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THE PHASE SPACE STRUCTURE IN MINISUPERSPACE COSMOLOGICAL MODELS

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Структура фазового пространства
в минисуперпространственных космологических моделях

Изучается структура фазового пространства (ФП) в минисуперпространственных космологических моделях с калибровочными полями. Для SO(n), $n > 3$, калибровочной группы показано, что физическое ФП отличается от обычной плоскости. Данное явление ведет к модификации представления континуального интеграла для волновой функции основного состояния Вселенной. Доказывается также, что благодаря нетривиальной структуре физического ФП калибровочных полей, меняется квантование размера "кротовой норы".

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The Phase Space Structure in Minisuperspace
Cosmological Models

A phase space (PC) structure in minisuperspace cosmological models with gauge fields is investigated. It is shown for the SO(n), $n > 3$, gauge group that the physical PS differs from an ordinary plane. Due to this phenomenon, the path integral representation gets modified for the ground state wave function of the Universe. It is also argued that the wormhole size quantization should change due to a non trivial physical PS structure of gauge fields.

The investigation has been performed at the Laboratory of Theoretical Physics, JINR.

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1. Recently much attention has been devoted to the study of the Einstein-Yang-Mills system in quantum cosmology. This system is rather difficult by itself. Following, however, Ref.[1] one may introduce a set of simplifying assumptions and consider closed cosmologies with an $\mathbf{R}\otimes\mathbf{S}^3$ topology. In this case gauge fields on homogeneous space are described by the $SO(4)$-invariant ansatz [2]–[4]. The reduced system (the minisuperspace model) contains only a finite number of degrees of freedom corresponding to gravitational and gauge fields. Nevertheless, it is believed that the model keeps some dynamical features of the original field theory. In particular, it has two local symmetry groups, the reparametrization and gauge ones.

It is known that models with a gauge symmetry may have a nontrivial phase space (PS) of physical degrees of freedom [5]–[7]. In the present letter we study the physical PS structure of the minisuperspace model. In particular, PS for one of physical variables in the case of the $SO(n)$ gauge group, $n > 3$, turns out to be a cone unfoldable into a half-plane. We also argue that this phenomenon gives rise to a modification of the path integral representation for the ground-state wave function of the Universe and, as a consequence, the quasiclassical approximation changes in the minisuperspace model. Then we consider wormholes. Wormholes can be treated as gravitational instanton type solutions, i.e. as solutions of the euclideanized classical field equations corresponding to tunnelling through classically inaccessible region [8], [2]–[4]. We observe that the wormhole quantization rule [2],[4] should be modified due to the physical PS reduction if a gauge group has a rank higher than one.

2. In the minisuperspace approach to the Einstein-Yang-Mills system, a metric has the $SO(4)$-invariant form. The most general form of such a metric, i.e. a metric which is spatially homogeneous and isotropic in a space $\mathbf{R}\otimes\mathbf{S}^3$ topology, is given by the Friedmann-Robertson-Walken ansatz

$$g_{\mu\nu}dx^\mu dx^\nu = \frac{2G}{3\pi}\left(-N^2(t)dt^2 + \varrho^2(t)\omega^i\omega^i\right) \tag{1}$$

where $N(t)$ and $\varrho(t)$ are arbitrary non-vanishing functions of time; G is the gravitational constant and ω^i are the left-invariant one-forms ($i = 1,2,3$) on the three-sphere satisfying the condition $d\omega^i = -\varepsilon_{ijk}\omega^j\wedge\omega^k$.

The ansatz for gauge fields in the metric (1) was suggested in Refs.[4], [9]. Gauge fields with the $SO(n)$ group, $n > 3$, are described by a scalar

$\chi = \chi(t) \in \mathbf{R}$, a vector $\mathbf{x} = \mathbf{x}(t) \in \mathbf{R}^l$, $l = n-3$, and a real antisymmteric $l \times l$ matrix $y = y(t)$, i.e. $y = y_a T^a$, $y_a \in \mathbf{R}$, T^a are generators of $SO(l)$, so that the effective minisuperspace action of the Einstein-Yang-Mills system reads

$$S = \frac{1}{2}\int_{t_1}^{t_2} dt \frac{N}{\varrho}\left[-\left(\frac{\varrho}{N}\partial_t \varrho\right)^2 + \varrho^2 - \lambda^2\varrho^4 + \right.$$
$$\left. + \left(\frac{\varrho}{N}\partial_t \chi\right)^2 + \left(\frac{\varrho}{N}D_t \mathbf{x}\right)^2 - 2V\right] \qquad (2)$$

where $D_t = \partial_t + y$ is the covariant derivative, $\lambda^2 = 2G\Lambda/9\pi$, Λ is the cosmology constant;

$$V = \frac{\alpha}{3\pi}\left[\left(\chi^2 + \mathbf{x}^2 - \frac{3\pi}{2\alpha}\right)^2 + 4\chi^2\mathbf{x}^2\right] \qquad (3)$$

is the potential of the Yang-Mills fields, $\alpha = g^2/4\pi$ is the fine structure constant.

The action (2) is invariant under two local groups, the reparametrizaton one

$$t \to t'(t), \qquad N(t) \to N(t')\frac{dt'}{dt} \qquad (4)$$

and the $SO(l)$ gauge group, under which only variables $\mathbf{x}$ and y transform as follows

$$\mathbf{x} \to \Omega\mathbf{x}, \qquad y \to \Omega y \Omega^T + \Omega\partial_t\Omega^T\ , \qquad (5)$$

where $\Omega = \exp\omega_a T^a \in SO(l)$, $\Omega\Omega^T = \Omega^T\Omega = 1$ and $\omega_a = \omega_a(t)$ are arbitrary functions of time; other variables remain unchanged. Due to these local symmetries, variables N and y turn out to be pure unphysical because their canonical momenta vanish,

$$p_N = \frac{\delta S}{\delta \dot{N}} = 0, \qquad \pi_a = \frac{\delta S}{\delta \dot{y}_a} = 0, \qquad (6)$$

here the dot means the time derivative. So, N and y play a role of the Lagrangian multipliers.

Determining canonical momenta $p_\varrho, \mathbf{p}$ and p_χ of the remaining variables $\varrho, \mathbf{x}$ and χ, respectively, we may find the canonical Hamiltonian in the standard way,

$$\begin{aligned} H &= \frac{N}{2\varrho}\left[-p_\varrho^2 - \varrho^2 + \lambda^2\varrho^4 + p_\chi^2 + \mathbf{p}^2 + 2V - y_a\sigma^a\right] \equiv \\ &\equiv \frac{N}{\varrho}\left[H_{UD} - y_a\sigma^a\right] \end{aligned} \qquad (7)$$

where $\sigma^a = \mathbf{p}T^a\mathbf{x}$. However, the system has the primary constraints (6) [10] therefore it is necessary to find all the secondary constraints calculating the Poisson brackets of H with the constraints (6) and putting the results equal to zero [10]. In so doing, we find that all the secondary constraints are equivalent to the following

$$H_{UD} = 0, \qquad (8)$$
$$\sigma^a = 0. \qquad (9)$$

As a consequence, the Hamiltonian vanishes, which is always the case for systems with a reparametrization symmetry. It is easily to be convinced that all the constraints are the first-class ones [10].

Equation (8) is the classical Wheeler-DeWitt equation for the minisuperspace model. The constraints (9) generate the $SO(l)$ gauge transformations of the canonically conjugated variables $\mathbf{x}$ and $\mathbf{p}$. However, not all of them are independent. The number of independent constraints is $l-1$ since any vector in $\mathbf{R}^l$ has a stationary subgroup $SO(l-1)$ (a vector $\mathbf{x} \in \mathbf{R}^l$ can be always directed along one of the coordinate axes by a gauge transformation). Therefore a "partilce" described by a vector $\mathbf{x}$ has only one physical degree of freedom. Really, it is a radial motion because the quantities σ^a coincide with components of the particle angular momentum in $\mathbf{R}^l$.

3. Consider now the physical PS structure of gauge field degrees of freedom. Obviously, the total PS of these variables consists of points $(\mathbf{x}, \mathbf{p}) \in \mathbf{R}^{2l}$ and $(\chi, p_\chi) \in \mathbf{R}^2$ (we ignore the pure unphysical degrees of freedom $(y_a, \pi_a = 0)$). The physical PS is a subspace of the total PS picked out by the constraints $\sigma_a = 0$ and an identification of all points connected by gauge transformations on the surface of the constraints (the latter points correspond to the same physical state of the system).

Variables χ and p_χ are gauge-invariant therefore their PS is a usual plane, $\mathbf{R}^2$. The general solution of Eq.(9) is $\mathbf{p} = \xi\mathbf{x}$ where a function of time ξ is determined by dynamics (by the potential V). This solution means that only radial excitations are admissible. Further, one may always direct a vector $\mathbf{x}$ along one of the coordinate axes with the help a gauge transformation (the unitary gauge), for example, we put $x_i = \delta_{i1}x$. As a consequence, $p_i = \delta_{i1}p, \ \ p = \xi x$. However, this is not the end. There remain residual gauge transformations forming the $\mathbf{Z}_2$ gauge group with the help of which one may change the sign of $x \ : \ x \to \pm x$ (the gauge rotations through the angle π).

The residual gauge group cannot decrease a number of physical degrees of freedom, but it reduces their PS. Indeed, the sign of p should change simultaneously with the one of x due to the equality $p = \xi x$. Hence the points (x,p) and $(-x,-p)$ on the phase plane $\mathbf{R}^2$ are gauge equivalent and should be identified. The phase plane turns into a cone unfoldable into a half-plane that is just the physical PS because the gauge arbitrariness is exhausted. The same result may be found by considering the motion in the gauge-invariant canonical variables $r = |\mathbf{x}|, \ \ p_r = (\mathbf{p},\mathbf{x})/r$ [7].

A modification of the physical PS leads to some consequences in the corresponding quantum theory. For example, the path integral approach differs from the usual one [6],[7],[11] and, as a consequence, a quasi-classical approximation changes [7],[12]. Quantum Green functions have unusual analytical properties [13]. Therefore we may expect the appearance of analogous features in our system. As we show below, that is really the case.

4. According to the Dirac quantization method [10] of systems with the first-class constraints, to get a quantum theory corresponding to the classical one, we change all the canonical variables q_α and p_α, where $q_\alpha, \ p_\alpha$ mean the sets $(\mathbf{x},\varrho,\chi), \ (\mathbf{p},p_\varrho,p_\chi)$, respectively [1], by operators with the canonical commutation relations

$$[\hat{q}_\alpha, \hat{p}_\beta] = i\hbar\delta_{\alpha\beta}; \tag{10}$$

the operators of constraints (8), (9) must annihilate physical states. The equation corresponding to Eq.(8) is the Wheeler-DeWitt equation. In the coordinate representation it reads [9]

$$\frac{1}{2}\left[\hbar^2\varrho^{-p}\left(\partial_\varrho \circ \varrho^p\partial_\varrho\right) - \varrho^2 + \lambda^2\varrho^4 - \right.$$

[1]We omit again the pure unphysical degrees of freedom y_a and N [14],[7].

$$\left. - \ \hbar^2\partial_\chi^2 - \hbar^2\partial_{\mathbf{x}}^2 + 2V\right]\psi_{ph}(q) = E\psi_{ph}(q) \tag{11}$$

where E is an arbitrary constant arising from the matter-energy renormalization [1]. The real number $p > 0$ is usually introduced in order to take into account a curvilinearity of the minisuperspace (it reflects also the operator ordering problem here) [1]. The quantum version of Eq.(9) reads

$$\hat{\sigma}^a\psi_{ph}(q) = 0 \tag{12}$$

where $\hat{\mathbf{p}} = -i\hbar\partial_{\mathbf{x}}$. The functions ψ_{ph} are normalizable by the following condition

$$\int_0^\infty d\varrho \ \varrho^p \int_{-\infty}^{\infty} d\chi \int_{\mathbf{R}^l} d\mathbf{x}|\psi_{ph}(q)|^2 = 1. \tag{13}$$

The operator $\hat{H}_{WD}$ in (11) is Hermitian with respect to this scalar product.

Equation (12) means that physical states are invariant under $SO(l)$-rotations of the vector $\mathbf{x}$, i.e., they are s-states $\psi_{ph}(\mathbf{x},\chi,\varrho) = \phi(r,\chi,\varrho) \equiv \phi(z)$ where $r = |\mathbf{x}|$. Moreover, these s-states should be even,

$$\phi(r,\chi,\varrho) = \phi(-r,\chi,\varrho) \tag{14}$$

because the potential V is an analytical function. One may also prove the same property for the variable ϱ

$$\phi(r,\chi,\varrho) = \phi(r,\chi,-\varrho). \tag{15}$$

Indeed, due to the parity of H_{UD} in ϱ we may divide all solutions of Eq.(11) into the even and odd ones. However, only the even ones have a regular behavior at $\varrho = 0$ (wave functions should be regular [15], [1]). When $\varrho \to 0$, the first term in $\hat{H}_{WD}$ is only essential, therefore, $\phi \sim \varrho^\nu J_\nu, \ \nu = (p-1)/2, \ J_\nu$ being the Bessel function. We must select from all solutions of (11) only those which possess the asymptotics $\varrho^\nu J_\nu, \ \varrho \to 0$; the odd wave functions, obviously, have not it.

To formulate our quantum theory only via physical variables, we introduce the spherical coordinates. Since ϕ are independent of the angular variables, we get instead of (11)

$$\frac{1}{2}\left[-\hat{p}_\varrho^2 - \varrho^2 + \lambda^2\varrho^4 - \hbar^2\partial_\chi^2 + \right.$$

$$+ \ \hat{p}_r^2 + 2V + 2V^{eff}\big] \, \phi_E(z) = E\phi_E(z); \qquad (16)$$

$$V^{eff} = -\frac{\hbar^2 p(p-2)}{8\varrho^2} + \frac{\hbar^2 (l-1)(l-3)}{8r^2} \qquad (17)$$

where $\hat{p}_a = -i\hbar\varrho^{-\nu}\partial_\varrho \circ \varrho^\nu$,$\nu = (p-1)/2$, $\hat{p}_r = -i\hbar r^{-\nu'}\partial_r \circ r^{\nu'}$, $\nu' = (l-2)/2$ are the Hermitian momentum operators. The corresponding scalar product reads

$$\int_0^\infty dr d\varrho \int_{-\infty}^\infty d\chi \mu(z)\phi_E^*(z)\phi_{E'}(z) = \delta_{EE'} \qquad (18)$$

where $\mu(z) = r^{(l-1)}\varrho^p$; we include the total solid angle into the norm of ϕ_E.

Let us turn now directly to describe the quantum theory in terms of the of path integral (PI). The following amplitude

$$U_\eta^{ph}(z,z') = \langle z| \exp\left(-\frac{1}{\hbar}\eta \hat{H}_{WD}\right)|z'\rangle \ , \qquad (19)$$

where η is the Euclidean conformal time ($d\eta = d\tau/\varrho$, $\tau = -it$), is the standard object in this approach because in the limit $\eta \to \infty$ this kernel gives the ground-state wave function of the Universe [1].

When deriving PI for the amplitude (19) in the usual way based on iterations of infinitesimal kernels U_ϵ^{ph}, $\epsilon \to 0$, we meet certain difficulties. The first is the existence of the non-trivial measure $\mu(z)$ in the scalar product (18). The reduction of the integration region in (18) represents the second, more serious problem for calculating the iterations. Indeed, according to (18) a convolution of two infinitesimal amplitudes (18) contains integrals over a semiaxis ($r > 0$, $\varrho > 0$). So, in the limit $\epsilon \to 0$ we get a PI on a semiaxis and its calculation is indefinite even for the simplest systems such as a free particle and an oscillator (we cannot calculate an infinite dimensional Gaussian integral in semi-infinite limits). The latter, in fact, reflects the physical PS structure in the theory. Note that in contrast with the variables r, the PS of which is reduced due to the gauge symmetry [2], the PS of ϱ is reduced at the very beginning (see (13)) because only $\varrho > 0$ have a physical meaning.

[2]One should not, however, think that the PS of the radial variable in the corresponding non-gauge model ($y \equiv 0$ in (2)) is a half-plane. A careful analysis shows that PS of each variable in the spherical coordinate system is a complete plane [7].

Nevertheless, the properties (14), (15) allow us to avoid these difficulties and to get PI with a standard measure where integration is carried out over all variables in infinite limits. We do not give here its derivation because it may be easily done by the method suggested in [16]. The final formulas read

$$U_\eta^{ph}(z,z') = \int_{-\infty}^{\infty} \frac{dz''}{(\mu(z)\mu(z''))^{1/2}} U_\eta^{eff}(z,z'')\tilde{Q}(z'',z'), \qquad (20)$$

$$\tilde{Q}(z,z') = \delta(\chi-\chi')Q(\varrho,\varrho')Q(r,r'), \qquad (21)$$

$$Q(r,r') = \delta(r-r') + \delta(r+r') \qquad (22)$$

where $dz \equiv dr d\varrho d\chi$, the kernel $Q(\varrho,\varrho')$ coincides with the one in (22) and

$$U_\eta^{eff}(z',z'') = \int Dz \exp\left(-\frac{1}{\hbar}S^{eff}[z]\right), \qquad (23)$$

$$S^{eff}[z] = \frac{1}{2}\int_{\eta'}^{\eta''} d\eta \left[-\dot{\varrho}^2 - \varrho^2 + \lambda^2\varrho^4 + \right.$$
$$\left. + \ \dot{\chi}^2 + \dot{r}^2 + 2(V+V^{eff})\right]. \qquad (24)$$

Here in (23) the symbol $Dz = D\varrho D r D\chi$ means the standard measure of the Lagrangian PI and initial conditions are defined as $z(\eta') = z'$, $z(\eta'') = z''$; the dot in the effective action S^{eff} denotes the derivative with respect to the conformal time η.

The operator $\tilde{Q}$ entering into (20) shows that together with a direct trajectory connecting points ϱ, ϱ' or r, r' one should take into account contributions into the transition amplitude of trajectories going from $-\varrho'$ to ϱ and from $-r'$ to r. It resembles the motion on a semiaxis restricted by the impenetrable barrier at zero when the trajectory reflected from the "wall" contribute to the transition amplitude as does the direct trajectory [17]. The difference, however, is that in the latter case there exists the boundary condition $\psi|_{r=0}$ (or $\psi|_{\varrho=0}$) and because of it a contribution of the reflected trajectory is taken with the opposite sign. Therefore the operator $Q(r,r') = \delta(r-r') - \delta(r+r')$ is antisymmetric in r and r'. There is no "wall" at zero in our theory, but the existence of the residual gauge group (see (14),(15)) gives effectively the boundary condition $\partial_\varrho\phi|_{\varrho=0} =$

$\partial_r\phi|_{r=0} = 0$ that leads to the operator Q (22), symmetrical in both arguments. It resembles also a reflection of a quantum particle without any change in its phase.

Thus, we have found that PI for the minisuperspace model in a quantum cosmology is modified due to the physical phase (configurational) space reduction. The amplitude (20) gives the ground-state wave function of the Universe in the limit $\eta \to \infty$. Therefore, we may expect that its quasiclassical calculation turns out to be also modified.

5. When calculating the integral (23) quasiclassically, one should find a stationary trajectory z_{st} satisfying the equation

$$\delta S^{eff}[z] = 0, \qquad z(\eta) = z_{st}(\eta). \tag{25}$$

The condition

$$S^{eff}[z_{st}] \ll \hbar \tag{26}$$

is assumed to be valid for this trajectory. Since the effective quantum correction V^{eff} to the potential is proportional to $\hbar^2$, we may approximate (25) by the classical equations of motion $\delta S = 0$. However, a contribution of the effective quantum correction taken on the classical trajectory may be infinite due to the singularity at $\varrho = r = 0$. Hence, the condition (26) is broken if a classical trajectory goes too near the points $\varrho = 0$, $r = 0$. This singularity cannot be eliminated by using trajectories satisfying (25) because they have a gap at $r = 0$ and $\varrho = 0$. Therefore, the quasiclassical approach in neighbourhoods of these points is forbidden. It is necessary to solve the exact quantum problem.

The exceptional case arises at $p = 2$ and $l = 3$ when $V^{eff} \equiv 0$ and a quasiclassical approach is correct in neighbourhoods of $\varrho = 0$ and $r = 0$. The singularity of the multiplier $(\mu(z)\mu(z'))^{-1/2} = (\varrho\varrho' rr')^{-1}$ in the kernel (20) cancels because of the operator $\hat{Q}$ action.

If we are interested in the wave function behavior far from points $\varrho = 0$, $r = 0$ (when the effective quantum correction to the classical action is much smaller than the action itself), then Eq.(20) orders only the symmetrization of the quasiclassical kernel (23) with respect to the group $\mathbf{Z}_2 \otimes \mathbf{Z}_2$ ($\varrho \to \pm\varrho$, $r \to \pm r$) [3]. We may also neglect the measure μ contribution in (20). Since the scalar product contains μ, we may

[3] The symmetrization in ϱ is trivial here because the quasiclassical amplitude (23) depends analytically on ϱ^2 [1].

consider the kernel μU^{ph}_η eliminating then μ from the scalar product. The multiplier $(\mu(z')/\mu(z''))^{1/2}$ is equivalent to an additional term ($\sim \hbar$) in the system action because

$$\left(\frac{\mu(z')}{\mu(z'')}\right)^{1/2} = \exp\frac{1}{2}\int_{\eta'}^{\eta''} d\eta\,\mu^{-1}\partial_\eta\mu \tag{27}$$

where $z(\eta') = z'$ and $z(\eta'') = z''$.

Thus, the measure μ as well as V^{eff} are essential only for calculating quantum corrections to the leading term of a quasiclassical series, but the $\mathbf{Z}_2$ symmetrization in r should be always done as it is ordered by Eq.(20).

6. Another example where the physical PS structure of gauge fields plays an important role is the wormhole quantization, first encountered in Ref.[2] for the gauge group $SU(2)$, but in this minisuperspace model the PS of gauge fields is a plane because $SU(2) \sim SO(3)$, i.e. $l = 0$. The case of an arbitrary gauge group was considered in [4] where, however, the non-trivial structure of the physical PS was not taken into account.

Consider the Euclidean version of the equation of motion for the system (2) ($-it = \tau$, $y \to iy$) and introduce the Euclidean conformal time η. Due to the $SO(l)$ gauge invariance of the equation of motion we may always put $x_i(\eta) = \delta_{i1}x(\eta)$, $i = 1, 2, ..., l$. It means that the physical state changes are described by x varying along the first axis (the unitary gauge). In other words, one may always choose arbitrary functions $y_a(\eta)$ (choose a gauge) so that $x_i(\eta) = 0$, $i = 2, 3, ..., l$. It was shown in Ref.[4] that there exist periodic solutions $x(\eta)$, $\varrho(\eta)$ and $\chi(\eta)$ with periods T_x, T_ϱ and T_χ, respectively. If we interpret the solution $\varrho(\eta)$ as a wormhole connecting two points in the same space, the gauge fields should be the same at both the sides. Since $\chi(\eta)$ and $x(\eta)$ are periodic, the period T_ϱ (the time between two ϱ-maxima) should be an integer multiple of their periods [2], i.e.

$$T_\varrho = nT_\chi = mT_x \tag{28}$$

where n and m are numbers. The relation (28) leads to the exponential quantization of a wormhole size [2].

The relation (28) is valid if the physical PS of gauge fields is assumed to be a plane. However, as we have shown above, that is not the case for the variable x. Its PS is a cone unfoldable into a half-plane. Since x oscillates around $x = 0$ [4] with a period T_x, we find for the physical period $T_x^{ph} = T_x/2$ (points $x < 0$ are gauge equivalent to points $x > 0$; T_x^{ph} is the time during which the system returns to an initial physical state. Therefore the quantization rule of wormholes (28) should be changed,

$$T_\varrho = nT_\chi = mT_x^{ph} = \frac{m}{2}T_x. \tag{29}$$

As a consequence, the quantization of the wormhole size is also modified.

If the theory contains other fields realizing a certain gauge group representation, periods of their physical oscillations would be defined by powers of the independent Casimir operators for a given representation [7].

7. We have considered the simplest case of the gauge group $SO(l+3)$. In principle, the case of an arbitrary gauge group changes nothing in our investigation, only technical details are complicated. We have also not included fermion fields into the minisuperspace model. Fermion degrees of freedom may also have a non-trivial PS structure due to a gauge symmetry [7] and, moreover, the corresponding quantum description has specific features as compared with the bosonic case [18]. We will study such models elsewhere.

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