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CONSERVATION LAWS IN THE THEORY OF GRAVITATION WITH THE BACKGROUND CONNECTION

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I. Introduction

Attempts of solving the problem of localization of the energy-momentum characteristics of the gravitational field take their start from the early works of founders of General Relativity. However, until now there is no common opinion of all specialists about this question. For solving any physical problem connected with the gravitation the Einstein equations are as a rule used. Most difficulties of the classical General Relativity result from the invariance of the Einstein equations with respect to the group of diffeomorphisms.

Mathematical constructions corresponding to the physical observables are often ill-defined as pseudotensors and integrals of the spatial components of tensor densities. Especially, this concerns such conceptions as the energy and momentum.

In the present work, the structure of energy-momentum characteristics of the gravitational field is investigated. It is found that those characteristics may be defined in principle but a more detail investigation shows that these constructions have no concern to the energy and momentum.

To prove these statements, we proceed as follows, We begin with a more careful consideration of the Noether theorems and then we see that in certain cases the Noether algorithms do not define the conserved quantities describing the local dynamics of fields. Further, we attempt to apply these algorithms to the Einstein equations. As it has been shown in papers /I-3/ for constructing the gravitational Lagrangian it is necessary to introduce the background connection into the theory (see also /4/). If the background connection does not permit the group of motions, then the conservation laws are absent. But it would be logical if the background connection has the maximal mobility /4/. The presence of the group of motions of the background connection leads to the existence of conserved Noether currents. But the structure of these currents is such that the conservation laws become improper and the group of action invariance can be extended to the infinite-parameter pseudogroup. In the last part of the paper the question about this extension will be investigated in detail.

2. Noehter's theorem and the structure of the conserved currents

Let us consider a system described by the fields φ^A where A is the collective index. Let equations for φ^A follow from the condition of action functional

$$S=\int L\, d^4x, \tag{I}$$

where L is the Lagrangian, being stationary.

The statement known as the first Noether theorem was formulated in the first section of the famous Noether paper [5]: If the action is invariant under the τ -parameter Lie group G_τ, then τ linearly independent combinations of the variational derivatives turn into divergences, i.e.

$$\partial J^j_{(\lambda)}=\sum_A \Psi_A X^A_{(\lambda)}, \quad \lambda=1,\dots,\tau, \tag{2}$$

where $J^j_{(\lambda)}$ are expressions named the Noether currents, $\Psi_A=\dfrac{\delta S}{\delta \varphi^A}$ are variational derivatives, $X^A_{(\lambda)}$ are the representation generators corresponding to the transformations of φ^A under G_τ.

Let the action (I) be invariant under a continuous group which may be parameterized by p arbitrary functions of the coordinates. We shall denote this group as $G_{p\infty}$. If one singles out a subgroup G_τ from the group $G_{p\infty}$, then according to the first Noether theorem, τ local conservation laws will take place.

In Sect.6 of paper [5] it has been formulated and proved that if G_τ is a subgroup of the group $G_{p\infty}$, all currents $J^j_{(\lambda)}$ may be represented in the form :

$$J^j_{(\lambda)}=A^j_{(\lambda)}+B^j_{(\lambda)}, \tag{3}$$

where $A^j_{(\lambda)}=0$ if $\Psi_A=0$, and $B^j_{(\lambda)}$ satisfies the condition $\partial_j B^j_{(\lambda)}\equiv 0$. These currents were named improper currents by Noether. The improper currents can be represented by a derivative of an antisymmetrical potential [6].

Let there be any group G_Σ, which includes groups G_τ and $G_{p\infty}$ as subgroups and G_τ transformations of the dynamical variables must be derived from $G_{p\infty}$ ones by giving a concrete expression to the group parameters, i.e. the "representation" of G_τ in the φ^A "space" is the "subrepresentation" of $G_{p\infty}$.

A General structure of the Noether current is as follows :

$$J^j_{(\lambda)}=A^j_{(\lambda)}+B^j_{(\lambda)}+C^j_{(\lambda)} \tag{4}$$

where $A^j_{(\lambda)}$ and $B^j_{(\lambda)}$ were defined earlier and $C^j_{(\lambda)}$ is a proper component of the current. The field charge is a first integral of equations of motion corresponding to a one - parameter group of the action invariance. As it appears from (4), if $C^j_{(\lambda)}=0$, i.e.

if the current is improper, then the formally calculated charge becomes a trivial since it does not depend on equations of motion.

For the nontrivial charge to exist it is necessary for the term $C^j_{(\lambda)}$ be present. It is a term that does not permits the group of the dynamical invariance to be extended to the infinite - parameter one. It is more exact to say that the following statemant can be valid:

If $C^j_{(\lambda)}=0$, then the group G_τ can be included in the group G_Σ as a subgroup of the action invariance. If $C^j_{(\lambda)}\neq 0$, then the group G_τ cannot be included in the group G_Σ as a subgroup of the action invariance.

Further we shall see that we may take as an example the gravitational field. But for investigation of the Einstein eguations it is necessary to use the correct variational principle.

3. Variational principle and the theory of gravitation with the background connection

Let us consider vacuum Einstein's equations

$$G_{jk}=0, \tag{5}$$

where $G_{jk}=R_{jk}-\frac{1}{2}Rg_{jk}$ is the Einstein tensor, $R=R_{\alpha\beta}g^{\alpha\beta}$ is the curvature scalar, $R_{jk}=R^\rho_{\rho jk}$ is the Ricci tensor,

$$R^p_{jik}=\partial_j\Gamma^p_{ik}-\partial_i\Gamma^p_{jk}+\Gamma^p_{js}\Gamma^s_{ik}-\Gamma^p_{is}\Gamma^s_{jk}$$

is the Rieman tensor,

$$\Gamma^p_{ik}=\frac{1}{2}g^{p\alpha}(\partial_i g_{\alpha k}+\partial_k g_{\alpha i}-\partial_\alpha g_{ik})$$

is the Christoffel's symbol.

For using Noether algorithm it is necessary to find a suitable Lagrangian. But if we will do it, unsurmountable difficulties occur. For avoiding them, the background connection must be introduced into the theory [4]. It is assumed that the background affine connection coefficients $\check{\Gamma}^i_{km}$ are symmetric, $\check{\Gamma}^i_{km}=\check{\Gamma}^i_{mk}$. The difference

$$P^i_{km}=\check{\Gamma}^i_{km}-\Gamma^i_{km} \tag{6}$$

is a tensor. It is called the affine-deformation tensor. Being varied with respect to g_{ik}, the action

$$\tilde{S}=\int \tilde{L}\, d^4x \tag{7}$$

with the Lagrangian

$$\tilde{L}=\sqrt{-g}\, g^{mn}(P^a_{mb}P^b_{an}-P^a_{ba}P^b_{mn}) \tag{8}$$

leads to the equations which coincide with the Einstein ones, if the background connection satisfies the conditions

$$\check{R}_{(ik)}=0, \tag{9}$$

where $\check{R}_{ik}$ is the Ricci tensor of the background connection.

Let us consider a general Lagrangian

$$L=L(g_{mn};\partial_k g_{mn};\check{\Gamma}^k_{mn}). \tag{10}$$

It is supposed that the background connection is symmetric. We denote by $\check{\nabla}_k$ the covariant derivative with respect to $\check{\Gamma}^k_{mn}$. Let the following terms be defined as

$$t^k_a=\frac{\partial L}{\partial g_{mn,k}}\check{\nabla}_a g_{mn}-L\delta^k_a; \tag{11}$$

$$\sigma^{jk}_a=\frac{\partial L}{\partial g_{mn,j}}(g_{ma}\delta^k_n+g_{na}\delta^k_m); \tag{12}$$

$$\Psi^{mn}=2\frac{\delta S}{\delta g_{mn}}=2\left(\frac{\partial L}{\partial g_{mn}}-\partial_j\frac{\partial L}{\partial g_{mn,j}}\right); \tag{13}$$

$$\Theta^{mn}_k=\frac{\delta S}{\delta \check{\Gamma}^k_{mn}}=\frac{\partial L}{\partial\check{\Gamma}^k_{mn}}; \tag{14}$$

where

$$S=\int L\, d^4x \tag{15}$$

is the action functional; comma before index means the partial derivative. What do these terms mean see /4/ or /7/.

The action S is invariant under the Lie variations with an arbitrary vector field

$$\delta x^j=\varepsilon\xi^j; \tag{16}$$

$$\delta\check{\Gamma}^k_{mn}=-(\check{\nabla}_m\check{\nabla}_n(\varepsilon\xi^k)+\check{R}^k_{amn}\varepsilon\xi^a); \tag{17}$$

$$\delta g_{mn}=-(g_{ma}\check{\nabla}_n(\varepsilon\xi^a)+g_{na}\check{\nabla}_m(\varepsilon\xi^a)+\varepsilon\xi^a\check{\nabla}_a g_{mn}). \tag{18}$$

Here ε is an infinitesimal parameter, $\check{R}^k_{amn}$ is the curvature tensor of the background connection.

But more exactly, this action invariance is to be called the covariance because changing the functional type of the Lagrangian by the group of symmetry whose action on the nondynamical fields, i.e. on the background connection, is not trivial.

Let the background connection permits the γ -parameter group of motion, and let $\xi^j_{(\lambda)},\lambda=1,\dots,\gamma$, generate this group i.e. the equations

$$\check{\nabla}_m\check{\nabla}_n\xi^k_{(\lambda)}+\check{R}^k_{amn}\xi^a_{(\lambda)}=0 \tag{19}$$

are satisfied. Then infinitesimal transformations of the group invariance are

$$\delta x^j=\varepsilon^{(\lambda)}\xi^j_{(\lambda)}; \tag{20}$$

$$\delta g_{mn}=-(g_{ma}\check{\nabla}_n(\varepsilon^{(\lambda)}\xi^a_{(\lambda)})+g_{na}\check{\nabla}_m(\varepsilon^{(\lambda)}\xi^a_{(\lambda)})+\varepsilon^{(\lambda)}\xi^a_{(\lambda)}\check{\nabla}_a g_{mn}). \tag{21}$$

According to the first Noether theorem the following identities take place /4/:

$$\partial_j J^j_{(\lambda)}=X_{mn(\lambda)}\Psi^{mn}, \tag{22}$$

where

$$J^j_{(\lambda)}=\sigma^{jk}_a\check{\nabla}_k\xi^a_{(\lambda)}+t^j_a\xi^a_{(\lambda)}, \tag{23}$$

$$X_{mn(\lambda)}=-\frac{1}{2}\xi^a_{(\lambda)}\check{\nabla}_a g_{mn}-g_{ma}\check{\nabla}_n\xi^a_{(\lambda)}. \tag{24}$$

4. The groups of invariance and the structure of currents

Let the action be invariant not only under (20)-(21), but also under the group generated by the following infinitesimal transformations of the dynamical fields :

$$\delta_K g_{mn} = -\left(g_{ma}\check{\nabla}_n(\delta\nu^{(\lambda)}\xi^{a}_{(\lambda)}) + g_{na}\check{\nabla}_m(\delta\nu^{(\lambda)}\xi^{a}_{(\lambda)}) + \delta\nu^{(\lambda)}\xi^{a}_{(\lambda)}\check{\nabla}_a g_{mn}\right), \qquad (25)$$

where $\delta\nu^{(\lambda)}$ are arbitrary infinitesimal functions of the coordinates vanisching at the integration limits. Then

$$\delta_K S = \int \frac{\delta S}{\delta g_{mn}}\,\delta_K g_{mn}\, d^4x = 0. \qquad (26)$$

Now we substitute the terms of $\delta_K g_{mn}$ into this formula. Then, we obtain

$$\int \Psi^{mn}\left(\frac{1}{2}\delta\nu^{(\lambda)}\xi^{s}_{(\lambda)}\check{\nabla}_s g_{mn} - g_{ms}\check{\nabla}_n(\delta\nu^{(\lambda)}\xi^{s}_{(\lambda)})\right) d^4x = 0. \qquad (27)$$

To transform the integrand we shall use the identity that can be verified easily:

$$\Psi^{mn}\left(\left(\frac{1}{2}\delta\nu^{(\lambda)}\xi^{s}_{(\lambda)}\check{\nabla}_s g_{mn} - g_{ms}\check{\nabla}_n(\delta\nu^{(\lambda)}\xi^{s}_{(\lambda)})\right)\right) = \qquad (28)$$

$$= \delta\nu^{(\lambda)}\xi^{n}_{(\lambda)}\nabla_m\Psi^{m}_{n} - \check{\nabla}_m(\Psi^{m}_{n}\delta\nu^{(\lambda)}\xi^{n}_{(\lambda)}),$$

where

$$\Psi^{k}_{\alpha} = \Psi^{km}g_{m\alpha}, \qquad (29)$$

∇_m is a covariant derivative with respect to the Christoffel symbols. Since Ψ^{k}_{α} is a tensor density of weight one, the last term in the right-hand side of (28) is an ordinary divergence and it may be discarded because $\delta\nu^{(\lambda)} = 0$ at the integration limits. Then we are only left with

$$\int \delta\nu^{(\lambda)}\xi^{j}_{(\lambda)}\nabla_\alpha\Psi^{\alpha}_{j}\, d^4x = 0. \qquad (30)$$

Since $\delta\nu^{(\lambda)}$ is arbitrary, it follows from (30) that

$$\xi^{j}_{(\lambda)}\nabla_\alpha\Psi^{\alpha}_{j} = 0 \qquad (31)$$

If it is assumed that $\delta\nu^{(\lambda)} \equiv 1$ in (28), then the left-hand side of (28) coincides with the right-hand side of the (22). Consequently,

$$\partial_j J^{j}_{(\lambda)} = \xi^{j}_{(\lambda)}\nabla_\alpha\Psi^{\alpha}_{j} - \check{\nabla}_k(\Psi^{k}_{\alpha}\xi^{\alpha}_{(\lambda)}), \qquad (32)$$

and because (31) is true, we obtain

$$\partial_j J^{j}_{(\lambda)} = -\check{\nabla}_j(\Psi^{j}_{\alpha}\xi^{\alpha}_{(\lambda)}) \equiv \partial_j(-\Psi^{j}_{\alpha}\xi^{\alpha}_{(\lambda)}). \qquad (33)$$

Hence,

$$J^{j}_{(\lambda)} = -\Psi^{j}_{\alpha}\xi^{\alpha}_{(\lambda)} + B^{j}_{(\lambda)}, \qquad (34)$$

where

$$\partial_j B^{j}_{(\lambda)} \equiv 0.$$

Thus, from the action invariance with respect to (25) it follows that the Noether currents corresponding to (20)-(21) are improper.

Now we shall investigate the inversion of this statement. Let us consider the common Noether current (12). Let $A^{j}_{(\lambda)} = -\Psi^{j}_{\alpha}\xi^{\alpha}_{(\lambda)}$. Let us show that if $C^{j}_{(\lambda)} = 0$, then the group of the action invariance which is G_r can be extended to the group G_Σ defined at the end of sect 2.

By virtue of (28) the expression (22) can be transformed into (32). We have

$$\partial_j J^{j}_{(\lambda)} = \xi^{\alpha}_{(\lambda)}\nabla_k\Psi^{k}_{\alpha} - \partial_j(\Psi^{j}_{\alpha}\xi^{\alpha}_{(\lambda)}), \qquad (35)$$

or

$$\partial_j(B^{j}_{(\lambda)} - \Psi^{j}_{\alpha}\xi^{\alpha}_{(\lambda)} + C^{j}_{(\lambda)}) = \xi^{\alpha}_{(\lambda)}\nabla_k\Psi^{k}_{\alpha} - \partial_j(\Psi^{j}_{\alpha}\xi^{\alpha}_{(\lambda)}). \qquad (36)$$

Since $\partial_j B^{j}_{(\lambda)} = 0$, if $C^{j}_{(\lambda)} = 0$, then the expression (36) turns into

$$\xi^{\alpha}_{(\lambda)}\nabla_k\Psi^{k}_{\alpha} = 0. \qquad (37)$$

It means that there are r identities among the equations $\Psi^{mn} = 0$. These identities can be symbolically written down as

$$\int \Psi^{mn}(x')\Lambda_{mn(\lambda)}(x',x)\,d^4x'=0, \tag{38}$$

where $\Lambda_{mn(\lambda)}$ are generators

$$\Lambda_{mn(\lambda)}(x',x)=-\xi_{(\lambda)(m}(x)\nabla_{n)}\delta(x'-x). \tag{39}$$

Here $\xi_{(\lambda)m}=g_{ma}\xi^a_{(\lambda)}$; $\nabla_n\delta(x'-x)$ is a covariant derivative of the four-dimensional δ -function with respect to x'. Let us consider the infinitesimal transformations

$$\delta_\Lambda g_{mn}(x)=\int\Lambda_{mn(\lambda)}(x,x')\,\delta\nu^{(\lambda)}(x')\,d^4x', \tag{40}$$

where $\delta\nu^{(\lambda)}$ are arbitrary infinitesimal functions of coordinates vanishing at the boundary of the range of integration. Let us substitute (39) into (40) and perform the integration. Then we obtain

$$\delta_\Lambda g_{mn}=-\frac{1}{2}\left(g_{m\alpha}\nabla_n(\delta\nu^{(\lambda)}\xi^a_{(\lambda)})+g_{n\alpha}\nabla_m(\delta\nu^{(\lambda)}\xi^\alpha_{(\lambda)})\right). \tag{4I}$$

Now we shall find the action variation

$$\delta_\Lambda S=\int\frac{1}{2}\Psi^{mn}\delta_\Lambda g_{mn}\,d^4x. \tag{42}$$

If we substitute (40) into (42), then we get

$$\delta_\Lambda S=\int\delta\nu^{(\lambda)}(x')d^4x'\int\frac{1}{2}\Psi^{mn}(x)\Lambda_{mn(\lambda)}(x,x')\,d^4x=0.$$

It means that the action is invariant with respect to the group generated by (4I). But generators (39) are not independent, and not all of the parameters $\delta\nu^{(\lambda)}$ are essential. For generators to be independent, the system of equations

$$\int\Lambda_{mn(\lambda)}(x,x')\,\delta\nu^{(\lambda)}(x')\,d^4x'=0 \tag{43}$$

must have a single solution $\delta\nu^{(\lambda)}=0$ for arbitrary g_{mn}.

If we substitute the definition (39) into (43) and perform integration, we obtain

$$\nabla_{(n}(\xi_{(\lambda)m)}\delta\nu^{(\lambda)})=0. \tag{44}$$

It is clear, that the left-hand side of (44) up to a factor coincides with the right-hand side of (4I). Consequently, the condition that all parameters in (41) are essential coincides with the condition that the solution $\delta\nu^{(\lambda)}=0$ of the system (44) is single.

Let us consider an arbitrary point M within the range of integration. Let the orbit of the point M , i.e. the multitude of the points of the area which can be transferred to the point M by the transformations of the group G_τ , be denoted by the term Q_M.

Let among the vector fields $\xi_{(\lambda)}$ there be exactly m fields which are zero fields in M . It can be assumed without loss of generality that the zero fields are $\xi_{(\rho)}$, $\rho=1,\ldots,m$. It means that $\xi_{(\rho)}$ form the Lie algebra of the stability subgroup of the point M . In differential geometry the stability subgroup is more often called the group of isotropy of M . Let us denote this group by the symbol H_M.

So, the vector fields $\xi_{(\gamma)}$, $\gamma=m+1,\ldots,\tau$, are not equal to zero in M . It should be remarked that these fields do not generally form the Lie algebra. Let us prove that in any neighborhood of the point M they form a set of basis fields of the orbit Q_M .

Indeed, according to the Frobenious theorem/8/ the integral curves of the fields $\xi_{(\lambda)}$ compose a family of the submanifolds of the initial manifold, because they form the Lie algebra. Each of the points of the initial manifold belongs to one of these submanifolds which are orbits of these points. A linear envelope spanned over $\xi_{(\lambda)}$ at the point M is a tangent space for the Q_M. Let it be denoted by T_M . Since $\xi_{(\rho)}|_M=0$, then T_M coincides with the linear envelope of $\xi_{(\gamma)}|_M$. Then, Q_M is homogeneous under action of the G_τ by definition. Therefore, Q_M is isomorphic to G_τ/H_M which is a factor space of the group of motion to the group of isotropy. Hence, $\dim Q_M=\dim G_\tau-\dim H_M=\tau-m=p$. Consequently, the dimension of the linear envelope $\xi_{(\gamma)}|_M$ is equal to the number of the vectors ξ , therefore, these vectors in M form a basis set of T_M .

Vector fields $\xi_{(\gamma)}$ are assumed to be differentiable, therefore there is any neighborhood U_M of the point M in which these fields remain linearly independent, and because their integral curves completely belong to Q_M , then in U_M the vector fields $\xi_{(\gamma)}$ form a basis set of Q_M .

Let us consider a vector field $\eta^m=\xi^m_{(\gamma)}\delta\nu^{(\gamma)}$. In U_M an arbitrary vector field tangent to Q_M can be decomposed over the fields $\xi_{(\gamma)}$ with any variable coefficients. Consequently, in a neighborhood of M any a priori given tangent to Q_M vector field can be obtained from η^m by a suitable choice of $\delta\nu^{(\gamma)}$. It means, the generator of an arbitrary diffeomorphism of Q_M at the point M has the form $\xi_{(\gamma)}\delta\nu^{(\gamma)}$.

Now we return to (44). It has been shown that in the neighborhood of M an arbitrary, tangent to Q_M, vector field can be decomposed over the fields $\xi_{(\gamma)}$. The field $\xi_{(\lambda)}\delta\nu^{(\lambda)}$ for arbitrary $\delta\nu^{(\lambda)}$ is tangent to Q_M since all $\xi_{(\lambda)}$ are tangent to Q_M. Therefore, for any $\delta\nu^{(\lambda)}$, $\delta\overset{*}{\nu}{}^{(\gamma)}$ can be picked out such that in some neighborhood of M, $\xi_{(\lambda)}\delta\nu^{(\lambda)}=\xi_{(\gamma)}\delta\overset{*}{\nu}{}^{(\gamma)}$. Then (44) transforms to the form

$$\nabla_{(n}(\xi_{(\gamma)m)}\delta\overset{*}{\nu}{}^{(\gamma)})=0. \tag{45}$$

But (45) is the Killing equation for the covector field $\overset{*}{\eta}_m=\xi_{(\gamma)m}\delta\overset{*}{\nu}{}^{(\gamma)}$. As an arbitrary metric tensor has no Killing vectors, then the only solution of (45) is $\overset{*}{\eta}_m=0$, and since $\xi_{(\gamma)m}$ are linearly independent, we obtain $\delta\overset{*}{\nu}{}^{(\gamma)}=0$.

Summarizing we conclude that the group generated by the infinitesimal transformations (4I) have p essential parameters depending on coordinates, i.e. it is the group $G_{p\infty}$. It is the group of the metric transformations corresponding to arbitrary diffeomorphisms of the orbits. The diffeomorphism of the orbits is such a diffeomorphism of the whole manifold that integral curves of the generating vector fields do not leave the orbits. It must be emphasized that the elements of group $G_{p\infty}$ are not diffeomorphisms because they do not act on the coordinates. It is exactly the metric maps.

So, if $C^{j}_{(\lambda)}=0$, then the group G_r can be extended to the group G_Σ including G_r as a subgroup. It is easy to be convinced of that $G_\Sigma=G_r\otimes G_{p\infty}$, where the symbol $\otimes$ means semidirect product. Really, $G_{p\infty}$ is invariant in G_Σ, the Cartesian product $G_r\times G_{p\infty}$ is supplied by the multiplication with the system of the automorphisms of $G_{p\infty}$ depending on G_r, and the groups G_r and $G_{p\infty}$ as subgroups of G_Σ can be intersected only in the unit element because $G_{p\infty}$ does not act on the coordinates.

The Cartesian product G_r and $G_{p\infty}$ is a multitude of all ordered pairs $(g;g_\infty)$, where g is an element of G_r, and g_∞ is an element of $G_{p\infty}$. Let g_1 and g_2 be elements of G_r, and $g_{1\infty}$ and $g_{2\infty}$ be elements of $G_{p\infty}$. Action of the pair $(g;g_\infty)$ on $\{x^i;\ g_{mn}(x)\}$ consist on successive application of the operations g_∞ and g to $\{x^i;\ g_{mn}(x)\}$. Therefore to obtain the group structure the Cartesian product $G_r\times G_{p\infty}$ must be supplied by the multiplication via the scheme

$$(g_1;g_{1\infty})\times(g_2;g_{2\infty})=(g_1g_2;\ g_{1\infty}^{g_2}g_{2\infty}), \tag{46}$$

where $g_{1\infty}^{g_2}=g_2^{-1}g_{1\infty}g_2$ is internal automorphism of $G_{p\infty}$. It is just (46) that means the Cartesian product being semidirect product.

Einstein's equations (5) are obtained by varying the action with the Lagrangian (8) when the condition (9) on the background connection holds. The presence of the group of motions of the background connection leads to the conservation laws connected with the space-time symmetries. But as it was shown in /4/, the corresponding currents are improper, consequently these conservation laws are degenerated into the trivial laws. When this takes place, then action of the group of motions on the dynamical variables can be extended to the infinite-parameter group of transformations. If we want to have, among the conservation laws, those which may be interpreted as the conservation of the energy-momentum, then the group of motions must contain four abelian subgroups. It means that the action of this group is transitive. The orbit of an arbitrary point appears to be the whole manifold, and the infinite-parameter extension of the group of motions appears to act on the g_{mn} of arbitrary diffeomorphisms. It is a well-known dynamical invariance of the Einstein equations, which is usually understood as a consequence of its general covariance.

5. Concluding remarks

Despite the well-known dynamical invariance of the Einstein equations, the question of constructing the conservation laws remain open because for applying the Noether algorithms the suitable Lagrangian must be picked out. After clearing up that the background connection must be introduced into the Lagrangian, the hope has been arisen that the problem of localization of the energy-momentum characteristic of the gravitational field will be solved. As it is seen, these hopes have not justified oneself: The $G_{4\infty}$ invariance of the Einstein equations remains to be true for the action, which leads to the conserved quantities being improper.

The question about the physical meaning of the background connection remains to be open, but in recent time there have been undertaken attempts to give a physical intepretation for this object /9,IO/.

References

I. N.A. Chernikov, Communication JINR P2-87-683, Dubna, 1987, in Russian.
2. N.A. Chernikov, Preprint JINR P2-88-27, Dubna, 1988, in Russian.
3. N.A. Chernikov, Preprint JINR P2-88-778, Dubna, 1988, in Russian.
4. M.N. Tentyukov, Acta Phys. Pol. B20, No II, p. 9II (1989).
5. E. Noether, Nachr. Konigl. Ges. Wiss. Gottingen, Math.-Phys. KI, 1918, Heft 2, pp. 235-258.
6. B.M. Barbashov, V.V. Nesterenko, Fortsch. Phys., 3I, 10, p. 535. (1983).
7. M.N. Tentykov, Communication JINR P2-88-182, Dubna, 1988. in Russian.
8. B.F. Schutz, Geometrical Methods of Mathematical Physics, Cambridge University Press, 1982.
9. E. Kapuscik, in The Gravity Energy and the Gravity Waves, JINR P2-89-138, p. 24, Dubna, 1989, in Russian.
10. E. Kapuscik, in Proceedings of the Workshop on Building of Gravitational Wave Transmitter and Detector, JINR D4-89-221, p. 18, Dubna, 1989, in Russian.

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Тентюков М.Н. E2-91-319
Законы сохранения в теории гравитации с фоновой связностью

Корректность лагранжева подхода к уравнениям Эйнштейна требует введения в теорию фоновой аффинной связности. Существование группы движений фоновой связности приводит к наличию сохраняющихся токов Нетер. Но структура этих токов такова, что законы сохранения становятся несобственными, а группа инвариантности действия может быть расширена до бесконечно-параметрической псевдогруппы Ли. Подробно исследовано построение такого расширения.

Работа выполнена в Лаборатории теоретической физики ОИЯИ.

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Tentyukov M.N. E2-91-319
Conservation Laws in the Theory of Gravitation with the Background Connection

A correct Lagrangian approach to the Einstein equations requires introduction of the background affine connection. The presence of the group of motions of the background connection leads to the existence of conserved Noether's currents. But the structure of these currents is such that the conservation laws become improper and the group of the action invariance can be extended to the infinite-parameter Lie pseudogroup. The construction of this extension is investigated in detail.

The investigation has been performed at the Laboratory of Theoretical Physics, JINR.

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