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NONRENORMALIZABILITY OF THE MASSIVE N = 2 SUPER-YANG-MILLS THEORY

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Неперенормируемость массивной N = 2 супер-Янг-Миллсовой теории

Исследуется массивная N = 2 суперсимметричная теория Янга-Миллса. Использован формализм Штюкельберга в N = 2 гармоническом суперпространстве. Установлено, что неперенормируемость теории проявляется, начиная с четвертого порядка теории возмущения. Она обусловлена неперенормируемостью нелинейной сигма модели, образованной штюкельберговскими полями.

Работа выполнена в Лаборатории теоретической физики ОИЯИ.

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Nonrenormalizability of the Massive N = 2 Super-Yang-Mills Theory

The massive N = 2 supersymmetric Yang-Mills theory is investigated. Its nonrenormalizability is revealed starting from the fourth order of the perturbation theory. The N = 2 harmonic superspace approach and the Stueckelberg-like formalism are used. The Stueckelberg fields form some nonlinear sigma model. Nonrenormalizability of the latter produces nonrenormalizability of the N = 2 supersymmetric Yang-Mills theory.

The investigation has been performed at the Laboratory of Theoretical Physics, JINR.

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1 Introduction

A lot of papers [1-7] were devoted to the investigation of the massive Yang-Mills theories. It was shown there that the mass term $\mathrm{Tr}(m^2 A_\mu^2)$ makes the theory either nonrenormalizable or nonunitary.

As is well-known, supersymmetry considerably improves the ultra-violet behavior of theories. From this point of view, it seems interesting to know the effect of the mass within the $N = 2$ super-Yang-Mills theory [8]. In the present paper we consider the massive Yang-Mills theory in the $N = 2$ harmonic superspace approach. As in [1-6], we use the Stueckelberg formalism which allows us to work with a transversal propagator for the Yang-Mills field without breaking the S-matrix unitarity. We give the appropriate Feynman rules and obtain the following result: The massive $N = 2$ super-Yang-Mills theory is nonrenormalizable. The stumbling-block of nonrenormalizability is the self-interaction of the Stueckelberg fields. This self-interaction is characterized by a coupling constant λ^2 of dimension $[m^{-2}]$ and form a nonlinear σ-model. In the case of the ordinary massive Yang-Mills theory this σ-model is well-known [6]:

$$\lambda^2 \mathrm{Tr} \int d^4x (e^{i\omega}\partial_\mu e^{-i\omega})(e^{i\omega}\partial^\mu e^{-i\omega}). \tag{1.1}$$

In our example the $N = 2$ supersymmetric extension of this σ-model arises that differs from (1.1) by a change of the space-time derivative ∂_μ to the harmonic one D^{++}, see below eq.(4.15).

Nonrenormalizabililty of $N = 2$ Yang-Mills theory is revealed apparently in the fourth order of perturbation theory.

It is proved that in the $N = 2$ supersymmetric Yang-Mills theory there is no renormalization of the mass of vector field (in the ordinary Yang-Mills theory ($N = 0$)[6,7] this mass has to be renormalized).

The plan of paper is as follows: In Sec.2 we begin with a brief review of the $N = 2$ harmonic superspace and then the Stueckelberg formalism is developed for the massive $N = 2$ super-Yang-Mills theory. In Sec.3 we give the Feynman rules for our theory. In Sec.4 quantum calculations are performed. The results obtained are compared to those of the ordinary massive Yang-Mills theory [6,7]. Our notation and conventions in this paper are mostly the same as in Refs. [9] and [11].

2 The Stueckelberg Formalism for the Massive N=2 Super-Yang-Mills Theory

As mentioned in the Introduction, we consider the massive Yang-Mills field in the $N = 2$ harmonic superspace [9,10] which is obtained from the standard one by adding the sphere $S^2 = SU(2)_A/U(1)$, where $SU(2)$ is the automorphism group of the $N = 2$ superalgebra. The harmonics $u_i^\pm$ are coordinates on this sphere and have $SU(2)$ index i and $U(1)$ charge ± 1, respectively. The analytic subspace of the harmonic superspace plays a very important role (see Appendix).

The physical composition of the off-shell $N=2$ Yang-Mills hypermultiplet is well-known: the vector field $A_\mu(x)$, scalar complex field $M(x)+iN(x)$, Majorana isodoublet $\psi^i_\alpha(x), \bar{\psi}'_{\dot{\alpha}i}(x)$ and triplet of the scalar auxiliary fields $D^{ij}(x)$. The analytic superfield describing this supermultiplet is $V^{++}(z,u)$. The action for it was given in [11-13]:

$$S^{N=2}_{SYM} = \frac{1}{g^2}\mathrm{Tr}\sum_{n=2}^{\infty}\frac{(-1)^n}{n}\int d^{12}z du_1 \ldots du_n \frac{V^{++}(z,u_1)\ldots V^{++}(z,u_n)}{(u_1^+u_2^+)(u_2^+u_3^+)\ldots(u_n^+u_1^+)} \cdot \tag{2.1}$$

It is invariant under the following gauge transformations:

$$(V^{++})' = e^{i\omega}(V^{++} - iD^{++})e^{-i\omega}, \qquad V^{++} = V^{++}_a T_a,$$
$$\omega = \omega(\zeta,u) = \tilde{\omega}(\zeta,u), \qquad \omega = \omega_a T_a. \tag{2.2}$$

Here T_a are generators in the adjoint representation of the Yang-Mills group

$$[T_a, T_b] = i f_{abc} T_c, \qquad \mathrm{Tr}(T_a T_b) = \delta_{ab}\,. \tag{2.3}$$

Gauge transformations (2.2) copy those of the ordinary Yang-Mills theory ($N=0$) with the space-time derivative ∂_μ changed to the harmonic one D^{++}. The symmetry parameters $\omega(\zeta,u)$ now are localized in the analytic harmonic superspace. Correspondingly the number of gauge degrees of freedom now is infinite and $\omega(\zeta,u)$ is a real analytic hypermultiplet [9,14]

The mass term for $V^{++}(\zeta,u)$ that breaks gauge invariance (2.2) has the following form:

$$S^{N=2}_m = \frac{1}{2g^2}\int d\zeta^{(-4)} du \mathrm{Tr}[m^2(V^{++})^2] \tag{2.4}$$

where $d\zeta^{(-4)}du = d^4x d^2\theta^+ d^2\bar{\theta}^+ du$ is the integration measure over the analytic superspace. Now, as in the $N=0$ case we construct the corresponding Stueckelberg formalism [6] for our theory. Let us substitute (2.2) into (2.1) and (2.4). Due to its gauge invariance the pure Yang-Mills part (2.1) of the action will not be affected. As for the massive term (2.4), after some algebra it can be written as (cr. [6])

$$S^{N=2'}_m = \frac{m^2}{2g^2}\mathrm{Tr}\int d\zeta^{(-4)}du[e^{i\omega}(V^{++} - iD^{++})e^{++}]^2 =$$
$$= \frac{m^2}{2g^2}\int d\zeta^{(-4)}du D^{++}\omega P^{++}(\omega, V^{++}) + \frac{m^2}{2g^2}\mathrm{Tr}\int d\zeta^{(-4)}du(V^{++})^2, \tag{2.5}$$

where the expansion of $P^{++}(\omega,V^{++})$ in powers of ω has the form:

$$P^{++}(\omega, V^{++}) = \sum_{n=0}^{\infty}{}' \frac{2}{(n+2)!}[\ldots[\mathcal{D}^{++}\omega,\omega]\ldots]_n \tag{2.6}$$

with the covariant harmonic derivative

$$\mathcal{D}^{++} = D^{++} + iV^{++}\,. \tag{2.7}$$

Repeated commutators symbols in (2.6) are defined as

$$[\ldots[\mathcal{D}^{++}\omega,\omega]\ldots]_1 = [\mathcal{D}^{++}\omega,\omega],$$
$$[\ldots[\mathcal{D}^{++}\omega,\omega]\ldots]_2 = [[\mathcal{D}^{++}\omega,\omega],\omega],$$
$$[\ldots[\mathcal{D}^{++}\omega,\omega]\ldots]_3 = [[[\mathcal{D}^{++}\omega,\omega],\omega],\omega],$$
$$\ldots\ldots\ldots\ldots\ldots\ldots\ldots\ldots$$

Due to the trace in the expression (2.5), there are no terms containing an odd number of the Stueckelberg fields - ω. This means that we have no vertices with the odd number of the Stueckelberg fields.

The expression $S = S^{N=2}_{SYM} + S^{N=2}_M$ is by construction invariant under the following gauge transformations:

$$(V^{++})' = e^{i\xi}(V^{++} - iD^{++})e^{-i\xi} \tag{2.8}$$

$$e^{i\omega'} = e^{i\omega}e^{-i\xi}\,. \tag{2.9}$$

We have to fix gauge and carry out the Faddeev-Popov procedure [11]. We choose the Fermi-Feynman like gauge

$$D^{++}V^{++} = 0$$

(see [11] for details]).

Finally, our action takes the following form

$$S^{N=2} = S^{N=2}_{SYM} + S^{N=2'}_m + S^{N=2}_{GF} + S^{N=2}_{F.P} =$$
$$= \frac{1}{2g^2}\mathrm{Tr}\int d\zeta^{(-4)}du_1du_2 V^{++}(\zeta,u_1)(D_1^+)^4\frac{1}{(u_1u_2)^2}V^{++}(\zeta,u_2) +$$
$$+\frac{1}{g^2}\mathrm{Tr}\sum_{n=3}^{\infty}\frac{(-i)^n}{n}\int d^{12}zdu_1\ldots du_n\frac{V^{++}(z,u_1)\ldots V^{++}(z,u_n)}{(u_1^+u_2^+)\ldots(u_n^+u_1^+)} +$$
$$+\frac{m^2}{2g^2}\mathrm{Tr}\int d\zeta^{(-4)}duD^{++}\omega P^{++}(\omega,V^{++}) + \frac{m^2}{2g^2}\mathrm{Tr}\int d\zeta^{(-4)}dum^2(V^{++})^2 +$$
$$+\frac{1}{2g^2}(1+\frac{1}{\alpha})\mathrm{Tr}\int d\zeta^{(-4)}du_1du_2V^{++}(\zeta,u_1)(D_1^{++})^4\frac{1}{(u_1^+u_2^+)^2}V^{++}(\zeta,u_2) +$$
$$+\frac{i}{g^2}\mathrm{Tr}\int d\zeta^{(-4)}duFD^{++}(D^{++}+iV^{++})P \tag{2.10}$$

here α is the gauge fixing parameter.

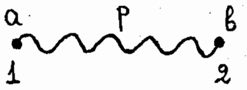
Now, let us consider the Feynman rules.

3 Quantization of the Massive $N=2$ Super-Yang-Mills Theory

The Feynman rules are obtained in the usual way using the functional integral [11,12]:

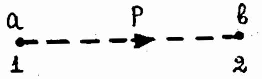
$$Z = N\int DV^{++}D\omega DFDPe^{iS^{N=2}}, \tag{3.1}$$

Without going into details, we give the form of the propagators and vertices.
The Yang-Mills field propagator $< V^{++}(1)V^{++}(2) >$ in the Fermi-Feynman gauge is:



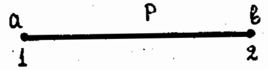
$$\frac{i}{p^2-m^2}(D_1^+)^4\delta^8(\theta_1-\theta_2)\delta^{(2,-2)}(u_1,u_2)\delta_{ab} \qquad (3.2)$$

while the Faddeev-Popov ghosts propagator $< F_a(1)P_b(2) >$ and the Stueckelberg field propagator $< \omega_a(1)\omega_b(2) >$ are written as



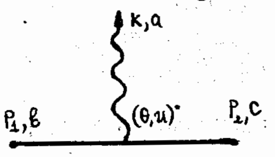
$$\frac{-1}{p^2}(D_1^+)^4(D_2^+)^4\delta^8(\theta_1-\theta_2)\frac{u_1^-u_2^-}{(u_1^+u_2^+)^3}\delta_{ab} \qquad (3.3)$$

and

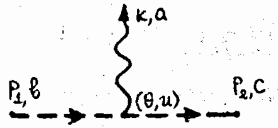


$$\frac{i}{p^2}(D_1^+)^4(D_2^+)^4\delta^8(\theta_1-\theta_2)\frac{u_1^-u_2^-}{(u_1^+u_2^+)^3}\delta_{ab} \qquad (3.4)$$

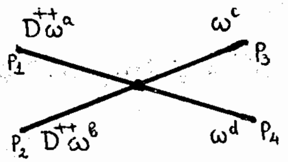
respectively,where p is the momentum.
The vertices are: integration is meant over the analytic superspace $\frac{1}{(2\pi)^4}\int d^4pd^4\theta^+du$,



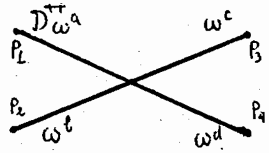
$$-igf^{abc}(D^{++}_{(b)}-D^{++}_{(c)})(2\pi)^4\delta(p_1-p_2-k) \qquad (3.5)$$



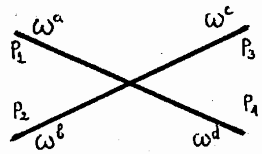
$$gf^{abc}D^{++}_{(b)}(2\pi)^4\delta(p_1-p_2-k) \qquad (3.6)$$



$$\frac{-1}{6}A^{abcd}\lambda^2 \qquad (3.7)$$



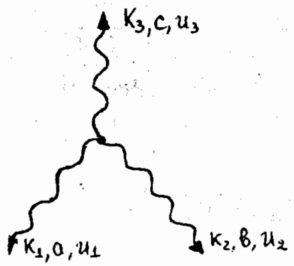
$$\frac{1}{3}A^{abcd}\lambda^2(D^{++}_{(c)}-D^{++}_{(d)}) \qquad (3.8)$$



$$\frac{-1}{24}A^{abcd}\lambda^2D^{++}D^{++} \qquad (3.9)$$

where $(A^{abcd}=\mathrm{Tr}(T_aT_bT_cT_d))$

Of importance is that there appears a coupling constant λ^2 of dimension $[m^2]$. As mentioned above, we have no vertices corresponding to interactions of an odd number of the Stueckelberg fields. Besides these vertices we have an infinite number of vertecis corresponding to the Yang-Mills field interaction. Note that the configuration-space integral at such vertex is $\int d^8\theta du_1du_2du_3$, so the Grassmann integration measure is already complete. For instance, the three-particle vertex is:



$$\frac{igf^{abc}}{(u_1^+u_2^+)(u_1^+u_3^+)(u_2^+u_3^+)}(2\pi)^4\delta(k_1+k_2+k_3)\ . \qquad (3.10)$$

We have also an infinite number of the vertices corresponding to the Stueckelberg field self-interaction and their interaction with Yang-Mills fields. We do not give them here, because for argumentation below the vertices (3.5-3.10) are quite enough.

Surely, in each vertex the integration is implemented over the analytic superspace

$$\frac{1}{(2\pi)^4}\int d^4pd^4\theta^+du.$$

We shall prove that the theory is nonrenomalizable starting from the g^4 order of perturbation theory.

4 Quantum Calculations in the $N=2$ Super-Yang-Mills Theory

It is a well-known fact that in the massive $N=0$ Yang-Mills theory quantum corrections appear in the g^2 order of perturbation theory. They originate from the following diagrams shown on Fig.1. and Fig.2.

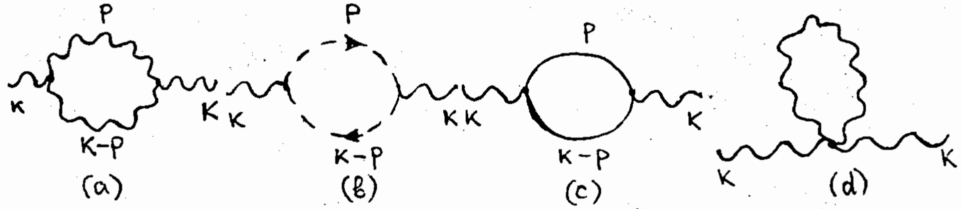


Fig.1.

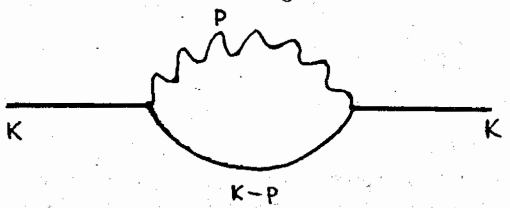


Fig.2.

Let us consider the corresponding diagrams in $N=2$ supersymmetric theory. According to the above Feynman rules we have:

$$\Gamma_a^{YM}=g^2\int\frac{d^4kd^4p}{(2\pi)^8(p^2-m^2)[(k-p)^2-m^2]}d^8\theta d^8\eta du_1du_2du_3dw_1dw_2dw_3$$
$$\frac{V_a^{++}V_a^{++}}{(u_1^+u_2^+)(u_1^+u_3^+)(u_2^+u_3^+)(w_1^+w_2^+)(w_1^+w_3^+)(w_2^+w_3^+)}$$
$$(D_\theta^+(u_2))^4\delta^{(2,-2)}(u_2,w_2)(D_\eta^+(u_3))^4\delta^8(\theta-\eta)\delta^{(-2,2)}(u_3,w_3)\,.$$

According to [11], this expression takes the form:

$$\Gamma_a^{YM}=2g^2\int\frac{d^4kd^4pd^8\theta du_1du_2}{(2\pi)^4(p^2-m^2)[(k-p)^2-m^2]}\frac{u_1^-u_2^-}{u_1^+u_2^+}V_a^{++}(1)V_a^{++}(2)\,. \qquad (4.1)$$

For diagram in Fig.1(b) we find:

$$\Gamma_b^{F.P.}=2g^2\int\frac{d^4kd^4p}{(2\pi)^8p^2(k-p)^2}d^4\theta_1^+d^4\theta_2^+du_1du_2V_a^{++}(1)V_a^{++}(2)(D_1^+)^4(D_2^+)^4$$
$$\delta^8(\theta_1-\theta_2)D_1^{++}\frac{u_1^-u_2^-}{(u_1^+u_2^+)^3}(D_1^+)^4(D_2^+)^4\delta^8(\theta_1-\theta_2)D_2^{++}\frac{u_1^-u_2^-}{(u_1^+u_2^+)^3}=$$
$$=2g^2\int\frac{d^4kd^4p}{(2\pi)^8p^2(k-p)^2}d^8\theta du_1du_2\frac{(u_1^+u_2^-)(u_1^-u^+)}{(u_1^+u_2^+)^2}V_a^{++}(1)V_a^{++}(2)\,. \qquad (4.2)$$

The contribution from diagram in Fig.1(c) has the following form:

$$\Gamma_c^{St}=2g^2\int\frac{d^4kd^4p}{(2\pi)^8p^2(k-p)^2}d^4\theta_1^+d^4\theta_2^+V_a^{++}(1)V_a^{++}(2)(D_1^+)^4(D_2^+)^4\delta^8(\theta_1-\theta_2)$$
$$(D_1^+)^4(D_2^+)^4\delta^8(\theta_1-\theta_2)\frac{u_1^-u_2^-}{(u_1^+u_2^+)^3}D_1^{++}D_2^{++}\frac{u_1^-u_2^-}{(u_1^+u_2^+)^3}-$$

$$-D_1^{++}\frac{u_1^-u_2^-}{(u_1^+u_2^+)^3}D_2^{++}\frac{u_1^-u_2^-}{(u_1^+u_2^+)^3}=$$
$$=2g^2\int\frac{d^4kd^4p}{((2\pi)^8p^2(k-p)^2}d^8\theta du_1du^2\frac{V_a^{++}(1)V_a^{++}(2)}{(u_1^+u_2^+)^2}\,. \qquad (4.3)$$

Let us sum up the infinite parts of (4.1) and (4.2). Using the useful identity

$$(u_1^+u_2^+)(u^-u_2^-)=1+(u_1^+u_2^-)(u_1^-u_2^+) \qquad (4.4)$$

we arrive at:

$$\Gamma^{YM}_{(a)\infty}+\Gamma^{F.P}_{(b)\infty}=-2g^2c^\infty\int d^{12}zdu_1du_2\frac{V_a^{++}(1)V_a^{++}(2)}{(u_1^+u_2^+)^2}\,. \qquad (4.5)$$

Here c^∞ contains logarithmical divergence. The result just cancels the infinite part of (4.2). Let us consider the diagram in Fig.2. The contribution of this diagram is equal to zero due to the $N=2$ supersymmetry. In fact, according to the Feynman rules it contains the expression $\delta^8(\theta_1-\theta_2)(D^+)^4\delta^8(\theta_1-\theta_2)$ that vanishes because Grassmann δ-functions have the following property:

$$\delta^8(\theta_1-\theta_2)(D^+)^m\delta^8(\theta_1-\theta_2)=0, \qquad if \qquad m<8\,. \qquad (4.6)$$

The contribution of the tadpole diagram in Fig.1(d) is equal to zero for the same reason. Therefore we come to the following conclusion: Massive $N=2$ super-Yang-Mills theory is finite in the g^2 order of perturbation theory. The general nonrenormalization theorem [15,16] says: if theory contain dimensionless parameters only and if it is finite in one loop, then it will be finite in all loops. Therefore we should look for infinite corrections in diagrams which contain the dimensionful coupling constant λ^2. The first possible lowest-order diagram containing the coupling constant λ^2 is shown on the Fig.3.

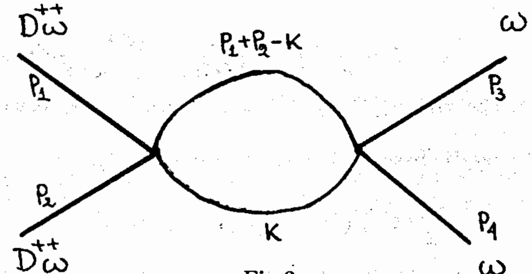


Fig.3.

According to the Feynman rules the contribution of this diagram is

$$\Gamma\sim\lambda^4\int\frac{d^4p_1...d^4p_4d^4k}{(2\pi)^{16}k^2(p_1+p_2-k)^2}d^4\theta_1^+d^4\theta_2^+du_1du_2\delta(p_1+p_2-p_3-p_4)$$
$$\left[(D_2^{++})^2\left(\frac{u_1^-u_2^-}{(u_1^+u_2^+)^3}\right)\frac{u_1^-u_2^-}{(u_1^+u_2^+)^3}+D_2^{++}\left(\frac{u_1^-u_2^-}{(u_1^+u_2^+)^3}\right)D_2^{++}\left(\frac{u_1^-u_2^-}{(u_1^+u_2^+)^3}\right)\right]$$
$$\delta^8(\theta_1-\theta_2)(D_1^+)^4(D_2^+)^4$$
$$\delta^8(\theta_1-\theta_2)(D_1^+)^4(D_2^+)^4D_1^{++}\omega(p_1,\theta_1,u_1)D_1^{++}\omega(p_2,\theta_1,u_1)\omega(p_3,\theta_2,u_2)\omega(p_4,\theta_2,u_2)\,.$$

Using the formula [11]

$$\delta^8(\theta_1-\theta_2)(D_1^+)^4(D_2^+)^4\delta^8(\theta_1-\theta_2)=(u_1^+u_2^+)^4\delta^8(\theta_1-\theta_2) \tag{4.7}$$

and taking then into consideration the identity for generalized harmonic functions [11]

$$D_1^{++}\frac{1}{(u_1^+u_2^+)^n}=\frac{1}{(n-1)!}(D_1^{--})^{(n-1)}\delta^{(n,-n)}(u_1,u_2), \tag{4.8}$$

and the property of the harmonic $\delta^{(q,-q)}(u_1,u_2)$ function

$$(u_1^+u_2^+)\delta^{(q,-q)}(u_1,u_2)=(u_1^-u_2^-)\delta^{(q,-q)}(u_1,u_2)=0, \tag{4.9}$$

we transform (4.7) to the following form:

$$\Gamma\sim\lambda^4\int\frac{d^4p_1...d^4p_4d^4k}{(2\pi)^{16}k^2(p_1+p_2-k)^2}d^8\theta du_1du_2\delta(p_1+p_2-p_3-p_4)$$
$$\left[\frac{(u_1^+u_2^-)^2}{(u_1^+u_2^+)^2}\right]D_1^{++}\omega(p_1,\theta,u_1)D_1^{++}\omega(p_2,\theta,u_1)\omega(p_3,\theta,u_2)\omega(p_4,\theta,u_2) \tag{4.10}$$

If the external lines are put on-shell ($(D^{++})^2\omega=0$), it will be possible to prove the following useful formula:

$$D^{++}\omega D^{++}\omega=\frac{1}{4}D^{++}D^{--}(D^{++}\omega D^{++}\omega). \tag{4.11}$$

(We use the algebra of the harmonic derivatives D^{++}, D^{--} and D^0, see appendix)

Let us use (4.8),(4.10) and (4.11) and transform (4.10) to the following form

$$\Gamma\sim\frac{1}{4}\lambda^4\int\frac{d^4p_1...d^4p_4d^4k}{(2\pi)^{16}k^2(p_1+p_2-k)^2}d^8\theta du$$
$$\omega(p_3,\theta,u)\omega(p_4,\theta,u)(D^{--})^2[D^{++}\omega(p_1,\theta,u)D^{++}\omega(p_2,\theta,u)] \tag{4.12}$$

Now we pass to the analytic basis and recall the formula [11]:

$$\frac{-1}{2}(D^+)^4(D^{--})^2\phi(\zeta,u)=\Box\phi(\zeta,u) \tag{4.13}$$

for any analytic superfield ϕ. Finally, the infinite correction to the action takes the form:

$$\Gamma^\infty\sim c^\infty\lambda^4\int d\zeta^{(-4)}du\omega^2\Box(D^{++}\omega)^2 \tag{4.14}$$

Obviously, Γ^∞ differs from all the terms of the initial action (2.10) and thus the theory is proven to be nonrenormalizable.

The diagrams of Figs.4,5 give a vanishing contributions. One can easily check this statement in the same manneras we used above when calculating of the diagram Fig.3 contribution.

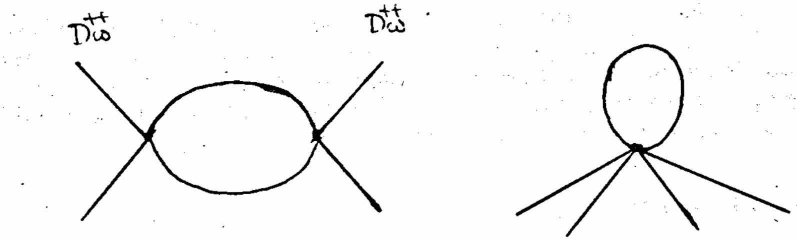


Fig.4.

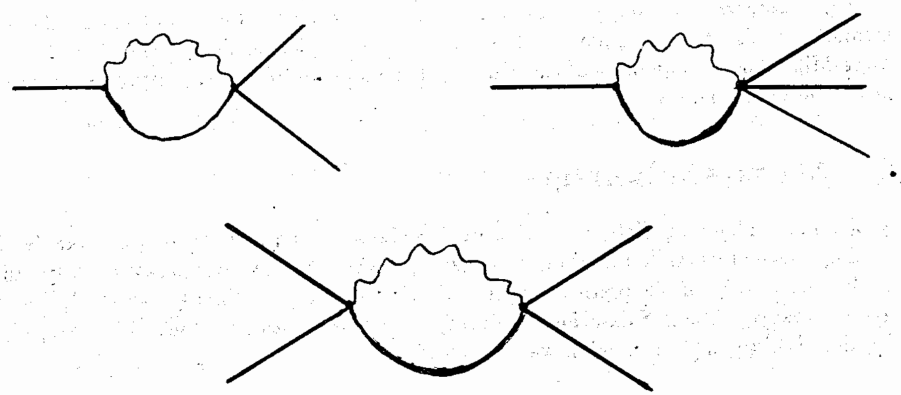


Fig.5.

Note that diagram in Fig.3. describes the Stueckelberg field self-interaction in the g^4 order of perturbation theory. As mentioned above, this Stueckelberg part of the action corresponds to the $N=2$ nonlinear σ-model:

$$\lambda^2\mathrm{Tr}\int d\zeta^{(-4)}du(e^{i\omega}D^{++}e^{-i\omega})(e^{i\omega}D^{++}e^{-i\omega}) \tag{4.15}$$

that is the $N=2$ supersymmetrization of the remarkable ordinary σ model (1.1).

Note that we can have proven also that $N=2$ sigma-model (4.15) is nonrenormalizable itself.

In contrast to $N=2$ theory, the mass of the Yang-Mills field in $N=2$ supersymmetric case is not renormalized. This is direct consequence of the general nonrenormalization theorem [15,16]. According to this theorem, in $N=2$ super-Yang-Mills theory all two- and more-loop quantum corrections can be expressed as integrals over the full $N=2$ superspace integration measure. Proceeding from the mass term for $N=2$ super-Yang-Mills field is the integral over the analytic superspace, the counterterm for mass should be the integral over the same measure. It is forbidden by above mentioned general nonrenormalization theorem. As for the one-loop level, usually the tadpole diagrams corresponds to the mass divergences. As we have proved above, such diagrams in $N = 2$ supersymmetry are equal to zero (Fig.1.(d)). In other words, the mass of the Yang-Mills field in the $N = 2$ supersymmetric case is not renormalized.

5 Conclusion

Summing up the results, we can state that the massive $N = 2$ supersymmetric Yang-Mills theory is nonrenormalizable. As in $N = 0$ case, the properties of nonrenormalizable theories in the $N = 2$ supersymmetric case become apparent in the g^4 order of perturbation theory. Nonrenormalizability of the nonlinear σ-model that appears in the Stueckelberg formalism is the stumbling-block of the nonrenormalizability of the massive $N = 2$ super-Yang-Mills theory. The mass of the Yang-Mills field in the $N = 2$ supersymmetric case is not to be renormalized.

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7 Appendix

For convenience of reading we give the relations between central and analytic bases of the $N=2$ harmonic superspace [9-11]:

Central basis:

$$[x^m, \theta_{\alpha i}, \bar\theta^i_{\dot\alpha}, u^+_i, u^-_i] = [z^M, u^+_i, u^-_i] \qquad i = 1,2$$

$$D^i_\alpha = \frac{\partial}{\partial\theta^\alpha_i} + i\partial_{\alpha\dot\alpha}\bar\theta^{\dot\alpha i},$$

$$\bar D_{\dot\alpha i} = -\frac{\partial}{\partial\bar\theta^{\dot\alpha i}} - i\theta^\alpha_i\partial_{\alpha\dot\alpha},$$

$$D^{++} = u^{+i}\frac{\partial}{\partial u^{-i}}, \qquad D^{--} = u^{-i}\frac{\partial}{\partial u^{+i}},$$

$$D^0 = [D^{++}, D^{--}] = u^{+i}\frac{\partial}{\partial u^{+i}} - u^{-i}\frac{\partial}{\partial u^{-i}}$$

Analytic basis:

$$[x^m_A, \theta^+_\alpha, \bar\theta^+_{\dot\alpha}, u^\pm_i, \theta^-_\alpha, \bar\theta^-_{\dot\alpha}] = [(\zeta^M, u^\pm_i), \theta^-_\alpha, \bar\theta^-_{\dot\alpha}]$$

$$x^m_A = x^m - 2i\theta^{(i}\sigma^m\theta^{j)}u^+_i u^-_j, \qquad \theta^\pm_\alpha = \theta^i_\alpha u^\pm_i, \qquad \bar\theta^\pm_{\dot\alpha} = \bar\theta^i_\alpha u^\pm_i,$$

$$D^+_\alpha = u^+_i D^i_\alpha = \frac{\partial}{\partial\theta^{-\alpha}}, \qquad D^+_{\dot\alpha} = u^+_i \bar D^i_{\dot\alpha} = \frac{\partial}{\partial\bar\theta^{-\dot\alpha}},$$

$$D^-_\alpha = u^-_i D^i_\alpha = -\frac{\partial}{\partial\theta^{+\alpha}} + 2i\bar\theta^{-\dot\alpha}\partial_{\alpha\dot\alpha}, \qquad \bar D^-_{\dot\alpha} = u^-_i \bar D^i_{\dot\alpha} = -\frac{\partial}{\partial\bar\theta^{+\dot\alpha}} - 2i\theta^{-\alpha}\partial_{\alpha\dot\alpha},$$

$$D^{++} = u^{+i}\frac{\partial}{\partial u^{-i}} - 2i\theta^+\sigma^m\bar\theta^+\frac{\partial}{\partial x^m_a} + \theta^{+\alpha}\frac{\partial}{\partial\theta^{-\alpha}} + \bar\theta^{+\dot\alpha}\frac{\partial}{\partial\bar\theta^{-\dot\alpha}},$$

$$D^{--} = u^{-i}\frac{\partial}{\partial u^{+i}} - 2i\theta^-\sigma^m\bar\theta^+\frac{\partial}{\partial x^m_a} + \bar\theta^{-\alpha}\frac{\partial}{\partial\theta^{+\alpha}} + \bar\theta^{-\dot\alpha}\frac{\partial}{\partial\bar\theta^{+\dot\alpha}},$$

$$(7.16)$$

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