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H^+H^- INTERACTION UP TO HIGHER ORDERS OF PERTURBATION THEORY IN THE MODEL WITH TWO HIGGS DOUBLETS (COUNTER TERMS)

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The paper is devoted to a calculation of the counterterms needed for the renormalization of the H^+H^- amplitude performed on the mass shell and in the unitary gauge. This work is the continuation of our previous paper /1/ where we presented the general expressions for $\Gamma^{Ren.}$ and $i\mathcal{M}^{Ren.}$:

$$\Gamma^{c.t.}_{H^\pm H^\pm A}(p^2, q^2, t) = -ie\left(Z_{H^\pm} - 1\right), \tag{1}$$

$$\Gamma^{c.t}_{H^\pm H^\pm Z}(p^2, q^2, t) = -\frac{ig(2R-1)}{2\sqrt{R}}\left[\left(Z_{H^\pm} - 1\right) + \frac{1}{2}\left(Z_Z - 1\right) + \frac{\delta g}{g} + \frac{2R+1}{2(2R-1)}\,\frac{\delta R}{R}\right] - ie \times \left[\left(Z_{H^\pm} - 1\right) + \frac{1}{2}\left(Z_M - 1\right) - \frac{1}{2}\left(Z_A - 1\right)\right] \tag{2}$$

and
$$\Gamma^{c.t}_{H^\pm H^\pm H^0_{1,2}}(p^2, q^2, t) = -igM_W\left[\begin{Bmatrix}\cos(\beta-\alpha)\\ \sin(\beta-\alpha)\end{Bmatrix} \mp \frac{1}{2R} \times \cos 2\beta \begin{Bmatrix}\cos(\beta+\alpha)\\ \sin(\beta+\alpha)\end{Bmatrix}\right] \times \left[\left(Z_{H^\pm} - 1\right) + \frac{1}{2}\left(Z_{H^0_{1,2}} - 1\right) + \frac{\delta g}{g} + \frac{1}{2}\,\frac{\delta M_W^2}{M_W^2}\right] - igM_W\left[\pm\frac{1}{2R}\cos 2\beta\begin{Bmatrix}\cos(\beta+\alpha)\\ \sin(\beta+\alpha)\end{Bmatrix}\right] \times \frac{\delta R}{R} - igM_W\left[\begin{Bmatrix}\sin(\beta-\alpha)\\ \cos(\beta-\alpha)\end{Bmatrix} \pm \frac{1}{2R}\cos 2\beta\begin{Bmatrix}\sin(\beta+\alpha)\\ \cos(\beta+\alpha)\end{Bmatrix}\right] \times \left[\left(Z_{H^\pm} - 1\right) + \frac{1}{2}\left(Z_{M2,M1} - 1\right) + \frac{\delta g}{g} + \frac{1}{2}\,\frac{\delta M_W^2}{M_W^2}\right] - igM_W\left[\mp\frac{1}{2R}\cos 2\beta\begin{Bmatrix}\sin(\beta+\alpha)\\ \cos(\beta+\alpha)\end{Bmatrix}\right]\frac{\delta R}{R}. \tag{3}$$

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Besides

$$iM^{c.t.} = -\frac{g^2\cos^2 2\beta}{2\cos^2\Theta_W}\left[2\frac{\delta g}{g}+2(Z_{H^\pm}-1)-\frac{\delta R}{R}\right].^{(4)}$$

So we need to find the next counter terms : $\delta M_W^2/M_W^2$, $\delta M_{H^\pm}^2/M_{H^\pm}^2$, $\delta M_Z^2/M_Z^2$, Z_A-1, Z_Z-1, Z_M-1, $Z_{H_1^0}-1$, $Z_{H_2^0}-1$, $Z_{H^\pm}-1$, $Z_{M1}-1$, $Z_{M2}-1$.

The mass counter terms are

$$\frac{\delta M_W^2}{M_W^2} = \frac{ig^2}{16\pi^2}\left\{\left[\frac{34}{3}-\frac{3}{2}\frac{1}{R}-\frac{1}{3}N_f - r_+ - r_1 - r_2 + \frac{1}{M_W^2}\times\right.\right.$$

$\times\, Tr\, m_i^2\Big] P$ + finite part of SM with the exception of Higgs terms

$$-\frac{5}{18}+\frac{r_+}{2}+\frac{r_1(1+\sin^2(\beta-\alpha))+r_2(1+\cos^2(\beta-\alpha))}{4}-$$

$$-\frac{r_1\cos^2(\beta-\alpha)+r_2\sin^2(\beta-\alpha)}{12}-\frac{r_1^2\cos 2(\beta-\alpha)-r_2^2\cos 2(\beta-\alpha)-r_3^2}{12}-$$

$$-r_+\frac{r_1\sin^2(\beta-\alpha)+r_2\cos^2(\beta-\alpha)-r_3}{6}+\left(\frac{r_1\sin^2(\beta-\alpha)+r_2\cos^2(\beta-\alpha)-r_3}{12}-\right.$$

$$\left.-\frac{1}{2}\right)r_+\ln r_+ + \left(\frac{r_+-r_1-1}{12}\sin^2(\beta-\alpha)+\frac{r_1}{12}\cos^2(\beta-\alpha)-\frac{1}{4}\right)\times$$

$$\times r_1\ln r_1 + \left(\frac{r_+-r_2-1}{12}\cos^2(\beta-\alpha)+\frac{r_2}{12}\sin^2(\beta-\alpha)-\frac{1}{4}\right)r_2\ln r_2 +$$

$$+\left(\frac{r_3-r_++1}{12}-\frac{1}{4}\right)r_3\ln r_3 - \cos^2(\beta-\alpha)\left(1-\frac{1}{3}r_1+\frac{1}{12}r_1^2\right)\times$$

$$\times I_0(-M_W^2, M_W^2, M_{H_1^0}^2) - \sin^2(\beta-\alpha)\left(1-\frac{1}{3}r_2+\frac{1}{12}r_2^2\right)I_0(-M_W^2,$$

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$$M_W^2, M_{H_2^0}^2\Big) + \frac{(r_+ - r_1 - 1)^2 - 4r_1}{12}\sin^2(\beta-\alpha)\,\bar{I}_0(-M_W^2, M_{H^\pm}^2, M_{H_1^0}^2) +$$

$$+\cos^2(\beta-\alpha)\,\bar{I}_0(-M_W^2, M_{H^\pm}^2, M_{H_2^0}^2)\frac{(r_+ - r_2 - 1)^2 - 4r_2}{12} + \frac{1}{3}r_3 \times$$

$$\times\,\bar{I}_0(-M_W^2, M_{H^\pm}^2, M_{H_3^0}^2)\Big\} \qquad (5)$$

and

$$\frac{\delta M_Z^2}{M_Z^2} = \frac{ig^2}{16\pi^2}\Big\{\Big[-\frac{5}{3} + \frac{40}{3}R - \frac{13}{6}\frac{1}{R} - \frac{1}{3}\Big(2 - \frac{1}{R}\Big)N_f -$$

$$-\frac{8}{3}\frac{(1-R)^2}{R}Tr\,Q_i^2 + \frac{1}{M_W^2}Tr\,m_i^2\Big]P$$ + finite part of SM

with the exception of Higgs terms $\frac{2}{9} + \frac{2}{9}R - \frac{1}{6}\frac{1}{R} -$

$$-R\left(\frac{r_1^2 + r_2^2 + r_3^2}{12} - r_3\frac{r_1\sin^2(\beta-\alpha) + r_2\cos^2(\beta-\alpha)}{6}\right) +$$

$$+\frac{r_1\cos^2(\beta-\alpha) + r_2\sin^2(\beta-\alpha)}{6} + r_+\ln r_+\Big(-\frac{1}{3} + \frac{4}{3}R - \frac{4}{3}R^2\Big) -$$

$$-\frac{\ln R}{R}\left(\frac{1}{6} - R\frac{r_1\cos^2(\beta-\alpha) + r_2\sin^2(\beta-\alpha)}{12}\right) + r_1\ln r_1\left(\frac{\sin^2(\beta-\alpha)}{12} -\right.$$

$$\left.-\frac{1}{4}\right) + r_2\ln r_2\left(\frac{\cos^2(\beta-\alpha)}{12} - \frac{1}{4}\right) + \frac{r_1^2R}{12}\ln r_1 + \frac{r_2^2R}{12}\ln r_2 -$$

$$-r_3\ln r_3\left(\frac{1}{6} - \frac{r_3R}{12} + R\frac{r_1\sin^2(\beta-\alpha) + r_2\cos^2(\beta-\alpha)}{12}\right) - r_3R\,\times$$

$$\times\frac{r_1\sin^2(\beta-\alpha)\ln r_1 + r_2\cos^2(\beta-\alpha)\ln r_2}{12} + \Big(\frac{1}{3} - \frac{1}{12}\frac{1}{R} - \frac{1}{3}R +$$

$$+\frac{1}{3}r_+ - \frac{4}{3}r_+R + \frac{4}{3}r_+R^2\Big)\bar{I}_0(-M_Z^2, M_{H^\pm}^2, M_{H^\pm}^2) + \sin^2(\beta-\alpha)\,\times$$

$$\times\left(\frac{1}{3}r_1 - \frac{(r_3R - r_1R - 1)^2}{12R}\right)\bar{I}_0(-M_Z^2, M_{H_1^0}^2, M_{H_3^0}^2) + \cos^2(\beta-\alpha)\,\times$$

$$\times\left(\frac{1}{3}r_2 - \frac{(r_3R - r_2R - 1)^2}{12R}\right)\bar{I}_0(-M_Z^2, M_{H_2^0}^2, M_{H_3^0}^2) - \cos^2(\beta-\alpha)\,\times$$

$$\times\left(\frac{1}{R} - \frac{1}{3}r_1 + \frac{1}{12}r_1^2R\right)\bar{I}_0(-M_Z^2, M_Z^2, M_{H_1^0}^2) - \sin^2(\beta-\alpha)\Big(\frac{1}{R} -$$

$$-\frac{1}{3}r_2 + \frac{1}{12}r_2^2R\Big)\bar{I}_0(-M_Z^2, M_Z^2, M_{H_2^0}^2)\Big\}. \qquad (6)$$

The counter terms for a renormalization of the Higgs ($H^\pm$) self-energy are

$$\frac{\delta M_{H^\pm}^2}{M_{H^\pm}^2} = \frac{ig^2}{16\pi^2}\Big\{\Big[\frac{1}{R}\Big(\frac{3}{2} - \cos^2 2\beta\Big) - \big(r_1\sin^2(\beta-\alpha) + r_2\cos^2(\beta-\alpha) -$$

$$-r_3\big) - \frac{1}{r_+}\left(11 + \frac{r_1\cos^2(\beta-\alpha) + r_2\sin^2(\beta-\alpha)}{2} - \frac{r_1^2\sin^2(\beta-\alpha) + r_2^2\cos^2(\beta-\alpha) - r_3^2}{2}\right) +$$

$$+\frac{1}{r_+R}\left(6 + 2\cos^2 2\beta + \frac{(r_1 - r_2)\cos 2\beta\cos 2\alpha + r_3\cos^2 2\beta}{4}\right) - \frac{1}{r_+R^2}\,\times$$

$$\times\Big(\frac{3}{2} + \frac{1}{2}\cos^2 2\beta\Big) - Tr\,\frac{m_u^2\,ctg^2\beta + m_d^2\,tg^2\beta}{M_W^2} + \frac{2}{M_W^2M_{H^\pm}^2}Tr\big(m_u^2m_d^2 +$$

$$+(m_u^2 + m_d^2)(m_u^2\,ctg^2\beta + m_d^2\,tg^2\beta)\big)\Big]P + \frac{1}{R}\Big(\frac{1}{4} + \frac{\cos^2 2\beta}{2}\Big) +$$

$$+\frac{2}{r_+}\Big(1 - \frac{1}{R} + \frac{1}{4R^2}\Big)\ln R - \frac{1}{R}\Big(\frac{1}{4} + \frac{\cos^2 2\beta}{2}\Big)\ln r_+ +$$

$$+\frac{1}{r_+}\left(\frac{1}{2} + \frac{r_1\cos^2(\beta-\alpha) + r_2\sin^2(\beta-\alpha)}{4} - \frac{r_1\sin^2(\beta-\alpha) + r_2\cos^2(\beta-\alpha) - r_3}{2} -\right.$$

$$-\frac{r_1 \ln r_1 \cos 2(\beta-\alpha) - r_2 \ln r_2 \cos 2(\beta-\alpha) + r_3 \ln r_3}{4}\Big) - \frac{1}{r_+ R}\cdot$$

$$\cdot\Big(\frac{(r_1-r_2)\cos 2\beta\cos 2\alpha + r_3\cos^2 2\beta}{8} - \frac{(r_1\ln r_1 - r_2 \ln r_2)\cos 2\beta\cos 2\alpha}{8} -$$

$$-\frac{r_3 \ln r_3 \cos^2 2\beta}{8}\Big) - \frac{1}{M_W^2}\mathrm{Tr}\Big(\frac{m_u^2 \mathrm{ctg}^2\beta + m_d^2 \mathrm{tg}^2\beta}{4} + \frac{m_u^2 m_d^2}{M_{H^\pm}^2} +$$

$$+\frac{m_u^2 \mathrm{ctg}^2\beta + m_d^2 \mathrm{tg}^2\beta}{2M_{H^\pm}^2}\Big(m_u^2 \ln\frac{m_u^2}{M_W^2} + m_d^2\ln\frac{m_d^2}{M_W^2}\Big)\Big) + 4(1-R)\cdot$$

$$\cdot I_0(-M_{H^\pm}^2, 0, M_{H^\pm}^2) - \Big(4 - 4R - \frac{1}{R} + \frac{1}{r_+} - \frac{1}{r_+R} + \frac{1}{4r_+R^2}\Big)\cdot$$

$$\cdot I_0(-M_{H^\pm}^2, M_Z^2, M_{H^\pm}^2) + \Big(\frac{(r_1 - r_+ - 1)^2}{r_+} - 4\Big)\frac{\sin^2(\beta-\alpha)}{4} I_0(-M_{H^\pm}^2,$$

$$M_W^2, M_{H_1^0}^2) + \Big(\frac{(r_2-r_+-1)^2}{r_+} - 4\Big)\frac{\cos^2(\beta-\alpha)}{4} I_0(-M_{H^\pm}^2, M_W^2, M_{H_2^0}^2) -$$

$$-\Big(\frac{1}{r_+} - 1\Big) I_0(-M_{H^\pm}^2, M_W^2, M_{H_3^0}^2) - \frac{1}{r_+}\Big(\cos(\beta-\alpha) - \frac{\cos 2\beta\cos(\beta+\alpha)}{2R}\Big)^2\cdot$$

$$\cdot I_0(-M_{H^\pm}^2, M_{H^\pm}^2, M_{H_1^0}^2) - \frac{1}{r_+}\Big(\sin(\beta-\alpha) + \frac{\cos 2\beta\sin(\beta+\alpha)}{2R}\Big) I_0(-M_{H^\pm}^2,$$

$$M_{H^\pm}^2, M_{H_2^0}^2) - \frac{1}{2M_W^2}\mathrm{Tr}\Big[m_u^2 \mathrm{ctg}^2\beta + m_d^2 \mathrm{tg}^2\beta - \frac{4m_u^2 m_d^2}{M_{H^\pm}^2} -$$

$$-\frac{(m_u^2+m_d^2)(m_u^2 \mathrm{ctg}^2\beta + m_d^2 \mathrm{tg}^2\beta)}{M_{H^\pm}^2}\Big] I_0(-M_{H^\pm}^2, m_u^2, m_d^2)\Big\}. \qquad (7)$$

and

$$Z_{H^\pm} - 1 = \frac{ig^2}{16\pi^2}\Big\{\Big[2 - 2R - \frac{3}{2}\frac{1}{R} + r_1\sin^2(\beta-\alpha) + r_2\cos^2(\beta-\alpha) -$$

$$-r_3 + \frac{1}{M_W^2}\mathrm{Tr}(m_u^2 \mathrm{ctg}^2\beta + m_d^2 \mathrm{tg}^2\beta)\Big]P + 4(1-R)P_{IR} - 5 + 5R +$$

$$+\frac{1}{4R} + \Big(R - 1 + \frac{1}{4R}\Big)\ln R + 2(1-R)\ln r_+ + \frac{1}{4M_W^2}\cdot$$

$$\cdot\mathrm{Tr}(m_u^2 \mathrm{ctg}^2\beta + m_d^2 \mathrm{tg}^2\beta) - 2(1-R) I_0(-M_{H^\pm}^2, 0, M_{H^\pm}^2) -$$

$$-2\Big(R-1+\frac{1}{4R}\Big) I_0(-M_{H^\pm}^2, M_Z^2, M_{H^\pm}^2) + \frac{\sin^2(\beta-\alpha)}{2}(1 - r_+ +$$

$$+r_1) I_0(-M_{H^\pm}^2, M_W^2, M_{H_1^0}^2) + \frac{\cos^2(\beta-\alpha)}{2}(1-r_+ + r_2) I_0(-M_{H^\pm}^2,$$

$$M_W^2, M_{H_2^0}^2) - \frac{1-r_+ + r_3}{2}\cdot I_0(-M_{H^\pm}^2, M_W^2, M_{H_3^0}^2) + \frac{1}{2M_W^2}\mathrm{Tr}(m_u^2\cdot$$

$$\cdot \mathrm{ctg}^2\beta + m_d^2 \mathrm{tg}^2\beta) I_0(-M_{H^\pm}^2, m_u^2, m_d^2) + \Big(4R - 4 + \frac{1}{R} - \frac{1}{r_+} +$$

$$+\frac{1}{r_+R} - \frac{1}{4r_+R^2}\Big)M_{H^\pm}^2\frac{\partial I_0(q^2=-M_{H^\pm}^2, M_Z^2, M_{H^\pm}^2)}{\partial q^2} + \frac{\sin^2(\beta-\alpha)}{4}\cdot$$

$$\cdot\Big(\frac{(r_1-r_+-1)^2}{r_+} - 4\Big)M_{H^\pm}^2\frac{\partial I_0(q^2=-M_{H^\pm}^2, M_W^2, M_{H_1^0}^2)}{\partial q^2} + \frac{\cos^2(\beta-\alpha)}{4}\cdot$$

$$\cdot\Big(\frac{(r_2-r_+-1)^2}{r_+}-4\Big)M_{H^\pm}^2\frac{\partial I_0(q^2=-M_{H^\pm}^2, M_W^2, M_{H_2^0}^2)}{\partial q^2} - \Big(\frac{1}{r_+}-1\Big)\cdot$$

$$\cdot M_{H^\pm}^2\frac{\partial I_0(q^2=-M_{H^\pm}^2, M_W^2, M_{H_3^0}^2)}{\partial q^2} - \frac{1}{r_+}\Big(\cos(\beta-\alpha) - \frac{1}{2R}\cos 2\beta\cdot$$

$$\cdot\cos(\beta+\alpha)\Big)^2 M_{H^\pm}^2\frac{\partial I_0(q^2=-M_{H^\pm}^2, M_{H^\pm}^2, M_{H_1^0}^2)}{\partial q^2} - \frac{1}{r_+}\Big(\sin(\beta-\alpha) +$$

$$+\frac{1}{2R}\cos 2\beta\sin(\beta+\alpha)\Big)^2 M_{H^\pm}^2\frac{\partial I_0(q^2=-M_{H^\pm}^2, M_{H^\pm}^2, M_{H_2^0}^2)}{\partial q^2} -$$

$$-\frac{r_+}{2}Tr\left[m_u^2\, ctg^2\beta + m_d^2\, tg^2\beta - \frac{(m_u^2+m_d^2)(m_u^2\, ctg^2\beta + m_d^2\, tg^2\beta)}{M_{H^\pm}^2} - \frac{4m_u^2 m_d^2}{M_{H^\pm}^2}\right]\frac{\partial \bar{I}_0(q^2=-M_{H^\pm}^2, m_u^2, m_d^2)}{\partial q^2}\Bigg\}. \quad (8)$$

The other counter terms are

$$Z_A - 1 = \frac{ie^2}{16\pi^2}\left\{\left[-\frac{40}{3}+\frac{8}{3}Tr\, Q_i^2\right]P + \frac{1}{3}\ln r_+ + \frac{2}{3}\left(1 + Tr\, Q_i^2\right) + \frac{4}{3} Tr\, Q_i^2 \ln\frac{m_i^2}{M_W^2}\right\}. \quad (9)$$

$$Z_Z - 1 = \frac{ig^2}{16\pi^2}\left\{\left[4 - \frac{40}{3}R + \frac{1}{R} + \frac{1}{3}\left(2-\frac{1}{R}\right)N_f + \frac{8}{3}\frac{(1-R)^2}{R}Tr\, Q_i^2\right]P\right.$$

+ finite part of SM with the exception of Higgs terms

$$+\frac{2}{9} - \frac{2}{9}R - \frac{1}{4}\frac{1}{R} - \frac{R}{12}(r_1^2+r_2^2+r_3^2) - \frac{1}{12}\ln R\left(\frac{1}{R} - r_1\cos^2(\beta-\alpha) - r_2\sin^2(\beta-\alpha)\right) + \frac{r_1\cos^2(\beta-\alpha)+r_2\sin^2(\beta-\alpha)}{6} -$$

$$-\frac{r_1\ln r_1\cos^2(\beta-\alpha)+r_2\ln r_2\sin^2(\beta-\alpha)}{12} + \frac{R}{12}\left(r_1^2\ln r_1 + r_2^2\ln r_2 + r_3^2\ln r_3\right) + \frac{r_3R}{6}\left(r_1\sin^2(\beta-\alpha)+r_2\cos^2(\beta-\alpha)\right) - \frac{r_3R}{12}\left(r_1\ln r_1 \times \sin^2(\beta-\alpha) + r_2\ln r_2\cos^2(\beta-\alpha)\right) - \frac{r_3R\ln r_3}{12}\left(r_1\sin^2(\beta-\alpha) + r_2\cos^2(\beta-\alpha)\right) + \frac{1}{12}\left(4R-4+\frac{1}{R}\right)\bar{I}_0(-M_Z^2,M_{H^\pm}^2,M_{H^\pm}^2) -$$

$$-\frac{\sin^2(\beta-\alpha)}{12}\left(\frac{1}{R} - R(r_3-r_1)^2\right)\bar{I}_0(-M_Z^2, M_{H_1^0}^2, M_{H_3^0}^2) - \frac{\cos^2(\beta-\alpha)}{12}\times$$

$$\times\left(\frac{1}{R} - R(r_3-r_2)^2\right)\bar{I}_0(-M_Z^2,M_{H_2^0}^2,M_{H_3^0}^2) + \frac{r_1}{6}\left(1-\frac{r_1R}{2}\right)\cos^2(\beta-\alpha)\times \bar{I}_0(-M_Z^2,M_Z^2,M_{H_1^0}^2) + \frac{r_2}{6}\left(1-\frac{r_2R}{2}\right)\sin^2(\beta-\alpha)\bar{I}_0(-M_Z^2,M_Z^2,M_{H_2^0}^2) +$$

$$+\frac{1}{3}\left(1 - R + \frac{r_+}{2} - \frac{1}{4R} - 2r_+R - 2r_+R^2\right)M_Z^2\frac{\partial \bar{I}_0(q^2=-M_Z^2,M_{H^\pm}^2,M_{H^\pm}^2)}{\partial q^2} -$$

$$+\frac{\sin^2(\beta-\alpha)}{3}\left[r_1 - \frac{R}{4}\left(r_3-r_1-\frac{1}{R}\right)^2\right]M_Z^2\frac{\partial\bar{I}_0(q^2=-M_Z^2,M_{H_1^0}^2,M_{H_3^0}^2)}{\partial q^2} -$$

$$+\frac{\cos^2(\beta-\alpha)}{3}\left[r_2 - \frac{R}{4}\left(r_3-r_2-\frac{1}{R}\right)^2\right]M_Z^2\frac{\partial \bar{I}_0(q^2=-M_Z^2,M_{H_2^0}^2,M_{H_3^0}^2)}{\partial q^2} -$$

$$-\frac{\cos^2(\beta-\alpha)}{R}\left[1-\frac{r_1R}{3}+\frac{r_1^2R^2}{12}\right]M_Z^2\frac{\partial\bar{I}_0(q^2=-M_Z^2,M_Z^2,M_{H_1^0}^2)}{\partial q^2} -$$

$$-\frac{\sin^2(\beta-\alpha)}{R}\left[1-\frac{r_2R}{3}+\frac{r_2^2R^2}{12}\right]M_Z^2\frac{\partial\bar{I}_0(q^2=-M_Z^2,M_Z^2,M_{H_2^0}^2)}{\partial q^2}\Bigg\}. \quad (10)$$

The counter term $\sqrt{Z_M}$ is calculated from the ZA – mixing diagrams.

$$\sqrt{Z_M} = \frac{ieg}{16\pi^2}R^{-1/2}\left\{\left[\frac{44}{3}R - \frac{8}{3} - \frac{1}{6}\frac{1}{R} - \frac{1}{3}N_f + \frac{8}{3}(1-R)Tr\, Q_f^2\right]P\right.$$

+ finite part of SM with the exception of Higgs terms (11)

$$+\frac{1}{9} - \frac{2}{9}R - \frac{2}{3}Rr_+\ln r_+(1-2R) - \left(\frac{1}{6} - \frac{1}{3}R + \frac{2}{3}r_+R - \frac{4}{3}r_+R^2\right)\bar{I}_0(-M_Z^2,M_{H^\pm}^2,M_{H^\pm}^2)\Bigg\}.$$

For neutral Higgs particles we have

$$Z_{H^0_{1,2}} - 1 = \frac{ig^2}{16\pi^2}\Big\{\Big[\Big[\begin{Bmatrix}\sin^2(\beta-\alpha)\\ \cos^2(\beta-\alpha)\end{Bmatrix}\cdot\Big(\frac{3}{2}-\frac{3}{2R}+r_+-r_3\Big) -$$

$$- \begin{Bmatrix}\cos^2(\beta-\alpha)\\ \sin^2(\beta-\alpha)\end{Bmatrix}\Big(3+\frac{3}{2R}-\frac{3}{2}r_{1,2}\Big) + \frac{1}{4M_W^2}\mathrm{Tr}\Big(\frac{m_u^2}{\sin^2\beta}\cdot$$

$$\cdot\begin{Bmatrix}\sin^2\alpha\\ \cos^2\alpha\end{Bmatrix} + \frac{m_d^2}{\cos^2\beta}\begin{Bmatrix}\cos^2\alpha\\ \sin^2\alpha\end{Bmatrix}\Big)\Big]P - \frac{1}{4} + \frac{3}{4}\begin{Bmatrix}\cos^2(\beta-\alpha)\\ \sin^2(\beta-\alpha)\end{Bmatrix} +$$

$$+\frac{1}{4R}(1+\ln R) + \frac{1}{16M_W^2}\mathrm{Tr}\Big(\frac{m_u^2}{\sin^2\beta}\begin{Bmatrix}\sin^2\alpha\\ \cos^2\alpha\end{Bmatrix} +$$

$$+\frac{m_d^2}{\cos^2\beta}\begin{Bmatrix}\cos^2\alpha\\ \sin^2\alpha\end{Bmatrix}\Big) + \frac{1}{2}\begin{Bmatrix}\sin^2(\beta-\alpha)\\ \cos^2(\beta-\alpha)\end{Bmatrix}(r_+ - r_{1,2}+1)\,\bar{I}_0(-M^2_{H^0_{1,2}},$$

$$M_W^2, M_{H^\pm}^2) - \frac{1}{2}\begin{Bmatrix}\sin^2(\beta-\alpha)\\ \cos^2(\beta-\alpha)\end{Bmatrix}\Big(r_3 - r_{1,2}+\frac{1}{R}\Big)\bar{I}_0(-M^2_{H^0_{1,2}},$$

$$M_Z^2, M^2_{H^0_3}) - \begin{Bmatrix}\cos^2(\beta-\alpha)\\ \sin^2(\beta-\alpha)\end{Bmatrix}\Big(1-\frac{1}{2}r_{1,2}\Big)\bar{I}_0(-M^2_{H^0_{1,2}}, M_W^2, M_W^2) -$$

$$- \begin{Bmatrix}\cos^2(\beta-\alpha)\\ \sin^2(\beta-\alpha)\end{Bmatrix}\frac{1}{2R}\Big(1-\frac{1}{2}r_{1,2}R\Big)\bar{I}_0(-M^2_{H^0_{1,2}}, M_Z^2, M_Z^2) + \frac{1}{8M_W^2}\cdot$$

$$\cdot\mathrm{Tr}\Big[\frac{m_u^2}{\sin^2\beta}\begin{Bmatrix}\sin^2\alpha\\ \cos^2\alpha\end{Bmatrix}\bar{I}_0(-M^2_{H^0_{1,2}}, m_u^2, m_u^2)\Big] + \frac{1}{8M_W^2}\mathrm{Tr}\Big[\frac{m_d^2}{\cos^2\beta}\cdot$$

$$\cdot\begin{Bmatrix}\cos^2\alpha\\ \sin^2\alpha\end{Bmatrix}\bar{I}_0(-M^2_{H^0_{1,2}}, m_d^2, m_d^2)\Big] + \Big(\frac{(r_+ - r_{1,2}-1)^2}{4r_{1,2}} - 1\Big)\cdot M^2_{H^0_{1,2}}\cdot$$

$$\cdot\begin{Bmatrix}\sin^2(\beta-\alpha)\\ \cos^2(\beta-\alpha)\end{Bmatrix}\frac{\partial\bar{I}_0(q^2=-M^2_{H^0_{1,2}}, M_W^2, M^2_{H^\pm})}{\partial q^2} - \Big(\frac{(r_3 - r_{1,2}-1/R)^2}{4r_{1,2}} -$$

$$-\frac{1}{R}\Big)\begin{Bmatrix}\sin^2(\beta-\alpha)\\ \cos^2(\beta-\alpha)\end{Bmatrix}M^2_{H^0_{1,2}}\frac{\partial\bar{I}_0(q^2=-M^2_{H^0_{1,2}}, M_Z^2, M^2_{H^0_3})}{\partial q^2} + \Big(1 -$$

$$-\frac{r_{1,2}}{4} - 3\frac{1}{r_{1,2}}\Big)M^2_{H^0_{1,2}}\frac{\partial\bar{I}_0(q^2=-M^2_{H_{1,2}}, M_W^2, M_W^2)}{\partial q^2}\begin{Bmatrix}\cos^2(\beta-\alpha)\\ \sin^2(\beta-\alpha)\end{Bmatrix}$$

$$+\Big(\frac{1}{2}\frac{1}{R} - \frac{r_{1,2}}{8} - \frac{3}{2}\frac{1}{R^2 r_{1,2}}\Big)\begin{Bmatrix}\cos^2(\beta-\alpha)\\ \sin^2(\beta-\alpha)\end{Bmatrix}M^2_{H^0_{1,2}}\cdot$$

$$\cdot\frac{\partial\bar{I}_0(q^2=-M^2_{H^0_{1,2}}, M_Z^2, M_Z^2)}{\partial q^2} - \frac{1}{r_{1,2}}\Big[\begin{Bmatrix}\cos(\beta-\alpha)\\ \sin(\beta-\alpha)\end{Bmatrix}\mp\frac{\cos 2\beta}{2R}\cdot$$

$$\cdot\begin{Bmatrix}\cos(\beta+\alpha)\\ \sin(\beta+\alpha)\end{Bmatrix}\Big]^2 M^2_{H^0_{1,2}}\frac{\partial\bar{I}_0(q^2=-M^2_{H^0_{1,2}}, M^2_{H^\pm}, M^2_{H^\pm})}{\partial q^2} - \frac{9}{8}\frac{1}{R^2 r_{1,2}}\cdot$$

$$\cdot\cos^2 2\alpha\begin{Bmatrix}\cos^2(\beta+\alpha)\\ \sin^2(\beta+\alpha)\end{Bmatrix}M^2_{H^0_{1,2}}\frac{\partial\bar{I}_0(q^2=-M^2_{H^0_{1,2}}, M^2_{H^0_{1,2}}, M^2_{H^0_{1,2}})}{\partial q^2} -$$

$$-\frac{1}{8}\frac{1}{R^2 r_{1,2}}\Big[2\sin 2\alpha\begin{Bmatrix}\sin(\beta+\alpha)\\ \cos(\beta+\alpha)\end{Bmatrix}\mp\cos 2\alpha\begin{Bmatrix}\cos(\beta+\alpha)\\ \sin(\beta+\alpha)\end{Bmatrix}\Big]^2\cdot M^2_{H^0_{1,2}}\cdot$$

$$\cdot\frac{\partial\bar{I}_0(q^2=-M^2_{H^0_{1,2}}, M^2_{H^0_{2,1}}, M^2_{H^0_{2,1}})}{\partial q^2} - \frac{1}{4}\frac{1}{R^2 r_{1,2}}\Big[2\sin 2\alpha\begin{Bmatrix}\cos(\beta+\alpha)\\ \sin(\beta+\alpha)\end{Bmatrix}$$

$$\pm\cos 2\alpha\begin{Bmatrix}\sin(\beta+\alpha)\\ \cos(\beta+\alpha)\end{Bmatrix}\Big]^2 M^2_{H^0_{1,2}}\frac{\partial\bar{I}_0(q^2=-M^2_{H^0_{1,2}}, M^2_{H^0_{1,2}}, M^2_{H^0_{2,1}})}{\partial q^2} - \frac{1}{8}\frac{\cos^2 2\beta}{R^2 r_{1,2}}\cdot$$

$$\cdot\begin{Bmatrix}\cos^2(\beta+\alpha)\\ \sin^2(\beta+\alpha)\end{Bmatrix}M^2_{H^0_{1,2}}\frac{\partial\bar{I}_0(q^2=-M^2_{H^0_{1,2}}, M^2_{H^0_3}, M^2_{H^0_3})}{\partial q^2} - \frac{r_{1,2}}{8\sin^2\beta}\begin{Bmatrix}\sin^2\alpha\\ \cos^2\alpha\end{Bmatrix}\cdot$$

$$\cdot\mathrm{Tr}\Big(m_u^2 - \frac{2m_u^4}{M^2_{H^0_{1,2}}}\Big)\frac{\partial\bar{I}_0(q^2=-M^2_{H^0_{1,2}}, m_u^2, m_u^2)}{\partial q^2} - \frac{r_{1,2}}{8\cos^2\beta}\begin{Bmatrix}\cos^2\alpha\\ \sin^2\alpha\end{Bmatrix}\cdot$$

$$Tr\left(m_d^2 - \frac{2m_d^2}{M^2_{H^0_{1,2}}}\right)\frac{\partial I_0(q^2=-M^2_{H^0_{1,2}}, m_d^2, m_d^2)}{\partial q^2} \qquad (12)$$

and for $H_1^0 H_2^0$- mixing the appropriate counter term is

$$\sqrt{Z_{M1,M2}} = \frac{ig^2}{16\pi^2}\Big\{\Big[-\frac{11}{4}\sin 2(\beta-\alpha) + \frac{3}{8} r_{1,2}\sin 2(\beta-\alpha) +$$

$$+\frac{1}{r_{1,2}}\Big(4\sin 2(\beta-\alpha) + \frac{r_+}{4}(\sin 2\beta\cos 2\alpha - 3\sin 2\alpha\cos 2\beta) +$$

$$+\frac{r_3}{4}\sin 2(\beta-\alpha)\Big) + \frac{1}{r_{1,2}R}\Big(\frac{3}{2}r_+\sin 2\alpha\cos 2\beta - \frac{3}{8}(r_1-r_2)\times$$

$$\times \sin 4\alpha\Big) + \frac{1}{r_{1,2}R^2}\Big(\frac{3}{2}\sin 2(\beta-\alpha) - \frac14\sin 4\alpha\cos 2(\beta+\alpha) -$$

$$-\sin 2(\beta+\alpha)\Big(\sin^2 2\alpha + \frac{\cos^2 2\alpha}{2} + \frac38\cos^2 2\beta\Big)\Big) + \frac{\sin 2\alpha}{2M_W^2\sin^2\beta}\times$$

$$\times Tr\, m_u^2\Big(1-\frac{6m_u^2}{M^2_{H^0_{1,2}}}\Big) - \frac{\sin 2\alpha}{2M_W^2\cos^2\beta}Tr\, m_d^2\Big(1-\frac{6m_d^2}{M^2_{H^0_{1,2}}}\Big)\Big]P +$$

$$+\frac38\sin 2(\beta-\alpha) + \frac{1}{r_{1,2}}\Big(\frac98\sin 2(\beta-\alpha) + \frac{r_+}{4}(2\sin 2\alpha\cos 2\beta -$$

$$-\sin 2\beta\cos 2\alpha) - \frac{r_+\ln r_+}{8}(3\sin 2\alpha\cos 2\beta - \sin 2\beta\cos 2\alpha)\Big) +$$

$$+\frac{1}{r_{1,2}R}\Big(-\frac{r_+}{4}\cos 2\beta\sin 2\alpha + \frac{r_3}{8}(2\sin 2\beta\cos 2\alpha - 3\sin 2\alpha\times$$

$$\times\cos 2\beta) + \frac{r_+\ln r_+}{4}\cos 2\beta\sin 2\alpha + \frac{r_3\ln r_3}{8}\times$$

$$\times(2\sin 2\alpha\cos 2\beta - \sin 2\beta\cos 2\alpha) +$$

$$+\frac{3}{16}\sin 4\alpha(r_1-r_2) - \frac{3}{16}r_1\ln r_1\sin 4\alpha + \frac{3}{16}r_2\ln r_2\sin 4\alpha\Big) +$$

$$+\frac{1}{r_{1,2}R}\ln R\Big(\frac{r_3}{8}\sin 2(\beta-\alpha) - \frac{1}{8R}\sin 2(\beta-\alpha)\Big) + \frac{1}{r_{1,2}R^2}\frac38\times$$

$$\times\sin 2(\beta-\alpha) + \frac{\sin 2\alpha}{8M_W^2\sin^2\beta}Tr\, m_u^2\Big(1-\frac{2m_u^2}{M^2_{H^0_{1,2}}}\Big) - \frac{\sin 2\alpha}{8M_W^2\cos^2\beta}\times$$

$$\times Tr\, m_d^2\Big(1-\frac{2m_d^2}{M^2_{H^0_{1,2}}}\Big) - \frac{\sin 2\alpha}{2M_W^2\sin^2\beta}Tr\frac{m_u^4}{M^2_{H^0_{1,2}}}\ln\frac{m_u^2}{M_W^2} +$$

$$+\frac{\sin 2\alpha}{2M_W^2\cos^2\beta}Tr\frac{m_d^4}{M^2_{H^0_{1,2}}}\ln\frac{m_d^2}{M_W^2} + \frac18\Big(\frac{(r_+-r_{1,2}-1)^2}{r_{1,2}} - 4\Big)\sin 2(\beta-\alpha)\times$$

$$\times I_0(-M^2_{H^0_{1,2}}, M_W^2, M^2_{H^\pm}) - \frac18\Big(\frac{(r_3-r_{1,2}-1/R)^2}{r_{1,2}} - \frac4R\Big)\sin 2(\beta-\alpha)\times$$

$$\times I_0(-M^2_{H^0_{1,2}}, M_Z^2, M^2_{H^0_3}) - \Big(\frac12 - \frac{r_{1,2}}{8} - \frac{3}{2r_{1,2}}\Big)\sin 2(\beta-\alpha) I_0(-M^2_{H^0_{1,2}},$$

$$M_W^2, M_W^2) - \Big(\frac{1}{4R} - \frac{r_{1,2}}{16} - \frac{3}{4r_{1,2}R^2}\Big)\sin 2(\beta-\alpha) I_0(-M^2_{H^0_{1,2}}, M_Z^2,$$

$$M_Z^2) + \frac{1}{r_{1,2}}\Big(\cos(\beta-\alpha)\sin(\beta-\alpha) + \frac{1}{2R}\sin 2\alpha\cos 2\beta - \frac{1}{4R^2}\cos^2 2\beta\times$$

$$\times\cos(\beta+\alpha)\sin(\beta+\alpha)\Big) I_0(-M^2_{H^0_{1,2}}, M^2_{H^\pm}, M^2_{H^\pm}) - \frac{1}{8R^2 r_{1,2}}\cos^2 2\beta\times$$

$$\times\cos(\beta+\alpha)\sin(\beta+\alpha) I_0(-M^2_{H^0_{1,2}}, M^2_{H^0_3}, M^2_{H^0_3}) - \frac{3\cos 2\alpha}{8R^2 r_{1,2}}(2\sin 2\alpha\times$$

$$\times\cos^2(\beta+\alpha) + \frac12\cos 2\alpha\sin 2(\beta+\alpha)) I_0(-M^2_{H^0_{1,2}}, M^2_{H^0_1}, M^2_{H^0_1}) + \frac{3\cos 2\alpha}{8R^2 r_{1,2}}\times$$

$$\left(2\sin 2\alpha \sin^2(\beta+\alpha) - \frac{1}{2}\cos 2\alpha \sin 2(\beta+\alpha)\right)\bar{I}_0\left(-M^2_{H^0_{1,2}}, M^2_{H^0_2}, M^2_{H^0_2}\right) +$$

$$-\frac{1}{4R^2 r_{1,2}}\left(2\sin^2 2\alpha \sin 2(\beta+\alpha) - \sin 4\alpha \cos 2(\beta+\alpha) - \frac{1}{2}\cos^2 2\alpha \times\right.$$

$$\left.\times \sin 2(\beta+\alpha)\right)\bar{I}_0\left(-M^2_{H^0_{1,2}}, M^2_{H^0_1}, M^2_{H^0_2}\right) + \frac{\sin 2\alpha}{4M_W^2 \sin^2\beta}\,\mathrm{Tr}\, m_u^2\left(1 -\right.$$

$$\left.-\frac{4m_u^2}{M^2_{H^0_{1,2}}}\right)\bar{I}_0\left(-M^2_{H^0_{1,2}}, m_u^2, m_u^2\right) - \frac{\sin 2\alpha}{4M_W^2\cos^2\beta}\,\mathrm{Tr}\, m_d^2\left(1 - \frac{4m_d^2}{M^2_{H^0_{1,2}}}\right)\times$$

$$\left.\times \bar{I}_0\left(-M^2_{H^0_{1,2}}, m_d^2, m_d^2\right)\right\} \qquad (13)$$

The above expressions contain next abbreviations: $r_+ = M^2_{H^\pm}/M^2_W$, $r_{H^0_i} = M^2_{H^0_i}/M^2_W$, $R = M_W^2/M_Z^2$, $tg\,\beta = v_2/v_1$. M_i are masses of particles in the SUSY model; v_1 and v_2 are expectation values; α is a mixing angle in the neutral scalar sector.

So we calculated the counter terms for a renormalization in the unitary gauge. The analogous problem was solved in /2/, but we didn't use any approximations as contrasted to the author of the ref. paper.

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Двоеглазов В.В., Скачков Н.Б. E2-91-179

H^+H^--взаимодействие с учетом высших порядков теории возмущений в модели с двумя хиггсовскими дублетами /контрчлены/

Вычислены контрчлены, необходимые для перенормировки однопетлевых диаграмм, которые используются для построения амплитуды H^+H^- взаимодействия в рамках модели с минимальным суперсимметричным расширением стандартной модели.

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Dvoeglazov V.V., Skachkov N.B. E2-91-179

H^+H^- Interaction up to Higher Orders of Perturbation Theory in the Model with Two Higgs Doublets /Counter Terms/

The counter terms for the one-loop renormalization of the H^+H^- amplitude in the framework of the minimal SUSY extension of the standard model are calculated.

The investigation has been performed at the Laboratory of Theoretical Physics, JINR.

Communication of the Joint Institute for Nuclear Research. Dubna 1991