\ \delta^0_L(\frac{1}{\partial^2}) = -2\epsilon_k\frac{1}{\partial^2}\partial_k\partial_0\frac{1}{\partial^2} \end{aligned} \qquad (2.14)$$

the functional (2.9) of the transversal variables "changes" its "gauge" automatically,

$$A^T_k[A + \delta^0_L A] - A^T_k = \delta^0_L A^T_k + \partial_k\Lambda(x)\ ;\ (A^T_0 = 0)\ . \qquad (2.15)$$

Here $\Lambda(x)$ is the special gauge transformation, depending on the fields

$$\Lambda(x) = \epsilon_k\frac{1}{\partial^2}\partial_0 A^T_k\ . \qquad (2.16)$$

In terms of the "minimal method" this transformation means a "change of variables" from (2.9) to transversal ones with respect to the new time-axis $\eta'_\mu = \eta_\mu + \delta^0_L\eta_\mu$; $(\eta_\mu = (1,0,0,0))$.

These classical relativistic transformation (2.15) coincide with the quantum ones at the level of operators;

$$i\,\epsilon_i\left[M_{0i}\ ,\ A^T_k\right] = \delta^0_L A^T_k + \partial_k\Lambda\ , \qquad (2.17)$$

where

$$M_{0k} = \int d\mathbf{x}(x_k T^G_{00} - x_0 T^G_{0k}) \qquad (2.18)$$

is the boost operator. (The minimal method is the unique one where the classical and quantum transformations completely coincide).

In the same way we can consider the minimal quantization of electrodynamics

$$\mathcal{L} = -\frac{1}{4}F^2_{\mu\nu} + \bar\psi(i\not\partial - m^0)\psi + A_\mu j^\mu\ ;\ j_\mu = e\bar\psi\gamma_\mu\psi\ . \qquad (2.19)$$

Upon the solution of the Gauss equation,

$$A_0 = \frac{1}{\partial^2}(\partial_i\partial_0 A_i + j_0)\ . \qquad (2.20)$$

Lagrangian (2.19) has the form [17, 18]

$$\mathcal{L} = \frac{1}{2}(\partial_\mu A^T_i)^2 + \frac{1}{2}j^T_0\frac{1}{\partial^2}j^T_0 - j^T_i A^T_i + \bar\psi^T(i\not\partial - m^0)\psi^T\ , \qquad (2.21)$$

where A^T, ψ^T are the transversal variables;

$$A^T_k = v^T[A](A_k + i\frac{1}{e}\partial_k)(v^T[A])^{-1}\ ;\ \psi^T = v^T[A]\psi\ ; \qquad (2.22)$$

$$v^T[A] = \exp\{ie\frac{1}{\partial^2}\partial_j A_j\}\ , \qquad (2.23)$$

with the classical Heisenberg- Pauli transformation of the type (2.15)

$$\begin{aligned} \psi^T[A + \delta^0_L A, \psi + \delta^0_L\psi] - \psi^T[A,\psi] &= \delta^0_L\psi^T + ie\Lambda(x')\psi^T \\ (\delta^0_L\psi^T &= \epsilon_i(x'_i\partial_{0'} - x'_0\partial_{i'})\psi^T(x') + \frac{1}{4}\epsilon_k[\gamma_0,\gamma_k]\psi^T(x')) \end{aligned} \qquad (2.24)$$

$$\begin{aligned} A^T_k[A_i + \delta^0_L A] - A^T_k[A] &= \delta^0_L A^T_k + \partial_k\Lambda \\ (\delta^0_L A^T_k &= \epsilon_i(x'_i\partial_{0'} - x'_0\partial_{i'})A^T_k(x') + \epsilon_k\frac{1}{\partial^2}j^T_0) \end{aligned} \qquad (2.25)$$

with

$$\Lambda = \epsilon_k\frac{1}{\partial^2}(\partial_0 A^T_k + \partial_k\frac{1}{\partial^2}j^T_0)\ . \qquad (2.26)$$

The classical transformation coincides with the quantum one given by

$$i\,\epsilon_k[M_{0k}, \hat\psi^T] = \delta^0_L\hat\psi^T + ie\Lambda\hat\psi^T\ ;\ M_{0k} = \int d\mathbf{x}\,[x_k T_{00} - x_0 T_{0k}] \qquad (2.27)$$

if, firstly, as we have discussed above the boost operator M_{0k} is constructed with the Belinfante energy- momentum tensor

$$
\begin{aligned}
T_{\mu\nu} &= F_{\mu\alpha}F_{\nu}^{\alpha} + \bar{\psi}(ie\partial_{\mu} - eA_{\mu})\gamma_{\nu}\psi - \delta_{\mu\nu}\mathcal{L} + \frac{i}{2}\partial_{\lambda}(\bar{\psi}\Gamma_{\lambda\mu\nu}\psi) \; ; \\
\Gamma_{\lambda\mu\nu} &= \frac{1}{2}[\gamma_{\lambda},\gamma_{\mu}]\gamma_{\nu} + \delta_{\nu\mu}\gamma_{\lambda} - \delta_{\lambda\nu}\gamma_{\mu} \; ,
\end{aligned}
\tag{2.28}
$$

and, secondly, if the classical functional ψ^T (2.22) as a whole corresponds to the local quantum field $\hat{\psi}^T$ with the commutation relation

$$\{\hat{\psi}^{T+}(\mathbf{x},t), \hat{\psi}^{T}(\mathbf{y},t)\} = \delta(\mathbf{x}-\mathbf{y}) \; . \tag{2.29}$$

We call the transformations (2.24) and (2.25) Heisenberg- Pauli transformations as they were discussed at first in [12] with a reference to an unpublished remark J.von Neumann.

Their explicit form was found by B.Zumino [22] (see also ref. [6]).

However, the real meaning of the Heisenberg- Pauli transformation is impossible to understand in terms of the Dirac quantization. The main difference between the "Coulomb gauge" and the "minimal method" consists in the proof and interpretation of these transformations.

The formulation of the Coulomb gauge in the frame-work of the Dirac method leads to the usual canonical Hamiltonian that differs from the Belinfante one (2.28) by a total derivative which contributes to the first term of the boost operator (2.27).

Strictly speaking, the Coulomb gauge leads to another gauge functional Λ in (2.2), instead of (2.26), which breaks the usual relativistic covariance of matrix elements of the type of Green functions $< \psi(x) \ldots \bar{\psi} >$.

According to the interpretation of the Dirac method the term Λ in eqs. (2.25), (2.26) is treated as the gauge transformation which does not influence the physical results. But as we have discussed, this interpretation cannot be applied to off- mass-shell amplitudes and bound state physics. In the minimal method, the new type diagrams $< \psi(x)\Lambda(x) \ldots \bar{\psi} >$ with the gauge functional $\Lambda(x)$ defined by (2.26) restore the conventional relativistic properties of the Green functions in each order of radiation correction [18]- [20]. This means that the new spurious diagrams induced by the boost transformation (2.27) lead simultaneously to a change of the Coulomb interaction and of the transversal photon propagator $\mathbf{T}$ in (1.1).

$$\mathbf{C} \longrightarrow \mathbf{C}' \; , \; \mathbf{T} \longrightarrow \mathbf{T}' \; , \tag{2.30}$$

whereas in the old gauge theory version only the sum $(\mathbf{C}+\mathbf{T}) = (\mathbf{C}'+\mathbf{T}')$ is relativistic covariant.

Thus, the Coulomb interaction is really relativized by Λ in the lowest order of radiation correction $\mathbf{T}$.

In the old gauge theory version $\mathbf{T}$ is considered as a relativistic correction, and $\mathbf{C}$ is only nonrelativistic approximation. In the minimal method the potential model can be relativistic and we shall convince the reader in that in the next Lecture.

The minimal method of the relativistic covariance allows one to conserve the Markov- Yukawa principle ($\eta \sim \mathcal{P}$) for the potential picture of bound states in any relativistic frame of reference, and it allows us to see the dependence of bound state physics on the choice of the time- axis under complete gauge invariance of physical results (see Lecture 1).

3 LECTURE

ATOMS IN *QED* AND PHENOMENOLOGY OF HADRONIZATION.

It is widely believed that the potential models are related only to the nonrelativistic approximation. This opinion, however, is based on the experience of solving scattering and dissociation problems in *QED*, where the Coulomb propagator corresponding to transversal photon exchange, converts into the relativistic invariant Feynman propagator up to the longitudinal part vanishing on the mass shell. Such a "relativistic method" of the potential model is incorrect for the description of bound states where elementary particles are off their mass shells and the longitudinal part of the propagator differs from zero.

From the experience of the description of atoms in *QED* one knows that the bound state (atom) is formed by an instantaneous interaction (Coulomb) potential (with a singularity on the time axis) whereas the transversal photon exchange plays the role of a correction which does not destroy the instantaneous property of the bound state wave function. However, in this case, unlike scattering and dissociation processes, the sum of the Coulomb and transversal propagators is not relativistic - invariant due to nonzero longitudinal photon contributions. It should be stressed that the propagators in manifestly covariant gauges (with a singularity on the light cone) themselves can not describe the bound states conserving their instantaneous property.

So to treat the bound states it is important to choose such transversal variables for which the instantaneous Coulomb potential separates.

In practice the relativistic description of the spectrum of a moving atom is done by means of the Coulomb potential moving together with the atom. This correspondens to the transformation to new transversal variables refering to a new time axis (η_μ) which is parallel to atom's 4-momentum. To such a relativization there corresponds the following effective action [23, 24] :

$$W_{eff}[\psi,\bar\psi] = \int d^4x[\bar\psi(x)(i\partial\!\!\!/ - m^0)\psi(x) + \frac{1}{2}\int d^4y(\psi(y)\bar\psi(x))\mathcal{K}^{(\eta)}(z^\perp \mid X)(\psi(x)\bar\psi(y))]. \tag{3.1}$$

Here $\partial\!\!\!/ = \partial^\mu\gamma_\mu$, $\mathcal{K}^{(\eta)}$ is the kernel

$$\mathcal{K}^{(\eta)}(z^\perp \mid X) = \eta\!\!\!/ V(z^\perp)\delta(z\cdot\eta)\eta\!\!\!/,\ (\eta\!\!\!/ = \eta^\mu\gamma_\mu), \tag{3.2}$$

where z and X are the relative and total coordinates defined, respectively, as

$$z = x - y,\ X = \frac{x+y}{2}, \tag{3.3}$$

and $V(z^\perp)$ is the potential depending on the transversal (with respect to the time axis η) component of the relative coordinate, $z^\perp_\mu = z_\mu - \eta_\mu(z\cdot\eta)$.

Notice that just such a relativistic transformation law of the fields with a simultaneous rotation of the time - axis has been used in the historically first formulation of *QED* [12] (a consistent construction of the gauge theories with such properties has been proposed recently [18]).

The next question is how to choose the time - axis in (3.1) for describing interacting atoms. It has been suggested in ref. [10] to take in this case as a time - axis the unit vector which is proportional to the eigenvalue of the bound state total momentum operator, i.e.

$$\eta_\mu \sim i\frac{\partial}{\partial X_\mu}. \tag{3.4}$$

When this requirement is satisfied the bound state wave functions automatically belong to the irreducible representation of the Poincare group [11].

It should be stressed that in the relativistic theory the decomposition (3.3) is independ both of the variation of the quark masses and of the bound state.

The expression (3.1) with the kernel (3.2) and with the time - axis defined by (3.4) represents a relativistic covariant action. It seems that a most straightforward way for constructing a theory of bound states is the redefinition of action (3.1) in terms of bilocal fields by means of the Legendre transformation [24] - [27]

$$\begin{aligned}\frac{1}{2}&\int d^4xd^4y(\psi(y)\bar\psi(x))\mathcal{K}(x,y)(\psi(x)\bar\psi(y)) = \\ = -\frac{1}{2}&\int d^4xd^4y\mathcal{M}(x,y)\mathcal{K}^{-1}(x,y)\mathcal{M}(x,y) + \\ +&\int d^4xd^4y(\psi(x)\bar\psi(y))\mathcal{M}(x,y)\end{aligned} \tag{3.5}$$

where $\mathcal{K}^{-1}$ is the inverse of the kernel (3.2). Following ref. [25],[26] we introduce the short- hand notation

$$\begin{aligned}\int d^4x\bar\psi(x)(i\partial\!\!\!/ - m^0)\psi(x) &\equiv \int d^4xd^4y\psi(y)\bar\psi(x)(i\partial\!\!\!/ - m^0)\delta(x-y) = \\ &= (\psi\bar\psi, -G_0^{-1}),\\ \int d^4xd^4y(\psi(x)\bar\psi(y))\mathcal{M}(x,y) &= (\psi\bar\psi,\mathcal{M}).\end{aligned}$$

Then, from action (3.1) we get

$$W_{eff}[\mathcal{M}] = (\psi\bar\psi,(-G_0^{-1}+\mathcal{M})) - \frac{1}{2}(\mathcal{M},\mathcal{K}^{-1}\mathcal{M}). \tag{3.6}$$

After quantization (or integration) over N_c fermion fields and normal ordering, this action takes the form

$$W_{eff}[\mathcal{M}] = -\frac{1}{2}N_c(\mathcal{M},\mathcal{K}^{-1}\mathcal{M}) + iN_c\sum_{n=1}^{\infty}\frac{1}{n}\Phi^n. \tag{3.7}$$

Here $\Phi \equiv G_0\mathcal{M}, \Phi^2, \Phi^3$ etc. mean the following expressions

$$\Phi(x,y) \equiv G_0\mathcal{M} = \int d^4z G_0(x,z)\mathcal{M}(z,y),$$
$$\Phi^2 = \int d^4x d^4y \Phi(x,y)\Phi(y,x), \qquad (3.8)$$
$$\Phi^3 = \int d^4x d^4y d^4z \Phi(x,y)\Phi(y,z)\Phi(z,x) \,, \ldots \; etc$$

As a result of such quantization, only the the contributions with inner fermionic lines (but no scattering and dissociation channel contribution) are included in the effective action since we are interested only in the bound states.

The requirement for the choice of the time axis (3.4) in bilocal dynamics is equivalent to Markov - Yukawa condition [10]

$$z_\mu \cdot i\frac{\partial \mathcal{M}(z,X)}{\partial X_\mu} = 0 \qquad (3.9)$$

where $z_\mu = (x-y)_\mu$ and $X_\mu = (1/2)(x+y)_\mu$ are relative and total coordinates.

The first step to the quantization of the action (3.7) is the determination of its minimum

$$N_c^{-1}\frac{\delta W_Q(\mathcal{M})}{\delta \mathcal{M}} == -\mathcal{K}^{-1}\mathcal{M} + i\sum_{n=1}^{\infty} G_0(\mathcal{M}G_0)^n \equiv -\mathcal{K}^{-1}\mathcal{M} + \frac{i}{G_0^{-1} - \mathcal{M}} = 0. \quad (3.10)$$

We denote the corresponding classical solution for the bilocal field by $\Sigma(x-y)$. It depends only on the differrence $x-y$ because of translation invariance of vacuum solutions.

The next step is the expansion of the action (3.1) around the point of minimum $\mathcal{M} = \Sigma + \mathcal{M}'$,

$$W_Q(\Sigma + \mathcal{M}') = W_Q(\Sigma) + N_c[-\frac{1}{2}\mathcal{M}'\mathcal{K}^{-1}\mathcal{M}' + \frac{i}{2}(G_\Sigma \mathcal{M}')^2] +$$
$$+ \; iN_c\sum_{n=3}^{\infty}\frac{1}{n}(G_\Sigma\mathcal{M}')^n, \;\; (G_\Sigma = (G_0^{-1} - \Sigma)^{-1}), \qquad (3.11)$$

and the representation of the small fluctuations $\mathcal{M}'$ as a sum over the complete set of classical solutions Γ,

$$\frac{\delta^2 W_Q(\Sigma + \mathcal{M}')}{\delta \mathcal{M}^2}|_{\mathcal{M}'=0} \cdot \Gamma = 0. \qquad (3.12)$$

Using the definitions (3.8) and (3.11) it is easy to obtain the standard form of equations (3.10) and (3.12) :

$$\Sigma(x-y) = m^0\delta^{(4)}(x-y) + i\mathcal{K}(x,y)G_\Sigma(x-y), \qquad (3.13)$$

$$\Gamma = i\mathcal{K}(x,y)\int d^4z_1 d^4z_2 G_\Sigma(x-z_1)\Gamma(z_1,z_2)G_\Sigma(z_2-y) \qquad (3.14)$$

which are, respectively, the Schwinger - Dyson (SD) and Bethe - Salpeter (BS) equations. They describe the spectrum of Dirac particles and the spectrum of the bound states, respectively.

In the momentum space we obtain with

$$\underline{\Sigma}(k) = \int d^4x \Sigma(x) e^{ikx},$$
$$\underline{\Gamma}(q|\mathcal{P}) = \int d^4x d^4y e^{i\frac{x+y}{2}\mathcal{P}} e^{i(x-y)q}\Gamma(x,y)$$

for the kernel (3.2) the following equation for the mass operator ($\underline{\Sigma}$) and the vertex function ($\underline{\Gamma}$)

$$\underline{\Sigma}(k) = m^0 + i\int \frac{d^4q}{(2\pi)^4}\underline{V}(k^\perp - q^\perp)\not\eta \underline{G_\Sigma}(q)\not\eta, \qquad (3.15)$$

$$\underline{\Gamma}(k,\mathcal{P}) = i\int \frac{d^4q}{(2\pi)^4}\underline{V}(k^\perp - q^\perp)\not\eta[\underline{G_\Sigma}(q + \frac{\mathcal{P}}{2})\Gamma(q|\mathcal{P})\underline{G_\Sigma}(q - \frac{\mathcal{P}}{2})]\not\eta \qquad (3.16)$$

where $G_\Sigma(q) = (\not q - \underline{\Sigma}(q))^{-1}, \underline{V}(k^\perp)$ means the Fourier transform of the potential, $k_\mu^\perp = k_\mu - \eta_\mu(k\cdot\eta)$ is the transversal with respect to η_μ relative momentum, $\mathcal{P}_\mu$ is the total momentum.

The quantities $\underline{\Sigma}$ and $\underline{\Gamma}$ depend only on the transversal momentum

$$\underline{\Sigma}(k) = \underline{\Sigma}(k^\perp), \underline{\Gamma}(k|\mathcal{P}) = \underline{\Gamma}(k^\perp|\mathcal{P}),$$

because of the instantaneous form of the potential $\underline{V}(k^\perp)$ in any frame.

Therefore, we may integrate in (3.15) and (3.16) over the longitudinal momentum $q_0 = (q\cdot\eta)$ using the representation

$$\underline{\Sigma}_a(q) = \not q^\perp + E_a(q^\perp)S_a^{-2}(q^\perp) \qquad (3.17)$$

for the self - energy with

$$S_a^{-2}(q^\perp) = exp\{-\hat{\not q}^\perp 2v_a(q^\perp)\}, \;\; \hat{q}_\mu^\perp = q_\mu^\perp/|q^\perp| \qquad (3.18)$$

where S_a is the Foldy - Wouthuysen type transformation matrix with the parameter v_a.

Then, one has

$$\underline{G_{\Sigma_a}} = [q_0\not\eta - E_a(q^\perp)S_a^{-2}(q^\perp)]^{-1} =$$
$$= [\frac{\Lambda^{(\eta)}_{(+)a}(q^\perp)}{q_0 - E_a(q^\perp) + i\epsilon} + \frac{\Lambda^{(\eta)}_{(-)a}(q^\perp)}{q_0 + E_a(q^\perp) + i\epsilon}]\not\eta \qquad (3.19)$$

where

$$\Lambda^{(\eta)}_{(\pm)a}(q^\perp) = S_a(q^\perp)\Lambda^{(\eta)}_{(\pm)}(0)S_a^{-1}(q^\perp), \;\; \Lambda^{(\eta)}_{(\pm)}(0) = (1 \pm \not\eta)/2 \qquad (3.20)$$

are the operators separating the states with positive ($+E_a$) and negative ($-E_a$) energies.

As a result, we obtain the following equations for the one - particles energy E and the angle v :

$$E_a(k^\perp)\cos 2v(k^\perp) = m_a^0 + \frac{1}{2}\int \frac{d^3q^\perp}{(2\pi)^3}\underline{V}(k^\perp - q^\perp)\cos 2v(q^\perp) \qquad (3.21)$$

$$E_a(k^\perp)\sin 2v(k^\perp) = |k^\perp| + \frac{1}{2}\int \frac{d^3q^\perp}{(2\pi)^3}\underline{V}(k^\perp - q^\perp)|\hat{k}^\perp\cdot\hat{q}^\perp|\sin 2v(q^\perp) \qquad (3.22)$$

Let us consider the Bethe - Salpeter equation (3.16) after integration over the longitudinal momentum q_0. The vertex function is given by

$$\Gamma_{ab}(k^\perp|\mathcal{P}) = \int \frac{d^3q^\perp}{(2\pi)^3}\underline{V}(k^\perp - q^\perp)\not\eta\psi_{ab}(q^\perp)\not\eta, \qquad (3.23)$$

where the bound state wavefunction ψ_{ab} is given by

$$\psi_{ab}(q^\perp) = \not\eta[\frac{\bar{\Lambda}_{(+)a}(q^\perp\Gamma_{ab}(q^\perp|\mathcal{P})\Lambda_{(-)b}(q^\perp)}{E_T - \sqrt{\mathcal{P}^2} + i\epsilon} + \frac{\bar{\Lambda}_{(-)a}(q^\perp\Gamma_{ab}(q^\perp|\mathcal{P})\Lambda_{(+)b}(q^\perp)}{E_T + \sqrt{\mathcal{P}^2} - i\epsilon}]\not\eta \qquad (3.24)$$

$E_T = E_a + E_b$ means the sum of one - particles energies of the two particles (a) and (b) defined by (3.21, 3.22) and the notation

$$\bar{\Lambda}_{(\pm)}(q^\perp) = S^{-1}(q^\perp)\Lambda_{(\pm)}(0)S(q^\perp) = \Lambda_{(\pm)}(-q^\perp) \qquad (3.25)$$

has been introduced.

Acting with the operators (3.20) and (3.25) on equation (3.23) one gets the equations for the wavefunction ψ in an arbitrary moving reference frame

$$\begin{aligned}(\quad & E_T(k^\perp) \mp \sqrt{\mathcal{P}^2})\Lambda^{(\eta)}_{(\pm)a}(k^\perp)\psi_{ab}(k^\perp)\Lambda^{(\eta)}_{(\mp)b}(-k^\perp) = \\ = \quad & \Lambda^{(\eta)}_{(\pm)a}(k^\perp)[\int \frac{d^3q^\perp}{(2\pi)^3}\underline{V}(k^\perp - q^\perp)\psi_{ab}(q^\perp)]\Lambda^{(\eta)}_{(\mp)b}(-k^\perp). \qquad (3.26)\end{aligned}$$

All these equations (3.21, 3.22) and (3.26) have been derived without any assumption about the smallness of the relative momentum $|k^\perp|$ and for an arbitrary total momentum $\mathcal{P}_\mu = (\sqrt{M_A^2 + \vec{\mathcal{P}}^2}, \vec{\mathcal{P}} \neq 0)$.

If the atom is at rest ($\mathcal{P}_\mu = (M_A, 0, 0, 0)$) equation (3.26) coincides with the Salpeter equation [28]. If one assumes that the current mass m^0 is much larger than the relative momentum $|q^\perp|$ then the coupled equations (3.21, 3.22) and (3.26) turn into the Schrödinger equation. In the rest frame ($\mathcal{P}_0 = M_A$) equations (3.21, 3.22) for a large mass ($m^0/|q^\perp| \to \infty$) describe a nonrelativistic particle

$$E_a(\mathrm{k}) = \sqrt{(m_a^0)^2 + \mathrm{k}^2} \simeq m_a^0 + \frac{1}{2}\frac{\mathrm{k}^2}{m_a^0},$$

$$\tan 2v = \frac{k}{m^0} \to 0;\; S(\mathrm{k}) \simeq 1;\; \Lambda_{(\pm)} \simeq \frac{1 \pm \gamma_0}{2}.$$

Then, in equation (3.26) only the state with positive energy remains

$$\Lambda_{(+)}\psi\Lambda_{(-)} \simeq \psi_{Sch},\;\; \Lambda_{(-)}\psi\Lambda_{(+)} \simeq 0,$$

and finally the Schrödinger equation results in

$$[\frac{1}{2\mu}\mathrm{k}^2 + (m_a^0 + m_b^0 - M_A)]\psi_{Sch}(\mathrm{k}) = \int \frac{d\mathrm{q}}{(2\pi)^3}\underline{V}(\mathrm{k} - \mathrm{q})\psi_{Sch}(\mathrm{q}), \qquad (3.27)$$

where $\mu = m_a \cdot m_b/(m_a + m_b)$. For an arbitrary total momentum $\mathcal{P}_\mu$ equation (3.26) takes the form

$$[-\frac{1}{2\mu}(k_\nu^\perp)^2 + (m_a^0 + m_b^0 - \sqrt{\mathcal{P}^2})]\psi_{Sch}(k^\perp) = \int \frac{d^3q^\perp}{(2\pi)^3}\underline{V}(k^\perp - q^\perp)\psi_{Sch}(q^\perp), \qquad (3.28)$$

and describes a relativistic atom with nonrelativistic relative momentum $|k^\perp| << m^0_{a,b}$. In the framework of such a derivation of the Schrödinger equation it is sufficient to define the total coordinate according to (3.3), $X = (x + y)/2$, independently of the magnitude of the masses of the two particles forming an atom.

Now we consider the opposite case of massless particles, $m_a^0 = m_b^0 \to 0$. Suppose that in this case equations (3.21, 3.22)

$$2E_a(k^\perp)\cos 2v(k^\perp) = \int \frac{d^3q^\perp}{(2\pi)^3}\underline{V}(k^\perp - q^\perp)\cos 2v(q^\perp) \qquad (3.29)$$

$$2E_a(k^\perp)\sin 2v(k^\perp) = |k^\perp| + \int \frac{d^3q^\perp}{(2\pi)^3}\underline{V}(k^\perp - q^\perp)|\hat{k}^\perp\cdot\hat{q}^\perp|\sin 2v(q^\perp) \qquad (3.30)$$

have a nontrivial solution $v(k^\perp) \neq 0$. This solution describes the spontaneous breakdown of chiral symmetry [9, 23, 24], [29]- [33].

It can easily be seen that equations (29,30) are identical with (3.26) for the bound state wavefunction with zero eigenvalue, $\mathcal{P}_\mu^2 = 0$ and

$$\Lambda_{(+)}\psi\bar{\Lambda}_{(-)} = \Lambda_{(-)}\psi\bar{\Lambda}_{(+)} \equiv \psi$$

$$2E_a(k^\perp)\psi(k^\perp) = \int \frac{d^3q^\perp}{(2\pi)^3}\underline{V}(k^\perp - q^\perp)\psi(q^\perp). \qquad (3.31)$$

Therefore,

$$\psi = \cos 2v(k^\perp)/F \qquad (3.32)$$

where F is a proportionality constant.

In this way, the coupled equations (3.21, 3.22) and (3.26) describe the pure relativistic effect of the appearance of the Goldstone mode due to spontaneous breakdown of chiral symmetry. Thus in the framework of instantaneous action (3.1) we get the proof of the Goldstone theorem in the bilocal variant.

Just this example represents a model for the construction of a low - energy theory of light mesons, in which the pion is considered in two different ways, as a quark -

antiquark bound state and as a Goldstone particle. So it turns out that our relativistic instantaneous model for bound states can, in the lowest order of radiative corrections, also describes mesons.

Indeed, there is a number of paper (cf. [9, 23, 24, 25], [29]-[31] and references therein) where equations (3.21, 3.22) and (3.26) are used for the calculation of the mass spectrum of light mesons, the constituent quark masses and the meson decay constants. In the papers the potentials are determined from the spectroscopy of heavy quarkonia as sums of rising and Coulomb potentials, for instance

$$V(r) = \frac{\alpha_S}{r} - V_0 r^2, V_0^{1/3} \simeq 250MeV, \alpha_S \simeq 0.3 . \qquad (3.33)$$

Thereby, the heavy quarkonia ($m^0 >> 250MeV$) themselves are described by Schrödinger equation (3.28) which, as has been shown, can be derived from eq. (3.26) in the limit of large masses. In this limit, the effect of spontaneous breakdown of chiral symmetry also disappeares, and the constituent quark masses are identical to the current ones [32].

The advantage of such a potential approach compared with all other ones consists in the first constructive connection between the fundamental parameters for physics at short distances (the parameters of rising and Coulomb potentials and the current quark masses) with those of hadron physics for long distances (the pion mass and its weak decay constant F_π).

The shortcomings of this approach were the following: the nonrelativistic formulation (in the rest frame) of the bound state, the absence of an relativistic meson interaction Hamiltonian, and the open problem of the status of radiative QCD corrections. The first two disadvantages are absent in the new relativistic potential model [23, 24, 33] considered here. This model represents a logical interpretation of relativistic atomic physics, i.e. an interpretation of the "atomization" of QED.

From this point of view the "hadronization" of QCD qualitatively differs only by the short - range property of the quark - antiquark interaction potential for light quarkonia. Furthermore, the effective action for light mesons must be an action for a chiral Lagrangian. The proof of the fact that (3.11) leads to a chiral Lagrangian has been performed in [33] with the help of the separable approximation which can be used just for short - range potentials. For low orbital momenta such potentials can be represented with good accuracy as a product of two factors

$$< l = 0|V(\mathbf{p} - \mathbf{q})|l = 0 >= f(p^\perp)f(q^\perp).$$

The underlying model (3.11) becomes equivalent to one of versions of the Nambu - Jona - Lasinio model [25, 34, 35] with explicit indication of the formfactor $f^2(p)$ for the ultraviolet regularization. It is well known [25, 34, 35], that this model leads to chiral Lagrangians.

The validity of the separable approximation for short - range potentials explains the fact of the weak dependence of low - energy physics for light mesons on the form of the potential. Therefore, there exists a number of models yielding a satisfactory description of the experimental data.

Here, one should mention also papers dealing with the derivation of nonlinear chiral Lagrangians from QCD (cf.ref. in [35]). The essence of those proofs consists in a formal derivation of the determinant (3.7), (3.11) by means of chiral transformation which are parametrized by the meson field (without the derivation of the equation for the meson spectrum).The main aim of these papers is to find the coefficients in higher order terms in the expansion of chiral Lagrangians in meson momenta and to establish the description of baryons in the form of "skyrmions" [36]. All these papers concerning the justification of chiral Lagrangian from QCD are not devoted to the determination of essential parameters of the low - energy physics (F_π, F_K, m_π, ...) from QCD.

The relativistic model for atoms and hadrons (3.1) - (3.26) as compared with the above - mentioned popular non - relativistic [9] and nonlinear [35] approaches unifies aspects of both approaches and gives a constructive generalization of chiral Lagrangians to heavy quarkonium physics, i.e. it allows to describe decays of heavy quarkonia into light ones in the framework of one unified action of the type (3.11) with a minimal number of parameters, defined in the short - range region where the perturbation theory begins to work.

The construction of such a quantum relativistic hadron theory on the basis of the action (3.11) has been given in papers [23, 24].

4 LECTURE

MINIMAL QUANTIZATION OF NONABELIAN THEORY.

Construction of physical variables, collective excitation of gluons, the topological degeneration, anomalies, confinement.

We have seen above that the description of instantaneous bound states demands the relativistic covariant separation of all interactions of gauge theories into the instantaneous and retardation parts. The consistent way of this separation is the minimal approach to gauge field quantization [17]- [20], [21]. According to this approach we should choose the time-axis of quantization (η_μ) and construct the minimal set of physical variables with the help of explicit solution of the equation for the time component of the gauge field $A_0 = (\eta \cdot A)$.

Let us apply this minimal approach to non-Abelian theory (following papers [17]- [20]):

$$\begin{aligned} W &= \int dx\, \mathcal{L}(x) \\ \mathcal{L}(x) &= -\frac{1}{4}(F^2_{\mu\nu})^2 + \bar\psi(i \not\nabla - m^0)\psi \ , \end{aligned} \tag{4.1}$$

where

$$\begin{aligned} \nabla_\mu &= \partial_\mu + \hat A_\mu \ ; \ \hat A_\mu = e\frac{A^a_\mu \tau^a}{2i} \ ; \\ F^a_{\mu\nu} &= \partial_\mu A^a_\nu - \partial_\nu A^a_\mu + e\epsilon^{abc} A^b_\mu A^c_\nu \ ; \\ \nabla^{ab}_i(A) &= \delta^{ab}\partial_i + e\epsilon^{abc}A^c_i \ ; \ j^a_\mu = e\,\bar\psi\gamma_\mu\frac{\tau^a}{2}\psi \ . \end{aligned} \tag{4.2}$$

We will consider $SU(2)$-theory with the Pauli matrices τ^a (a=1, 2, 3).

At first, we construct physical variables solving the Gauss equation for the time component $\delta W/\delta A_0 = 0$:

$$[\nabla^{ab}_i(A)\nabla^{bc}_i(A)A^c_0] = \nabla^{ab}_i(A)\partial_0 A^b_i + j^a_0 \ . \tag{4.3}$$

The Lagrangian (4.1) and equation (4.3) invariant under the gauge transformation

$$\hat A^g_\mu = g(\hat A_\mu + \partial_\mu)g^{-1} \ ; \ \psi^g = g\,\psi \tag{4.4}$$

Here $g(x)$ is a matrix with values in $SU(2)$. The formal explicit solution of equation (4.3) may be represented by

$$A^a_0(x) = O^a(x) + a^a_0 + \left[\frac{1}{(\nabla_i(A))^2}\right]^{ab} j^b_0 \ , \tag{4.5}$$

where a^a_0 is the functional over A_i

$$a^a_0 = \left[\frac{1}{(\nabla_i(A))^2}\nabla_j(A)\partial_0 A_j\right]^a \ . \tag{4.6}$$

O^a is the zero mode of the differential operator $(\nabla_i(A))^2$ if it exists.

At first, we consider the case $O^a \equiv 0$. Then, we can introduce the analogs of gauge-invariant variables in QED

$$\begin{aligned} \hat a^T_i[A] &= \mathcal{V}_T[A](\hat A_i + \partial_i)(\mathcal{V}_T[A])^{-1} \\ \psi^T[A,\psi] &= \mathcal{V}_T[A]\psi \end{aligned} \tag{4.7}$$

which the matrix $\mathcal{V}_T[A]$ satisfies the equation

$$\partial_0\mathcal{V}_T[A] = \mathcal{V}_T[A]\hat a_0[A] \ , \tag{4.8}$$

The formal solution of equation (4.8) is represented in the form of a time-ordered exponent

$$\mathcal{V}_T[A] = T\exp\left\{\int^t dt'\hat a_0[A(\mathrm{x},t')]\right\} \tag{4.9}$$

where $\int^t dt' f(t') = F(t)$ is an indefinite integral.

It is easy to check [17, 18] that the matrix $\mathcal{V}_T[A]$ transforms under the gauge transformations (4.4) by the rule

$$\mathcal{V}_T[A^g] = \mathcal{V}_T[A]g^{-1}$$

and the functionals (4.7) are gauge- invariant

$$\psi^T[A^g,\psi^g] = \mathcal{V}_T[A^g]g\psi = \mathcal{V}_T g^{-1}g\psi = \mathcal{V}_T\psi = \psi^T[A,\psi]$$

In terms of the variables (4.7) the Lagrangian (4.1) has the form

$$\begin{aligned} \mathcal{L}^T(a^T,\psi^T) &= \frac{1}{2}(\partial_0(a^b_i)^T)^2 - \frac{1}{4}F^2_{ij}(a^T) - (j^a_i)^T(a^a_i)^T + \\ &+ \frac{1}{2}(j^a_0)^T\left[\frac{1}{(\nabla_k(a^T))^2}\right]^{ab}(j^b_0)^T + \bar\psi^T[i\not\partial - m^0]\psi^T \ . \end{aligned} \tag{4.10}$$

The variables a^T_i identically satisfy the gauge conditions

$$\nabla_i(a^T)\partial_0 a^T_i \equiv 0 \ , \tag{4.11}$$

which are a generalization of the Coulomb gauge in QED.

The canonical quantization of a physical system of the type of (4.10),(4.11) (without spinor fields) was considered in paper [37]). The result of this quantization can be represented in the form of a continual integral for the generating functional of the Green functions with the field sources $J, \eta, \bar{\eta}$:

$$Z(\eta^T, \eta^T, J^T) = \int d^3 a_i(x) (\det \nabla_i \nabla_i)^{\frac{1}{2}} \delta(\nabla_i(a) \partial_0 a_i) \int d\psi d\bar{\psi} \exp\{i \int dx [\mathcal{L}^T(A, \psi) + \bar{\psi}\eta^T + \bar{\eta}^T \psi + a_i^T J_i^T]\} \quad ,(4.12)$$

where $\int dA(x), \delta(f(x))$ mean the continual limits of the multiple integral and δ- function (see, for example [26]). Expression (4.12) was also obtained [17] by the Faddeev-Popov method [15].

The quantization (4.12) is still to be researched on the level of the Schwinger method of quantization in the Coulomb gauge [38, 18].

Let us consider another case when there is a nontrivial zero mode O^a of the operator $(\nabla(A))^2$ in equation (4.5). In terms of the gauge-invariant variables (4.7) the functional O^a satisfies the equation

$$[\nabla_i^2(a_i^T)]^{ab}(O^T)^b = 0 \quad (4.13)$$

$$e(O^T)^b \frac{\tau^b}{2i} = \hat{O}^T = \mathcal{V}_T[A]\hat{O}(\mathcal{V}_T[A])^{-1}$$

and the initial Lagrangian (4.1) has the additional terms

$$\mathcal{L} = \mathcal{L}^T(a^T, \psi^T) + \Delta\mathcal{L}(a^T, \psi^T, O^T) \quad (4.14)$$

where

$$\Delta\mathcal{L} = \frac{1}{2}(\nabla_i^T O^T)^2 + (\nabla_i^T O^T)^a (\nabla_i^T \frac{1}{(\nabla_i^T)^2} j_0)^a + (O^T)^a (j^T)^a - (\nabla_i^T O^T)^a \partial_0 (a_i^T)^a \ ; \quad (4.15)$$

$$(\nabla^T \equiv \nabla(a^T)) \ .$$

Using the transversality conditions (4.11), (4.13) and the identity

$$(\nabla_i A) \cdot B + A \cdot (\nabla_i B) \equiv \partial_i (A \cdot B) \ .$$

We can represent (4.15) in the form of the total derivative

$$\Delta\mathcal{L} = \partial_i\{\frac{1}{4}\partial_i((O^T)^a)^2 + (O^T)a[\nabla_i^T \frac{1}{(\nabla_i^T)^2} j_0 - \partial_0 a_i^T]^2\} \quad (4.16)$$

This means that the Lagrangian (4.14) does not locally depend on the field O^T and the last describes the excitation of the system as the whole. The main problem consists in the definition of physical variables of this excitation. In the presence of the

zero-mode O^T there is another method of defining the physical field variables classically equivalent to (4.7)

$$\hat{A}_i^{ph} = w(\hat{a}_i^T + \partial_i)w^{-1} \ ; \ \psi^{ph} = w\psi^T \ , \quad (4.17)$$

where the matrix w satisfies the equation

$$w^{-1}\partial_0 w = O^T \ ; \ w = T\exp\{\int^t dt' \hat{O}^T(t')\} \ . \quad (4.18)$$

It is easy to check that due to eq. (4.13) the variables (4.7) and (4.17) satisfy the same constraint (4.11)

$$\nabla_i(A^{ph})\partial_0 \hat{A}^{ph} \equiv w[\nabla_i^T(\partial_0 \hat{a}^T + \nabla_i^T O^T)]w^{-1} = 0 \ .$$

According to the initial Weyl idea one can give the physical meaning to the phase factor w (4.18) instead of the phase O^T, and represent the definition (4.17) as a gauge covariant separation of the "large" variable (w) from the bare quasiparticle excitations a^T, ψ^T.

The quantum field theory for the variables (4.17) differs from the result of the first quantization (4.12) by the new spurious diagrams that go out from the physical field sources of the type

$$\bar{\psi}^{ph}\eta^{ph} + \bar{\eta}^{ph}\psi^{ph} = \bar{\psi}(w^{-1}\eta^{ph}) + (\bar{\eta}^{ph}w)\psi \ , \quad (4.19)$$

and by the non-Abelian U(1) -anomaly terms[40] arising from the Dirac vacuum of the physical fermions[17]. In just the terms of the physical fermions we should make the procedure of hadronization (see Lecture 2)

$$\frac{1}{2}\int dxdy (j_0^{ph} \frac{1}{\nabla^2(A^{ph})} j_0^{ph}) \Rightarrow (\psi^{ph}\bar{\psi}^{ph}, \mathcal{M}) - \frac{1}{2}(\mathcal{M}, \mathcal{K}^{-1}(A^{ph})\mathcal{M}) \quad (4.20)$$

and should quantize the Lagrangian

$$\int dxdy \bar{\psi}^{ph}(x)\{[i\partial\!\!\!/_x - m^0 - \gamma_i \hat{A}_i^{ph}(x)]\delta(x-y) - \mathcal{M}(x,y)\}\psi^{ph}(y) \ . \quad (4.21)$$

The result of this quantization, the effective action

$$i\int dxdy tr\{\log\{[i\partial\!\!\!/_x - m^0 - \gamma_i \hat{A}_i^{ph}(x)]\delta(x-y) - \mathcal{M}(x,y)\}\} \quad (4.22)$$

explicitly depends on the phase factor w under the transformation (4.17) due to the non-Abelian anomaly. In the present time this U(1) -anomaly was calculated only for the local approximation of hadron fields $\mathcal{M}(x,y) \simeq \mathcal{M}^L(x)\delta(x-y)$. For example [40], one of the anomaly terms has the form

$$\Delta_{Anom.} = N_c \int dx [arg \mathcal{M}^L(x)] \partial_0 K_0(x) \quad (4.23)$$

where

$$K_0(x) = \frac{1}{24\pi^2} \epsilon_{ijk} tr(\hat{V}_i \hat{V}_j \hat{V}_k) \ ; \ \hat{V}_k = w^{-1}\partial_i w \ . \quad (4.24)$$

As the final result, instead of the generating functional (4.12) we have the expression

$$\begin{aligned} Z(\eta^T, \eta^T, J^T) = & \int [dw] \int d^3 a_i(x) (\det \nabla_i \nabla_i)^{\frac{1}{2}} \delta(\nabla_i(a)\partial_0 a_i) \\ & \int d\psi d\bar{\psi} \exp\{ i \int dx [\mathcal{L}^T(A,\psi) + \Delta\mathcal{L}(w^{-1}\partial_0 w, a, \psi) + \\ & + \Delta_{Anom.} + \bar{\psi}(w^{-1}\eta) + (\bar{\eta} w)\psi + a_i^a (w^{-1}\hat{J}_i w)^a] \} \ , \end{aligned} \quad (4.25)$$

where the measure of integration over the Weyl phase factor is defined by the explicit solution of eq. (4.13), and the operator $\nabla^2(a)$ under the sign of the integral in (4.25) is defined on the function without zero-mode.

It is very important to emphasize that the Weyl interpretation of the zero-mode of the gauge-covariant Laplace operator ($\nabla_i^2(a)$) leads really to nontrivial solutions of eq. (4.13) with the trivial boundary condition on the Weyl factor

$$\lim_{|\mathbf{x}| \to R} w(\mathbf{x}, t = \pm\frac{T}{2}) = 1 \ ; \ |\mathbf{x}| \leq R \ ; \ -\frac{T}{2} \leq t \leq \frac{T}{2} \ , \quad (4.26)$$

if we consider the finite space-time to describe the quantum properties of the zero-mode excitation. While the zero boundary condition on the phase

$$\lim_{|\mathbf{x}| \to R} O^T(\mathbf{x}, t = \pm\frac{T}{2}) = 0 \quad (4.27)$$

leads only to the solution $O^T \equiv 0$ and to the strictly positive Laplace operator ($\nabla^2 > 0$) [37]. The cause of this difference consists in the topological properties of the Weyl factors. To consider the example of a topologically nontrivial solution in the pure form, we can neglect the local excitation of gauge field ($a^T = 0$). One of this solutions can be the expression [17, 18]

$$w(\mathbf{x}) = e^{i\frac{x^a \tau^a}{R}\pi n} \ ; \ n = 0, \pm1, \pm2, \dots \ , \quad (4.28)$$

which is characterized by the topological index of mapping of the space $R(3)(|\mathbf{x}| \leq R)$ into $SU(2)$

$$n = \int_{|\mathbf{x}| \leq R} d\mathbf{x} K_0(\mathbf{x}) \ , \quad (4.29)$$

where $K_0(x)$ is given by eq.(4.24).

The existence of such a solution indicates that physical variables (4.17) are degenerated in the space of finite volume. However, the degeneration index n (4.28) does not disappear even in the limit of infinite space ($R \to \infty$) and represents an example of topological quantum anomalies of the type of the axial current divergence $\partial_\mu j_\mu^5 \sim F_{\mu\nu}\tilde{F}^{\mu\nu}$ (4.23). Both these quantities, the index n and divergence $\partial_\mu j_\mu^5$, after removing the regularization are not equal to zero despite the disappearance of the initial elements of their construction: the fields $V_i = w^{-1}\partial_i w$ or the Pauli-Willars propagators.

Degeneration is removed by the averaging of the Green function generating functional over all degeneration parameters [17]. This means that instead of the integral over w in eq.(4.25) we have the sum over the degeneration index (n). Thus,expression (4.25) up to the terms disappearing in the limit of infinite space turns out into the Green function generating functional

$$Z(\eta, \bar{\eta}, J) = \lim_{R,T \to \infty} \lim_{L \to \infty} \frac{1}{2L} \sum_{n=-L}^{n=L} Z_{R,T}^T [w^{-1}\eta, \bar{\eta} w, w\hat{J}_i w^{-1}] \ , \quad (4.30)$$

where $Z_{R,T}^T$ is the usual Faddeev-Popov integral in the finite space-time. Since all the computable physical quantities in quantum field theory such as cross-sections, probabilities of decays, etc., are normalized to the finite space-time, the transition to the infinite space should be performed after removing the degeneration.

In quantum field theory (4.29) we have two types of colour particles (like in the t'Hooft model of QCD_2 [41]): the "bare", internal states as the usual elements of the perturbation theory (a^T, ψ^T), and the "dressed", external physical states (A^{ph}, ψ^{ph}).

The physical quarks and gluons cannot propagate

$$\lim_{L \to \infty} \frac{1}{2L} \sum_{n=-L}^{n=L} e^{i\frac{x^a \tau^a}{R}\pi n} = \begin{cases} 1, & |\mathbf{x}| = 0 \\ 0, & |\mathbf{x}| \neq 0 \end{cases} \quad (4.31)$$

even in a space of any volume ($R \to \infty$) due to the destructive interference of the infinite number of phase factors of the topological degeneration.

At the same time, colourless correlations of electromagnetic and weak currents are expressed in terms of propagators of "bare" quarks and gluons, and they coincide with the usual expression of perturbative QCD. As has been shown in refs. [41, 42, 43], situation does not contradict the unitarity of the S- matrix and leads just to the principle of quark- hadron duality in the Minkowski space widely used in the parton phenomenology.

The topological mechanism of dressing physical quarks explains the main hypothesis of this phenomenology from the first gauge principles in the Weyl form: the probability of hadronization being unity and the independence of hadronization of the confinement. This confinement may be realized in any non-Abelian theory constructed on the semi-simple group G_0. In the mapping of the space $R(3)$ onto a group space, it is important, to pick out the $SU(2)$ subgroup from G in minimal manner.

"Minimality" means that the fundamental representation G is simultaneously an irreducible representation of the $SU(2)$ subgroup. For example, in the case of $SU(3)$ the Gell-Mann matrices $\lambda_2, \lambda_5, \lambda_7$ coinciding with the adjoint $SU(2)$ representation are the generators of the minimal subgroup.

5 LECTURE

TOWARDS THEORY OF HADRONS.

Physical program of modern accelerators and hadron perturbation theory. The "logic" of quantum theory, the operator derivation of bound state equations, gluon spectrum, and "asymptotic freedom". Quark- hadron duality.

5.1 Hadron perturbation theory in QCD.

Nowadays one expects essential progress in the development of accelerators with essentially higher luminosity of beams, the so- called $c - \tau$- factories. (Especially we have in mind a class of machines with typical energies in the centre of mass system $\sqrt{s} = 3 \div 5 Gev.$ and luminosity $> 10^{33} cm^{-2} s^{-1}$.)

The main achievement of the future factories should be the increase by two orders (10^2 times) of the accuracy of measurement of hadron processes which to a marked degree is in excess of the accuracy of the modern conventional theoretical methods of QCD calculations and estimations of hadron interaction amplitudes (lattice calculations, sum rules, the parton- QCD model, low- energy theorems et.al.)

Therefore, the problem arises to formulate the QCD perturbative theory for hadrons as relativistic bound states.

One may expect that an experimental progress in this energy region will serve as a strong stimulation for theoretical activities in this field in the nearest future, like the famous J/ψ experiment very much helped to establish QCD as the theory of strong interactions.

The conventional QCD has been created mainly by the parton phenomenology [44] and by the asymptotic freedom formula [45]

$$\alpha(Q^2) = [b \ln(\frac{Q^2}{\Lambda^2})]^{-1} \; ; \; b = \frac{1}{4\pi}(11 - \frac{2}{3}N_f) \; , \qquad (5.1)$$

where Λ is the parameter of the boundary condition. This formula is a main argument for justification of the perturbation theory in the deep- Euclidean region of the quark and gluon momenta, $Q^2 > \Lambda^2$, where these colour particles are considered as free partons. But this formula (5.1) cannot answer up to now the question: What is the vacuum and other physical states of QCD?

The parton QCD turns over the usual logic of quantum theory where at first we should define the physical states and then calculate the weak amplitudes transitions between them.

When we are constructing the QCD perturbation theory for hadron at first we should define the bilocal (1.12) or N- local (1.22) hadron states as irreducible representations of the Lorentz group (with mass and spin) and then quantize the chromodynamics according to this definition.

The main peculiarity of such hadron states is their covariant instantaneity (1.19), (1.22) with respect to the eigen- values of the bound state total momentum operator $\mathcal{P}_\mu$. In Lectures 2 and 4 we discussed the minimal approach for the construction of physical variables of gauge theory, based on the explicit solution of the classical equation for the "time"- component of gauge field ($A_0 = (\eta \cdot A)$; $\eta_\mu \sim \mathcal{P}_\mu$; $\eta^2 = 1$) where the gluon exchange interaction between quarks is naturally divided into two parts: the instantaneous and retardation contributions.

If we define any hadron bound state by formulae (1.19) and (1.22) as an irreducible nonlocal representation of the Poincare group, it is easy to understand that the covariant instantaneous gluon interaction (4.10)

$$W_{Ins.}(a^\perp) = \int dx_1 dx_2 \frac{1}{2} j^a_\eta(x_1) \left[\frac{1}{[\nabla^\perp_\mu(a^\perp_\mu)]^2}\right] j^a_\eta(x_2) \delta(\eta \cdot z) \; , \qquad (5.2)$$

(where $j^a_\eta = e\bar{q}\frac{\lambda_a}{2}\not{\eta} q$, $z = x_1 - x_2$, $a^\perp_\mu = a_\mu - \eta_\mu(a \cdot \eta)$) at the point of existence of a bound state $\eta \cdot (x_1 - x_2) = 0$ is greater than the remaining retardated part of this interaction in QCD. This minimal approach allows one to formulate the hadron perturbation theory [24], the lowest order of which is the potential model defined as the gluon vacuum average of

$$< 0|W_{Ins.}(a^\perp)|0 >= \int dx_1 dx_2 \frac{1}{2} j^a_\eta(x_1) V_{eff}(z^\perp) j^a_\eta(x_2) \delta(\eta \cdot z_0) \; , \qquad (5.3)$$

The calculation of this average is one of the main tasks of the hadron QCD. On the recent phenomenological level of QCD we can choose the quark- quark potential in the form of the sum of the rising and Coulomb ones (3.33) using the heavy quarkonia spectroscopy as the analogy of the Coulomb experiment in electrodynamics.

5.2 The spontaneous breakdown of chiral symmetry and the physical vacuum.

As result we get in the rest frame $\eta = (1,0,0,0)$ the potential model

$$\mathcal{H} = \int d\mathrm{x} \bar{q}(i\partial_i\gamma_i + m^0)q + \frac{1}{2}\int d\mathrm{x} d\mathrm{y} (q^+_i(\mathrm{x})\frac{\lambda^a_{ij}}{2}q_j(\mathrm{x}))V(\mathrm{x}-\mathrm{y})(q^+_k(\mathrm{y})\frac{\lambda^a_{kl}}{2}q_l(\mathrm{y})). \quad (5.4)$$

which is widely used now for description of the spectroscopy of heavy and light quarkonia [9, 23, 24] [29]- [33], [46]- [48].

The instantaneous singularities, forming the bound states, cannot be reproduced by a relativistic gauge where all gauge field propagators have singularities only on the light cone. All modern QCD approaches, including the lattice calculations, do not take into account these peculiarities of the problem of bound states.

Model (5.4) gives a good illustration of the "quantum logic". Let us consider the physical vacuum and the spontaneous breakdown of chiral symmetry in this model by the operator approach.

The first step for constructing the physical states consists in the definition of the one - quasi - particle creation (a^+, b^+) and annihilation (a, b) operators with the help of the Bogolubov fermion expansion

$$q_\alpha(\mathbf{x}) = \sum_s \int \frac{d\mathbf{q}}{(2\pi)^{3/2}} e^{i\mathbf{q}\mathbf{x}}[a_s(\mathbf{q})\mu_\alpha(\mathbf{q}, s) + b_s^+(-\mathbf{q})\nu_\alpha(-\mathbf{q}, s)]. \tag{5.5}$$

Here $\mu_\alpha(\mathbf{q}, s)$ and $\nu_\alpha(-\mathbf{q}, s)$ are the coefficients determined from the Schrödinger equation for the one - particles energy

$$< a_s(\mathbf{q})|\hat{H}|a_s^+(\mathbf{q}') >= E(\mathbf{q}) < 0|a_s(\mathbf{q})a_s^+(\mathbf{q}')|0 > . \tag{5.6}$$

They can be represented via the Foldy - Wouthuysen matrix (18) as

$$\mu_\alpha(\mathbf{q}, s) = S(\mathbf{q})_{\alpha\beta}\mu_\beta(0, s); \;\; \nu_\alpha(-\mathbf{q}, s) = S(-\mathbf{q})_{\alpha\beta}\nu_\beta(0, s)$$

with

$$S_{\alpha\alpha'}(\mathbf{q})[\sum_s \mu_{\alpha'}(0, s)\mu^+_{\beta'}(0, s)]S^{-1}_{\beta'\beta}(\mathbf{q}) = (S\frac{1+\gamma_0}{2}S^{-1})_{\alpha\beta} \equiv (\Lambda^0_+(\mathbf{q}))_{\alpha\beta},$$

$$S_{\alpha\alpha'}(-\mathbf{q})[\sum_s \nu_{\alpha'}(0, s)\nu^+_{\beta'}(0, s)]S^{-1}_{\beta'\beta}(-\mathbf{q}) = (S\frac{1-\gamma_0}{2}S^{-1})_{\alpha\beta} \equiv (\Lambda^0_-(-\mathbf{q}))_{\alpha\beta}.$$

Λ^0_+ and Λ^0_- are projection operators on states with positive, resp., negative energy. Then, equation (5.6) takes the form of the Schwinger - Dyson equation (3.29,3.30) which can compactly be written as

$$E(p)S^{-2}(\mathbf{p}) = m^0 + p_i\gamma_i + \frac{2}{3}\hat{I}_{\mathbf{pq}}S^{-2}(\mathbf{q}), \tag{5.7}$$

where $\hat{I}_{\mathbf{pq}}$ is a short- hand notation for the integral operator

$$\hat{I}_{\mathbf{pq}} = \int \frac{d\mathbf{q}}{(2\pi)^3}\underline{V}(\mathbf{p}-\mathbf{q}) \tag{5.8}$$

After inserting (5.5) into (5.4) the Hamiltonian can be given in the following manner:

$$\begin{aligned}
\mathcal{H} &= E_0 + H_1 + :H_4:,\\
E_0 &= < 0|\mathcal{H}|0 >,\\
H_1 &= \sum_{(1)} E(\mathbf{p}_1)(a_1^+a_1 + b_1^+b_1),\\
:H_4: &= \frac{2}{3}\sum_{1,2,3,4,} \delta^{(4)}(p_1 - p_2 + p_3 - p_4)\underline{V}(\mathbf{p}_1 - \mathbf{p}_3)\\
&\{ a_1^+b_2^+a_3^+b_4^+\mu_1^*\nu_2^*\mu_3^*\nu_4^* + a_1^+b_2^+b_3a_1\mu_1^*\nu_2\nu_3^*\mu_4 +\\
&+ \; b_1a_2a_3^+b_4^+\nu_1^*\mu_2\mu_3^*\nu_4 + b_1a_2b_3a_4\nu_1^*\mu_2\nu_3^*\mu_4 + \; ...\} + \; ...
\end{aligned} \tag{5.9}$$

In $:H_4:$ only terms forming colourless mesons as pair correlation [49] - [51] have been kept. The following abbreviations have been used in (5.9):

$$\sum_I = \sum_{s_I}\int \frac{d\mathbf{p}_I}{(2\pi)^{3/2}}, \; \{I\} = \{p_I, s_I\}, \; \{\hat{I}\} = \{-p_I, -s_I\}, \; I = 1, 2, 3, 4.$$

For diagonalizing the Hamiltonian (5.9) with respect to pair correlations $(a_1^+b_2^+)$, (b_3a_4) one defines a new vacuum as the coherent state

$$|0>>_\alpha = \exp\{\sum_{1,2,3,4} \alpha(1, \hat{2}, \hat{3}, 4)[(a_1^{+i_1}b_2^{+i_1})(b_3^{+j_1}a_4^{+j_1})]\}|0> \tag{5.10}$$

and the creation operator for the bound state (of pair correlation)

$$B^+(n) = \sum_{1,2}\delta(\mathbf{p}_1 - \mathbf{p}_2)[X_+(1, \hat{2})a^{+i}(1)b^{+i}(\hat{2}) - X_-(\hat{1}, 2)b^i(\hat{1})a^i(1)]. \tag{5.11}$$

The coefficient X_+ and X_- are determined from the Schrödinger equation for the two - particle energy M_B ,

$$_\alpha << 0|B(n)(H_1 + H_4)B^+(n)|0 >>_\alpha = M_B \, _\alpha << 0|B(n)B^+(n)|0 >>_\alpha, \tag{5.12}$$

and the parameter α in (5.10) is given with the help of the definition of the annihilation operator $B(n)$ for the pair correlation

$$B(n)|0 >>_\alpha = 0. \tag{5.13}$$

Equation (5.12) coincides with equation (3.26) in the rest frame (the Salpeter equation) for the meson spectrum

$$(E_1(\mathbf{p}) + E_2(\mathbf{p}) \mp M_B)\psi_{\pm\pm}(\mathbf{p}) = \frac{4}{3}\Lambda_\pm(\mathbf{p})[\hat{I}_{\mathbf{pq}}(\psi_{++}(\mathbf{q}) + \psi_{--}(\mathbf{q}))]\Lambda_\pm(-\mathbf{p}) \tag{5.14}$$

up to the notation

$$\begin{aligned}
\psi &= \psi_{++} + \psi_{--}; \; \psi_{\pm\pm} = \Lambda_\pm\psi\Lambda_\mp;\\
\psi_{++}(\mathbf{p})_{\alpha\beta} &= \sum_{s_1,s_2} X_+(\mathbf{p}, \mathbf{p}, s_1, s_2)\mu^+_\alpha(\mathbf{p}, s_1)\nu_\beta(\mathbf{p}, s_2),\\
\psi_{--}(\mathbf{p})_{\alpha\beta} &= \sum_{s_1,s_2} X_-(\mathbf{p}, \mathbf{p}, s_1, s_2)\nu^+_\alpha(\mathbf{p}, s_1)\mu_\beta(\mathbf{p}, s_2).
\end{aligned} \tag{5.15}$$

The one - particle energies $E_1(\mathbf{p})$, $E_2(\mathbf{p})$ in (5.14) are defined via the Schwinger - Dyson equation (5.7).

Notice that equations of the type (5.7), (5.14) are well - known from the nonrelativistic many - body theory (Landau's theory of fermi liquids [49], Randon Phase Approximation [50]) and play an essential role in the description of elementary excitation in atomic nuclei [51]. Their relativistic analogues describing the Goldstone pion and the constituent masses of the light quarks are equations (3.21,3.22) and (3.26) [9, 24], [46]- [48].

The Green function method discussed in sect.2, and the operator approach lead to one and the same equations and complement each other. The first allows one to make easily the relativistic generalization and to construct the effective meson interaction Lagrangian, whereas the second yields an adequate interpretation of quantum states and enables one to describe more complicated system, e.g. baryons and other many - quark states [52].

5.3 The relativistic equation for many - quark systems.

Let us construct by means of the quasiparticle operator method the relativistic equation for the baryon as a three - quark system. In the meson "coherent" vacuum (5.10) the baryon creation operator consists not only of creation operators for quarks (a^+) but also of annihilation operator for antiquarks (b) with the same quantum numbers

$$\underline{B}^+ = \sum_{1,2,3}\delta(p_1+p_2+p_3)[X_{+++}(1,2,3)a^{i(+)}(1)a^{j(+)}(2)a^{l(+)}(3) + X_{--+}(1,2,3)b^{i(+)}(1)b^{i(+)}(2)b^{i(+)}(3) + interchange\ of\ (1,2,3)]\epsilon^{ijk} \quad (5.16)$$

The baryon functions are as follows:

$$\psi_{+++}(1,2,3)_{\alpha\beta\gamma} = \sum_{s_1s_2s_3}\mu^+_\alpha(1)\mu^+_\beta(2)\mu^+_\gamma(3)X_{+++}(1,2,3),$$
$$\psi_{--+}(1,2,3)_{\alpha\beta\gamma} = \sum_{s_1s_2s_3}\nu^+_\alpha(1)\nu^+_\beta(2)\nu^+_\gamma(3)X_{--+}(1,2,3),$$

etc.

Then, the eigenvalue equation for the Hamiltonian operator,

$$_\alpha \ll 0|\underline{B}H\underline{B}^+|0\gg_\alpha = M_B\ {}_\alpha\ll 0|\underline{B}\ \underline{B}^+|0\gg_\alpha, \quad (5.17)$$

is equivalent to the following system for the baryons wave functions $\psi_{+++},\psi_{--+},\psi_{-+-},\psi_{+--}$.

$$[\begin{pmatrix}+\\+\\+\\-\end{pmatrix}E(1)\begin{pmatrix}+\\+\\-\\+\end{pmatrix}E(2)\begin{pmatrix}+\\-\\+\\+\end{pmatrix}E(3)\begin{pmatrix}-\\+\\+\\+\end{pmatrix}M_B]\psi_{\begin{pmatrix}+++\\--+\\-+-\\+--\end{pmatrix}}(1,2,3) =$$
$$= \frac{2}{3}\Lambda_{\begin{pmatrix}+\\-\\-\\+\end{pmatrix}}(1)\Lambda_{\begin{pmatrix}+\\-\\+\\-\end{pmatrix}}(2)\Lambda_{\begin{pmatrix}+\\+\\-\\-\end{pmatrix}}(3)$$

$$\{\ \hat{I}_{1,2}[\psi_{\begin{pmatrix}+++\\--+\\-+-\\+--\end{pmatrix}}(\underline{1},\underline{2},3)+\psi_{\begin{pmatrix}--+\\+++\\+--\\-+-\end{pmatrix}}(\underline{1},\underline{2},3)] +$$
$$+\ \hat{I}_{2,3}[\psi_{\begin{pmatrix}+++\\--+\\-+-\\+--\end{pmatrix}}(1,\underline{2},\underline{3})+\psi_{\begin{pmatrix}+--\\-+-\\--+\\+++\end{pmatrix}}(1,\underline{2},\underline{3})] + \quad (5.18)$$
$$+\ \hat{I}_{1,3}[\psi_{\begin{pmatrix}+++\\--+\\-+-\\+--\end{pmatrix}}(\underline{1},2,\underline{3})+\psi_{\begin{pmatrix}-+-\\+--\\+++\\--+\end{pmatrix}}(\underline{1},2,\underline{3})]\}$$

where

$$I_{1,2}\psi(\underline{1},\underline{2},3) = \int\frac{dq}{(2\pi)^3}\underline{V}(q)\psi(p_1-q,p_2+q,p_3)$$
$$p_1 + p_2 + p_3 = 0. \quad (5.19)$$

Equation (5.18) is the analogue of the Salpeter equation (5.14) for a bound state consisting of three particles. In the same notations eq. (5.14) has the form

$$[\begin{pmatrix}+\\+\end{pmatrix}E_1(1)\begin{pmatrix}+\\+\end{pmatrix}E_2(\hat{2})\begin{pmatrix}-\\+\end{pmatrix}M_B]\bar{\psi}_{\pm\pm}(1,\hat{2}) =$$
$$= \frac{4}{3}\Lambda_\pm(1)\{\hat{I}_{1,\hat{2}}[\psi_{++}(\underline{1},\underline{\hat{2}})+\psi_{--}(\underline{1},\underline{\hat{2}})]\}\Lambda_\pm(\hat{2})$$

with the condition $p_1 = p_2 = p$ and with the taking into account the identities

$$\int dq\underline{V}(p-q)\psi(q) =$$
$$= \int dq\underline{V}(q)\psi(q+p) =$$
$$= \int dq\underline{V}(q)\psi(p_1+q,-p_2-q)|_{p_1=p_2=p}$$
$$\psi\ (p_1,-p_2) = \psi(1,\hat{2})$$

The nonrelativistic reduction from (3.26) to the Schrödinger equation (3.27),

$$E_a(p) \simeq \sqrt{m_a^2+p^2} \simeq m_a + \frac{1}{2}\frac{p^2}{m_a},$$
$$S_a(p) \simeq 1,\ \psi_{+++} \equiv \psi \gg \psi_{\begin{pmatrix}+--\\-+-\\--+\end{pmatrix}},$$

leads in our case to the well - known nonrelativistic equation for the wave function of three particle bound states [53]

$$[\frac{p_1^2}{2m_1}+\frac{p_2^2}{2m_2}+\frac{p_3^2}{2m_3}-(M_{\underline{B}}-m_1-m_2-m_3)]\psi(p_1,p_2,p_3) =$$
$$= \frac{2}{3}[\hat{I}_{1,2}\psi(\underline{p}_1,\underline{p}_2,p_3)+\hat{I}_{2,3}\psi(p_1,\underline{p}_2,\underline{p}_3)+\hat{I}_{1,3}\psi(\underline{p}_1,p_2,\underline{p}_3)]. \quad (5.20)$$

Here, the condition (5.19), which means the choice of the rest frame $\mathcal{P}_\mu = (M_{\underline{B}}, 0, 0, 0)$, has to be fulfilled.

Notice that the Jacobi coordinates, which allow to write the Hamiltonian in the term of two relative momenta, have sense only in the nonrelativistic limit.

For the description of baryons in an arbitrary relativistic reference frame, one needs to generalize the Markov - Yukawa condition (3.9) for the bilocal field to the N- local field $\Phi(x_1, x_2, ..., x_N)$. [2]

By the example of a bilocal system we have seen that the definition of the total and relative coordinate $X = (x_1 + x_2)/2$ and $z = x_1 - x_2$ respectively is universal for quarks with arbitrary mass, including also constituent masses depending on momenta. By analogy, we introduce for the N- local field the total and relative coordinates

$$X_\mu = \frac{1}{N}\sum_{i=1}^{N} x_{i\mu}\,,\quad z_\mu^{(i)} = x_{i\mu} - X_\mu \qquad (5.21)$$

which are connected by the identity

$$\sum_{i=1}^{N} z_\mu^{(i)} = 0.$$

Then, the generalization of the Markov - Yukawa condition takes the form

$$z_\mu^{(i)}\frac{\partial}{\partial X_\mu}\Phi(z_\mu^{(1)}, z_\mu^{(2)}, ..., z_\mu^{(N)}) = 0 \quad (i = 1, 2, ... N). \qquad (5.22)$$

Let $\mathcal{P}_\mu$ be the eigenvalue of the operator for the total 4-momentum, and η_μ be the unit vector in the direction $\mathcal{P}(\eta_\mu \sim \mathcal{P}_\mu)$. Owing to the condition (5.22) the N- local function $\underline{\Phi}(p_\mu^{\perp(1)}, p_\mu^{\perp(2)} ... , p_\mu^{\perp(N)}|\mathcal{P})$, being the Fourier transform of $\Phi(z_\mu^{(1)}, X_\mu)$ with respect to all coordinates, depends only on the transversal relative momenta

$$p_\mu^{(i)\perp} = p_\mu^{(i)} - \eta_\mu(p^{(i)}\cdot\eta);\quad \sum_{i=1}^{N} p_\mu^{(i)\perp} = 0. \qquad (5.23)$$

To describe the baryon in an arbitrary reference frame it is sufficient to substitute in (5.18) all relative momenta $\mathbf{p}_i$ by the transversal ones, $p_\mu^{\perp(i)}$, and the projection operators $\Lambda_\pm(\mathbf{p})$ by the operators

$$\Lambda_\pm(\mathbf{p}^\perp) = S(\mathbf{p}^\perp)\frac{M_{\underline{B}} \pm \mathcal{P}}{2M_{\underline{B}}}S(\mathbf{p}^\perp)^{-1}.$$

In the same way one can generalize the equation (5.18) and its relativization for an arbitrary N- quark state.

[2] See also the generalization of the Markov - Yukawa condition for three - local and N - local [54] cases.

The method for constructing relativistic wave functions of many - quark system explained above unambiguously enables one to build from the nonrelativistic bound state wave function

$$\chi_{\alpha_1,\alpha_2,...\alpha_N}\cdot e^{iMX_0}\cdot\Phi_{\alpha_1,\alpha_2,...\alpha_N}(\mathbf{p}^1, \mathbf{p}^2, ..., \mathbf{p}^N),\quad \sum_i \mathbf{p}(i) = 0$$

relativistic wave functions for the same bound states with total momentum $\mathcal{P}_\mu = (\omega = \sqrt{\mathbf{P}^2 + M^2}, \mathbf{P})$,

$$\begin{aligned}\chi_{\alpha_1,\alpha_2,...\alpha_N}\quad\cdot\ & e^{i\mathcal{P}X}\cdot\Lambda_{+\alpha_1\alpha_1'}(p^{(1)\perp})\Lambda_{+\alpha_2\alpha_2'}(p^{(2)\perp})\ ...\ \Lambda_{+\alpha_N\alpha_N'}(p^{(N)\perp})\cdot\\ \cdot\ & \Phi_{\alpha_1',\alpha_2',...\alpha_N'}(p^{(1)\perp}, p^{(2)\perp}\ ...\ p^{(N)\perp}),\\ & (\ \sum_i p_\mu^{(i)\perp} = 0).\end{aligned}$$

Here $\chi_{\alpha_1,\alpha_2,...\alpha_N}$ is the matrix selecting one or another representation of the Lorentz group with a definite spin. (A representation of the Poincare group that preserves the one - time dependence of wave functions see in ref. [54]).

5.4 Relativistic equations for gluonic systems.

The quantization method considered above for fermion systems can also be used to calculate the parameters of gluon states described by the QCD Hamiltonian [18, 38] in the Schwinger operator approach

$$\begin{aligned}\mathcal{H}_{YM} &= \int dx\frac{1}{2}[(E_i^{Ta})^2 + (B_i^a)^2] +\\ &+ \frac{1}{2}\int dx\int dy f^{b_1c_1d_1}E_i^{Tc_1}(\mathbf{x})A_i^{Td_1}(\mathbf{x})V^{b_1b_2}(A|\mathbf{x}-\mathbf{y})f^{b_2c_2d_2}E_j^{Tc_2}(\mathbf{y})A_j^{Td_2}(\mathbf{y}) +\\ &+ \textit{Schwinger terms}.\end{aligned} \qquad (5.24)$$

Here $V(A|\mathbf{x})$ denotes the potential satisfying the equation

$$(\nabla_i(A)\partial_i)\frac{1}{\partial^2}(\nabla_j(A)\partial_j)V(A|\mathbf{x}) = -g^2\delta(\mathbf{x}),\ \nabla_i = \partial_i + gA_i^a\frac{\lambda^a}{2},$$

and the Schwinger terms are defined from the Lorentz covariance condition. It is important to note that the field operators in the Hamiltonian (5.24) are Weyl - ordered [18, 38] due to the condition of relativistic invariance.

Let us represent the gluon fields as Bogolubov expansion in creation and annihilation operators

$$\begin{aligned}E_i^{Tb}(\mathbf{x}) &= i\int\frac{d\mathbf{p}}{(2\pi)^{3/2}}\sqrt{\frac{\phi(\mathbf{p})}{2}}[a_\alpha^{(+)b}(\mathbf{p})e_i^\alpha - a_\alpha^{(-)b}(\mathbf{p})e_i^\alpha]e^{i\mathbf{p}\mathbf{x}},\\ A_i^{Tb}(\mathbf{x}) &= \int\frac{d\mathbf{p}}{(2\pi)^{3/2}}\frac{1}{\sqrt{2\phi(\mathbf{p})}}[a_\alpha^{(+)b}(\mathbf{p})e_i^\alpha + a_\alpha^{(-)b}(\mathbf{p})e_i^\alpha]e^{-i\mathbf{p}\mathbf{x}},\end{aligned} \qquad (5.25)$$

where the function $\phi(p)$ is to be calculated from the diagonalization condition of the Hamiltonian (5.24) with respect to the operators $a^{(+)}, a^{(-)}$.

In the formal form, the gluon Hamiltonian reads as

$$H = E_0 + \int dk[\ (\ a^{(+)}_{\alpha_1}(-k)a^{(+)}_{\alpha_2}(k) + a^{(-)}_{\alpha_1}(k)a^{(-)}_{\alpha_2}(-k))\frac{C^{\alpha_1\alpha_2}(\phi)}{2} + $$
$$+ a^{(+)}_{\alpha_1}a^{(-)}_{\alpha_2}\omega^{\alpha_1\alpha_2}(\phi) + O(a^4)]. \tag{5.26}$$

Here $C(\phi)$ and $\omega(\phi)$ are some defined functions the dependence on ϕ of which is determined from the expression of the Hamiltonian (5.24).

The diagonalization condition for the Hamiltonian (5.24) means that the coefficients $C(\phi)$ vanish

$$C(\phi) = 0 \tag{5.27}$$

For the solutions (5.27) the function $\omega(\phi)$ defines the gluon energy spectrum in the same way as the Schwinger-Dyson equation (5.7) defines the energy spectrum for the quarks.The Green function for the transversal gluon A^T_i corresponding to equations (5.25) and (5.26) is given by

$$\mathcal{D}_{ij}(q_0, q) = \frac{\omega(q)}{\phi(q)}\frac{1}{q_0^2 - \omega^2(q) - i\epsilon}(\delta_{ij} - q_i\frac{1}{q^2}q_j). \tag{5.28}$$

From its meaning the quantity $\frac{\omega}{\phi} = Z(q)$ can be called the infrared renormalization constant of the wave function.

The Green functions for the quarks, eq. (3.19) and gluons, (5.28), are elements of a new quasiparticle perturbation theory in terms of which all matrix elements are calculated including the "running" coupling constant.

The phenomenon of dimensional transmutation appearing in the "running" coupling constant should be investigated in accordance with the logic of quantum theory at the stage of defining the quark and gluon energy spectrum and their one-particle Green functions.

Let us illustrate this scheme by calculating the one-particle energy of the gluon and its bound states for the simplest example of the theory (5.24) where the operator for the potential is substituted by an effective potential. This means we consider the sum of the free Hamiltonian

$$H_0 = \frac{1}{2}\int dx[(E^T_i)^2 + (\partial_i A^T_j)^2],$$

and the Hamiltonian of potential interaction between colour gluon currents

$$H_I = -\frac{1}{8}\int dp_1 dq_1 dp_2 dq_2 \delta(p_1 - p_2 + q_1 - q_2)\frac{\underline{V}(p_1 - p_2)}{(2\pi)^3}\sqrt{\frac{\phi(p_1)\phi(q_1)}{\phi(p_2)\phi(q_2)}}\cdot$$
$$\cdot\ f^{ab_1b_2}f^{ac_1c_2}[a^{(+)}(-1) - a^{(-)}(-1)][a^{(+)}(2) + a^{(-)}(-2)]\cdot$$
$$\cdot\ [a^{(+)}(-1') - a^{(-)}(1')][a^{(+)}(2') + a^{(-)}(-2')].$$

Here, the short - hand notation $(\pm p_1, b_1, i_1) = (\pm 1), (\pm q_1, c_1, j_1) = (\pm 2)$, $(\pm p_2, b_2, i_2) = (\pm 1'), (\pm q_2, c_2, j_2) = (\pm 2')$ has been used. In our case, the coefficients C and ω are given by the expressions

$$C^{\alpha_1\alpha_2}(\phi) = [\frac{k^2\delta_{ij} + (\mathcal{M}^2(k))^{ij}}{2\phi} - \frac{\phi}{2}z^{ij}]e^{\alpha_1}_i(k)e^{\alpha_2}_j(-k) = 0,$$
$$\omega^{\alpha_1\alpha_2}(\phi) = [\frac{k^2\delta_{ij} + (\mathcal{M}^2(k))^{ij}}{2\phi} + \frac{\phi}{2}z^{ij}]e^{\alpha_1}_i(k)e^{\alpha_2}_j(k) \tag{5.29}$$

with

$$\mathcal{M}^2(k))^{ij} = \frac{N_c}{2}\int\frac{dq}{(2\pi)^3}\underline{V}(k-q)\phi(q)(\delta_{ij} - q_i\frac{1}{q^2}q_j),$$
$$z^{ij}(p) = \delta_{ij} + \frac{N_c}{2}\int\frac{dq}{(2\pi)^3}\underline{V}(k-q)\frac{1}{\phi(q)}(\delta_{ij} - q_i\frac{1}{q^2}q_j). \tag{5.30}$$

Since two gluons form the simplest bound state, by analogy with the mesons (cf. (5.11)), we introduce a glueball creation operator

$$G^+ = \sum_b\int dk\ [\ X^{(++)}_{\gamma_1\gamma_2}(k)e^{\gamma_1}_i(k)e^{\gamma_2}_j(-k)a^{(+)b}_{\gamma_1}(k)a^{(+)b}_{\gamma_2}(-k) +$$
$$+\ X^{(--)}_{\gamma_1\gamma_2}(k)e^{\gamma_1}_i(k)e^{\gamma_2}_j(-k)a^{(-)b}_{\gamma_1}(k)a^{(-)b}_{\gamma_2}(-k)]$$

and a "coherent" vacuum

$$|0>>_\alpha = \exp\{\sum_c\ \int\ dk_1dk_2\alpha(k_1,k_2)(a^{(+)c}_{\gamma_1}(k_1)a^{(+)c}_{\gamma_2}(k_2))\cdot$$
$$\cdot\ (a^{(+)b}_{\gamma_1}(k_1)a^{(+)b}_{\gamma_2}(k_2))\}|0>.$$

Then, the Schrödinger equation for eigenvalues of the Hamiltonian operator

$$_\alpha << 0|GHG^+|0>>_\alpha = M_G\ _\alpha << 0|GG^+|0>>_\alpha, \tag{5.31}$$

is equivalent to the equation for the glueball wave function

$$(2\omega(k) - M_G)X^{(++)}_{ij}(k)$$
$$= \frac{N_c}{4}\hat{I}_{kq}\{(W^+(q|k))^2X^{(++)}_{ij}(q) - (W^-(q|k))^2X^{(++)}_{ij}(q)\} \tag{5.32}$$

$$(2\omega(k) + M_G)X^{(--)}_{ij}(k)$$
$$= \frac{N_c}{4}\hat{I}_{kq}\{(W^+(q|k))^2X^{(--)}_{ij}(q) - (W^-(q|k))^2X^{(--)}_{ij}(q)\} \tag{5.33}$$

with

$$W^\pm(q|k) = [\sqrt{\frac{\phi(q)}{\phi(k)}} \pm \sqrt{\frac{\phi(k)}{\phi(q)}}],$$
$$\hat{I}_{kq}f(q) = \int\frac{dq}{(2\pi)^3}\underline{V}(k-q)f(q). \tag{5.34}$$

Furthermore, one can write by analogy with the baryon (5.18) also the equation for the wave function of a antisymmetric three - gluon state $\psi_{+++}(\psi_{+--}, \psi_{-++}, \psi_{-+-})$ with eigenvalues M_{BG}

$$\begin{aligned} [\omega(1) + \omega(2) + \omega(3) - M_{BG}]\psi_{+++}(1,2,3) = \\ = \frac{N_c}{2}\{\hat{I}_{1,\underline{2}}(\omega^{+}_{1\underline{1}}\omega^{+}_{2\underline{2}}\psi_{+++}(\underline{1},\underline{2},3) + \omega^{-}_{1\underline{1}}\omega^{-}_{2\underline{2}}\psi_{--+}(\underline{1},\underline{2},3)) + \dots\}. \end{aligned} \tag{5.35}$$

For an estimate of the solution to equation (62,63) we make use of the separable approximation for the potential

$$\int \frac{d\mathbf{q}}{(2\pi)^3}\underline{V}(\mathbf{k}-\mathbf{q})\phi(\mathbf{q})(\delta_{ij} - q_i\frac{1}{\mathbf{q}^2}q_j) \simeq \frac{2}{3}\frac{1}{\mu^2_{QCD}}\int_0^L \frac{d\mathbf{q}}{(2\pi)^3}\phi(\mathbf{q})\delta_{ij}, \tag{5.36}$$

with the parameters describing the physics of light quarkonia ($L = 1.6GeV, \mu_{QCD} = 0.35GeV$). Equation (5.30) take the form

$$\begin{aligned} \sqrt{Z}m_g^2 &= \frac{1}{\mu^2_{QCD}}\int_0^L \frac{d\mathbf{p}}{(2\pi)^3}\sqrt{\mathbf{p}^2 + m_g^2}, \\ Z &= 1 + \frac{\sqrt{Z}}{\mu^2_{QCD}}\int_0^L \frac{d\mathbf{p}}{(2\pi)^3}\frac{1}{\sqrt{\mathbf{p}^2 + m_g^2}}, \\ \omega(\mathbf{p}) &= \sqrt{Z}\sqrt{\mathbf{p}^2 - m_g^2} \end{aligned} \tag{5.37}$$

and have the solutions $m_g \simeq 0.8GeV, \sqrt{Z} \simeq 1.18$. For the scalar glueball mass as an eigenvalue of equation (62,63) in the separable approximation

$$\begin{aligned} [2\sqrt{Z}\sqrt{\mathbf{k}^2 + m_g^2} &- M_G]X = \\ &= \frac{N_c}{4\mu^2_{QCD}}\int^L \frac{d\mathbf{q}}{(2\pi)^3}[(X - Y)W(\mathbf{k}|\mathbf{q}) + 2(X + Y)], \\ [2\sqrt{Z}\sqrt{\mathbf{k}^2 + m_g^2} &+ M_G]X = \\ &= \frac{N_c}{4\mu^2_{QCD}}\int^L \frac{d\mathbf{q}}{(2\pi)^3}[(Y - X)W(\mathbf{k}|\mathbf{q}) + 2(X + Y)], \\ W(\mathbf{k}|\mathbf{q}) &= \sqrt{\frac{\mathbf{k}^2 + m_g^2}{\mathbf{q}^2 + m_g^2}} + \sqrt{\frac{\mathbf{q}^2 + m_g^2}{\mathbf{k}^2 + m_g^2}}, \end{aligned} \tag{5.38}$$

one obtains the value $M_G \simeq 1.6GeV$.

The appearance of constituent masses for quarks and gluons does of course influence the determination of the "running" coupling constant which in the new theory cannot have any singularities in the whole Euclidean region of the transferred momenta, among them also at $\mathbf{q}^2 = 0$ [24, 55].

5.5 The problem of confinement.

5.5.1 Phenomenology.

The confinement of quarks in phenomenology means their nonobservation, or the fact that the cross-section for the creation of quarks in a free state is equal to zero.

When we want to prove to the nonobservation of any physical object, we first of all need to describe constructively the experiments testifying to the existence of this object. In recent physics the proof of "the existence" of a physical object means the experimental measurement of all its quantum number.

It is worth recalling that the quark idea follows from the classification group of hadron. But even at this step of "classification" quarks have been considered rather as an auxiliary mathematical construction than as a reality. Quarks became "real object" only when the first experiments appeared on the deep-inelastic measurement of the parton quantum numbers which coincided with the ones of the Gell-Mann- Zweig hypothesis quarks.

The parton interpretation of the deep- inelastic experiments on the phenomenological level has been developed by Feynman and Bjorken [44] and its essence is that the sum over all hadron final states of probabilities of processes $e^+e^- \to$ hadron $e^+e^-p \to$ hadrons etc., is described as an imaginary part of the corresponding elastic amplitude:

$$\sum_{(hadrons)} {}_{phys}< f|T|h >_{phys} \; {}_{phys}< h|T^*|i >_{phys} = 2Im_{phys} < f|T|i >_{phys}, \tag{5.39}$$

constructed from quark- gluon diagrams of the QCD perturbation theory

$$2Im_{phys} < f|T|i >_{phys} \simeq \sum_{(partons)} {}_{phys}< f|T|p >_{phys} \; {}_{phys}< p|T^*|i >_{phys} . \tag{5.40}$$

The resulting relation

$$\begin{aligned} \sum\nolimits_{(hadrons)} {}_{phys}< f|T|h >_{phys} \; {}_{phys}< h|T^*|i >_{phys} \simeq \\ \simeq \sum_{(partons)} {}_{phys}< f|T|p >_{phys} \; {}_{phys}< p|T^*|i >_{phys} \end{aligned} \tag{5.41}$$

is called quark- hadron duality (QHD), and is used in phenomenology as the energy averaging (global QHD) and without averaging, in the energy region far from resonances (local QHD). For example, the cross- section of the process $e^+e^- \to$ hadrons in the nonresonance energy region coincides not only on the average but also at points with the imaginary part of quark loops. The local QHD means that the perturbation theory really is used in Minkowski space.

The ordinary unitarity for the S- matrix:

$$S = 1 + iT \; ; \; S \cdot S^+ = 1 \;\to\; T \cdot T^* = 2ImT \tag{5.42}$$

differs from (5.13) by the Feynman supposition that the hadrons form the complete set of physical states

$$T \cdot T^* = \sum_{(hadron)} T|h >_{phys} \; {}_{phys}< h|T^* \tag{5.43}$$

The last equation means that the probability of hadronization is equal to unity, and, correspondingly, the probability of creation of colour particles is equal to zero, i.e. equation (5.43) means the confinement of colour particles (c)

$$_{phys} < i|T|c >_{phys} = 0 \; ; \; |c>_{phys} \neq |p> \qquad (5.44)$$

Thus, for the measurement of quantum numbers of colour particles by QHD (5.46) one uses the confinement hypothesis (5.46) and two different types of states: physical hadron states (in the left- hand side of [Eq.(5.46)] which are detected directly by experiments and parton states (in the left hand side) which reflect particular analytical properties of the "elastic" *hadron amplitude* reproducing the imaginary parts of quark diagrams.

In usual field theory, for instance in QCD, physical and "parton" states coincide in the framework of perturbation theory, thus indicating that the theoretical "observation" of quarks as their parton images in local QHD [Eq.(5.46)] and their experimental nonobservation [Eq.(5.44)] take place in one and the same energy region of Minkowski space.

So, the very physical procedure of the measurement of the colour particle quantum number is contradictory, if it does not imply the existence of some mechanism of "dressing" partons as a result of which they transform into nonobservable physical quarks $|c>_{phys}$ not losing their quantum numbers.

One of the formulations of the confinement problem is as follows: What does the coincidence of physical and parton states not occur in QCD? Another formulation: Why is the probability of colour particle production to zero [Eq.(5.44)] while the probability of hadronization is equal to unity?

Up to now the answer to these questions is the confinement hypothesis. However, the parton QCD high- energy phenomenology [44] does not practically depend on the realization of this hypothesis. The same situation is now observed in QCD low- energy phenomenology. For example, chiral Lagrangians are obtained from one- loop quark diagrams [24] and at the same time quarks are removed from the unitary condition corresponding to an implicit use of the hypothesis [Eq.(5.44)] in the absence of any dynamics substantiating this hypothesis. The quark hadronization process in low- energy phenomenology is not connected with the confinement process. The same conclusion is made in the phenomenology of the sum rule method [56], where confinement effects do not in any way influence the description of experimental data.

Thus, in the recent quark high and low-energy phenomenology there is a tendency to decouple completely the hadronization process from confinement, and the term of "confinement" means the specific procedure of measurement of colour particle quantum numbers which uses two sorts of colour states: physical and parton ones in one and the same energy region [Eq.(5.44)].

5.5.2 Theory.

QCD arose as the substantiation of parton phenomenology with the help of the asymptotic freedom phenomenon [Eq.(5.1)] [45].

There is a set of peculiarities of the conventional parton QCD that prevent us from recognizing the phenomenological statement of confinement problem.

Their main peculiarities are the conviction that the formula [Eq.(5.1)] explains the parton phenomenology and ordinary proof of unitary relation in perturbative theory.

In agreement with asymptotic freedom, perturbation theory in QCD is valid only in Euclidean space. The correspondence of theoretical quantities with the relativistic experimentally measurable cross- sections in Minkowski space is established with the help of dispersion relations.

However, in this way one can explain only the global quark- hadron duality but cannot explain the local QHD, i.e. the fact that the cross- section for the process e^+e^- onto hadrons in the energy region far from resonances coincides with the imaginary part of a quark loop at every point. To explain the local QHD and perturbation theory in the Minkowski space, we must understand why in the same energy region quarks play roles simultaneously of nonobservable physical states [the left- hand side of Eq.(5.46)] and of observable free partons [the right- hand side of Eq.(5.46)].

This fact is also difficult to understand in the potential version of confinement, where argumentation is based on different regimes of the quark behaviour in different energy regions (long and short distances). On the other hand, all confinement- potential attempts to explain the nonobservability of individual quarks by solving the Dyson-Schwinger equation for the quark propagator have led not to confinement, but rather to the spontaneous breaking of chiral symmetry. At present, confinement potentials are succesfully used as potentials of hadronization [9, 24], [46]- [48].

We see that in QCD theory is a tendency to consider the dynamics which is traditionally connected with confinement only as the dynamics of hadronization.

The main problem of how the Minkowski space perturbation theory (with a nonzero imaginary part of the quark- gluon diagrams) can be made consistent with the confinement hypothesis [Eq.(5.46)] remains open; its answer is given neither by asymptotic freedom nor by the confinement potential.

One of the first attempts to explain the phenomenon of confinement with two sorts of quarks [Eq.(5.46)] was the version of the two- dimensional chromodynamics suggested by 't Hooft [57]. In this model, physical (dressed) quark and parton (bare) quarks are distinguished. As a consequence of the infrared divergences, all physical quarks have infinite masses whereas colourless amplitudes are expressed in terms of bare quark propagators with finite masses without infrared divergences.

The absence of the amplitude for colour particle production does not contradict the unitary relation. The point is that when bound states are present, the unitarity relation should not be understood as an identity, but rather as one of the self- consistency conditions of the theory used for normalizing the bound states and their interaction constants [58]. (It is worth recalling that in quantum mechanics one and the same Hamiltonian allows different laws of conservation of probability which are considered as additional suppositions). If for some reason the probability of the colour channels disappears, the probability of other channels increases so that the total probability is equal to unity. From this point of view, the perturbation theory for bare quarks in the Minkowski space does not contradict the confinement of physical quarks, which is explained not by the interaction potential but by the process of dressing bare quarks.

The similar procedure of dressing bare quarks explaining the phenomenological confinement has been found in the minimal version of quantization of conventional chromodynamics [17, 18].

6 LECTURE

THE UNIVERSE AS A "BOUND STATE" IN THE EINSTEIN THEORY.

The spectrum of physical excitation of gauge theories, the "Heaven" and "Sky" spaces in general relativity theory, excitation of the "Sky" space, the inflation scenario, the solution of the hidden mass problem and the Hubble constant.

6.1 Statement of problem.

In Lectures 2,4 we convinced that the minimal approach to quantization of gauge theories is convenient for definition of the spectrum of physical excitations of the theory.

In *QED* the minimal approach means that we consider the equation for the time component A_0

$$\partial_i^2 A_0 = j_0 + \partial_i \partial_0 A_i \ ,$$

as a constraint which we should solve explicitly to construct the independent physical variables. The result of this construction, roughly speaking, consists in the separation of this equation in two parts: the homogeneous ($\partial_i\partial_0 A_i = 0$) and inhomogeneous ($\partial_i^2 A_0 = j_0$). The first part is the transversality constraint for a "radiation", which is treated as a "correction" by perturbation theory, the second- leads to the instantaneous (Coulomb) interaction and, in the end, to bound states.

It is wonderful that this method of the estimation of the physical spectrum up to now has not been applied to the general relativity theory (*GRT*)

$$\mathcal{W} = \int d^4x \sqrt{-g}\, \mathcal{L} \ ; \ \mathcal{L} = -\frac{1}{2\kappa} R + \mathcal{L}_M \ , \tag{6.1}$$

and, in particular, to solve the Einstein equations

$$R^\mu_\nu - \frac{1}{2}\delta^\mu_\nu R = \kappa T^\mu_\nu(M) \ ,$$

(where M means matter fields, R is the curvature in the metric of four- dimensional space- time $g_{\mu\nu}$). In the recent treatment of *GRT* the metric $g_{\mu\nu}$ is considered as the gauge field in the abstract mathematical space x_μ. We shall call this space- time the "Heaven" to emphasize its difference from the physical observable space- time $\xi^\mu = (\tau, \xi^i)$: $d^4\xi = d^4x\sqrt{-g}$, which we shall call the "Sky".

We shall construct the "Sky" space and the physical variables of *GRT* by the minimal approach. The main differences of this construction from others [59] is the choice of invariant dynamical variables.

6.2 The the choice of variables and coordinates.

We choose the *ADM* metric [59, 60]

$$\begin{pmatrix} \alpha^2 - \beta^2 & \beta_i \\ \beta_j & -\gamma_{ij} \end{pmatrix} \ ; \ \beta^i = \gamma^{ij}\beta_j \ ; \ \beta^2 = \beta_i \cdot \beta^i \ ; \ \sqrt{-g} = \alpha\sqrt{\gamma} \ . \tag{6.2}$$

In this case the Lagrangian (6.1) does not depend on the time derivative of the fields α and β_i

$$\mathcal{L} = \mathcal{L}(\alpha, \beta_i, \gamma_{ij}, \partial_0\gamma_{ij}, \cdots)$$

and the corresponding momenta p_α, p_{β_i} are equal to zero. There are two ways of quantization of a theory like that:

i). to consider the equation $p_\alpha = 0, p_\beta = 0$ as constraints on the dynamical variables and to quantize all variables (α, β, γ) by the Dirac method, or

ii). to construct the minimal set of classical dynamical variables on the explicit solutions of the classical equation

$$\frac{\delta \mathcal{W}}{\delta \alpha} = 0 \ ; \frac{\delta \mathcal{W}}{\delta \beta_i} = 0 \ ,$$

and to quantize only these variables (The physical meaning of this "minimal approach" is discussed in detail in Lectures 2,4 and ref.s [17, 18]).

In the first method we are forced to fix simultaneously the fields α, β_i and their momenta, which contradicts the Heisenberg relation in quantum theory. Therefore, we use here the second way, and solve the classical equation for fields α and β_i explicitly.

In particular, we should solve exactly the invariant Einstein equation ($\partial\mathcal{W}/\delta\beta i = 0$) :

$$R_i^0 \equiv \frac{1}{2}[\frac{1}{\alpha}\pi_i^k]_{;k} - \frac{1}{2}[\frac{1}{\alpha}\pi_k^k]_{;i} = T_i^0(M)\kappa \tag{6.3}$$

where

$$\pi_{ki} = \partial_0\gamma_{ki} - \beta_{k;i} - \beta_{i;k} \ ,$$

$(;i)$ is a covariant derivative in the metric γ_{ij}. $T_i^0(M)$ are the matter energy- momentum tensor components.

We can devide the field β_k into two parts with respect to the transitiveness of a general covariant group transformation

$$\beta_i = \beta_i^{tr} + \beta_i^M \ ; \ (R_i^0 = R_i^0(\beta^{tr}) + R_i^0(\beta^M)) \ .$$

It is clear that the nontransitive part β^M is defined by the matter

$$[\frac{1}{\alpha}\beta^{Mk}{}_{;k}]_{;i} - \frac{1}{2}[\frac{1}{\alpha}(\beta^{Mk}{}_{;i} + \beta_i^{M\,;k})]_{;k} = T_i^0(M)\kappa \ , \tag{6.4}$$

and β_i^{tr} is defined by the fields $(\partial_0\gamma_{ki})$ according to the equation of the type of (6.3) without the matter tensor

$$R_i^0(\beta^{tr}) = 0 .$$

The explicit solution of this equation means that instead of six fields γ_{ij} we get only three dynamical independent variables γ_{ij}^T satisfying the identity:

$$[\frac{1}{\alpha}(\gamma^T)^{ki}(\partial_0\gamma^T)_{il}]_{;k} - [\frac{1}{\alpha}(\gamma^T)^{ki}(\partial_0\gamma^T)_{ik}]_{;l} \equiv 0 \quad (6.5)$$

(Like in QED the explicit solution of the same equation

$$\partial_i\partial_0 A_i - \partial_i^2 A_0 = 0 \ ; \ (A_0 = \frac{1}{\partial^2}\partial_i\partial_0 A_i)$$

leads to the gauge-invariant transversal variables

$$A_i^T = (\delta_{ij} - \partial_i\frac{1}{\partial^2}\partial_j)A_j$$

satisfying the identity $\partial_i A^T \equiv 0$).

For purposes of the definition of an observable "Sky" space and the proof of the Newton law we separate also the variable $\sqrt{\gamma}$ by the definition of a new metric h_{kl}

$$\gamma_{kl}^T = a^2(x^\mu)h_{kl}(x^\mu) \ ; \ \sqrt{\gamma} = a^3(x^\mu) , \quad (6.6)$$

with the condition

$$det\ (h) = 1 \ ; \ (h^{ik}\delta h_{ik} = 0) . \quad (6.7)$$

Just in this case the constraint (6.5) has a very simple form

$$p^k_{l;k} = 4\ \partial_l(\frac{\dot{a}}{a}) , \quad (6.8)$$

where

$$p_l^k = h^{kl}\,p_{il} \ ; \ p_{kl} = \frac{1}{\alpha}\partial_0 h_{kl} = \frac{\partial}{\partial\tau}h_{kl} \equiv \dot{h}_{kl} \quad (6.9)$$

and $d\tau = \alpha dx^0$ is the "Sky"-time differential.

The Einstein equations

$$R_0^0 - \frac{1}{2}R = \kappa T_0^0(M) \ ; \ R_0^0 = \frac{\kappa}{2}(T_0^0(M) - T_k^k(M))$$

in terms of these variables have the forms

$$\frac{1}{2}[\mathcal{K} + {}^3\mathcal{R}] = \kappa T_0^0(M) \quad (6.10)$$

$$\mathcal{K} + \Sigma = \frac{\kappa}{2}(T_0^0(M) - T_k^k(M)) \quad (6.11)$$

where ${}^3\mathcal{R}$ is the curvature for the metric $\gamma_{ij} = a^2 h_{ij}$

$${}^3\mathcal{R} = \mathcal{R}_a - \mathcal{R}_h . \quad (6.12)$$

(The explicit expression of ${}^3\mathcal{R}(\gamma)$ is given in ref. [61], see also Appendix A, where we give also the complete expressions for $\mathcal{K}, \Sigma$).
$\mathcal{K}$ is the kinetic part

$$\mathcal{K} = 6(\frac{\dot{a}}{a})^2 - \frac{1}{4}(p_l^k p_k^l) . \quad (6.13)$$

Σ becomes the total derivative under the sign of the integral ($\int d^4x\sqrt{-g}$)

$$\Sigma = \frac{1}{\alpha}\alpha^{;k}_{;k} - \frac{1}{a^3}\frac{\partial^2}{\partial\tau^2}(a^3) . \quad (6.14)$$

We introduce also the coordinates of the "Sky" space

$$\alpha dx^0 = d\tau \ ; \ a dx^i = d\xi^i \ ; \ \xi^\mu = (\tau, \xi^i) \ ; \ \int d^4x\sqrt{-g} = \int d^4\xi , \quad (6.15)$$

where we can make the synchronization of our watches [62] and define the procedure of measuring the intervals of physical time and space

$$(\tau_1 - \tau_2)^2 \ ; \ (\bar{\xi}_1 - \bar{\xi}_2)^2 .$$

So, the explicit solution of equation for β_i (6.3) leads to the natural separation of variables

$$\mathcal{W}(\alpha, \beta, \gamma, M)|_{\beta=\beta^{tr}(\gamma)+\beta^M} = \mathcal{W}(\alpha, a, h_{ij}, M) , \quad (6.16)$$

and to the definition of the physical "Sky"- space- time (6.15), in the terms of which the action (6.1) depends on only the variables

$$\log\ (a) = \sqrt{\kappa}\mu \ , \ h_{ij} \quad (6.17)$$

(or more correctly, on its derivatives $\dot{\mu}, \partial_{\xi^i}\mu$).

The equation for α (6.10) $\delta\mathcal{W}/\delta\alpha = 0$ in terms of the "Sky"- space turns out into the constraint for the dynamical fields (μ, h_{ij}, M). Equation (6.11) is the difference of equations for α and a $(\delta\mathcal{W}/\delta\alpha - \delta\mathcal{W}/\delta a) = 0$. The Newton law $\Delta\phi = (\kappa/2)M\delta(\mathrm{x})$ is the joint solution of equations for α and a in the approximation $a = 1 - \phi$, $\alpha = 1 + \phi$, $h_{ij} = \delta_{ij}$, $T_0^0(M) = M\delta(\mathrm{x})$.

6.3 Vacuum cosmological solutions.

The fundamental questions of modern cosmology are:

i). Why is the Universe expanding?

ii). How could we compensate in the Einstein equation the energy density of the Hubble expansion (the Hidden mass problem)?

Conventional answers to these questions are based on the hypotheses of the homogeneous space (Fridman) and the existence of unobservable and unknown form of matter the density of which is greater more than an order of magnitude than the density of the observable forms of matter (baryons, leptons, and gravitons i.e. all particle excitations with positive energy).

On the other hand, according to the observable data we can neglect in eqs. (6.8), (6.11), (6.12) all physical excitations with positive energy ($T^{\nu}_{\mu} = 0; h_{ik} = \delta_{ik}$) and try to solve exactly (without the hypothesis of homogeneity) the vacuum Einstein equations.

$$\begin{pmatrix} 0 \\ 0 \end{pmatrix} : \quad \frac{1}{\kappa}[3(\frac{\dot{a}}{a})^2 - \frac{4}{3}\frac{1}{a^{3/2}}\underline{\Delta}(a^{3/2})] = 0 , \qquad (6.18)$$

$$\begin{pmatrix} k \\ k \end{pmatrix} - \begin{pmatrix} 0 \\ 0 \end{pmatrix} : \quad -3\frac{\ddot{a}}{a} + \frac{1}{\alpha}[\underline{\Delta}\alpha + 2\underline{\partial}_i\alpha\underline{\partial}_i(\log(a))] = 0 , \qquad (6.19)$$

$$\begin{pmatrix} 0 \\ l \end{pmatrix} : \quad \underline{\partial}_l(\frac{\dot{a}}{a}) = 0 \ ;(\underline{\partial}_l = \frac{\partial}{\partial \xi_l} \ ; \ \underline{\Delta} = \underline{\partial}_l\underline{\partial}^l) \qquad (6.20)$$

This approximation in GRT corresponds to the description of bound states in QED in lowest order in radiation. To get eq.s.(6.18)-(6.20) we also neglected by gravity radiation, and consider the Universe in bound state approximation.

The general solution of the last eq.(6.20)

$$a(\tau, \bar{\xi}) = e^{\tau H_n} a^0_n(\bar{\xi}) \ ; \ (\frac{\dot{a}}{a} = H_n)$$

describes the "inflation" Hubble expansion of the "Sky"- space with the constant H_n. The density of the kinetic energy of this expansion in eq.(6.18) can be compensated not only by the density of the observable matter (which we neglected) but also by the weak nonhomogeneity of the "Sky" which can play the role of the "hidden matter".

This nonhomogeneity is described by the class of eigen- functions of the Laplace operator $\underline{\Delta}$

$$\underline{\Delta} f_n = H^2_n f_n(H_n\xi) \ , \ f_n(0) = 1 \ ,$$

characterized by the "quantum number", H_n and unit vector $\mathrm{n}, (\mathrm{n}^2 = 1)$ (or an orbital momentum l and its projectors m), for example

$$f_n(H_n\xi) = e^{H_n(\mathrm{n}\xi)} . \qquad (6.21)$$

We can represent the general solution of eq.(6.18) in the form

$$a(\tau, \xi) = e^{H_n\tau} a^0_n(\xi) \ ; \ a^0_n = [f_n(\frac{3}{2}H_n\xi)]^{2/3} . \qquad (6.22)$$

Eq.(6.19) reduces to an equation of the Schrödinger type

$$(\underline{\Delta} + V(\xi))\psi = \frac{9}{2}H^2_n\psi \ ; \ \psi = (a^0_n\alpha(\xi)) \ ; \ V = \frac{1}{2}(\underline{\partial}_i \log a^0_n)^2 . \qquad (6.23)$$

In particular, for (6.21) we get two solutions

$$\alpha = C_1 e^{H_n(\xi\mathrm{n})} + C_2 e^{-3H_n(\xi\mathrm{n})} \ , \ C_1 + C_2 = 1 . \qquad (6.24)$$

If H_R is equal to zero, we have only the "Minkowski" space solution $\alpha = a = 1$. (There is an opinion that this space is unstable [63].)

So, the vacuum excitations of "Sky"- metric can explain the "inflation scenario", without any matter fields and condensates and also the Hubble scale as the parameter of the boundary condition of the creation of the "Sky"- space.

We should like to note that the choice of the "Sky"- metric

$$(dx)^2 = \alpha^2(dx_0)^2 - a^2(dx)^2 = (d\tau)^2 - (d\xi)^2$$

does not mean that the field α and a are equal unite and disappear from the interaction. For example, the Fock Lagrangian [7] in the "Sky"- space has the form (in the approximation $h_{ij} = \delta_{ij}$)

$$W = \int d^4\xi\bar{\psi}[-i\gamma_0(\partial_\tau - \frac{1}{2}\partial_\tau \ln a^3) + i\gamma_k(\underline{\partial}_k - \frac{1}{2}\underline{\partial}_k \ln \alpha a^2)]\psi . \qquad (6.25)$$

We see that the zeroes of the solution $\alpha(\xi)$ (6.24), which correspond to the beginning of physical time, leads to the points of singularities where the creation of the matter is possible.

Next step is the definition of local deviations of the metric fields (a, α) induced by the matter and graviton energy- momentum tensors

$$\alpha(\xi) = \alpha_n(\xi) \cdot \alpha^{Local}[M, h] \ ; a(\xi) = a_n(\xi) \cdot a^{Local}[M, h] . \qquad (6.26)$$

If the observable density of matter now is much less than the critical one $\rho_{critical} = 3H^2/\kappa$ there is a possibility to calculate the local deviation of metrics by the perturbation theory with respect to the Newton constant κ

$$a^{Local} = 1 - \kappa\Phi_a + o(\kappa^2) \ , \alpha^{Local} = 1 + \kappa\Phi_\alpha + o(\kappa^2) \ ,$$

These local deviations lead to the Newton "potential" gravitational interactions of the matter fields and of gravitons (like the current- current potential interaction in the "minimal" QED [17]).

6.4 Conclusion.

There is a lot of ways of quantization of a gauge theory. In these Lectures I would like to demonstrate that the best way for finding the spectrum of elementary physical excitations and bound states of a gauge theory is the approach suggested by Heisenberg and Pauli in their papers [12] devoted to the first quantization of electrodynamics. The main idea of these papers [12] is to quantize only the physical degrees of freedom (two transversal fields) that do not correspond to any gauge (in terms of the Dirac approach) as the conventional relativistic properties of physical observables in the quantization are achieved only by change of gauge (more exactly, by change of the time-axis of quantization).

According to the Heisenberg-Pauli approach, the Coulomb potential C in QED or the J/ψ potential in QCD are relativistic covariant and depend on the time-axis (η_μ) parallel to the bound state total momentum operator (in other words the Coulomb field is always moving with the atom which is formed by this field).

In this case, it is very easy to prove that bilocal fields of atoms and hadrons are the irreducible representations of the Poincare group and to describe the very subtle effect of the chiral symmetry breaking accompanied by the bilocal Goldstone field (pion). We have shown that to construct on the classical level the physical transverse variables with the Heisenberg-Pauli relativistic transformational properties, it is enough to consider the gauge field action on explicit solution of the Gauss equation for the time component of the gauge field $A_0 = (\eta \cdot A)$:

$$\mathcal{W}_{QED}(A,\psi)|_{\frac{\delta\mathcal{W}}{\delta A_0}=0} = \mathcal{W}(\mathrm{C}, A^T[A], \psi^T[A,\psi]) \qquad (6.27)$$

This is the main formula of the minimal approach for finding the spectrum of elementary excitation of classical and quantum gauge theories and for construction of physical variables $A^T[A], \psi^T[A,\psi]$ as a gauge invariant functional from initial fields A_i, ψ. In this approach the gauge field becomes many-faced and is divided into the Coulomb field C and radiation $A_i^T[A]$.

Non- Abelian fields in the minimal approach contain also the Weyl phase factors of the topological generation of the physical variables, which can explain the phenomenon of confinement as the complete destructive interference of these phase factors.

In the Einstein General Relativistic Theory (GRT) we should construct not only invariant field variables, but also invariant physical coordinates ("Sky"- space ξ_μ), instead of the initial ones ("Heaven"- space x_μ):

$$\mathcal{W}(g_{\mu\nu}, \psi, ...|x_\mu)|_{\frac{\delta\mathcal{W}}{\delta g_{0\nu}}=0} = \mathcal{W}(\text{"Sky"}, \mathrm{N}, \gamma_{ij}^T, \psi^T, ...|\xi_\mu) \ . \qquad (6.28)$$

We have considered the model of physical coordinates $\sqrt{-g}d^4x = (\alpha dx_0)(a^3d^3x) = d\tau d^3\xi$ and have shown that in the spectrum of elementary excitation of GRT there is vacuum excitation of "Sky"- space which describes the very slow expansion of the Universe $a(\tau\xi) = e^{\tau H}a_0(\xi)$. The kinetic density of this expansion is completely compensated by the large- scale nonhomogeneity of the "Sky"- space $a_0(\xi)$, which plays the role of "hidden mass".

In the light of minimal approach the next step of definition of the modified Newton potential N and of the invariant field variables in eq.(6.28) is similar to the realization of the Bibble scenario.

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Appendix A.

We get the following components of the curvature $R = R_0^0 + R_k^k$ in the metric (6.2) with taking into account eq.s (6.3) and (6.5)

$$R_0^0 = \mathcal{K} + \Sigma \ ; \ R_k^k = -{}^3\mathcal{R} + \Sigma \ , \qquad (A.1)$$

where ${}^3\mathcal{R} = \mathcal{R}_a - \mathcal{R}_h$,

$$\mathcal{R}_a = -4[(\frac{\partial_k\partial_l a}{a^3}) - \frac{1}{2}(\frac{\partial_k a}{a^2})(\frac{\partial_l a}{a^2}) + (\frac{\partial_k a}{a^3})\partial_l]h^{kl} \ , \qquad (A.2)$$

$$\mathcal{R}_h = \frac{1}{a^2}[\partial_k\partial_l h^{kl} + \frac{1}{4}(\partial_i h_{mn})(\partial_j h_{ab})h^{ma}(h^{ij}h^{nb} - 2h^{bi}h^{nj})] \ , \qquad (A.3)$$

$$\Sigma = \frac{1}{\alpha}\frac{1}{a^3}\partial_k[ah^{kl}\partial_l\alpha] - \frac{1}{a^3}\partial_\tau(\partial_\tau - \underline{\nabla}_k J^k)a^3 + \frac{1}{a^3}\nabla_k[(3\frac{\dot{a}}{a} - \underline{\nabla}_l J^l)J^k], \qquad (A.4)$$

$$\mathcal{K} = 6(\frac{\dot{a}}{a})^2 - \frac{1}{4}(p_l^k p_k{}^l) - 4(\frac{\dot{a}}{a})(\underline{\nabla}_k J^k) + (\underline{\nabla}_k J^k)^2 - $$
$$- \frac{1}{2}[(\underline{\nabla}^k J_l)(\underline{\nabla}_k J^l) + (\underline{\nabla}^k J_l)(\underline{\nabla}^l J_k)] \ , \qquad (A.5)$$

$$J_l = (\Pi^{-1})_l^k T_k^0 \ ; \ \Pi_k^l = 2\nabla^l\underline{\nabla}_k - \nabla_k\underline{\nabla}^l - \nabla_j\underline{\nabla}^j\delta_k^l \ , \qquad (A.6)$$

$$\nabla_k J^l = \partial_k J^l + \Gamma_{ki}^l J^i \ , \ \underline{\nabla}_k J^l = \frac{1}{\alpha}\nabla_k[\alpha J^l] \ , \qquad (A.7)$$

$$\Gamma_{ki}^l = \frac{1}{2}\gamma^{lj}(\partial_k\gamma_{ij} + \partial_i\gamma_{jk} - \partial_j\gamma_{ki}) =$$
$$= \frac{1}{2}h^{lj}(\partial_k h_{ij} + \partial_i h_{jk} - \partial_j h_{ki}) + \qquad (A.8)$$
$$+ (\delta_k^l\partial_i + \delta_i^l\partial_k - h^{ln}h_{ki}\partial_n)\log a \ .$$

It is easy to be convinced that these expressions in the "Sky" space depend only on $\mu(\xi)$ (6.17).

For example, the Lagrangian for a scalar field has the form

$$\frac{1}{2}(g^{\mu\nu}\partial_\mu\partial_\nu + \frac{R}{6}\phi^2) = \frac{1}{2}[\dot{\phi}^2 - h^{ij}\partial_{\xi^i}\phi\partial_{\xi^j}\phi + \frac{\phi^2}{6}(\mathcal{K} + 2\Sigma - {}^3\mathcal{R})] \ . \qquad (A.9)$$

Expressions (A.2), (A.4) and (A.5) in the vacuum $h_{ij} = \delta_{ij}; T_\nu^\mu = 0$ can be represented in terms of the "Sky" coordinates

$$\mathcal{R}_a = -4[\frac{\partial_\xi^2 a}{a} + \frac{1}{2}(\frac{\partial_{\xi^i} a}{a})^2] \equiv -\frac{8}{3}\frac{1}{a^{3/2}}\partial_{\xi^i}^2[a^{3/2}] \ , \qquad (A.10)$$

$$\mathcal{K} + \Sigma = -3\frac{\ddot{a}}{a} + \frac{1}{\alpha}[(\partial_\xi^2\alpha) + 2(\partial_{\xi^i}\alpha)(\partial_{\xi^i}\log a)] \ . \qquad (A.11)$$

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