

# объединенный инСтитут ядерных исследований 

E2-91-267
A. V.Efremov, V.M.Karotkiyan *

# THE SPIN-FLIP pd ELASTIC SCATTERING AND GHOST OF THE U(1) PROBLEM 

Submitteed to "Ядерная физика"

[^0]Ефремов А.В., Кароткиян В.М.
Упругое рd рассеяние с переворотом спина и выделение духа U(1) проблемы

Анализируется вклад духового состояния, необходимого для решения $U(1)$ проблемы, в рd- упругое рассенние. Испопьзуется метод полюсов Редже. Показана возможность обнаружения существования духа в эксперименте. Вклад духа дает о́тличное от реджевского поведение сечения при больших s. Дана численная оценка зависимостей сечения pd pacсенния и ширины пика вперед not от $s$ при разных значениях константы связи духа с нуклоном GQNN。

Работа выполнена в Лаборатории теоретической физики ОИЯИ.

Препринт Объединенного института ядерных исследований. Дубна 1991

Efremov A. V., Karotkiyan V.M.
E2.91-267
The Spin-Flip pd Elastic Scattering and Ghost of the U(1) Problem

The contribution of the ghost state necessary to solve the U(1) problem to pd elastic scattering is analysed. The Regge poles method is used. A possibility to detect the ghost experimentally is shown. The ghost contribution leads to the non-Regge behaviour of the cross section at large $s$. The s-dependence of the cross section and width of a forward peak for different values of the ghostnucleon coupling constant 9QNN are given.

The investigation has been performed at the Laboratory of Theoretical Physics, JINR.

1. Introduction

As is known, solution of the $U(1)$ problem in QCD requires a massless ghost pole in the propagator $\left\langle K_{\mu} K_{\nu}\right\rangle_{0}$ :

$$
\begin{equation*}
\left.\lim _{q^{2} \rightarrow 0} q_{\mu} q_{\nu}<0\left|T\left(K_{\mu} K_{\nu}\right)\right| 0\right\rangle=-\lambda^{4}=-f_{\eta}^{2}, \Delta m_{\eta}^{2}, \quad[1] \tag{1}
\end{equation*}
$$

where

$$
K_{\mu}=N_{f} \frac{\alpha}{s} 2 \pi \varepsilon_{\mu \nu \alpha \beta} A_{\nu}^{a}\left(\partial_{\alpha} A_{\beta}^{a}-\frac{g}{3} f^{a b c} A_{\alpha}^{b} A_{\beta}^{c}\right),
$$

$N_{f}$ is the number of quark flavours and $f_{\eta^{\prime}}$ is the $\eta^{\prime}$-decay parameter $\langle 0| J_{\nu}^{5}\left|\eta^{\prime}\right\rangle=f_{\eta^{\prime}}, q_{\nu}$. The ghost cannot be a physical particle because the current $K_{\mu}$ is gauge non-invariant. However, its presence in the theory leads to observed effects. Mixing of the ghost with the Nambu-Goldstone boson $\eta_{0}$ gives it an additional mass, 958 Mev . It is a totally nonperturbative effect because the physical meaning of the ghost is a periodic dependence of the QCD potential on a collective variable $X=\int d^{3} \times K_{0}(\vec{x}, t)$ [2].

A new interest in the ghost had been stimulated by the so called "Spin crisis"[3]. The ghost pole contributes to the proton form factor of the axial gluon current $\tilde{G}_{2}\left(q^{2}\right)$ :

$$
\begin{align*}
& \left\langle p^{\prime}\right| K_{\mu}|p\rangle=\bar{u}\left(p^{\prime}\right)\left(\gamma_{\mu} \widetilde{G}_{1}\left(q^{2}\right)+q_{\mu} \tilde{G}_{2}\left(q^{2}\right)\right) \gamma_{5} u(p), \\
& \lim _{q^{2} \rightarrow 0} q^{2} \widetilde{G}_{2}\left(q^{2}\right)=\sqrt{\mathrm{N}} f_{f} f_{\eta^{\prime}}\left(\frac{\Delta m_{\eta^{\prime}}^{2}}{m_{\eta^{\prime}}^{2}} g_{\eta^{\prime} N N^{\prime}}-\Delta m_{\eta^{\prime}} g_{Q N N}\right) \tag{2}
\end{align*}
$$

where $g_{\eta^{\prime N N}}$ and $g_{\text {ONN }}$ are $\eta^{\prime}$ nucleon and ghost nucleon coupling constant, respectively, and the Adler-Bardeen relation

$$
a_{\nu} j_{\nu}^{5}=a_{\nu} K_{\nu}=N_{f} \frac{\alpha}{s \pi} F_{\alpha \beta}^{a} \widetilde{F}_{\alpha \beta}^{a}
$$

connects it with the quark contribution to the proton spin


$$
\begin{equation*}
\Delta \Sigma=G_{1}(0)-\tilde{G}_{1}(0)=\lim _{q^{2} \rightarrow 0} q^{2} \tilde{G}_{2}\left(q^{2}\right) \tag{3}
\end{equation*}
$$

where $j_{v}^{5}$ is the quark axial singlet current, and $G_{1}$ is the corresponding proton form factor of this current. However, the value of the coupling constant $g_{\text {QNN }}$ is completly unknown. Therefore it is interesting to propose an experiment that may provide a new information about the property of the contribution of the nonperturbative $Q C D$ connected with the quark contribution to the proton spin. The idea of one possible experiment was suggested in [4]. In this work we analyse pd elastic spin-flip scattering (the proton is longitudinally polarizated) and give a numerical estimation for the cross section and the width of the forward peak as functions of $g_{O N N}$.
2. Separation of the ghost contribution.

Let us turn to the Regge theory that predicts the energy behaviour of amplitudes in the region $s$ 》 $t$

$$
\begin{equation*}
A(s, t) \approx s^{\alpha(t)} \tag{4}
\end{equation*}
$$

where $\alpha(t)$ is the leading Regge trajectory. The ghost exchange in this region corresponds to $a \quad j=0$ fixed singularity assumed not to be reggeized, in accordance with its nature in contrast to other exchanges [4]. Consequently, the presence of the ghost will induce a non-Regge behaviour of the cross section. The exchange channel should obviously have quantum numbers of $\eta$ and $\eta^{\prime}$ mesons, since the ghost has the same (except spin) quantum numbers.

Contribution of the Regge poles to a helicity amplitude at $t \rightarrow 0$ has the form [5]
$\left.A_{\mu_{1} \mu_{2} \mu_{3} \mu_{4}^{(s, t)}(R)}^{(R)}-\frac{t}{s_{0}}\right)^{\frac{1}{2}}\left(\left|\mu_{1}-\mu_{3}\right|+\left|\mu_{2}-\mu_{4}\right|\right) \quad \times$

$$
\begin{equation*}
\times \sum_{j} \frac{\left.e^{-i \pi\left(\alpha_{j}\right.}(t)-v_{j}\right)}{2 \sin \left(\pi\left(\alpha_{j}(t)-\xi_{j}\right)\right)} \gamma_{\mu_{1} \mu_{2} \mu_{3} \mu_{4}} \quad\left(\frac{s}{s_{0}}\right)^{\alpha_{j}(t)} \tag{5}
\end{equation*}
$$

where $r_{j}(t)$ are residues of Regge poles, $\alpha_{1}(t)$ are Regge trajectories, $\xi_{j}$ are their signatures, $v_{j}=0$ when the spin of a particle which determines a trajectory is integer, and $v_{j}=1 / 2$ when the spin is half-integer. $s_{0}=1 \mathrm{Gev}^{2}$ is the empirical scaling parameter. Summation runs over trajectories which contribute to the given process.

We consider the elastic scattering of the longitudinal polarizated proton on the deuteron, and assume the polarization vector being parallel to the proton momentum in the initial state and antiparallel in the final state. only spin-flip amplitudes contribute to this process

$$
\begin{equation*}
\frac{\mathrm{d} \sigma}{\mathrm{dt}}+-=\frac{1}{3} \sum_{\mu_{2} \mu_{4}} \frac{\mathrm{~d} \sigma^{\mathrm{dt}}}{}+, \mu_{2}-, \mu_{4} \tag{6}
\end{equation*}
$$

where summation is over the deuteron helicities. Neglecting the interaction of the proton and neutron in the deuteron and the admixture of ${ }^{3} \mathrm{D}_{1}$ states we can. write

$$
\begin{align*}
& |d,+\rangle=|p,+>\otimes| n,+\rangle  \tag{7}\\
& |d, 0\rangle=\frac{1}{\sqrt{2}}(|p,+>\otimes| n,->+|p,->\otimes| n,+>) \tag{8}
\end{align*}
$$

where $\left|d, \mu^{\prime}\right\rangle,|p, \mu\rangle$ and $|n, \mu\rangle$ are deuteron, proton and neutron s-channel helicity states respectively. Using this we can decompose the helicity amplitude of pd scattering in helicity amplitudes of pp and pn elastic scattering. We shall assume that the Pomeron-nucleon vertex contains no a spinflip part and scalar trajectories are nonleading. So, only the double spin-flip amplitudes are present. Using the TPinvariance of strong interactions and the condition of factorization of residues [5]

$$
\begin{equation*}
\gamma_{\left.\mu_{1} \mu_{2} \mu_{3} \mu_{4}(t)=\gamma_{\mu_{1}} \mu_{3}^{(t)} \gamma_{\mu_{2}} \mu_{4}^{(t)}, \underline{t}\right)} \tag{9}
\end{equation*}
$$

one can obtain

$$
\begin{align*}
& \mathrm{A}_{++\prime--,}^{\mathrm{pd}}(\mathrm{~s}, \mathrm{t})=\mathrm{A}_{++--}^{\mathrm{pp}}\left(\frac{s}{2}, t\right)+\mathrm{A}_{++-}^{\mathrm{pn}}\left(\frac{\mathrm{~s}}{2}, t\right)  \tag{10a}\\
& \mathrm{A}_{++{ }^{\prime}-0^{\prime}}^{\mathrm{pd}}(s, t)=\frac{1}{\sqrt{2}}\left(A_{++--}^{\mathrm{pp}}\left(\frac{\mathrm{~s}}{2}, t\right)+\mathrm{A}_{++--}^{\mathrm{pn}}\left(\frac{\mathrm{~s}}{2}, t\right)\right)  \tag{10b}\\
& \mathrm{A}_{+0^{\prime}-0^{\prime}}^{\mathrm{pd}}(s, t)=0 \tag{10c}
\end{align*}
$$

where the primed indices are for deuteron helicities. Other helicity amplitudes are equal to (10) up to a phase. From (6) and (10) and taking account of normalization we have

$$
\begin{equation*}
\frac{\mathrm{d} \sigma_{+-}(s, t)}{\mathrm{dt}}=\frac{1}{48 \pi s q_{s}^{2}}\left|A_{++--}^{\mathrm{pp}}\left(\frac{\mathrm{~s}}{2}, \mathrm{t}\right)+{ }_{A_{++--}}^{\mathrm{pn}}\left(\frac{s}{2}, \mathrm{t}\right)\right|^{2} \tag{11}
\end{equation*}
$$

where $q_{s}^{2}$ is the 3 -dimensional momentum squared in the c.m.s. system for pd scattering

$$
\begin{equation*}
q_{s}^{2}=\frac{1}{4 s}\left(s^{2}-10 s m_{N}^{2}+9 m_{N}^{2}\right) \tag{12}
\end{equation*}
$$

Transferred quantum numbers of a t-channel are the same both for $p p$ and for pn elastic scattering. These are exchanges with $\Delta B$ (baryon charge) $=\Delta S$ (strangeness) $=\Delta Q$ (electric charge) $=0$ and the isospin transfer may be either 0 or 1 . There are no restrictions on G-parity and spin exchange. So, the following trajectories will contribute: P(pomeron), $\rho$, $A_{2}, H, f, \omega, A_{1}, \eta, \varepsilon, D, \eta^{\prime}, \delta, \pi, B$. The effective mesonnucleon vertex for mesons with isospin 1 contains the Pauli matrix $\tau_{3}$. For example, for the $\pi^{0}$ meson it has the form

$$
\begin{equation*}
L_{\pi N N}=g_{\pi N N} \bar{N} \gamma_{5} \tau_{3} N \pi^{0} \tag{13}
\end{equation*}
$$

It is clear that these residues should obey the relation

$$
\begin{equation*}
\gamma^{j p}(t)=-\gamma^{j n}(t) \tag{14}
\end{equation*}
$$

and from (11) one can see that the trajectories with isospin 1 do not contribute to pd scattering. At a high energy the vector and axial vector vertices conserve helicity of the
nucleon, therefore exchanges by $\omega$, $D$ and $H$ trajectories will not contribute to a spin-flip process. Also there are no convincing experimental evidences in favour of a noticable spin-flip part in the Pomeron and $\eta^{\prime}$-trajectory. There remain only exchanges by scalar and pseudoscalar trajectories. However, scalar trajectories are nonleading. For the estimation of the cross section behaviour it is sufficient to take account of $\eta$ and $\eta^{\prime}$ exchanges, and one may hope to observe the effect of the ghost contribution.


Fig. 1

Now we turn to the ghost exchange in the t-channel (Fig. 1). The effective ghost nucleon vertex can be written in the form

$$
\begin{equation*}
L_{\mathrm{QNN}}=\mathrm{g}_{\mathrm{QNN}} \stackrel{\rightharpoonup}{\mathrm{~N}} \partial_{\mu} \gamma_{5} N G_{\mu} . \tag{15}
\end{equation*}
$$

The derivative is here introduced for two reasons. First, the amplitude should not contain the unphysical zero mass pole corresponding to the ghost exchange. Second, the physical amplitude should be gauge-invariant. The ghost propagator in any covariant gauge has the form [4]

$$
\begin{equation*}
\Delta_{\mu \nu}(q)=-\left(g_{\mu \nu}+a \frac{q_{\mu} q_{v}}{q^{2}}\right) \frac{1}{q^{2}(a+1)} \tag{16}
\end{equation*}
$$

where $a$ is a gauge parameter. For the ghost contribution to the NN scattering amplitude one gets from (15) and (16)

$$
\begin{align*}
& A_{\mu_{1} \mu_{2} \mu_{3} \mu_{4}}^{(g) N N}=g_{Q N N}^{2} \bar{u}_{\mu_{3}}\left(k^{\prime}\right) q_{\alpha_{5}} \gamma_{\mu_{1}}(k) \Delta_{\alpha \beta}(q) \bar{u}_{\mu_{4}}\left(p^{\prime}\right) q_{\beta} \gamma_{s} u_{\mu_{2}}(p)= \\
& \quad=-g_{Q N N}^{2} \bar{u}_{\mu_{3}}\left(k^{\prime}\right) \gamma_{5} u_{\mu}(k) \bar{u}_{\mu_{4}}^{\left(p^{\prime}\right) \gamma_{5} u_{\mu}(p)} . \tag{17}
\end{align*}
$$

Thus, due to the structure of the vertex, the dependence on the gauge parameter and the pole $1 / q^{2}$ drop out. Notice that in expression (17) the dependence on $s$ is also absent. This contradicts the assumption made in paper [6] where the ghost exchange is connected with a new trajectory having a large intercept $\alpha_{0}=1-\varepsilon, \varepsilon « 1$. That connection would require the change of the derivative in the effective lagrangian (15) to the Dirac matrix $\gamma_{\mu}$, which would contradict the requirements of gauge invariance and absence of the unphysical pole in the amplitude (Even if one can escape the nonphysical pole at $t=0$ assuming the residue is equal to zero.)

Let's parametrize the contribution $A(g) N N$ in the form

where $\gamma^{g N}(t)$ have the form of Regge residues.
As for parametrization of the Regge pole residues, the helicity dependence in amplitudes (5) and (18) is explicitly separated, therefore one may omit helicity indices in the residuees. Due to the exponential behaviour of cross sections at small $t$, the residues can be choosen in the form

$$
\begin{align*}
& \gamma^{\eta d}(t)=\gamma^{\eta p}(t)+\gamma^{\eta n}(t)=2 g_{\eta N N} e^{a t}  \tag{19a}\\
& \gamma^{\eta^{\prime} d}(t)=2 g_{\eta^{\prime} N N} e^{a t}  \tag{19b}\\
& \gamma^{g d}(t)=\gamma^{g p}(t)+\gamma^{g n}(t)=2 g_{\mathrm{QNN}} e^{a t} \tag{19c}
\end{align*}
$$

where $a$ is expressed though the Regge radius squared dependent on the nucleon size. From (5), (11), (18) and (19)
one can get for the cross section

$$
\begin{align*}
& \frac{d \sigma_{+-}(s, t)}{d t}=\frac{1}{12 . \pi s q_{s}^{2}}\left(\frac{t}{s_{0}}\right)^{2} e^{4 a t} \times \\
& \times\left|\sum_{j=\eta, \eta^{2}} \frac{e^{-i \pi \alpha_{j}}(t)+1}{2 \sin \left(\pi\left(\alpha_{j}(t)\right)\right.} g_{j N N}^{2}\left(\frac{s}{2 s_{0}}\right)^{\alpha_{j}(t)}+s_{0} g_{Q N N}^{2}\right|^{2} . \tag{20}
\end{align*}
$$

It is clear now that the ghost contribution essentially differs from the contribution of the Regge poles. In the absence of the ghost the cross section behaves like $s^{2 \alpha(t)-2}$ in the limit $s \rightarrow \infty$. The presence of the ghost induces the behaviour like $1 / s^{2}$ because $\eta$ and $\eta^{\prime}$ trajectories have a negative intercept and they can be neglected in that limit, which means the absence of the Regge shrinkage of the cone for the ghost.
3. Estimation of the cross section and discussion.

For the estimation of the cross section let us take $a=$ $=2.5 \mathrm{Gev}^{-2}[7]$, which corresponds to a radius of an order of 1 Fm. Numerical evaluation for $g_{\eta N N}$ and $g_{\eta \prime N N}$ are taken from paper [8]: $g_{\eta N N}=6.8$ and $g_{\eta^{\prime} N N}=7.2$. We also assume for trajectories: $\alpha_{\eta}(t)=-m_{\eta}^{2}+t, \alpha_{\eta}(t)=-m_{\eta}^{2},+t$. The ghost nucleon coupling constant is not determined. The cross section (20) strongly depends on $g_{Q N N}$. Results of the numerical calculation are shown in Fig. 2.

We define the width of the forward peak in $t$

$$
\begin{equation*}
\Delta t=\left[\frac{\frac{d}{d t}\left(\frac{1}{t^{2}} \frac{d \sigma}{d t}\right)}{\frac{1}{t^{2}} \frac{d \sigma}{d t}}\right]^{-1} \tag{21}
\end{equation*}
$$

The Regge theory predicts that the width of the forward peak will decrease with increasing s. In the absence of the ghost contribution ( $g_{\mathrm{QNN}}=0$ ) we get from (20) and (21)

$$
\begin{equation*}
\Delta t(s, t) \underset{s \rightarrow \infty}{=} \frac{1}{4 a+F(t)+2 \ln \left(s / 2 s_{0}\right)} \tag{22}
\end{equation*}
$$



Fig. 2. s dependence of the pd scattering cross section. Dashed line corresponds to the behaviour of the cross section at absence of the ${ }_{1}$ ghost. Line, marked by $\square$, corresponds to $g_{Q N N}=4 \mathrm{Gev}^{-1},-\mathrm{g}_{\mathrm{QN}}=7 \mathrm{Gev}^{-1}$.


Fig. 3. $s$ dependence of the width of forward peak $\Delta t$. Dashed line correspond to the behaviour of $\Delta t$ at absence of the ghost. 1 Lines, marked by labels, correspond_to: $\mathrm{g}_{\mathrm{QNN}}=2 \mathrm{GeV}^{-1},+-\mathrm{g}_{\mathrm{QN}}=3 \mathrm{Gev}^{-1}, \Delta-\mathrm{g}_{\mathrm{QNN}}=4 \mathrm{GeV}^{-1}, \nabla-$ $\mathrm{g}_{\mathrm{QNN}}=5 \mathrm{Gev}^{-1}, \mathrm{CH}_{\mathrm{QNN}}=7 \mathrm{Gev}^{-1}$.

The existence of the ghost, however, changes this conclusion. When the value of $g_{\mathrm{ONN}}$ is large enough, the ghost exchange gives the main contribution in the limit $s \rightarrow \infty$.

$$
\begin{equation*}
\Delta t(s, t) \underset{s \rightarrow \infty}{=} \frac{1}{4 a} \tag{23}
\end{equation*}
$$

i.e. the shrinkage of the cone should be considerably smaller and at large $s$ the width of the forward peak tends to constant. However, this can occur at very high energies unattainable for experiment. It is interesting that in the region of intermediate energies the width of the forward peak can even increase. The numerical calculation of $\Delta t$ shows that it has a more complicated s dependence. In particular, $\Delta t$ has a peak at a certain value of $s=s_{p}$. The value of $s_{p}$ strongly depends on the value of $g_{\text {ONN }}$. In Fig. 3 s-dependences of $\Delta t$ for different values of $g_{Q N N}$ and $t=-0.17 \mathrm{Gev}^{2}$ are shown. The value of $t$ for the numerical estimation (Fig. 2 and 3) has been choosen with taking into account that this point is situated far from unphysical poles of the Regge amplitude to avoid further difficulties connected with a special parameterization of residues for canceling these poles.

Thus, we believe that the corresponding experiment the Dubna and Serpuhov energy region allows to establish not only the existence of the ghost, but also to estimate the value of the coupling constant $g_{Q N N}$.

References

1. Witten E. Nucl. Phys., 1979, v. 156B, p. 269. Veneziano G. Nucl. Phys., 1979, V. 159B, p. 213. Kogut J., Susskind. Phys. Rev., 1975, v. 11D, p. 3594. Thooft G. Phys. Rep., 1986, v. 142, p. 357.
2. Diaconov D. I., Eides M. V. JETP, 1981, v. 81, p. 434.
3. Veneziano G. Phys. Lett., 1990, v. 244B, p. 75.
4. Efremov A. V., Soffer J., Törnquist N. A. Marseille preprint CPT-90/P. 2457.
5. Collins P.D.B. An introduction to Regge theory and high
energy physics.
6. Dorokhov A. E., Kochelev N. I. Preprint JINR E2-91-95., 1991.
7. Beznogikh G. G. et al. Yad.Fis., 1973, v.18, p.348. Chernev H. M.. Yad. Fis., 1973, v.18, p. 403.
8. Dumbrajs O. et al. Nucl. Phys., 1983, v. 216B, p. 277.

[^0]:    ※Institute of Physics, Rostov State University, USSR

