91-249

сообщения
объединенного
института
ядерных
исследований
дубна

E2-91-249

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THE QUESTION IS: ARE FAST-MOVING SCALES CONTRACTED OR ELONGATED?

1991

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Спрашивается: сокращаются или удлиняются быстродвижущиеся масштабы?

Традиционное определение длины быстродвижущегося масштаба и концепция релятивистской длины рассмотрены с точки зрения условия релятивистской ковариантности. Показано, что этому условию удовлетворяет последняя концепция, тогда как традиционное определение ему противоречит ("синхронная" длина — не есть 4-вектор). Подчеркивается, что световые и запаздывающие расстояния и введенная фактически на их основе релятивистская длина служат базисом пространственно-временной картины локационной формулировки теории относительности. Следствием этого подхода является увеличение (а не сокращение) продольных размеров релятивистских объектов. Отмечается, что использование локационной длины (вместо "мгновенной") в известной трактовке опыта Майкельсона-Морли приводит к "формуле удлинения" для продольного плеча интерферометра.

Работа выполнена в Лаборатории высоких энергий ОИЯИ.

Сообщение Объединенного института ядерных исследований. Дубна 1991

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The Question is: Are Fast-Moving Scales Contracted or Elongated?

The traditional definition of the length of a fast-moving scale and the concept of relativistic length are considered from the point of view of the condition of relativistic covariance.

It is shown that the latter concept satisfies this condition whereas the traditional definition is in contrast with it ("synchronous length" is not a 4-vector). It is stressed that the light and retarded distances and the relativistic length introduced in fact on their base serve as the basis for the space-time picture of the radar formulation of relativistic theory. The consequence of this approach is lengthening (and not contraction) of longitudinal sizes of relativistic objects. It is noted that the use of radar length (instead of "instantaneous" one) in the known interpretation of the Michelson-Morley experiment leads to the elongation formula for a longitudinal arm of the interferometer.

The investigation has been performed at the Laboratory of High Energies, JINR.

Communication of the Joint Institute for Nuclear Research. Dubna 1991

1. INTRODUCTION

Relativity theory is apparently the most fundamental one of all existing physical theories. According to Dirac's figure of speech:[1] "...Lorentz transformations dominate in physics."

Relativity theory reflects the remarkable fact that nature laws can be formulated irrespective of a concrete (inertial) reference system (the principle of relativity). This concerns physical notions as well. Such a revolutionary theory of our century as quantum mechanics and then quantum field theory take into account requirements of relativity theory.

Physical laws are written down by mathematical formulae for quantities which can be finally expressed through space and time coordinated. It can be said that the space-time structure (picture) serves as the basis for physical theories. Interconnection of the corresponding pictures in different inertial reference systems is described by Lorentz transformations.

Relativity theory has established that the united space-time (Minkowski's space) is physical reality. All physical quantities are geometrical objects in this space (for example, 4-vectors).

The formulae written down through 4-quantities provide an automatic fulfilment of the mathematical covariance condition as a logic demand of non-contradiction of equations written in different coordinate systems.

Further taking into account the principle of relativity (as a physical principle) allows one to say already about relativistic covariance.

Understanding has come recently that there are as a matter of fact two approaches to the treatment of the space-time structure of relativity theory[2]. Their difference is associated with the space part. A new "radar" approach only deals with light and retarded distances. The concept of relativistic length (CRL)[3] based on the radar method of distance measurement can be thought to be its most striking expression.

Below we shall discuss these questions in detail and, in particular, show that just the CRL is in close interconnection with the logic structure of relativity theory itself. We shall pay our main attention to comparison of the generally accepted (Einstein's) definition of moving scale length and CRL: for example, their attitude to the demand of relativistic covariance. Moreover, we shall touch upon the interpretation of the Michelson-Morley experiment.

2. RELATIVISTIC LENGTH AS A "PHYSICAL BASE" OR RADAR FORMULATION

As is known, the generally accepted (Einstein's) definition of relativity theory operates with instantaneous (or synchronous) distances whereas radar formulation[4] deals with light or detarded distances directly observed in the experiment. Relativistic (or radar) length plays a key role in this approach. Recall that the relativistic length (the length of a fast-moving rod) is called the half-sum of distances covered by a light signal in direct and opposite directions along the rod. Let for simplicity the rod be oriented and move in the direction of the x-axis (from left to right) with velocity v. The signal is sent at the instant of flight of the left end. The light achieving the right end is reflected there and goes back to the left end. The 4-vector describing the process of light propagation, when it is going in the same direction with the rod ("run down" its right end) takes the form:

$$\ell_s^i \; [(1+\beta)\,\ell^*\gamma,\ (1+\beta)\,\ell^*\gamma,\ 0,\ 0]. \qquad (1)$$

Here ℓ^* is the length of the rod at rest, $\beta = v/c$ and γ is the Lorentz-factor. When the light signal (after reflection) goes in the direction opposite to the motion of the rod (to meet the left end), we have for the corresponding 4-vector

$$\ell_0^i \; [(1-\beta)\,\ell^*\gamma/c,\ -(1-\beta)\,\ell^*\gamma,\ 0,\ 0]. \qquad (2)$$

As a result, for the 4-vector of relativistic length $\ell_r^i = (\ell_s^i - \ell_0^i)/2$ we find

$$\ell_r^i \; (\beta\,\ell^*\gamma/c,\ \ell^*\gamma,\ 0,\ 0). \qquad (3)$$

In this case in the rest system it is obvious that

$$\ell_r^{i\,*} \; (0,\ \ell^*,\ 0,\ 0). \qquad (4)$$

On the basis of (3) we get for relativistic length $(\ell_r \equiv \ell_r^1)$.

$$\ell_r = \ell^*\gamma. \qquad (5)$$

It is evident that at $\beta \to 1$ the relativistic length will be simply defined by the half of ℓ_s^1. It should be stressed that the quantities ℓ_s^1 and ℓ_0^1 define initially distances between points taken at different time instants. They correspond exactly to two most typical modifications of retarded distances[5] (otherwise, retarded and advanced distances). Thus, one can say that CRL is an organic consequence of electrodynamics. However, the other could not be since the concept leans upon the radar method of distance measurement.

3. LORENTZ-COVARIANCE OF THE DEFINITION OF RELATIVISTIC LENGTH

3.1. Relativistic covariant definition is such a definition which can be formulated with the help of notions not concerned with a concrete reference system. Measuring procedures, which can be finally reduced to the set of simple events, are kept is mind here.

Let us consider from this viewpoint the generally accepted Einstein definition of the length ℓ_E of a moving scale (rod). As is known, it supposes[6] simultaneous ($\Delta t = 0$) position notches of the rod ends*. It is evident that in all other reference systems these two events (notches) will be already non-simultaneous. Therefore, at least one of them cannot be used in another reference system, and it is necessary to make one more "own" notch. But this means a direct connection of the generally accepted definition with a specific reference system what is incompatible with the principle of relativity**.

On the other hand, events of sending and arrival of a light signal in the radar method of relativistic length measurement can be used in any other reference system.

Thus, the generally accepted definition does not satisfy the covariance condition whereas the relativistic length is a covariante quantity.

3.2. Geometric scale representation. At first sight it seems that this question is solved very simply. However, relativity theory has established that as a matter of fact a material rod represents physically not a spatial object and a space-time configuration. This two-dimentional configuration is the world space-like strip in Minkowski's four-dimentional space. In the simplest case of Minkowski's flat space presented in the figure the world scale strip is vertical.

As already noted, in the frame of geometric representation the relativistic length is the value of a spatial part of the half-difference (X_r^i) of two 2-vectors (in this case) describing the processes of light propagation in the direct ($X_{t_1 e}$)

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* One can say that we deal with "synchronous length".

** Compare with Fermi's remark[7] that the usual approach (based on the condition t = const whence $\Delta t = 0$) evidently contradicts the principle of relativity.

and opposite (X_{et_2}) directions along the rod. In the figure these lines $t_1 e$ and et_2, t_1 and t_2 are the moments of the sending and receiving a light signal. In the S*-system we have

$$X^{i\,*}_{t_1 e}\,(\ell^*/c, \ell^*), \quad X^{i\,*}_{et_2}\,(\ell^*/c, -\ell^*) \cdot \qquad (6a,b)$$

where ℓ^* is the proper length of the rod (at rest). As a result, we find for the quantity X^i_r

$$X^{i*}_r\ (0, \ell^*). \qquad (4')$$

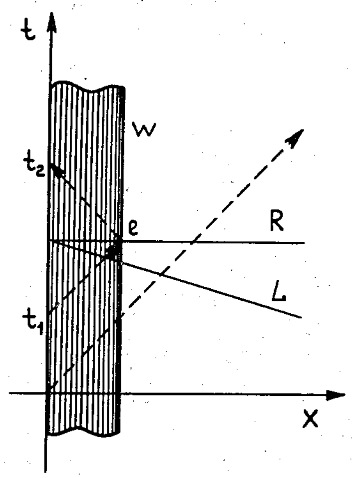

Fig. World strip of a rod.

In other words, the relativistic length corresponds to the normal section R of the world rod strip*.

Thus, there is a simple connection between the mutual positions of the world strip W and the line R being a geometric set of events which satisfy the definition of light simultaneity[8] relative to W: in any reference system the world lines W and the straight line R have equal Euclidean angles with the world line of a light signal.

But if the normal section R depends only on the world strip of the space W and not on the choice of reference system, this means that the definition of relativistic length is really covariant. Moreover, we have a complete analogy with the definition of relativistic time. On the other hand, to the traditional definition of fast-moving scale length there corresponds a number of sections L, each of which is defined by "its" reference system what is obvious.

3.3. "Synchronous length" is not a 4-vector[9]. The question on the length of a fast-moving rod by itself is not very simple as the distance between its ends depends on the fact what instants its positions are fixed at. However, the indicated demand that the set of difference coordinates* should be a 4-vector imposes rigid restrictions on the choice of definition of the notion of relativistic length.

* Here we have a complete analogy with the definition of perpendicularity in Euclidean geometry which is covariant with respect to linear transformations.

** Being a consequance of the measuring procedure.

As to the generally accepted definition of this quantity, it supposes a simultaneous (in the present reference system) notch of the position of the rod ends. For a moving (in a system S_1) rod with velocity v_1 along the x-axes we have

$$X_{E1}\ (0, \ell_{E1}, 0, 0), \quad s^2 = -\ell_{E1}^2 = -\ell^{*2}\,(1-\beta_1^2). \qquad (7a,b)$$

Here ℓ_{E1} is the rod length in motion; ℓ^*, its length at rest; and s, an interval. From the point of view of another S_2-system we find for this rod

$$X_{E2}\ (0, \ell_{E2}, 0, 0), \quad s^2 = -\ell_{E2}^2 = -\ell^{*2}\,(1-\beta_2^2). \qquad (8a,b)$$

Thus, the quantities s^2 calculated in these two cases are found to be unequal, However, if X_{E1} and X_{E2} are 4-vectors, then their squares have to be invariable when passing from one inertial reference system (S_1) to the other (S_2). The violation of this demand of invariance means that the set of difference coordinates X_{E1} and X_{E2} is not a 4-vector. In other words, the considered definition does not satisfy the covariance condition. For the relativistic length instead of (7) and (8) we have respectively

$$X_{r1}\ (\beta_1\,\ell_{r1}/c, \ell_{r1}, 0, 0), \quad s^2 = -\ell_{r1}^2\ (1-\beta_1^2), \qquad (9a,b)$$

$$X_{r2}\ (\beta_2\,\ell_{r2}/c, \ell_{r2}, 0, 0), \quad s^2 = -\ell_{r2}^2\ (1-\beta_2^2). \qquad (10a,b)$$

Taking into account "elongation formula" (5), we find easily that s^2 is invariant and consequently X_{r1} and X_{r2} are 4-vectors.

4. UNIQUENESS OF THE DEFINITION OF RELATIVISTIC LENGTH

4.1. The existing ambiguity of definition of the length of a moving scale. If we are distracted from the demand of the principle of relativity (relativistic covariance), we thus have two possible values of the length of a moving scale. In so doing, the relativistic length can be called "asynchronous" since $\Delta t \neq 0$ for it is in contradistinction to the traditional "synchronous length". But then there arises a natural question: why is the condition $\Delta t = 0$ usually preferred over, say, the condition $\Delta t = 1$ (which leads to the third definition) and so on?

On the other hand, the quantity having the meaning of the length of a moving rod can be introduced in the framework of the two existing definitions. These are mean geometric (proportional) quantities ℓ_E and ℓ_r

$$\ell_p = \sqrt{\ell_E \ell_r} = \ell^*. \qquad (11)$$

Thus, we return to the initial classical result: the rod length ℓ_p does not change due to motion. The careless treatment of the principle of relativity can lead to such a thing.

4.2. Rationality (simplicity) of the notion. This criterion which exact formulation presents great difficulties plays an important role in physics. It concerns what can be briefly though not quite clearly called "nature" or "logical simplicity" of the notion. In conformity with the case our interest this can mean the following.

Since the material rod is charecterized by a space-like vector, its time component is equal to zero in the only reference system. On the other hand, the proper reference system is also chosen among the set of inertial systems. Therefore, for reasons of rationality (simplicity) the definition should be chosen so that these systems might coincide/10/. The definition of relativistic length just satisfies this condition. At the same time we are in fact guided by the experiment which serves as a natural basis and CRL.

On the other hand, in the framework of the traditional definition it is nesessary to place and to synchronize the set of clocks. Then it is necessary to define at what points the beginning and the end of the measured rod (train) are at a definite instant of time. Further, the distance between these two points is measured by putting a standard scale. Of course, with this aim we use a radar now, since we return again to the definition of relativistic length. In so doing we do not need the set of unnecessary clocks and tiring procedure of their synchronization. Moreover, the main of applications of the relativistic theory is microworld where measuring procedures underlying the traditional definition are simply impossible. At the same time interactions occuring in microworld physics have a radar character. Therefore, the effective space sizes characterizing these interactions have necessarily to be defined just by relativistic length/11, 12/.

Here we want to dwell upon Goffman's discourses/13/ are based on that for light the fraction (covered distance)/(spent time) must have the same value independently of the reference system. Then he discusses as follows. Your clock is by a factor of γ slower than mine. However, I know that the value of the indicated fraction is equal to c. Consequently, if the "spent time" measured by your clock is γ times smaller, then the value obtained by me, I have to find out how many times your passed the distance, is smaller. If it is taken into account correctly now that the indicated clock is connected with the length at rest and I, an observer, measure the length in motion, then we come just to formula (5) but not to the Fitzgerald-Lorentz contraction as asserted in/13/.

5. "ELONGATION FORMULA" AS POSSIBLE RESULT OF TREATMENT OF MICHELSON-MORLEY EXPERIMENT

Considering usually this experiment (see, e.g./14/), the time of light propagation along the longitudinal arm of the interferometer is calculated. When the light propagates in the direction of interferometer motion, we have for the corresponding time t_1

$$ct_1 = \ell + vt_1 \quad \text{or} \quad t_1 = \frac{\ell}{c-v} = \frac{\ell}{c_1}. \qquad (12)$$

On its way back (for time t_2) the light travels only $\ell - vt_2$ and therefore we have now

$$ct_2 = \ell - vt_2 \quad \text{or} \quad t_2 = \frac{\ell}{c+v} = \frac{\ell}{c_2}. \qquad (13)$$

Then the total time is

$$t_{\parallel} = t_1 + t_2 = \frac{2\,\ell/c}{1 - v^2/c^2}. \qquad (14)$$

Calculating the time of light propagation $t_\perp$ along another (perpendicular) arm, one considers that the light goes along the hypothenuse of a right-angled triangle. From here is follows that

$$\left(\frac{ct_\perp}{2}\right)^2 = \ell^2 + \left(\frac{vt_\perp}{2}\right)^2 \qquad (15)$$

and

$$t_\perp = \frac{2\,\ell/c}{\sqrt{1 - v^2/c^2}}. \qquad (15')$$

To explain a negative result of the experiment indicating the equality of $t_\parallel$ and $t_\perp$, the hapothesis was proposed by Fitzgerald/15/ and Lorentz/16/ according to which longitudinal sizes of material bodies change, i.e. $\ell_\parallel$ should be written in formula (14) instead of ℓ. As a result, we have

$$\ell_\parallel = \ell\sqrt{1 - v^2/c^2} \quad \text{(contraction formula)}. \qquad (16)$$

Here, however, we want to pay attention to the following. In fact, when deriving formulae (12) and (13) it is implicitly supposed that the quantity ℓ (and then $\ell_\parallel$) is an "instantaneous length". If we use the radar method of distance measurement, we shall get another result to define the notion of the length of a moving rod.

As is known, according to this method, the rod length ℓ_r is defined by the half-sum of distances covered by a light (radio) signal along the rod (forward and backward)

$$\ell_r = \frac{1}{2} (\ell_f + \ell_b). \tag{17}$$

For the rod at rest distances ℓ_f and ℓ_b are equal. However, in the case of a moving rod (longitudinal arm) the signal travels a longer distance in the direction of motion. The difference of these distances (or the way of a glass plate in the discussed experiment) is $vt_{\parallel}$. If a certain analogy between the radar (as a source of light signals) and the charge (as a source of electromagnetic waves) is taken into account, the indicated distances can be rightfully treated as retarded (ℓ_{ret}) and advanced (ℓ_{adv}) ones, respectively[5].

Taking into account this, the radar length conformably already to the moving rod, is defined by the expression

$$\ell_r = \frac{1}{2} (\ell_{ret} + \ell_{adv}) = \frac{1}{2} ct_{\parallel} \tag{18}$$

and

$$\ell_{ret} - \ell_{adv} = vt_{\parallel} \tag{19}$$

Whence it follws that

$$\ell_{ret} = (1 + \frac{v}{c}) \ell_r \tag{20}$$

and

$$\ell_{adv} = (1 - \frac{v}{c}) \ell_r \tag{21}$$

On the other hand, it is obvious that $\ell_{ret} = ct_1$ and $\ell_{adv} = ct_2$, and so taking (20) and (21) instead of (12) and (13), we have

$$t_1 = \frac{\ell_r}{c^2} (c + v) = \frac{\ell_r}{c_1} \gamma^{-2}, \tag{22}$$

$$t_2 = \frac{\ell_r}{c^2} (c - v) = \frac{\ell_r}{c_2} \gamma^{-2}, \tag{23}$$

and

$$t_{\parallel} = 2 \frac{\ell_r}{c}. \tag{24}$$

Surely, for $c \to \infty$ "the nonsimultaneity degree"

$$\Delta t = \frac{1}{2} (t_1 - t_2) = \frac{v}{c^2} \ell_r \to 0 \tag{25}$$

and both definitions are practically coincident. However, taking into account the terms of the order v^2/c^2, their difference becomes essential. Equating (15′) and (24) (instead of (14)), we get the "elongation formula" (5).

6. CONCLUSION

The application of the relativistic covariance condition and, in particular the use of geometric scale representation, allows one to make an unambiguous choice between the two existing definitions of the moving scale length in favour of CRL. From the viewpoint of rationality the analysis of the indicated ambiguity of the notion also leads to the only definition, namely the definition of relativistic length. The relativistic (radar) length is expressed by light or retarded distances. All indicated quantities serve as the basis for the formulation of relativity theory. In the frame of this approach we have the increase of longitudinal sizes of relativistic objects.

The use of radar length (instead of "instantaneous length") in the known treatment of the Michelson-Morley experiment leads to the "elongation formula" for a longitudinal arm of the interferometer.

REFERENCES

1. Dirac P.A.M. — In: Some Strangeness in the Proportion: A Centennial Symposium to Celebrate the Achievements of Albert Einstein, Ed. Harry Woolf L. et al., Addison-Wesley Publ. Co., 1980, p.110.
2. Strel'tsov V.N. — JINR Comm., P2-90-426, Dubna, 1990.
3. Idem. Found. Phys., 1976, v.6, p.293.
4. Idem. JINR Comm. P2-90-484, Dubna, 1990.
5. Idem. JINR Comm. P2-89-772, Dubna, 1989.
6. Einstein A. — Ann. Phys., 1905, v.17, p.891.
7. Fermi E. — Z.Phys., 1922, v.23, p.340.
8. Burke W.L. Spacetime, Geometry, Cosmology, Univ.Sci.Books Mill Valley, California, 1980, §8.

9. Strel'tsov V.N. — JINR Comm. P2-84-843, Dubna, 1984.
10. Idem. JINR Comm. P2-10912, Dubna, 1977.
11. Idem. JINR Comm. P2-87-877, Dubna, 1987.
12. Idem. Particle and Nuclei, 1991, v.22, p.1129.
13. Hoffman B. — Relativity and Its Roots. Sci. Am. Books, NY, 1983, ch.4.
14. Feinman R.P., Leighton R.B., Sands M. — The Feynman Lectures on Physics, Addison-Wesley Publ. Co.,1963, v.1, p.15-3.
15. Fitzgerald G.F. — Science, 1889, v.13, p.390.
16. Lorentz H.A. — Versl.K.Akad.Wet., 1892, v.1, p.74.

Received by Publishing Department
on June 3, 1991.