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PARAMETERS OF LOW-ENERGY PHYSICS, RADIAL EXCITATIONS OF π, K, D MESONS AND QCD POTENTIAL

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Параметры низкоэнергетической физики, радиальные возбуждения π, K, D мезонов и потенциал КХД

Параметры низкоэнергетической физики π, K, D мезонов, массы их радиальных возбуждений, волновые функции и лептонные константы распадов $F_{\pi,\pi',K,K',D,D'}$ вычисляются в терминах фундаментальных параметров эффективного потенциала КХД в новой релятивистской потенциальной модели.

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Parameters of Low-Energy Physics, Radial Excitations of π, K, D Mesons and QCD Potential

The parameters of low-energy physics of π,K, D mesons masses of their radial excitations, wave functions, and leptonic decay constants $F_{\pi,\pi',K,K',D,D'}$ are calculated in terms of fundamental parameters of the QCD effective potential.

The investigation has been performed at the Laboratory of Theoretical Physics, JINR.

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1. Introduction

One of the main problems of QCD is the constructive description of low energy meson physics. There are two approaches to the solution of this problem: the relativistic local approximation of gluon interaction of the type of the Numbu-Jona-Lazinio model [1,2,3] (for the light quarkonia physics and chiral Lagrangians) and nonrelativistic instantaneous, but nonlocal, approximation of gluon interactions like the potential in QED (for description of heavy and light quarkonia spectroscopy [4-12]).

Recently, it has been supposed [3,13,14,15] that for unificalion of these opposite approaches it is enough to choose the time-axis of the nonrelativistic potential parallel to the total momentum of hadron: As the result, we get the new relativistic potential model (N.R.P.M.) with the relativistic invariant action

$$\begin{aligned} S_{eff} &= \int d^4x\bar{\psi}(x)(i\partial\!\!\!/ - m^0)\psi(x) \\ &- \frac{1}{2}\int d^4x d^4y \psi_{\beta_2}(y)\bar{\psi}_{\alpha_1}(x)[K(x-y)|\frac{x+y}{2})]_{\alpha_1\beta_1;\alpha_2\beta_2}\psi_{\beta_1}(x)\bar{\psi}_{\alpha_2}(y) \end{aligned} \quad (1)$$

and the "moving" potential kernel

$$K(z|X)_{\alpha_1\beta_1;\alpha_2\beta_2} = \eta\!\!\!/_{\alpha_1\beta_1}[V(z^{\perp})\delta(z\cdot\eta)]\eta\!\!\!/_{\alpha_2\beta_2}\ ; \quad \eta_\mu \sim \frac{1}{i}\frac{\partial}{\partial X_\mu} \quad (2)$$

N.R.P.M. is justified by the Heisenberg-Pauli minimal quantization of gauge theory [16] and the Markov-Yukava relativistic theory of bilocal fields ([13,14,15]). The separable approximation for N.R.P.M. with a short-range potential of light quarks leads to the Nambu-Jona-Lazinio model and chiral Lagrangian [3]. In the rest frame $\eta_\mu = (1,0,0,0)$ this model coincides with the nonrelativistic one [5-12] that describes the spontaneons symmetry breaking by the gap (Schwinger-Dyson) equation and the meson spectrum by the Salpeter equation in satisfactory agreement with experimental data.

N.R.P.M. can be considered as the lowest order of hadron perturbation theory [15] and it gives a possibility to find connection between the fundamental parameters of the quark potential (which are determinated from heavy quarkonia spectroscopy) and the parameters of low-energy physics like leptonic decay constant F_π.

Just this problem is discussed in the present paper, the purpose of which is to describe the static properties of pseudoscalar mesons π, K, D and their radial excitations in the framework of N.R.P.M.

In sect. 2, the Schwinger-Dyson (SD) equation with the Coulomb plus oscillator potential and nonzero current quark mass is solved. Sect. 3 devoted to the solution of the Bethe-Salpeter (BS) equation for pseudoscalar mesons and their radial excitations. In sect. 4 the leptonic decay constants of mesons are evaluated.

2. Quark self-energy

We consider the Coulomb plus oscillator potential

$$V(|\mathrm{p}|) = -\frac{4}{3}\left[\frac{4\pi\alpha_s}{p^2} + (2\pi)^3 V_0 \triangle_{\mathrm{p}}\delta(\mathrm{p})\right] \quad (3)$$

where α_s and V_0 are free parameters, and choose the momentum scale $(\frac{4}{3}V_0)^{1/3} = 1$ The Schwinger-Dyson equation for the model (1) has the form of two equation for the angle of the Foldy-Wouthuyzen transformation $\varphi(p)$ and for one-particl energy $E(p)$ [5-10]

$$\begin{aligned} E(p) = \quad & Z_p \cos\varphi(p) + Z_m m^0 \sin\varphi(p) \\ + \quad & \frac{1}{2}\int \frac{d^3q}{(2\pi)^3} V(\mid \mathbf{p}-\mathbf{q}\mid)\Big[\sin\varphi(q)\sin\varphi(p) + \hat{p}\hat{q}\cos\varphi(q)\cos\varphi(p)\Big] \end{aligned} \quad (4)$$

$$\begin{aligned} Z_p \sin\varphi(p) = Z_m m^0 \cos\varphi(p) + \frac{1}{2}\int \frac{d^3q}{(2\pi)^3} V(\mid \mathbf{p}-\mathbf{q}\mid)\Big[&\sin\varphi(q)\cos\varphi(p) \\ &-\hat{p}\hat{q}\cos\varphi(q)\sin\varphi(p)\Big] \end{aligned} \quad (5)$$

where Z and Z_m are the ultraviolet regulatization factors [5], m^0 is the current quark mass

$$Z = -\frac{4}{3}\int \frac{d^3q}{(2\pi)^3}\frac{4\pi\alpha_s}{\mid \mathbf{p}-\mathbf{q}\mid}\hat{p}\hat{q}\frac{q}{r(q)} \quad ,$$

$$Z_M = -\frac{4}{3}\int \frac{d^3q}{(2\pi)^3}\frac{4\pi\alpha_s}{\mid \mathbf{p}-\mathbf{q}\mid}\frac{m^0}{r(q)} \quad ; \qquad r(q) = \sqrt{q^2 + (m^0)^2}$$

Finally, we get the following equation for the angle $\varphi(p)$

$$\begin{aligned} F[\varphi(p)] + \frac{2\alpha_s}{3\pi}\int dq\frac{q}{p}\Big\{ \quad & l_1(p,q) \quad [\sin\varphi(q) - \frac{m^0}{r(q)}]\cos\varphi(p) \\ - \quad & l_2(p,q) \quad [\cos\varphi(q) - \frac{q}{r(q)}]\sin\varphi(p)\Big\} \end{aligned} \quad (6)$$

where

$$l_1(p,q) = \ln\mid\frac{p+q}{p-q}\mid \;, \quad l_2(p,q) = \frac{p^2+q^2}{2pq}l_1(p,q) - 1$$

$$F[\varphi(p)] = \varphi''(p) + \frac{2\varphi'(p)}{p} + \frac{\sin 2\varphi(p)}{p^2} - 2p\sin 2\varphi(p) + 2m^0\cos\varphi(p) \quad (7)$$

Equation (6) for the pure oscillator potential ($\alpha_s = 0$) takes the form of the differential equation

$$F[\varphi(p)] = 0 \quad (8)$$

with the external parameters m^0 , V_0. The solutions to this equation in the massless case (m^0) are obtained in ref [6],where the spontaneous breaking of chiral symmetry due to the rising potential is obtained. The massive case ($m^0 \neq 0$) is considered in refs[12,17], and it is shown that for heavy quark, $m^0 \geq (\frac{4}{3}V_0)^{1/3}$, the dynamical nature of the quark mass is negligible [17].

We have derived numerical solutions to the integro-differential equation (6) with the boundary conditions,

$$\varphi(0) = \frac{\pi}{2} \quad , \quad \varphi(\infty) = 0 \tag{9}$$

using the computational scheme developed in ref. [18]. This scheme consists the modification of the Newton iterations by combining the continuation over a parameter (m_0 or α_s) method. This scheme has a convenient algorithm for assignment of the initial conditions as functions of the external parameters (m_0 or α_s), due to a special choice of the iteration parameter as a step of the modified Newton iteration. So, in this scheme the solutions to problem (8),(9) are used as an input.

The numerical solutions to boundary problem (6),(9) and the quark energy for severals values of m^0 and α_s are shown in Figs 1 and 2. The values of free parameters m^0, V_0 and α_s belonging to the physical region will be defined from the solutions to the BS equation for mesons.

3. Spectrum of composite pseudoscalar mesons and their radial excitations

The BS equation for ${}^1S_0$ bound states of quark-antiquark pair (i.e. pseudoscalar mesons and their radial excitations) can be written in the following form [13]

$$ML_{\binom{2}{1}}(p) = E_T(p)L_{\binom{1}{2}}(p) - \int \frac{d^3q}{(2\pi)^3} V(|\mathbf{p}-\mathbf{q}|)\Big[c^{(\mp)}(p)c^{(\mp)}(q) + \hat{p}\hat{q}s^{(\mp)}(p)s^{(\mp)}(q)\Big]L_{\binom{1}{2}}(q) \tag{10}$$

where

$$c^{(\mp)} = \cos[\vartheta_{f_1}(p) \mp \vartheta_{f_2}(p)] \quad , \quad s^{(\mp)} = \sin[\vartheta_{f_1}(p) \mp \vartheta_{f_2}(p)] \ ,$$
$$\vartheta_f(p) = \frac{1}{2}\left[\frac{\pi}{2} - \varphi_f(p)\right] \ , \tag{11}$$

φ_f is the solution to the SD equation (6), and $E_T = E_{f_1} + E_{f_2}$ is the total energy of a quark-antiquark pair with flavors f_1 and f_2 ; M is the bound state energy (mass).

The functions $L_{\binom{1}{2}}$ satisfy the normalization condition

$$\int \frac{d^3q}{(2\pi)^3} L_1(q)L_2(q) = 1 \tag{12}$$

Before solving the equations we consider some its asymptotic properties. As it is shown in refs [5,13], equation (10) in the limit of zero current quark mass reduces to the SD equation yielding the Goldstone theorem for composite pseudoscalar mesons. The corresponding solutions are

$$L_1(p) \approx \frac{2\sin\varphi(p)}{F_\pi}\sqrt{\frac{N_c}{M_\pi}} \tag{13}$$

$$L_2(p) \approx \frac{4m^0}{M_\pi F_\pi}\sqrt{\frac{N_c}{M_\pi}} \tag{14}$$

where $N_c = 3$ is the colour number

$$\varphi = \frac{1}{2}(\varphi_{f_1} + \varphi_{f_2}) \quad , \quad m^0 = \frac{1}{2}(m^0_{f_1} + m^0_{f_2}) \tag{15}$$

In the other extremal limit, $m^0 \gg (\frac{4}{3}V_0)^{1/3}$, equation (10) takes the form of the Schrödinger equation

$$[M - 2r(p)]\tilde{L}_M(p) = \int \frac{d^3q}{(2\pi)^3} V(|\mathbf{p}-\mathbf{q}|)\tilde{L}_M(q) \tag{16}$$

where $r = (r_{f_1} + r_{f_2})/2$, $r_f = \sqrt{p^2 + m_f^2}$ and

$$\tilde{L}_M(p) = L_1(p) \simeq L_2(p) \tag{17}$$

Now consider a solution of equation (10) with potential (3). In this case the equation takes the form of integro-differential equations

$$U''_{\binom{1}{2}}(p) + \omega^{(\mp)}(p)U_{\binom{1}{2}}(p) + MU_{\binom{2}{1}}(p) + \alpha_s I^{(\mp)}_{\binom{1}{2}}(p) = 0 \tag{18}$$

where

$$\begin{aligned}
I^{(\mp)}_{\binom{1}{2}}(p) &= \int_0^\infty dq \mathcal{V}^{(\mp)}(p,q)U_{\binom{1}{2}}(q) \ , \\
\mathcal{V}^{(\mp)}(p,q) &= -\frac{1}{\pi}[l_1(p,q)c^{(\mp)}(p)c^{(\mp)}(q) + l_2(p,q)s^{(\mp)}(p)s^{(\mp)}(q)] \ , \\
\omega^{(\mp)}(p) &= -\{E_T(p) + [\varphi'^{(\mp)}(p)]^2 + \frac{2}{p^2}[s^{(\mp)}(p)]^2\} \ , \\
\varphi'^{(\mp)}(p) &= \frac{d}{dp}[\varphi_{f_1}(p) + \varphi_{f_2}(p)] \ , \quad U_{\binom{1}{2}}(p) = pL_{\binom{1}{2}}(p)
\end{aligned} \tag{19}$$

The boundary conditions are

$$U_{\binom{1}{2}}(0) = U_{\binom{1}{2}}(\infty) = 0 \tag{20}$$

The boundary problem (12),(18)-(20) in the case of pure oscillator potential (α_s for massless ($m^0_{f_1} = m^0_{f_2} = 0$) and massive ($m^0_{f_1} \neq 0$, $m^0_{f_2} \neq 0$) quarks composing a meson is considered in refs [6,12]. The main result of these works is the foundation of a large gap between masses of π and ρ mesons due to the effects of dynamical symmetry breakdown, without an additional spin-spin interaction

We have solved equations (12),(18)-(20) using the same computational scheme applied to the SD equation. The procedure is described in detail in ref [19]. In Fig.3, the dependence of the eigenvalues (M) on a current quark mass is shown.

The free parameters (m^0, V_0 and α_s) are fixed using the experimental values of the masses of π, K and D mesons (respectively , for $m_u^0 = m_d^0, m_s^0$ and m_c^0) , the determinations of ref. [6] (for V_0) and heavy quarkonia spectroscopy (for α_s) . They are

$$m_u^0 = m_d^0 \; = \; 0.021(\frac{4}{3}V_0)^{1/3} \;\; , \quad m_s^0 = 0.20(\frac{4}{3}V_0)^{1/3} \; , \quad m_c^0 = 5.0(\frac{4}{3}V_0)^{1/3} \; ,$$
$$(\frac{4}{3}V_0)^{1/3} = 289MeV \quad , \quad \alpha_s = 0.2 \tag{21}$$

The solutions to the BS equation with (21) give the masses and wave functions of bare radial excitations, As the masses of π, K and D mesons are fitted, in Table 1 we quoted only the masses of radial excited mesons (we restricted ourselves to the first excited levels). The wave functions of all the mesons are shown in Figs 3-9. In order to test the results we evaluated the leptonic decay constants of these mesons and compared them with the available data.

4. Leptonic decay constants of the mesons.

For the leptonic decay constants of composite pseudoscalar mesons in ref [13] the following expression is obtained :

$$F_M = \sqrt{\frac{4N_c}{M}} \int \frac{d^3q}{(2\pi)^3} L_2(q) \sin\varphi(q) \tag{22}$$

where φ is defined in (15). This equation is more accurate than ones used in literature. Indeed, in the limit of heavy quarks, $m^0 \gg (\frac{4}{3}V_0)^{1/3}$, from (5),(12) and (17) we have

$$\sin\varphi(p) \approx \frac{m^0}{r(p)} \quad , \quad L_1(p) \approx L_2(p) \tag{23}$$

Substituting (23) into (22) we obtain the nonrelativistic definition of the decay constants,

$$F_M(m^0 \sim \infty) \approx 2\sqrt{\frac{N_c}{M}} \int \frac{d^3q}{(2\pi)^3} \tilde{L}_M(q) \frac{m^0}{r(q)} \tag{24}$$

Expanding this expression in q/m^0 one can see that eq. (24) up to $O\Big((q/m^0)^2\Big)$ coincides with the known definition [20]

$$F_M = \sqrt{\frac{N_c}{\mu}} L_M(r = m_Q^{-1}) \tag{25}$$

where $\mu = m_q m_Q/(m_q + m_Q)$, m_q and m_Q are the masses of constituent quarks, and

$$\tilde{L}_M(r = m_Q^{-1}) = \int \frac{d^3q}{(2\pi)^3} e^{iq\bar{r}/m_Q} \tilde{L}_M(q)$$

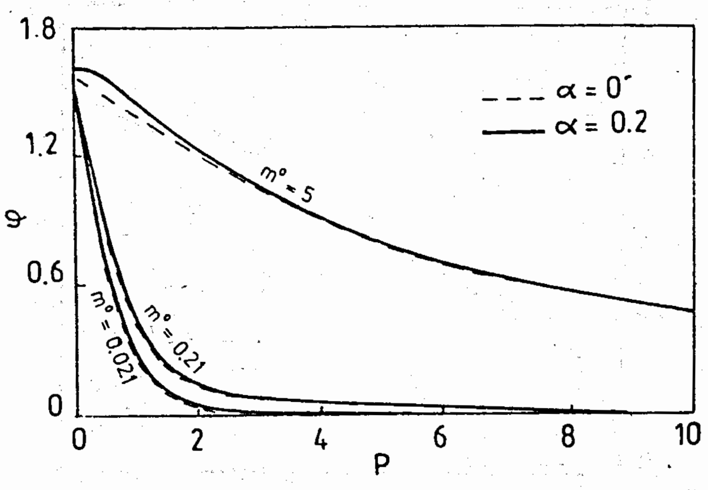

Fig. 1. The solutions SD equation (6) for quarks with the current masses.

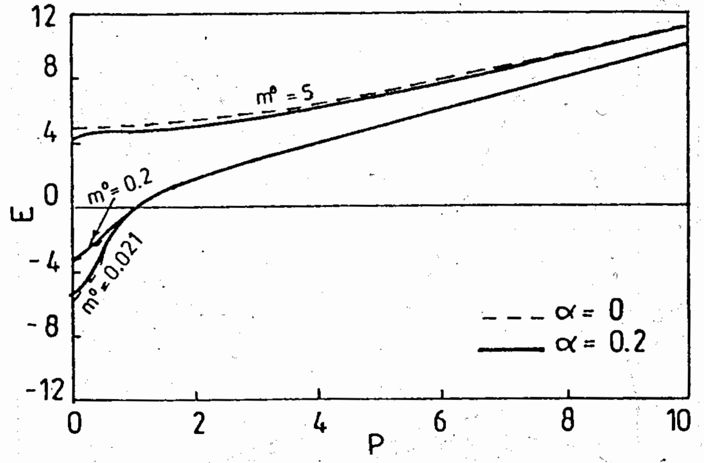

Fig. 2. The constituent quark energy (4) corresponding to the solutions of eq. (6).

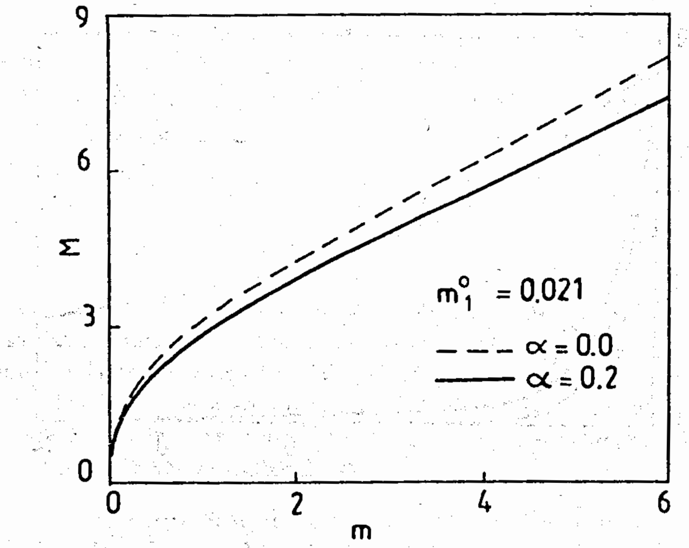

Fig. 3. The dependence of meson mass on the mass of second quark.

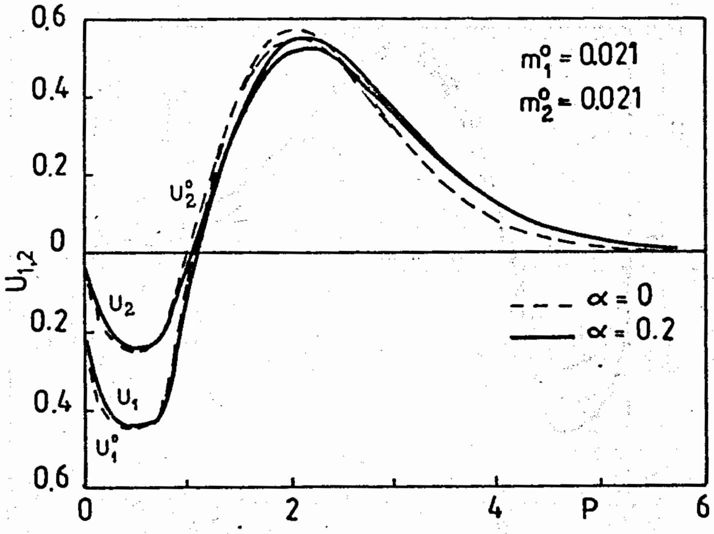

Fig. 5. The solution to BS equation for radial excitation of π meson.

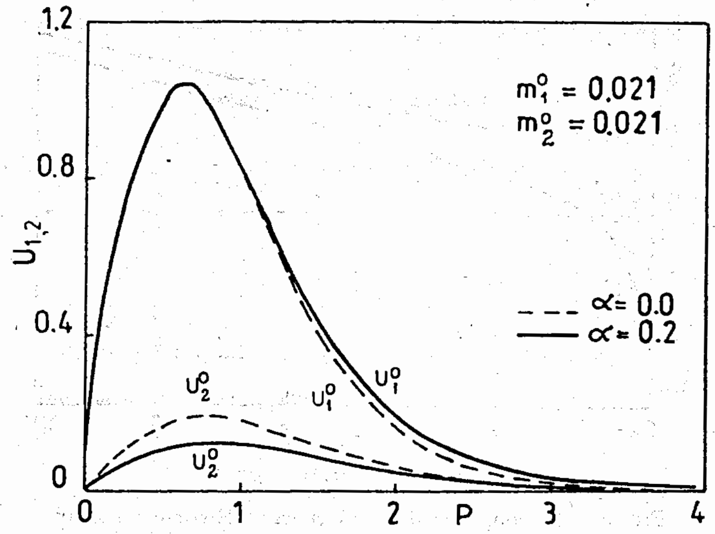

Fig. 4. The solution to BS equation for π meson.

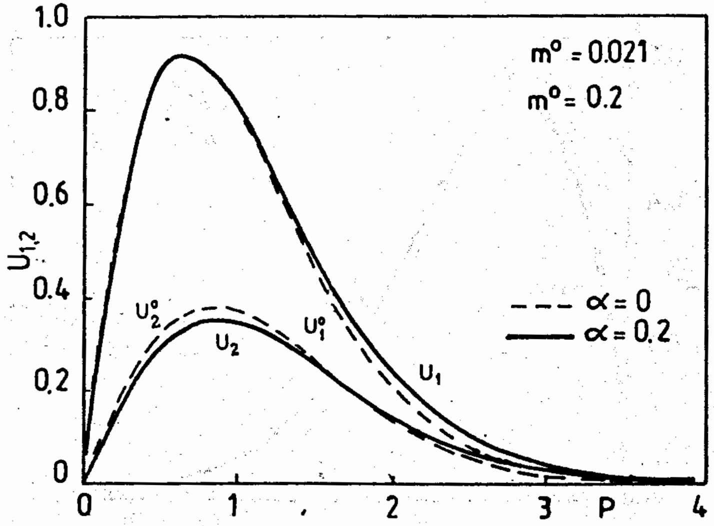

Fig. 6. The solution to BS equation for K meson.

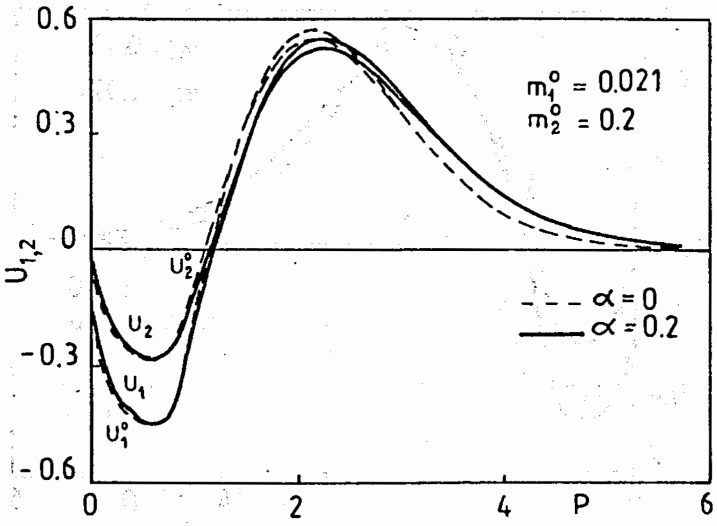

Fig. 7. Wave function of K meson radial excitation.

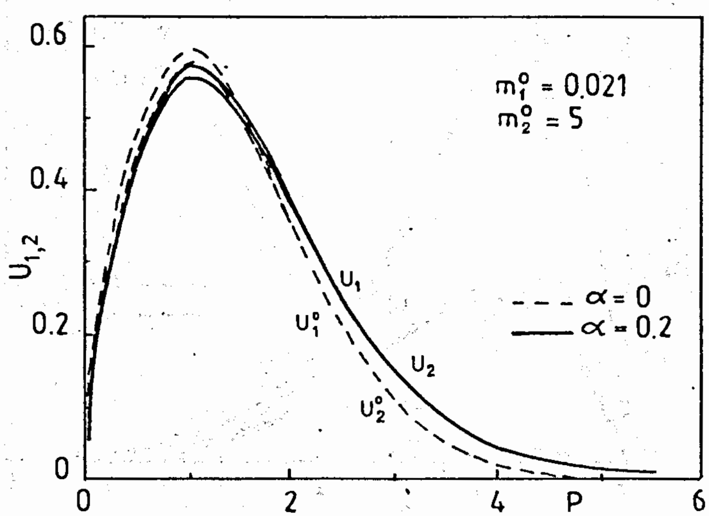

Fig. 8. Wave function of D meson.

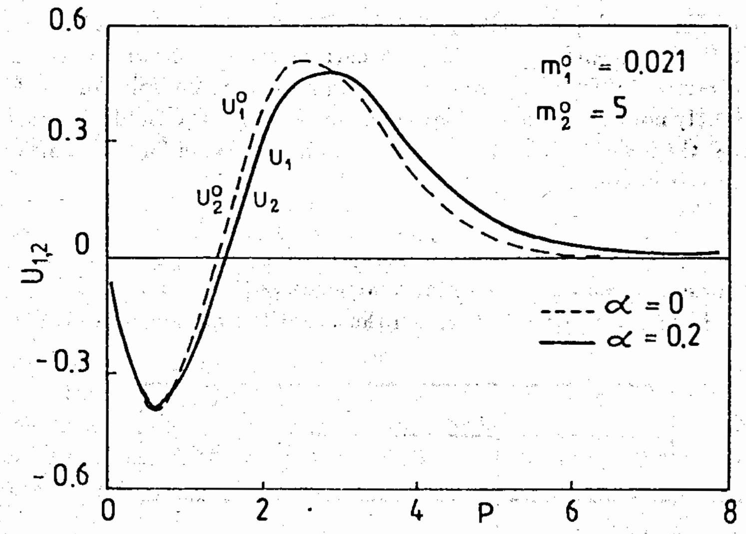

Fig. 9. Wave function of D meson radial excitation.

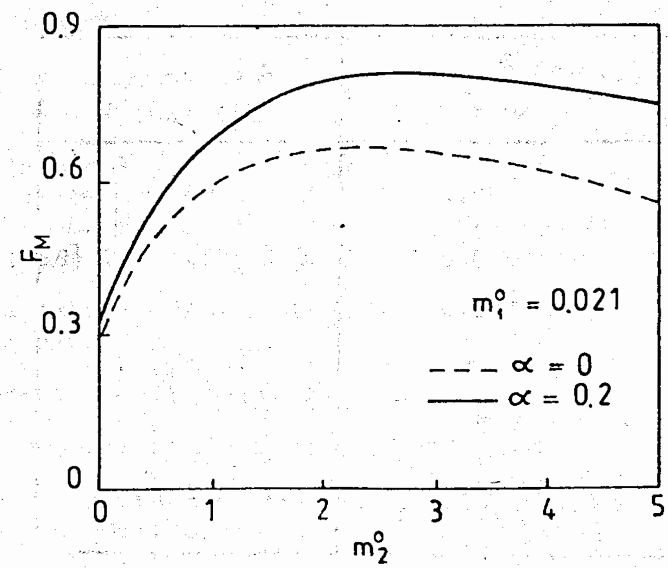

Fig. 10. The dependence of F_M on second quark mass.

The values of F_M calculated by using (21) and (22) for the mesons are quoted in Table 2. In Fig.10 the dependence of F_M on a current mass is shown. It should be noted that to correct evalution of F_M requires carefulness since the solution to the BS equation must satisfy not only normalization condition (12) but the Goldstone theorem (13) too. Perhaps, the neglect of just this requirement is the reason for too small values of F_π obtained in refs [6-10].

Table 1. The masses of meson (for current quark masses (for current quark masses $m_u^0 = 6.1, m_s^0 = 5\,8, m_c^0 = 1445$). All the quantities are given in MeV's.

M	a_s		Exp MeV
	0	0.2	
$M_{\pi'}$	1606.2	1483.6	1300
$M_{K'}$	1653.2	1520.	1460
$M_{D'}$	2753.	2490.8	

Table 2. Leptonic decay constants.

F_M	$(4V_0/3)^{1/3}$ = 289 MeV .6	Exp. or models
F_π	97.5	93
$F_{\pi'}$	3.6	6.2 [21], 4.7 [22
F_K	122.6	114
$F_{K'}$	21	51 [22]
F_D	216.4	
$F_{D'}$	208,	

5. Conclusions

We present here the result of the calculation of the Schwinger-Dyson and Bethe-Salpeter equations for model (1) with the potential (3). It is shown that in the region of heavy quarks, $m^0 \geq (\frac{4}{3}V_0)^{1/3}$, the dynamical nature of the quark inside the hadron becomes negligible, and the dependence of the meson mass on quark mass tends to linear law (see Figs 1-3). In this region, the effect of the Coulomb potential is considerable, unlike the region of light quarks. From Tables 1 and 2 we see that the calculated values of the meson masses and decay constants on the qualitativel level are in agreement with the available data. The value of $F_{\pi'}$ is close to the results of the sum rulles method ($F_{\pi'} = 6.2 \pm 2MeV$ [21] and $4.7MeV$ [22]) whereas $F_{K'}$ is smaller than $F_{K'} = 51MeV$ obtained in the sum rulles approach [22].

Thus, the potential model (1) gives good reproduction of the static properties of both the heavy and light quarkonia. Moreover, this approach yields new predictions for radial excited mesons which can be tested in current or planned experiments, for instance, charm-tau factories.

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