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ON TWO-DIMENSIONAL QUASICLASSICS

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1. INTRODUCTION

Quasiclassics is the branch of quantum mechanics, which studies the Schroedinger equation with a large and smooth enough potential. We shall write down this equation in the form

$$[\Delta + \lambda^2 f(x)]\psi(x,\lambda) = 0, \quad \lambda \to \infty. \tag{1.1}$$

Here $x = (x_1, x_2)$, $\Delta = \sum_i \partial^2/\partial x_i^2$ is the Laplace operator. Up to now one investigated mainly the one-dimensional quasiclassics, where one got a complete solution of the problem: one constructed the series

$$\psi(x,\lambda) = e^{i\lambda S(x)} \sum_{n=0}^{\infty} A_n(x)\,\lambda^{-n} \tag{1.2}$$

for the function ψ if the point x does not belong to a vicinity of a turning point $x_0(f(x_0) = 0)$ [1]. Main quasiclassical approximation to the wave function in a vicinity of a turning point, $\psi_0(x,\lambda)$, is determined by the Airy integral [1]. One also constructed a series for $\psi(x,\lambda)$

$$\psi(x,\lambda) = \psi_0(x,\lambda) + \psi_1(x,\lambda)/\lambda + \ldots$$

which gives a homogeneous (in powers of λ) decomposition of ψ in a vicinity of a turning point [2] and decompositions in inverse powers of λ for the bound-state energies (see the work [3] and references therein). From the results on many-dimensional quasiclassics we shall mention interesting result by Newell concerning the density of energy levels in an arbitrary potential [4]. This result enables one to prove (in quasiclassical limit) the microcanonical distribution via methods of quantum mechanics [5].

1. Here we shall study the two-dimensional quasiclassics: $x = (x_1, x_2)$. We shall study the potential V(x)

$$\lambda^2 f(x) = 2m(E - V(x)) \tag{1.3}$$

which does weakly depend on the azimuthal angle ϕ:

$$V(x) = V_0(r) + \epsilon V_1(r, \phi), \quad x_1 = r\cos\phi, \quad x_2 = r\sin\phi, \tag{1.4}$$

here ϵ is a small parameter. The change of variables $\rho = \ln r$ gives:

$$\Delta = \frac{1}{r}\frac{\partial}{\partial r} r\frac{\partial}{\partial r} + \frac{1}{r^2}\frac{\partial^2}{\partial\phi^2} = \frac{1}{r^2}\left(\frac{\partial^2}{\partial\rho^2} + \frac{\partial^2}{\partial\phi^2}\right). \tag{1.5}$$

Thus our Schroedinger equation assumes the form

$$\left[\frac{\partial^2}{\partial\rho^2} + \frac{\partial^2}{\partial\phi^2} + \lambda^2\eta(\rho, \phi, \epsilon)\right]\psi = 0, \tag{1.6}$$

$$\eta(\rho, \phi, \epsilon) = \eta_0(\rho) + \epsilon\eta_1(\rho, \phi), \tag{1.7}$$

$$\eta_1(\rho, \phi + 2\pi) = \eta_1(\rho, \phi).$$

If the point ρ, ϕ does not belong to a vicinity of turning curve (this curve is not determined by the equation $\eta(\rho, \phi, \epsilon) = 0$), one has again representation (1.2) of the function ψ. Here

$$(\nabla S(\rho, \phi))^2 = \eta(\rho, \phi, \epsilon), \tag{1.8}$$

$$\nabla(A_0\nabla S) + (\nabla A_0)\nabla S = 0, \tag{1.8a}$$

and so on; $\nabla = (\partial/\partial\rho, \partial/\partial\phi)$. In one dimension (there is no ϕ) one solves eq.(1.8) trivially /1/. But in two dimensions this equation is nontrivial.

1.1. In this work we shall construct some series in powers of ϵ which represent a solution $S(\rho, \phi)$ of eq.(1.8).

1.2. Let us first consider a radially symmetric potential: $\eta_1(\rho, \phi) = 0$. Then,

$$S(\rho, \phi) = R(\rho) + \omega\phi \tag{1.9}$$

where the function R satisfies the equation

$$\frac{dR}{d\rho} = \pm(\eta_0(\rho) - \omega^2)^{1/2}. \tag{1.10}$$

1.2.1. We shall suppose the continuous function $\eta_0(\rho) - \omega^2$, $\omega^2 < \omega_0^2$, to be positive within some interval

$$\alpha(\omega) < \rho < \beta(\omega) \tag{1.11}$$

and to be negative outside of this interval. Two zeroes of the function $\eta_0(\rho) - \omega^2$ will be denoted by $\alpha(\omega)$ and $\beta(\omega)$, $\alpha(\omega) < \beta(\omega)$ if $\omega^2 < \omega_0^2 \equiv \max_\rho \eta_0(\rho)$.

1.3. We shall generalize eqs.(1.9) and (1.10), if the potential depends on ϕ, as follows

$$\nabla S = \pm(a(\epsilon)^2 - U(\rho, \phi, \epsilon)^2)^{1/2}\nabla U(\rho, \phi, \epsilon) + \nabla\Phi(\rho, \phi, \epsilon), \tag{1.12}$$

$U(\rho, \phi + 2\pi, \epsilon) = U(\rho, \phi, \epsilon)$, see also eq.(1.18).
Here functions U and Φ and their first derivatives are continuous for all real values of ρ and ϕ, $a(\epsilon)$ is a constant, which depends only on ϵ. Equation (1.8) has to be satisfied for both signs $\pm$ in eq.(1.12). Thus, one has

$$\nabla U \nabla\Phi = 0. \tag{1.13}$$

One can satisfy eq. (1.13), setting

$$\nabla\Phi = M\tilde{\nabla}U, \quad M(\rho, \phi + 2\pi, \epsilon) = M(\rho, \phi, \epsilon). \tag{1.14}$$

where $\tilde{\nabla} = (-\partial/\partial\phi, \partial/\partial\rho)$ and function $M \equiv M(\rho, \phi, \epsilon)$ satisfies the equation

$$\nabla(M\nabla U) = 0 \tag{1.15}$$

which follows from the condition $\frac{\partial}{\partial\rho}\left(\frac{\partial\Phi}{\partial\phi}\right) = \frac{\partial}{\partial\phi}\left(\frac{\partial\Phi}{\partial\rho}\right)$ (see eq.(1.14)).

Thus we have reduced our problem to that of constructing the functions $M(\rho, \phi, \epsilon)$, $U(\rho, \phi, \epsilon)$ and the constant $a(\epsilon)$:

$$U = U_0(\rho) + \epsilon U_1(\rho, \phi) + \epsilon^2 U_2(\rho, \phi) + \dots \tag{1.16a}$$

$$M = \frac{\omega}{U_0'(\rho)} + \epsilon M_1(\rho, \phi) + \epsilon^2 M_2(\rho, \phi) + \dots \tag{1.16b}$$

$$a = a_0 + \epsilon a_1 + \epsilon^2 a_2 + \dots \tag{1.16c}$$

for the choice of the function M_0, $M_0 = \omega/U_0'(\rho)$ see Sec.2.

1.3.1. In Sec.2 we shall write down equations for determination of the functions U_n and M_n, n = 0,1,2,... These equations determine the functions U_n and M_n uniquely for any given value of a_n.

1.3.2. The condition that the n-th order correction does not change the value of ω^*, eq. (1.16b), enables one to determine the value of a_n. To prove this statement, we shall introduce decompositions

$$U_n(\rho,\phi) = \sum_k U_{nk}(\rho)\, e^{ik\phi}, \tag{1.17a}$$

$$M_n(\rho,\phi) = \sum_k M_{nk}(\rho)\, e^{ik\phi} \tag{1.17b}$$

and require

$$\Phi(\rho,\phi+2\pi) - \Phi(\rho,\phi) = 2\pi\omega. \tag{1.18}$$

Equations (1.14) and (1.18) give

$$B(\rho,a) \equiv \int_0^{2\pi} M(\rho,\phi,\epsilon)\, \frac{\partial U(\rho,\phi,\epsilon)}{\partial\rho}\, d\phi = 2\pi\omega. \tag{1.19}$$

It is easy to see that it follows from eqs.(1.17) and (1.15) that

$$\frac{dB(\rho,a)}{d\rho} = 0. \tag{1.20}$$

The procedure of Sec.2 shows that the functions

$$B_n(\rho,a) = \int_0^{2\pi} \sum_{\ell=0}^{n} M_\ell(\rho,\phi)\, \frac{\partial U_{n-\ell}(\rho,\phi)}{\partial\rho}\, d\phi$$

n = 1,2,..., depend on a_n linearly (see eq.(2.3)-(2.6)). This fact, eq.(1.20) and the representation

$$B(\rho,a) = \sum_0 B_n(\rho,a)\,\epsilon^n$$

prove that a) $B_n(\rho,a)$ does not depend on ρ and b) it can be eliminated by appropriately choosing a_n, n = 1,2,... Then eqs.(1.18) and (1.19) would be satisfied.

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*Quantity ω is determined via the condition (1.18).

1.4. When expressing M_n in terms of U_0, ... U_{n-1}, U_n, M_0,... M_{n-1}, a_0, a_1, ... a_n via eq.(2.13), item 2.2, we use, as a matter of fact, the condition that ω is not too small.

Sec.3 contains the extension of the Section´s 2 consideration to the case of small values of ω.

1.5. Sec.4 contains the solution to the Kramers - Airy problem (i.e. the consideration of the wave function in a vicinity of the turning curve). Our solution is not complete for we are not able to prove the property $C(\rho,\phi+2\pi) = C(\rho,\phi)$ of the function C (see eq.(4.3)). Note that the turning curve is determined by equation $U(\rho,\phi,\epsilon) = \pm a$.

2. THE ϵ-DECOMPOSITIONS, $\omega \gg \sqrt{\epsilon}$

Equations (1.8), (1.12) and (1.14) give

$$[a(\epsilon)^2 - U^2 + M^2](\nabla U)^2 = \eta(\rho,\phi,\epsilon). \tag{2.1}$$

Substituting here decompositions (1.16) one gets the equations

$$[a_0^2 - U_0^2 + M_0^2]\, U_0'(\rho)^2 = \eta_0(\rho), \quad U_0'(\rho) = \frac{dU_0(\rho)}{d\rho}, \tag{2.2}$$

$$LU_n + (U_0')^2 M_0 M_n = F_n, \quad n = 1,2,3,\dots . \tag{2.3}$$

Here

$$L = (a_0^2 - U_0^2)\, U_0' \frac{\partial}{\partial\rho} + M_0^2 U_0' \frac{\partial}{\partial\rho} - U_0 (U_0')^2, \tag{2.4}$$

$$F_1 = \eta_1(\rho,\phi)/2 - a_0 a_1 (U_0')^2, \tag{2.5}$$

$$F_2 = -(a_0a_2 + a_1^2/2 - U_1^2/2 - M_1^2/2)(U_0')^2 - (a_0^2 - U_0^2)(\nabla U_1)^2/2 - $$
$$- U_0' \frac{\partial U_1}{\partial\rho} 2(a_0a_1 - U_0U_1 + M_0M_1), \tag{2.6}$$

etc. The functions F_n (as well as functions T_n in eq.(2.8)) depend on U_0, U_1,... U_{n-1}, M_0, M_1,... M_{n-1} and a_0, a_1,... a_{n-1}, a_n.

Substitution of decompositions (1.16) into eq.(1.14) gives

$$\frac{\partial}{\partial\rho}(M_0 U_0'(\rho)) = 0, \tag{2.7}$$

$$\frac{\partial}{\partial\rho}(M_0\frac{\partial U_n}{\partial\rho}) + \frac{\partial}{\partial\phi}(M_0\frac{\partial U_n}{\partial\phi}) + \frac{\partial}{\partial\rho}(M_n U_0') = T_n, \quad n = 1,2,3,\ldots, \tag{2.8}$$

$$T_1 = 0, \tag{2.9}$$

$$T_2 = -\nabla(M_1 \nabla U_1), \tag{2.10}$$

etc.

2. It follows from eq.(2.7) that

$$M_0 = \omega / U_0' \tag{2.11}$$

see eq.(1.16b), here ω is a constant. Then eq.(2.2) reduces to the equation

$$(a_0^2 - U_0^2)(U_0')^2 = \eta_0(\rho) - \omega^2 \tag{2.12}$$

which uniquely determines the bounded together with its first derivative function $U_0(\rho)$ and constant a_0, $a_0 > 0$ (if $\max_\rho \eta_0(\rho) > \omega^2$), $U_0' > 0$. One has

$$U_0(\alpha(\omega)) = -a_0, \quad U_0(\beta(\omega)) = a_0 \tag{2.12a}$$

see item 1.2.1. and eq.(2.12). Equation (2.11) enables one to rewrite eq.(2.3) as

$$LU_n + \omega U_0' M_n = F_n. \tag{2.13}$$

Using this equation, one can eliminate M_n from eq.(2.8):

$$\frac{\partial}{\partial\rho}(LU_n - \frac{\omega^2}{U_0'}\frac{\partial U_n}{\partial\rho}) - \frac{\omega^2}{U_0'}\frac{\partial^2 U_n}{\partial\phi^2} = \frac{\partial F_n}{\partial\rho} - \omega T_n. \tag{2.14}$$

Using decomposition (1.17) one reduces eq.(2.14) to the form

$$\tilde{L}_k U_{nk} = \frac{dF_{nk}(\rho)}{d\rho} - \omega T_{nk}(\rho). \tag{2.15}$$

Here

$$\tilde{L}_k = \frac{d}{d\rho}[(a_0^2 - U_0^2) U_0' \frac{d}{d\rho} - U_0(U_0')^2] + \frac{k^2\omega^2}{U_0'}. \tag{2.16}$$

2.1. Consider the homogeneous equation

$$\tilde{L}_k v(\rho) = 0 \tag{2.17}$$

in a vicinity of the point $\rho = \alpha(\omega)$ (or the point $\rho = \beta(\omega)$), see eq. (2.12a). Two independent solutions of eq.(2.17) behave as $v_1 \sim$ const and $v_2(\rho) \sim [U_0(\rho) + a]^{-1/2}$ (or $v_2(\rho) \sim [U_0(\rho) - a]^{-1/2}$).

Let us denote by $w_\alpha(\rho)$ the solution of eq.(2.17) which is regular at the point $\rho = \beta(\omega)$ (and, in general, has the singularity $\sim[\rho - \alpha(\omega)]^{-1/2}$ at the point $\rho = \alpha(\omega)$)and $w_\beta(\rho)$ the solution which is regular at the point $\rho = \alpha(\omega)$ and has the singularity $\sim[\rho - \beta(\omega)]^{-1/2}$ at the point $\rho = \beta(\omega)$.

It follows from eqs.(2.17) and (2.16) that

$$w(\rho) \equiv \frac{dw_\alpha}{d\rho} w_\beta - \frac{dw_\beta}{d\rho} w_\alpha = \text{const}(a_0^2 - U_0^2)^{-3/2} (U_0')^{-1}. \tag{2.18}$$

2.2. Then the formula

$$U_{nk}(\rho) = w_\alpha(\rho) \int_{\alpha(\omega)}^{\rho} w_\beta(s)\, w(s)^{-1} ds\, [\frac{dF_{nk}(s)}{ds} - \omega T_{nk}(s)] - \tag{2.19}$$
$$- w_\beta(\rho) \int_{\beta(\omega)}^{\rho} w_\alpha(s)\, w(s)^{-1} ds\, [\frac{dF_{nk}(s)}{ds} - \omega T_{nk}(s)]$$

gives the solution to eq.(2.15) which is continuous together with its first two derivatives U_{nk}' and U_{nk}'' at points $\rho = \alpha(\omega)$ and $\rho = \beta(\omega)$. Equation (2.3) enables one to express the function $M_n(\rho, \phi)$ in terms of the functions U_0, $U_1, \ldots U_n$, M_0, M_1, $\ldots M_{n-1}$ and constants a_0, $a_1, \ldots, a_n$.

Thus, we have solved our problem: we have given the recurrent procedure for the subsequent determinations of the functions U_n, M_n, $n = 0,1,2,\ldots$

3. CONSIDERATION OF POSSIBILITY $\omega \sim \sqrt{\epsilon}$

The case of small values of ω is to be considered separately (see item 1.4). Here we shall consider the case

$$\omega = \gamma\sqrt{\epsilon}. \tag{3.1}$$

We shall use equations (1.16a), (1.16c) and equation

$$M = M_{1/2}\epsilon^{1/2} + M_{3/2}\epsilon^{3/2} + \dots \qquad (3.2)$$

(instead of eq. (1.16b)).
One gets (cf. eqs.(2.2) and (2.3))

$$[a_0^2 - U_0^2(\rho)][U_0'(\rho)]^2 = \eta_0(\rho), \qquad (3.3)$$

$$L_0 U_n = Q_n, \quad n = 1, 2, \dots \qquad (3.4)$$

here L_0 is the operator (2.4) with $M_0 \equiv 0$ and

$$Q_1 = F_1 - [U_0'(\rho) M_{1/2}(\rho, \phi)]^2/2, \qquad (3.5)$$

$$Q_2 = F_2|_{M_0=M_1=0} - (U_0')^2 M_{1/2} M_{3/2} - U_0' \frac{\partial U_1(\rho, \phi)}{\partial \rho}(M_{1/2})^2, \qquad (3.6)$$

etc. (see eqs.(2.5) and (2.6)).
Equation (1.15) gives

$$\frac{\partial}{\partial \rho}[M_{n+½}(\rho, \phi) U_0'(\rho)] = R_n, \quad n = 0, 1, 2, \dots, \qquad (3.7)$$

$$R_0 = 0, \qquad (3.8)$$

$$R_1 = -\nabla(M_{1/2}(\rho, \phi)\nabla U_1(\rho, \phi)), \qquad (3.9)$$

and so on.

3.1. Equation (3.3) uniquely determines continuous together with its derivative function $[U_0(\rho)]^2$, $U_0'(\rho) > 0$ and constant a_0^2. Then eq.(3.7) with n = 0 and eq.(3.8) give

$$M_{1/2}(\rho, \phi) = \alpha_{1/2}(\phi)/U_0'(\rho). \qquad (3.10)$$

3.1.2. A solution to equation $L_0 v = 0$ is $v(\rho) = [a_0^2 - U_0^2(\rho)]^{-1/2}$, thus equation

$$U_n(\rho) = [a_0^2 - U_0(\rho)^2]^{-1/2} \int_{\rho_-}^{\rho} q_n(s, \phi)\, ds, \qquad (3.11)$$

$$q_n(s, \phi) = [a_0^2 - U_0(s)^2]^{-1/2} U_0'(s)^{-1} Q_n(s, \phi), \qquad (3.12)$$

gives some solution to eq.(3.4). We shall choose $\rho_\pm$ as the roots of equation

$$U_0(\rho) = \pm a_0. \qquad (3.13)$$

Then the function (3.11) is bounded at the point $\rho = \rho_-$. We need this function to be bounded also at the point $\rho = \rho_+$. Then the condition

$$\int_{\rho_-}^{\rho_+} q(s, \phi)\, ds = 0 \qquad (3.14)$$

has to be fulfilled.

3.2. Let us first take n = 1. Equations (3.5), (3.10) and (3.12) show, that eq.(3.14) with n = 1 enables one to determine the function $[\alpha_{1/2}(\phi)]^2$ given the function $U_0(\rho)$ and constants a_0 and a_1.

For the function $B(\rho, a)$, see eqs.(1.19), (1.20), one has this time expansion

$$B(\rho, a, \epsilon) = \sum^{\infty} B_{n+½}(\rho, a)\epsilon^{n+½}.$$

Condition $\epsilon^{1/2} B_{1/2} = 2\pi\omega = 2\pi\gamma\sqrt{\epsilon}$ allows one to express a_1 in terms of γ (item 1.3.2).

3.2. Let us yet consider the case n = 2. Equation (3.7) with n = 1 and eq.(3.9) determine the function $M_{3/2}(\rho, \phi)$ (up to the term

$$\alpha_{3/2}(\phi)/U_0'(\rho) \qquad (3.15)$$

given the functions U_0, U_1 and $M_{1/2}$. Then eqs.(3.11), (3.6) give the function U_2. Equation (3.14) with n = 2 enables one to determine the function $\alpha_{3/2}(\phi)$.

3.3. The procedure outlined can be continued to any value of n, n = 3,4, ...

4. THE KRAMERS - AIRY PROBLEM

First of all note that while in one dimension

$$A_0(x) = \left(\frac{dS(x)}{dx}\right)^{-1/2} \sim (x - x_0)^{-1/4} \text{ as } x \to x_0, \; f(x_0) = 0, \qquad (4.1)$$

see[1], in two dimensions one has

$$A_0(x) = Z^{-1/4}D + Z^{1/4}G. \tag{4.2}$$

Here $Z = a^2 - U(x)^2$ and the functions D and G are regular in the real ρ, ϕ plane if so is the function η. Note also that eq.(1.8a) enables one to construct the function A_0 along the trajectory $dx(t)/dt = \nabla S(x(t))$.

4.1. Let us represent our function ψ in a vicinity of the turning curve in the form

$$\psi = e^{i\lambda\Phi} A(U, \lambda) C(\rho, \phi) + \dots \tag{4.3}$$

Then eq.(1.6) gives

$$\lambda^2[-(\nabla\Phi)^2 + \eta] AC + (\nabla U)^2 \frac{d^2A}{dU^2} C + i\lambda\Delta\Phi AC +$$
$$2i\lambda A\nabla\Phi\nabla C + 2\frac{dA}{dU}\nabla U\nabla C + \Delta U\frac{dA}{dU}C + \dots = 0, \quad \Delta = (\nabla)^2. \tag{4.4}$$

It follows from eqs.(1.8), (1.12) and (1.13) that $\eta - (\nabla\Phi)^2 = (\nabla U)^2(a^3 - U^2)$. Let us determine the function A by the equation

$$\frac{d^2A}{dU^2} + \lambda^2(a^2 - U^2) A = 0 \tag{4.5}$$

and consider, e.g., a vicinity of the point $U = a$. The change of the variable $U - a = y\lambda^{-2/3}$ reduces eq.(4.5) to the equation

$$\lambda^{4/3} [\frac{d^2A}{dy^2} - 2a yA] = 0 \tag{4.6}$$

whose solution is essentially the Airy function[1]. (See also eq.(4.2)). Thus we have taken account of terms $O(\lambda^{4/3})$ in eq. (4.4). Terms $O(\lambda)$ give

$$\Delta\Phi C + 2\nabla\Phi\nabla C = 0. \tag{4.7}$$

This equation determines the function C along a curve $U(\rho, \phi) = \text{const}$.

We do not know how to prove that $C(\rho, \phi + 2\pi) = C(\rho, \phi)$.

4.2 With this exception Sec.4 gives the complete solution of the lowest order (in powers of λ) Kramers - Airy problem[1].

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Note added in proof. Consideration of asymptotics of functions $u_{1k}(\rho)$ as $\rho \to -\infty$ shows that $u_{1k} \sim r^{-k} C_k$. It is impossible to eliminate all the constants C_k simultaneously. Thus, contrary to the statement of the abstract, our ϵ expansions are uniformly valid only outside of a small $O(\epsilon)$ - vicinity of the point $r = 0$. As for the constants a_n, $n = 1,2,\dots$, they are to be determined according to the procedure of item 1.3.2.

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К двумерной квазиклассике

Рассматривается квазиклассический предел двумерного уравнения Шредингера $[-\Delta/2m + V(x) - E]\psi(x) = 0$. Потенциал $V(x) \equiv V(x_1, x_2)$ берется слабо зависящим от азимутального угла $V(x) = V_0(r) + \epsilon V_1(r,\phi)$. Здесь ϵ - малый параметр. Построены ряды по степеням ϵ, представляющие функцию S (она определена уравнением $(\nabla S)^2 = 2m[E - V(x)]$) во всей области $0 < r < \infty$, $0 < \phi < 2\pi$. Рассмотрено поведение квазиклассической волновой функции в окрестности кривой поворота. Задача рассеяния не рассматривается.

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On Two-Dimensional Quasiclassics

We study quasiclassical limit of the two-dimensional Schroedinger equation $[-\Delta/2m + V(x) - E]\psi(x) = 0$. Potential $V(x)$ is supposed to weakly depend on the azimuthal angle ϕ: $V(\vec{x}) = V_0(r) + \epsilon V_1(r, \phi)$. Here ϵ is a small parameter. We have constructed the series in powers of ϵ for the function S, which is determined by the quasiclassical master-equation $(\nabla S)^2 = 2m[E - V(x)]$. Our decompositions are uniformly valid in the whole plane of two dimensions including the turning curve. We have considered also the behaviour of the quasiclassical wave function in a vicinity of turning curve. We consider mainly the bound state problem.

The investigation has been performed at the Laboratory of Theoretical Physics, JINR.

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