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ON THE PROBLEM OF ENHANCEMENT OF NEUTRINO OSCILLATION IN MATTER

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I. Introduction

At present, speaking about weak interaction, one can probably imply all processes with small coupling constants of specific properties, e.g. P–, CP–violation, etc.

On the other hand, one can speak about the electroweak interaction theory (implying the standard theory of electroweak interaction [1]). Now there are many experimental facts that go beyond the standard theory of the electroweak interaction. One of these facts is the existence of generations (e , μ , τ). Strictly following the standard theory of electroweak interaction, one finds two numbers conserved in this theory. They are the electric charge $j_\mu = e\bar{\Psi}_e\gamma_\mu\begin{pmatrix}1 & 0\\ 0 & 0\end{pmatrix}\Psi_e$ related to the U(I) subgroup and the lepton number $J_\mu = g\bar{\Psi}_e\gamma_\mu\Psi_e$ related to the SU(2) subgroup. So, these two conserved numbers must be present in each generation of particles. The question arises: will these two conserved numbers be identical for all generations?

Using general considerations one can easily prove that these two conserved numbers will be identical for each generation if these generations take part in electroweak interactions (universality of electroweak interaction).

Then, if the generations differ only in masses and charges, particles of different generations may convert into one another in the processes with charge conservation, or heavier particles will decay.

However, experiments show that these generations (e , μ , τ) differ in their conserved numbers l_e, l_μ, l_τ and weak transitions and decays occur with conservation of these numbers. It means that generation of particles must have same properties with regard to electroweak interaction (i.e. the charge and the SU(2) weak spin number I_w must be conserved). Now we pass to one of the main manifestations of transitions between different generations – neutrino oscillation [2].

The discussion of this topic has been recently stimulated by ref.[3,4] indicating a possibility of resonant enhancement of neutrino oscillation in matter.

The Wolfenstein mechanism of this enhancement is discussed. A mechanism of enhancement of neutrino oscillation in matter based on the so-called "horizontal" symmetry is proposed.

2. Wolfenstein equation and violation of the condition for enhancement of neutrino oscillation in matter

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Let us consider mixing of two neutrino types ν_e and ν_μ $\nu_f = (\nu_e, \nu_\mu)$. In the ultrarelativistic limit the evolution equation for $\Psi_f = (\Psi_e, \Psi_\mu)$ in matter has the form

$$i\frac{d\Psi_f}{dt} = (k\hat{I} + \frac{\hat{M}^2}{2k} + \hat{W})\Psi_f, \qquad (I)$$

where k, $\hat{M}^2$, $\hat{W}$ are the momentum, the neutrino mass matrix squared in vacuum, the matrix account for neutrino interaction with matter.

The mixing in (I) is due to the matrix M^2 being non-diagonal. If the neutrino mass is low $(S \ll G_F^{-1})$ and the layer of matter is smaller than the absorption length, the neutrino interaction is reduced to forward elastic scattering at an angle 0°. This effect results in appearance of the refractive index

$$(n_\alpha - 1) = \frac{2\pi W_\alpha}{k}, \quad \alpha = e, \mu$$

(W_α is an addition to the neutrino energy due to interaction).

Physical consequences for the oscillation are determined by the difference of diagonal elements in the matrices M^2 and W, namely

$$W = W_e - W_\mu = \frac{\Sigma \Delta f_i N_i}{k} \qquad (2)$$

$\Delta f_i = f_i^e(0) - f_i^\mu(0)$, N_i – is the concentration of electrons in matter. Here, if the interaction was the same for ν_e and ν_μ, the effect of the medium on the oscillation would disappear.

The medium must be nonsymmetric with regard to the oscillating components.

For ν_e-ν_μ and ν_e-ν_τ oscillations W depends on charged-current scattering of ν_e by electrons. For ν_e and ν_μ there is no interaction like this, and

$$\Delta f(0) = 2\sqrt{2}G_F k, \quad W = 2\sqrt{2}G_F N_e$$

the nonsymmetric character of an ordinary medium with regard to ν_e and ν_μ is due to presence of electrons and absence of muons in it.

In vacuum $\hat{W} = 0$ and (I) is reduced to

$$i\frac{d\Psi_f}{dt} = (k\hat{I} + \frac{\hat{M}^2}{2k})\Psi_f. \qquad (3)$$

The relation between ν_f and $\nu = (\nu_1, \nu_2)$ (ν – are eigenstates with certain masses) is established by diagonalization of $\hat{M}^2$

$$\nu_f = \hat{S}(\theta)\nu, \qquad (4)$$

where $\hat{S}(\theta) = \begin{pmatrix} \cos\theta & \sin\theta \\ -\sin\theta & \cos\theta \end{pmatrix}$,

$$S^+(\theta)\hat{M}^2 S(\theta) = \hat{M}^{2\,diag} = \mathrm{diag}(m_1^2, m_2^2),$$

here m_1 and m_2 are the masses of ν_1 and ν_2, θ – is the mixing angle in vacuum.

The mass matrix squared can be rewritten as

$$\hat{M}^2 = \frac{\Delta m^2}{2}\begin{pmatrix} \cos 2\theta & -\sin 2\theta \\ -\sin 2\theta & -\cos 2\theta \end{pmatrix}.$$

If the neutrino passes through matter, equation (I) has the form

$$i\frac{d\Psi_f}{dt} = \hat{H}\Psi_f \quad \text{where} \quad \hat{H} = \begin{vmatrix} H_e & \frac{1}{2}\bar{H} \\ \frac{1}{2}\bar{H} & H_\mu \end{vmatrix} \qquad \bar{H} = -\frac{\Delta m^2 \sin 2\theta}{2k} \qquad (5)$$

$$H = H_e - H_\mu = \frac{\Delta m^2 \cos 2\theta}{2k} + \frac{\Sigma \Delta f_i N_i}{2k}.$$

In a medium of varying density $N_i = N_i(x)$ and (5) is a set of differential equations with variable factors.

For a medium of constant density the new mixing angle θ_m involving the interaction has the form

$$\sin 2\theta_m = (\Delta m^2 \sin 2\theta)/A, \qquad \cos 2\theta_m = \frac{\Delta m^2 \cos 2\theta - 2\sqrt{2}G_F N_e k}{A} \qquad (6)$$

$$L_m = L\Delta m^2/A \qquad A = \sqrt{(\Delta m^2 \cos 2\theta - 2\sqrt{2}G_F N_e k)^2 + (\Delta m^2 \sin 2\theta)^2}$$

$$P_{\nu_e,\nu_e}(R) = 1-\frac{1}{2}\sin^2 2\theta_m(1-\cos 2\pi R/L_m).$$

What happens here? Because of difference in ν_e and ν_μ interaction with matter the effective mass of ν_e increases. This increase is due to interaction via the $W^\pm$ -boson. The mass variation is very small, but it is due to the $W^\pm$ -boson alone (no other small parameters related to the very weak interaction appear there). This change in the effective mass of ν_e results in enhancement of oscillation (because of interaction via $W^\pm$ -bosons), i.e. in nonconservation of the lepton numbers l_e, l_μ .

Two conditions must be satisfied for the oscillation to be amplified in matter:

1) interaction must result in a mass (or effective mass) change; as weak interaction is left-handed, the exchange of Z^0 and $W^\pm$ does not influence the effective mass of leptons and quarks (otherwise the Higgs mechanism is required);

2) this interaction must involve nonconservation of the relevant numbers (l_e, l_μ, l_τ). Experiments show that in weak interactions these numbers are well preserved (see the Table).

Table. Upper limits for the probability of some processes with lepton number nonconservation /6/

Reaction	Experimental data
$\mu^+ \to e^+\gamma$	$\frac{\Gamma(\mu^+ \to e^+\gamma)}{\Gamma(\mu^+ \to \mathrm{all})} < 1.7\cdot 10^{-10}$
$\mu^+ \to e^+e^-e^+$	$\frac{\Gamma(\mu^+ \to e^+e^-e^+)}{\Gamma(\mu^+ \to \mathrm{all})} < 2.4\cdot 10^{-12}$

Both conditions are not satisfied in weak interactions.

Thus, when ν_e passes through matter, there must be scattering but not enhancement of oscillation.

Let us consider example proving the above conclusions.

a) K^0 - $\tilde{K}^0$ oscillation.

Is it possible to predict K^0 - $\tilde{K}^0$ oscillations on the basis of general considerations?

The strong interaction does not allow this oscillation because strangeness and parity are conserved in strong interactions. Yet, if we take into account the presence of the weak interaction which violates strangeness and parity, the K^0 - $\tilde{K}^0$ oscillation becomes possible. Moreover, in the weak interaction /1/ there is no difference between K^0 and $\tilde{K}^0$, and they can be associated with the Z^0 -boson (on the appropriate mass shell). Then there will be an equal probability of "containing" K^0 and $\tilde{K}^0$ $\left\{ Z^0 \begin{array}{l} \nearrow K^0 \\ \searrow \tilde{K}^0 \end{array} \right\}$. The mixing angle will be 45°.

b) Oscillation of leptons

Similarly to K^0 - $\tilde{K}^0$ oscillations, one should consider oscillation of leptons bearing in mind that one deals with two different types of particles, fermions and bosons. Unlike the case with fermions, there are no gauge interaction theories for bosons. In the gauge theories there are conserved numbers whose violation is a nontrivial problem. We shall consider the scheme of violation of these conserved numbers in the gauge theory of weak interaction a bit later. Now we pass a second example.

Let us consider a weak decay of the Λ^0 -hyperon /7/ via $s \to d + Z^0$. The matrix element of this process is proportional to $\sin\theta$ (Cabibbo angle). Since the strange quark in the Λ^0 -hyperon is a constituent quark, the probability of the $s \to d$ transition must be higher ($\sin\theta \sim \sqrt{m_d/m_s}$) /8/ because $m_d \sim m_s$ when nuclear interaction is taken into account. But the experiment shows that there is no increase in the probability of the $s \leftrightarrow d$ transition and the mixing angle depends on the current quark masses. This result is obvious, because strong interaction does not violate strangeness and there must not be increase in the probability of the $s \leftrightarrow d$ transition because of strong interaction contribution to quark masses.

The above examples show that a mechanism violating conservation of the relevant numbers is necessary for oscillation to appear; and even if there is a mechanism like this, only its parameters violating conservation of these numbers (mass, coupling constants, etc) are responsible for the oscillation.

3. Oscillation of ν with allowance for "horizontal" symmetry

In the standard theory of electroweak interaction there are conserved vector currents which determine the presence of three lepton numbers l_e, l_μ, l_τ , when the existence of three generations is taken into account. As is pointed out above, there is no oscillation between neutrinos of the three generations because of weak interaction. If we want to consider oscillation similar to $K^0 \leftrightarrow \tilde{K}^0$, we must consider an interaction which violates conservation of the lepton numbers l_e, l_μ, l_τ (similarly to the strong interaction where conservation of flavour numbers is violated by the weak interaction). For oscillation of leptons it is necessary that there is an interaction with its own carriers which violate conservation of lepton numbers. A candidate for this interaction is the one related to the so-called "horizontal" symmetry /9/

$$\begin{array}{c|ccc|} & e & \mu & \tau \\ e & X_{ee} & X_{e\mu} & X_{e\tau} \\ \mu & X_{\mu e} & X_{\mu\mu} & X_{\mu\tau} \\ \tau & X_{\tau e} & X_{\tau\mu} & X_{\tau\tau} \end{array} \qquad \begin{array}{l} X_{ee} \simeq X_{\mu\mu} \simeq X_{\tau\tau} \simeq W^{\pm}Z^0 \\ X_{e\mu} \simeq X_{\mu e}, \quad X_{e\tau} \simeq X_{\tau e} . \\ X_{\mu\tau} \simeq X_{\tau\mu} \end{array} \tag{7}$$

The interaction carriers $X_{e\mu}, X_{e\tau}, X_{\mu\tau}$ in (7) violate conservation of the lepton numbers l_e, l_μ, l_τ. For this interaction formula (5) may have the form

$$i\frac{d\Psi_f}{dt} = \hat{H}\Psi_f , \qquad \hat{H} = \begin{vmatrix} H_e & \frac{1}{2}\bar{H} \\ \frac{1}{2}\bar{H} & H_\mu \end{vmatrix} , \qquad \bar{H} = -\frac{\Delta m^2 \sin 2\theta}{2k}$$

$$H = H_e - H_\mu = \frac{\Delta m^2 \cos 2\theta}{2k} + \frac{\Sigma F_i(0)N_i}{2k} . \tag{8}$$

$F_i(0)$ is the forward scattering amplitude (it appears owing to the fact that the initial flux consists of ν_e but after interaction via $X_{e\mu}$ a fraction of the initial ν_e converts into ν_μ and then this neutrino composition is stabilized if the medium has constant characteristics).

The form of expression (6) for the probability of the $\nu_e - \nu_\mu$ transition in matter does not change except for replacement

$$\Delta f_i \to F_i(0) , \qquad F_i(0) \simeq \frac{M_W^2}{M_{e\mu}^2} \Delta f_i .$$

The quantity that characterizes the degree of violation of lepton numbers in the amplitude of the processes is $\eta = M_{W,Z}^2 / M_{e\mu}^2 \left(M_{W,Z}^2 / \right)$

In the case with the Wolfenstein mechanism $\eta = 1$, in the case of the "horizontal" symmetry mechanism $\eta \ll 1$.

Conclusion

The Wolfenstein mechanism of enhancement of neutrino oscillation in matter has been discussed. The conclusion of inefficiency of this mechanism is drawn.

A mechanism of enhancement of neutrino oscillation based on the so-called "horizontal" symmetry is proposed. The efficiency of this mechanism will depend on $\Delta m^2 = m_2^2 - m_1^2$, θ and the density of matter through which neutrinos pass. Since the contribution of the "horizontal" symmetry mechanism to the effective mass (or effective energy) of the neutrino is "small", it is most important – for solving the problem of solar neutrinos – to study neutrino oscillation on the assumption that neutrinos have a magnetic moment which interacts with the magnetic field in the outer, convective layers of the Sun and causes precession of the neutrino spin changing its helicity /10/.

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К вопросу об усилении осцилляции нейтрино в веществе

Обсуждается механизм Вольфенштейна усиления осцилляции при прохождении нейтрино через вещество. Делается вывод о неэффективности этого механизма. Предлагается механизм осцилляции нейтрино при прохождении нейтрино через вещество, основанный на так называемой "горизонтальной" симметрии.

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On the Problem of Enhancement of Neutrino Oscillation in Matter

The Wolfenstein mechanism of enhancement of neutrino oscillation in matter is discussed. The conclusion that it is ineffective is drawn. A mechanism of enhancement of neutrino oscillation based on the so-called "horizontal" symmetry is proposed.

The investigation has been performed at the Laboratory of Nuclear Problems, JINR.

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