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INVARIANT STRUCTURES AND STATIC FORCES IN GAUGE THEORIES

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Инвариантные структуры и статические силы в калибровочных теориях

Проблема построения всех калибровочных инвариантов рассматривается в связи с конфайнментом. Показано, что любой калибровочный инвариант может быть построен из упорядоченных вдоль линии интегралов (струн) и локальных тензоров калибровочной группы. С этой точки зрения анализируется кулоново поле. Найдено, что в чистой SU(n) калибровочной теории (нет материи) потенциал взаимодействия статических источников линейно растет с расстоянием.

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Invariant Structures and Static Forces in Gauge Theories

The problem of finding all gauge invariants is considered in connection with confinement. It is shown that any gauge invariant may be built of the exponential line integrals (strings) and local gauge group tensors. The Coulomb field structure is analyzed from this point of view. In the pure SU(n) gauge theory (no matter), a potential of interacting static sources is found to be linearly rising with distance.

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1. The main peculiarity of gauge theories is the existence constraints, i.e. some relations between canonical variables [1]. The constraints do not contain the time derivatives, only the space ones, so they presuppose some static nonlocal structures to exist. These structures, appearing due to constraints, are preserved during the time evolution and manifests of a gauge invariance.

The well-known example of the nonlocal excitation is a physical electron, i.e. a charged fermion with its Coulomb field. If ψ describes an electron field then the nonlocal field $\Psi = \exp(-ie\Delta^{-1}(\nabla, \mathbf{A}))\psi$, where $\mathbf{A}$ is an electromagnetic vector potential, corresponds to the physical electron [2]. The exponential describes the Coulomb field [2]. The field Ψ is gauge-invariant.

The static fields surrounding a "gauge charge" and appearing due to constraints give rise to a static interaction of the charges (or static sources). For example, in electrody-namics the static interaction of charges described by the gauge-invariant field Ψ is the Coulomb one [2]. The example of electrodynamics teaches us that physical objects are described by some nonlocal gauge-invariant field configurations, and real dynamics should appear as motion and interaction of these objects (including, of course, local fields too if they are, like the field strength in electrodynamics, gauge-invariant). It suggests that before turning to dynamics, one should first "solve" constraints, i.e. one has to find all gauge invariant objects.

To find the static forces between charges in gauge theories, one should know all gauge-invariant structures. It is known, on the other hand, that gauge fields can be considered as connections in the principal fiber [3],[4], and the problem of finding gauge invariants becomes the pure geometrical one. In the present letter based on the geometrical approach we argue that the exponential line integrals are the only fundamental "gauge covariant" objects in gauge theories. All gauge-invariant configurations of fields, though complex, are made of them.

This observation is tightly connected with the problem of confinement. By itself this problem is very complex and has many different aspects. But one its feature – the existence of a linearly rising with distance potential – can be established for some models in a relatively simple way. Moreover, one can show that in gluodynamics for static quarks there could be no other forces. This comes from the fact that the only proper gauge-invariant object consists of the P-exponent (i.e. of the "string") connecting two opposite charges. However, in QCD the strings can branch. For this reason the interaction between real quarks is not so simple.

We demonstrate also how the Coulomb field may be built of the strings.

2. Let $P(M,G)$ be the principal fiber bundle with base M (the Minkowski space) and a compact group G [3]. The group G acts on P as a group of right translations: $ug' \equiv (x,g)g' = (x,gg')$, $u \in P$, $x \in M$, $g,g' \in G$, i.e. $M = P/G$. In P one defines a local cross section σ which may represented by the pair $\sigma(x) = (x, g(x))$, i.e. $\sigma(x)$ is a surface in P isomorphic to an open neighbourhood in M.

A gauge transformation in P is defined as a change of local cross sections $\sigma(x) \to \sigma_g(x) = \sigma(x)g(x)$ where $g(x)$ is a G-valued function on M [1].

Consider tensors $\psi(u)$ in P realizing a representation of G and having the following property $\psi(ug) = T_{g^{-1}}\psi(u)$, T_g is an element $g \in G$ in the corresponding representation; analogously for a conjugated representation: $\psi^*(ug) = \psi^*(u)T_g$. Matter fields on M are defined as $\psi_\sigma(x) = \psi(\sigma(x))$, $\psi^*_\sigma(x) = \psi^*(\sigma(x))$ for a certain cross section σ. So their gauge transformation law is

$$\psi'_\sigma = \psi(\sigma g) = T_{g^{-1}}\psi_\sigma \; , \qquad \psi'^*_\sigma = \psi^*(\sigma g) = \psi^*_\sigma T_g \; . \tag{1}$$

A construction of local invariants of ψ_σ, ψ^*_σ does not meet difficulties. Note also that in spite of the dependence of matter fields on σ, invariants built of them are independent of it because a change of σ is equivalent to a gauge transformation.

It is important, however, to know "nonlocal invariants" made of the fields $\psi_\sigma(x)$, $\psi^*_\sigma(x')$ taken at different points. To construct them one has to define a parallel translation of tensors from x' to x, i.e. to introduce a connection and a connection form ω [3],[4]. Consider a projection ω_σ of ω on a cross section σ

$$\omega_\sigma = A_\mu dx^\mu \tag{2}$$

[1] To define a gauge transformation globally, functions $g(x)$, each of which is defined on a certain neighbourhood in M, should satisfy supplementary conditions at points common for different neighbourhoods [3],[4]. However, it is not essential for what follows.

where coefficients A_μ are elements of the Lie algebra of G. This projection may be determined in a local coordinate system as a formal replace of du by $d\sigma(x)$ in ω and u by σ in its coefficients. Under gauge transformations the form ω_σ changes that induces gauge transformations of A_μ. One may show [3],[4] that

$$\omega_\sigma \to \omega_{\sigma g} = g^{-1}\omega_\sigma g + g^{-1}dg \equiv A^g_\mu dx^\mu \tag{3}$$
$$A_\mu \to A^g_\mu = g^{-1}A_\mu g + g^{-1}\partial_\mu g \; . \tag{4}$$

For this reason the coefficients A_μ are identified with the Yang-Mills potentials.

By definition translation of ψ_σ from x' to the neighbour point $x = x' + dx$ is given by

$$\tilde{\psi}_\sigma(x) = (1 + A_\mu dx^\mu)\psi_\sigma(x') \equiv P_{xx'}\psi_\sigma(x'). \tag{5}$$

According to Eq.(5) parallel translation of ψ_σ from x' to x along a curve $C(x,x')$ is given by

$$\tilde{\psi}_\sigma(x) = P[C(x,x')]\psi_\sigma(x'), \tag{6}$$

where the operator of translation along the curve $C(x,x')$

$$P[C(x,x')] = P\exp\Big(\int_{C(x,x')} A_\mu dx^\mu\Big) \equiv P_{xx'} \tag{7}$$

is the familiar P-exponent transforming as follows $P^g_{xx'} = T_{g^{-1}}(x)\, P_{xx'} \times T_g(x')$, i.e. $P_{xx'}$ is a "bilocal" tensor. Eqs.(6),(7) tell us: from geometrical point of view P-exponent, matter fields ψ, and invariant tensors (like the unit antisymmetric one $\epsilon_{\alpha\beta\ldots}$) are the only constituents of invariant structures, the simplest of them being

$$\psi^*_\sigma(x)P_{xx'}\psi_\sigma(x'). \tag{8}$$

It is the bilocal tensor $P_{xx'}$ in (8) that represents the external field associated with a charge (color). The significance of this circumstance is studied in the next sections.

Remark 1. One may construct a local tensor depending on A_μ. Obviously, any *local* tensor containing A_μ can be built of the covariant derivative $D_\mu = \partial_\mu + A_\mu \to g^{-1}D_\mu g$; it is linear in A_μ therefore any local tensor is polynomial of D_μ, for example, the strength tensor $F_{\mu\nu} = [D_\mu, D_\nu]$ etc. So, D_μ may be considered as a matter tensor when constructing invariants. It turns out also that any local tensor *nonlocally* depending on A_μ (a functional of A_μ) is expressed via the exponential line integral for a closed contour; it is an element of the holonomy group [5].

Remark 2. There are invariants containing the nonlocal operators $(\gamma_k D_k)^{-1}$ and D_k^{-2} where $k = 1,2,3$ and γ_μ are the Dirac matrices. However, it is known that the kernels of these integral operators may be expressed via the exponential line integral considering D_k^2 and $\gamma_k D_k$ as some Hamiltonians for quantum mechanical systems [6].

Remark 3. Note that our consideration is applicable to an Abelian theory too. It means, in particular, that the above mentioned tensor $\exp(-ie\Delta^{-1}(\nabla, \mathbf{A}))$ should be expressed via the exponential line integrals ("strings"). Let us show that this is really the case [7],[5]. Let us take a set of straight lines outgoing from a point x, attached to each line solid angle $4\pi/N$; the spherical angles φ_i, θ_j determine its direction so that $4\pi/N = \sin\theta_j \Delta\theta_j \Delta\varphi_i$ where $\Delta\theta_j$, $\Delta\varphi_i$ are angles between neighbouring lines. Consider the following product

$$P(x, N) = \prod_{j,i} \exp\left(-ig\int_{-\infty}^{x} A_\mu(y_{ij})\, dy^\mu_{ij}\right); \qquad (9)$$

here we introduce explicitly the coupling constant g. The product is taken over the set of lines. Let the angles $\Delta\theta_j$, $\Delta\varphi_i$ tend to zero when $N \to \infty$. Defining a new constant $e = gN$ and taking e fixed we obtain for (9)

$$\lim_{N\to\infty} P(x, N) = \lim_{N\to\infty} \exp\left[\frac{ie}{4\pi}\sum_{i,j} \sin\theta_j \Delta\theta_j \Delta\varphi_i \int_0^\infty dr\, A_r(r, \varphi_i, \theta_j)\right]$$

$$= \exp\left[\frac{ie}{4\pi}\int_0^{2\pi} d\varphi \int_0^{\pi} d\theta \sin\theta \int_0^\infty dr\, A_r\right] \equiv \exp(ieI) \qquad (10)$$

where $A_\mu dy^\mu_{ij} \equiv A_r(r, \varphi_i, \theta_j)dr$. The integral in the exponent can be rewritten in the form

$$I = \frac{1}{4\pi}\int d^3y\, r^{-2} A_r = -\frac{1}{4\pi}\int d^3y (\partial_r r^{-1})\, A_r =$$

$$= \frac{1}{4\pi}\int d^3y\, r^{-1}\left[r^{-2}\partial_r(r^2 A_r)\right] = -\Delta^{-1}(\nabla, \mathbf{A}). \qquad (11)$$

The last equality in Eq.(11) is due to the identity $(-1/4\pi)|\mathbf{x} - \mathbf{y}|^{-1} \equiv \Delta^{-1}(\mathbf{x}, \mathbf{y})$ and the formula for the divergence in the spherical coordinates $(\nabla, \mathbf{A}) = r^{-2}\partial_r(r^2 A_r) + (r\sin\theta)^{-1}[\partial_\theta(\sin\theta\ A_\theta) + \partial_\varphi A_\varphi]$, assuming that $A_\theta = A_\varphi = 0$. Thus, the factor $\exp(ieI)$ (see (10)) coincides with the one in $\Psi = \exp(-ie\Delta^{-1}(\nabla, \mathbf{A}))\psi$ describing the Coulomb field of a fermion. It completes an anatomy of the Coulomb field.

We see that according to the first principles a particle surrounded by the Coulomb field is not the simplest charged object. Rather, it is a very complex object, the simplest one being a particle with one line. The experimental consequences of the hypothesis that N in Eq.(9) is large but finite have been studied in [8],[9].

3. We have already mentioned above that external fields surrounding charged objects appear due to the first-class constraints and are responsible for the static interparticle forces. Here we study this aspect of the problem in detail.

Suppose that the theory is given by the standard Lagrangian density

$$\mathcal{L} = -\frac{1}{4} Tr\ F^2_{\mu\nu} + \bar{\psi}(i\hat{D} - m_q)\psi\ , \qquad (12)$$

where $F_{\mu\nu} = i(D_\mu D_\nu - D_\nu D_\mu)$, $D_\mu = \partial_\mu - iA^a_\mu \lambda_a \equiv \partial_\mu - iA_\mu$, $\hat{D} = \gamma_\mu D_\mu$; λ_a, γ_μ are the Gell-Mann and Dirac matrices, respectively, $Tr\lambda_a\lambda_b = \delta_{ab}$, $a, b = 1, 2, ..., N = \dim G = n^2 - 1$ ($G = SU(n)$) and m_q is a mass of the matter field ψ which may realize any non-trivial representation of G; we assume for certainty that ψ transforms according to the elementary one. The coupling constant g is put equal to unity in Eq.(12). If necessary, it can be introduced by substitution $A_\mu \to gA_\mu$, $\psi \to g\psi$, $\mathcal{L} \to \mathcal{L}' = \mathcal{L}/g^2$.

We are interested only in the field configuration of the lowest rank and energy. Of $\psi, \bar{\psi}, P_{xx'}$ and the invariant tensors $\epsilon_{\alpha\beta\gamma\cdots}, \epsilon^{\alpha\beta\gamma\cdots}$, (the unit totally antisymmetric ones of rank n), one can construct the following non-trivial invariants

$$\mathcal{G}_x = Tr\ P_{xx} \equiv (P_{xx})^\alpha_\alpha, \qquad (13)$$

$$\Theta(x, x') = \epsilon^{\alpha\beta\gamma\cdots}(P_{xx'})^{\alpha'}_\alpha (P'_{xx'})^{\beta'}_\beta (P''_{xx'})^{\gamma'}_\gamma \cdots \epsilon_{\alpha'\beta'\gamma'\cdots}, \qquad (14)$$

$$\mathcal{M}_{xx'} = \bar\psi(x)\, P_{xx'}\, \gamma_5\, \psi(x')\ , \quad (15)$$

$$\mathcal{B}_{x_1x_2x_3\cdots} = \epsilon^{\alpha\beta\gamma\cdots}(P_{xx_1})^{\alpha'}_{\alpha}(P_{xx_2})^{\beta'}_{\beta}(P_{xx_3})^{\gamma'}_{\gamma}\cdots\times \times\psi_{\alpha'}(x_1)\psi_{\beta'}(x_2)\psi_{\gamma'}(x_3)\cdots\ , \quad (16)$$

etc.; P, P' , P'' in Eq.(14) differ by contours. The fields (13) and (14) represent the simplest local and bilocal physical configurations of a pure gluonic field. The configuration (15) is usually referred to as a mesonic, while (16) – as a barionic field. Existence of the antisymmetric invariant tensors $\epsilon_{\alpha\beta\gamma\ldots}, \epsilon^{\alpha\beta\gamma\cdots}$ implies that there are infinitely many topologically nonequivalent bilinear in ψ, $\bar\psi$ invariants because strings in this case may branch with these tensors at vertices.

In contrast with the case $n > 2$ including chromodynamics, in the $SU(2)$-gauge theory the complex conjugate representation of the 2×2 U-matrices ($UU^+ = 1, \det U = 1$) is unitary equivalent to the original one. The invariant antisymmetric tensors $\epsilon^{\alpha\beta}$; $\epsilon_{\alpha\beta}$ transform spinors with the lower indices into those with the upper ones and vice versa. As a result, strings are not directed here. Contrary to the case $n > 2$ where with the string $(P_{xx'})^{\beta}_{\alpha}$ in (15) one may associate a directed line connecting points x, x'; an arrow being directed from the upper index to the lower one. Furthermore, the tensors $\epsilon^{\alpha\beta}$; $\epsilon_{\alpha\beta}$ have rank two , so the strings cannot branch. We conclude that a non-Abelian $SU(2)$ gauge theory differs drastically from any other theories with group $SU(n)$, $n > 2$. Rather, it is closer to electrodynamics, because strings there also do not branch [2].

For *elementary* representations a single string (possibly branched) is *the only allowed* field configuration; structures analogous to the Coulomb field are forbidden.

Question: can a string change its topology with time? The string topology changes if and only if strings can break or branch. Consider for simplicity the case $n = 3$. It is easily seen that in pure gluodynamics (no matter fields) strings in the fundamental representation preserve their topology. Indeed, strings in the elementary representation cannot break because in this theory there are no quarks, and open strings are forbidden by the gauge invariance [11]. Strings cannot branch either because, for

[2]Note that the gauge invariance does not forbid the existence of strings in the Abelian theory. The problem of the existence of objects of that sort in Nature remains open. Dirac [10] seriously considered this possibility.

example, the loop appearance on the string in (15) assumes the existence of a vertex trilinear in gluonic fields (to ensure transition $A \to A\, A$) made with the use of the product of the tensors $\epsilon^{\alpha\beta\gamma}\epsilon_{\alpha'\beta'\gamma'}$. Such a combination is absent in the gluonic Lagrangian and may appear only in the effective action as a trilinear vertex containing the invariant symmetrical tensor $d^{abc} \sim Tr(\lambda^a\{\lambda^b,\lambda^c\})$ (the sign $\sim$ means: equal up to a constant). This is easily seen from the equalities

$$\epsilon^{\alpha\beta\gamma}A^{\alpha'}_{\alpha}A^{\beta'}_{\beta}A^{\gamma'}_{\gamma}\epsilon_{\alpha'\beta'\gamma'} \sim Tr(A\{A,A\}) \sim d^{abc}A^aA^bA^c \quad (17)$$

(A is a certain matrix, $A = A^a\lambda^a$, $TrA = 0$; in the first equality we used the identity $\epsilon^{\alpha\beta\gamma}\epsilon_{\alpha'\beta'\gamma'} = \delta^{\alpha}_{\alpha'}\delta^{\beta}_{\beta'}\delta^{\gamma}_{\gamma'} + permutations\ with\ proper\ signs$). But effective vertices of this type never appear in pure gluodynamics. The latter from the very beginning contains only the structures constants f^{abc}, and from these constants one cannot construct the invariant symmetrical tensor d^{abc} [12] because $Tr(F^a\{F^b,F^c\}) = 0$, where $(F^a)_{bc} = f^{abc}$, i.e. in gluodynamics $\langle d^{abc}A^a_\mu(x)A^b_\nu(y)A^c_\rho(z)\rangle_0 = 0$. Hence, in pure gluodynamics strings in the fundamental representation cannot branch. We may expect appearance of effective d-vertices only after the introduction of quarks. It turns out however that even in QCD they are absent, at least in lowest orders of perturbation theory. Indeed, the simplest triangle Feynman graph *a* in Fig.

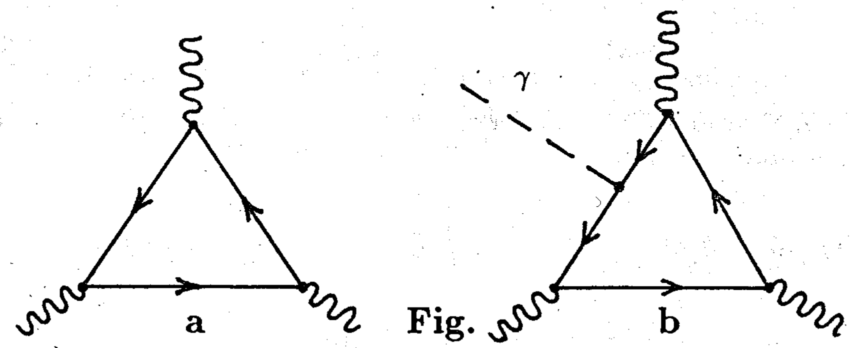

Fig.

does not give them because it is accompanied by a graph with opposite directions of arrows, so that their sum is proportional to $Tr(\lambda^a[\lambda^b,\lambda^c]) \sim f^{abc}$. This is the case for more complex graphs too (e.g. for graphs of order g^7 with six γ-vertices). In local limit such vertices vanish identically in all orders of perturbation theory simply because $d^{abc}F^a_{\mu\nu}F^b_{\nu\rho}F^c_{\rho\mu} \equiv 0$.

They appear only in the presence of some other vector fields (γ; Z_0) interacting with quarks. For example, effective d-vertices arise from graph b in Fig., as it has an even number of γ- vertices in the fermion loop; the extra line there represents a photon. Due to this circumstance the string branching is accompanied by emission of a photon, and each string vertex enters with the factor $g^{3/2}e^{1/2} \sim \alpha_s(\alpha/\alpha_s)^{1/4}$, where α is the QED fine structure constant. Note that a certain function of momenta also enters there (square root of the corresponding vertex function).

Let us turn now directly to finding static forces between charges in the simplest case of gauge-invariant excitation born by the operator

$$\begin{aligned} \hat{\mathcal{M}}_{xx'} &= \hat{\bar{\psi}}(x)\, \hat{P}_{xx'}\gamma_5\, \hat{\psi}(x') = \\ &= \hat{\bar{\psi}}(x) P \exp\left[i \int_0^1 \hat{A}^b_\mu(z(\sigma))\lambda_b(\sigma)\dot{z}^\mu(\sigma)d\sigma \right] \gamma_5 \hat{\psi}(x') \quad (18) \end{aligned}$$

where $\dot{z}^\mu(\sigma) = dz^\mu/d\sigma$, and $z(0) = x'$, $z(1) = x$; we introduced parameter-dependent λ-matrices [13] and instead of classical canonical variables took operators with the following equal time commutation relations ($x = (x_0, \mathbf{x})$)

$$\left[\hat{A}^a_j(\mathbf{x}) \,,\, \hat{E}^k_b(\mathbf{y}) \right] = i\delta^k_j \delta^a_b \delta(\mathbf{x}-\mathbf{y}), \quad (19)$$

$$\left[\hat{\psi}^+_\alpha(\mathbf{x}) \,,\, \hat{\psi}_\beta(\mathbf{y}) \right]_+ = \delta_{\alpha\beta}\delta(\mathbf{x}-\mathbf{y}); \quad (20)$$

indices α, β stand both for spinor and color indices. Using these commutation relations we find

$$\begin{aligned} \left[\hat{E}^j_a(\mathbf{y}) \,,\, \hat{P}_{xx'}\right] &= P\left[\hat{P}_{xx'} \int_0^1 \lambda_a(\tau)\dot{z}^j(\tau)\delta\left(\mathbf{z}(\tau)-\mathbf{y}\right) d\tau \right] \equiv \\ &\equiv P\left[\hat{P}_{xx'} I^j_a(x,x';\mathbf{y})\right]. \quad (21) \end{aligned}$$

With the help of (21) we may now calculate the average of the gluonic field energy operator $\hat{H}^g_0 = \int d^3x[\hat{\mathbf{E}}^2_a + \hat{\mathbf{H}}^2_a]/2$ in the state $\hat{\mathcal{M}}_{\mathbf{xx'}}|0\rangle$ where $|0\rangle$ is the physical vacuum state. To simplify formulae, we neglect irrelevant to the problem factors $2n[\delta^{(3)}(0)]^2$ appearing when $m_q \to \infty$. But one has to take into consideration one detail of this relation, appearance of the Kronecker symbol $\delta_{\alpha\beta}$, which allows us to represent the final expression for the gluonic average energy as a trace. We have in the limit of large quark masses $m_q \to \infty$:

$$\begin{aligned} \langle 0|\ \hat{\mathcal{M}}^+_{\mathbf{xx'}}\ \hat{H}^g_0\ \hat{\mathcal{M}}_{\mathbf{xx'}}\ |0\rangle &\doteq \langle \hat{H}^g_0\rangle_0 + \\ &+ \frac{1}{2n}\langle Tr\left\{ \tilde{P}\left[\int d^3y \hat{P}^+_{xx'} I^j_a(x,x';\mathbf{y})\right] P\left[\hat{P}_{xx'} I^j_a(x,x';\mathbf{y})\right]\right\}\rangle_0 \quad (22) \end{aligned}$$

where $\langle \hat{H}^g_0\rangle_0$ is the gluonic field vacuum energy. Here the equality $\doteq$ means: equal up to the normalization factor $2n[\delta^{(3)}(0)]^2$. In Eq.(22) we used the limits $\hat{\psi}^{(+)}|0\rangle \to 0$, $\hat{\bar{\psi}}^{(+)}|0\rangle \to 0$, $m_q \to \infty$, where $\hat{\psi}^{(+)}$ ($\hat{\bar{\psi}}^{(+)}$) contains the quark (antiquark) annihilation operators, and omitted the self-energy term. The symbol $\tilde{P}$ means an antiordering (relative to P). Due to this circumstance the last term in Eq.(22) (i.e. the interaction energy of charges) takes the extremely simple form

$$\begin{aligned} V_{xx'} &= \frac{1}{2n} Tr\lambda^2_a \int_0^1 d\tau \int_0^1 d\tau' \dot{z}^j(\tau)\dot{z}^j(\tau')\delta(\mathbf{z}(\tau) - \mathbf{z}(\tau')) = \\ &= \frac{1}{2n} Tr\lambda^2_a \delta_\perp(0)|\mathbf{x}-\mathbf{x}'|, \quad (23) \end{aligned}$$

the last equality being valid for the straight line contour. We conclude that the string-like external gluonic field given by the P-exponent (18) leads to the linearly rising potential.

The interaction energy for every bilocal multistring configuration with loops is given by the sum of functions of distance with proper weights. The general form of the potential V in this case is

$$V(r) = \sum_v w_v E_v(r) \quad (24)$$

where

$$E_v(r) = \sum_i c_{vi} l_i \quad (25)$$

is the energy of a multiloop string configuration with v vertices, l_i being the lengths of strings connecting vertices. The weights w_v are positive and normalized; $\sum_v w_v = 1$, while some coefficients c_{vi} in (25) may be negative. This complicates the problem of evaluation or estimation of the quark potential.

4. We have shown that the exponential line integral (string) is the only fundamental structure of the gauge theories (i.e. all the gauge-invariant structures are built of strings and local tensors), and that in pure gluodynamics, the interaction of static external sources (heavy quarks) is given by the linear potential. All the investigation is based on the following (trivial) assumptions: in classical theory only gauge-invariant configurations of fields are physical; in quantum theory physical operators and state vectors are gauge-invariant. The gauge invariance manifests itself in constraints. The existence of the first-class constraints is the very feature of gauge theories that makes them so different from the standard non-gauge ones. Constraints do not contain time derivatives of canonical variables (in contrast with the equations of motion), they are conditions on instantaneous configurations of fields. As a result, the gauge field excitations may appear only in the form of some 1-dimensional structures with a more or less complex topology – depending on the rank of the gauge group; every "charge" is accompanied by a static external field. It is due to these external fields that the instantaneous interaction of static charged objects takes place; the Coulomb interaction in electrodynamics is a well-known example of such forces. In pure gluodynamics, this immediately leads to linearly rising potentials, i.e. to confinement. It implies that confinement is a pure kinematical effect appearing as a consequence of the gauge invariance of a theory, i.e. following from the existence of the (secondary) first-class constraints.

Unfortunately, the above analysis says little about the real interquark potential. We learned that in QCD it cannot be a simple linear function of distance. To find the potential, one has to sum contributions of all the multistring graphs (see (24)). This aspect of the string physics is usually omitted in the hadron model construction.

A standard tool in the study of confinement is the Wilson P-exponent (P_W, the Wilson loop [14]). There is a principal difference between P_W and the P-exponents $P_{xx'}$ used in the text. The Wilson gauge-invariant P-exponent emerges from the QCD Lagrangian for massive quarks, and its integration contour has time-like sectors, while here the path-ordered exponents $P_{xx'}$ are fundamental objects of gauge field theories irrespective of the masses of quarks and the orientation of the integration contour.

The point of view that strings in QCD are built of chromoelectric lines of force squeezed into a tube due to the special structure of vacuum

("the monopole - antimonopole vacuum" [15]) is rather popular among physicists [16]. We see that gauge theories do not need this hypothesis for getting string-like objects. They are inborn entities of the gauge field theories. We have seen that the existence of strings follows from the first principles. Nevertheless, the vacuum structure plays an important role in QCD, partly because the theory is not closed, and some its physical parameters specifying the hadron physics (like quark masses) depend on the ground state of the dynamical system.

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