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$H^{+}H^{-}$ INTERACTION UP TO HIGHER ORDERS OF PERTURBATION THEORY IN THE MODEL WITH TWO HIGGS DOUBLETS (LAGRANGIAN AND ONE-LOOP AMPLITUDES)

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1. Introduction

The present paper continues the work [1] where the self-energy and vertex blocks necessary for finding the amplitude of Higgs-Higgs (H^+H^-) interaction up to the next-to-leading order were oalculated.

The paper is organized as follows. In Sect.2 the Lagrangian that corresponds to the model with two doublets of Higgs particles is presented in the unitary gauge. No approximations (like $v_1 \gg v_2$ [2]) are used. In Sect.3 the diagrams that define the amplitude of H^+H^- interaction are calculated. We have left (as in [1]) the vertex coupling constants not to be fixed in the diagrams that contain the exchange of two Higgses or two vector bosons. Appendix A contains the vertex constants that define the magnitude of Higgs-Higgs self-interaction (interaction between $H^\pm$, H_1^0, H_2^0 and H_3^0) in an arbitrary two-doublet model. In Appendix B the Lagrangian of self-interaction $\mathcal{L}_{4H}$ is presented for the case of the model with the minimal supersymmetrical extension of the Standard Model (MSSM) without the singlet of Higgs fields (compare with [3,4]).

2. The Lagrangian

As it has been shown in [1,2], the Lagrangian of the model considered by us has the form:

$$\mathcal{L}_H = \mathcal{L}_{GH} + \mathcal{L}_{FH} - V(\Phi_1, \Phi_2) . \qquad (2.1)$$

After the spontaneous breaking of the vacuum symmetry and eliminating of nonphysical fields we have (see also [5,6])

$$-\mathcal{L}_{GH} = \partial_\mu H^+ \partial_\mu H^- + \frac{1}{2}\left[(\partial_\mu H_1^0)^2 + (\partial_\mu H_2^0)^2 + (\partial_\mu H_3^0)\right] + \frac{g^2 v^2}{4} W_\mu^* W_\mu + \frac{g^2 v^2}{8\cos^2\theta_W} Z_\mu^2 +$$
$$+ \frac{g}{2} W_\mu \left\{ i\sin(\alpha-\beta)\left[H^- \partial_\mu H_1^0 - H_1^0 \partial_\mu H^-\right] + i\cos(\alpha-\beta) \times \right.$$

$$
\begin{aligned}
&\cdot\left[H^-\partial_\mu H_2^0 - H_2^0\partial_\mu H^-\right] - \left[H^-\partial_\mu H_3^0 - H_3^0\partial_\mu H^-\right]\Big\} + h.c. + \\
&+ g M_W W_\mu^* W_\mu \left[H_1^0\cos(\beta-\alpha) + H_2^0\sin(\beta-\alpha)\right] + ieA_\mu \cdot \\
&\cdot\left[H^-\partial_\mu H^+ - H^+\partial_\mu H^-\right] + \frac{g}{2\cos\theta_W} Z_\mu \Big\{ i\cos 2\theta_W \cdot \\
&\cdot\left[H^-\partial_\mu H^+ - H^+\partial_\mu H^-\right] + \sin(\alpha-\beta)\left[H_1^0\partial_\mu H_3^0 - \right. \\
&\left. - H_3^0\partial_\mu H_1^0\right] + \cos(\alpha-\beta)\left[H_2^0\partial_\mu H_3^0 - H_3^0\partial_\mu H_2^0\right]\Big\} + \qquad (2.2) \\
&+ \frac{g}{2\cos\theta_W} M_Z Z_\mu^2 \left[H_1^0\cos(\beta-\alpha) + H_2^0\sin(\beta-\alpha)\right] + \\
&+ \frac{g^2}{2} W_\mu^* W_\mu \Big\{H^+H^- + \frac{1}{2}\left[(H_1^0)^2 + (H_2^0)^2 + (H_3^0)^2\right]\Big\} + \\
&+ \frac{g^2}{4\cos^2\theta_W} Z_\mu^2 \Big\{H^+H^-\cos^2 2\theta_W + \frac{1}{2}\left[(H_1^0)^2 + (H_2^0)^2 + \right. \\
&\left. + (H_3^0)^2\right]\Big\} + e^2 A_\mu^2 H^+H^- + \frac{g^2}{4\cos^2\theta_W} Z_\mu A_\mu H^+H^- \sin 4\theta_W + \\
&+ \frac{g^2\sin 2\theta_W}{2} W_\mu A_\mu H^-\left[H_1^0\sin(\alpha-\beta) + H_2^0\cos(\alpha-\beta) + iH_3^0\right] + \\
&+ h.c. - \frac{g^2}{2\cos\theta_W} W_\mu Z_\mu H^-\sin^2\theta_W\left[H_1^0\sin(\alpha-\beta) + H_2^0\cos(\alpha-\beta) + \right. \\
&\left. + iH_3^0\right] + h.c.
\end{aligned}
$$

and

$$
\begin{aligned}
-\mathcal{L}_{FH} = \sum_{f=e,\mu,\tau} \Big\{ &\bar\nu_f \frac{1-\gamma_5}{2} f H^+(-g_1\sin\beta + g_2\cos\beta) + \\
&+ \bar f \frac{1+\gamma_5}{2} \nu_f H^-(-g_1\sin\beta + g_2\cos\beta) + \frac{1}{\sqrt{2}} \bar f f \cdot \\
&\cdot\left[g_1 v_1 + g_2 v_2 + (g_1\cos\alpha + g_2\sin\alpha)H_1^0 + (-g_1\sin\alpha + \right. \\
&\left. + g_2\cos\alpha)H_2^0\right] + \frac{1}{\sqrt{2}} i\bar f\gamma_5 f H_3^0 (g_1\sin\beta - \\
&- g_2\cos\beta)\Big\}. \qquad (2.3)
\end{aligned}
$$

If $g_1 = 0$, then $g_2 = g m_f/\sqrt{2} M_W \sin\beta$ and the Lagrangian of interaction of leptons with the Higgs field can be written as

$$
\begin{aligned}
-\mathcal{L}_{FH} = \sum_{f=e,\mu,\tau} m_f \bar f f + \frac{g m_f}{\sqrt{2} M_W} \Big\{ &\bar\nu_f \frac{1-\gamma_5}{2} f H^+ \operatorname{ctg}\beta + \\
&+ \bar f \frac{1+\gamma_5}{2} \nu_f H^- \operatorname{ctg}\beta + \frac{\bar f f}{\sqrt{2}\sin\beta}\left[H_1^0\sin\alpha + H_2^0\cos\alpha\right] - \\
&- \frac{i\bar f\gamma_5 f}{\sqrt{2}} H_3^0 \operatorname{ctg}\beta\Big\}. \qquad (2.4)
\end{aligned}
$$

In the case of interaction of quarks with the Higgs bosons one must start with the Lagrangian

$$
\begin{aligned}
\mathcal{L}_{QH} = &- \sum_q g_{1q}\left[\bar q_L \Phi_1^c q_R^u + \bar q_R^u \Phi_1^{+c} q_L\right] - \qquad (2.5) \\
&- \sum_q g_{2q}\left[\bar q_L \Phi_2 q_R^d + \bar q_R^d \Phi_2^+ q_L\right],
\end{aligned}
$$

or $$\mathcal{L}_{QH} = -\sum_q g_{1q}\left[\bar{q}_L \Phi_2^c q_R^u + \bar{q}_R^u \Phi_2^{c+} q_L\right] - \sum_q g_{2q}\left[\bar{q}_L \Phi_2 q_R^d + \bar{q}_R^d \Phi_2^+ q_L\right], \tag{2.6}$$

where $\Phi_1^c = i\sigma_2 \Phi_1^*$, $\Phi_2^c = i\sigma_2\Phi_2^*$.

In the first model the quarks with the charge $+2/3$ acquire their mass due to the interaction with the doublet Φ_1 and quarks with the charge $-1/3$ due to the interaction with the doublet Φ_2.

As a result $\left(m_u = \frac{g_{1q} v \cos\beta}{\sqrt{2}} = \frac{\sqrt{2} g_{1q} M_W \cos\beta}{g},\ m_d = \frac{g_{2q} v \sin\beta}{\sqrt{2}} = \frac{\sqrt{2} g_{2q} M_W \sin\beta}{g}\right)$:

$$\begin{aligned}\mathcal{L}_{QH} = &-\sum_q m_q \bar{q} q + \frac{g}{\sqrt{2} M_W}\Big\{\bar{q}^u\Big[m^u \frac{1+\gamma_5}{2} tg\beta + m^d \frac{1-\gamma_5}{2} ctg\beta\Big] q^d H^+ + \bar{q}^d\Big[m^u \frac{1-\gamma_5}{2} tg\beta + m^d\frac{1+\gamma_5}{2} ctg\beta\Big] q^u H^- + \\ &+\frac{m^u}{\sqrt{2}\cos\beta}\bar{q}^u q^u\left[H_1^0\cos\alpha - H_2^0 \sin\alpha\right] - \frac{i m_u tg\beta}{\sqrt{2}}\bar{q}^u \gamma_5 q^u H_3^0 + \\ &+\frac{m_d}{\sqrt{2}\sin\beta}\bar{q}^d q^d\left[H_1^0 \sin\alpha + H_2^0\cos\alpha\right] - \frac{i m_d ctg\beta}{\sqrt{2}}\bar{q}^d\gamma_5 q^d H_3^0\Big\}.\end{aligned} \tag{2.7}$$

The starting potential of self-interaction has the form:

$$\begin{aligned}V(\Phi_1,\Phi_2) = &-\mu_1^2 \Phi_1^+\Phi_1 - \mu_2^2 \Phi_2^+\Phi_2 + \lambda_1(\Phi_1^+\Phi_1)^2 + \\ &+\lambda_2(\Phi_2^+\Phi_2)^2 + \lambda_3(\Phi_1^+\Phi_1)(\Phi_2^+\Phi_2) + \lambda_4(\Phi_1^+\Phi_2)(\Phi_2^+\Phi_1) + \\ &+\frac{\lambda_5}{2}\left[(\Phi_1^+\Phi_2)^2 + (\Phi_2^+\Phi_1)^2\right].\end{aligned} \tag{2.8}$$

Due to the relations

$$tg\,\beta = v_2/v_1, \tag{2.9}$$

$$tg\,2\alpha = \frac{v_1 v_2(\lambda_3+\lambda_4+\lambda_5)}{\lambda_1 v_1^2 - \lambda_2 v_2^2} \tag{2.10}$$

and to the position of the potential minimum at the points

$$\begin{cases} 2\mu_1^2 = 2\lambda_1 v_1^2 + (\lambda_3+\lambda_4+\lambda_5) v_2^2 \\ 2\mu_2^2 = 2\lambda_2 v_2^2 + (\lambda_3+\lambda_4+\lambda_5) v_1^2\end{cases} \tag{2.11}$$

we get

$$\lambda_1 = \frac{1}{2v_1^2}\left[M_{H_1^0}^2 \sin^2\alpha + M_{H_2^0}^2\cos^2\alpha\right], \tag{2.12}$$

$$\lambda_2 = \frac{1}{2v_2^2}\left[M_{H_1^0}^2\cos^2\alpha + M_{H_2^0}^2\sin^2\alpha\right], \tag{2.13}$$

$$\lambda_3 = \frac{2M_{H^\pm}^2}{v^2} + \frac{M_{H_2^0}^2 - M_{H_1^0}^2}{2v_1 v_2}\sin 2\alpha, \tag{2.14}$$

$$\lambda_4 = \frac{M_{H_3^0}^2 - 2M_{H^\pm}^2}{v^2}, \quad \lambda_5 = -\frac{M_{H_3^0}^2}{v^2}. \tag{2.15}$$

With the help of the above formulae the potential $V(\Phi_1,\Phi_2)$ can be represented after the spontaneous symmetry breaking in the following form that contains the masses of Higgs particles:

$$\begin{aligned}V(H^\pm, H_1^0, H_2^0, H_3^0, \alpha, \beta) = &M_{H^\pm}^2 H^+H^- + \frac{1}{2}M_{H_1^0}^2 (H_1^0)^2 + \\ &+\frac{1}{2}M_{H_2^0}^2(H_2^0)^2 + \frac{1}{2}M_{H_3^0}^2(H_3^0)^2 + g_1 H^+H^-H_1^0 + \\ &+g_2 H^+H^-H_2^0 + g_3(H_1^0)^3 + g_4(H_1^0)^2 H_2^0 + g_5 H_1^0(H_2^0)^2 + \\ &+g_6(H_2^0)^3 + g_7 H_1^0(H_3^0)^2 + g_8 H_2^0(H_3^0)^2 + \end{aligned} \tag{2.16}$$

$$+ g_9 (H^+H^-)^2 + g_{10} H^+H^-(H_1^0)^2 + g_{11} H^+H^-(H_2^0)^2 +$$

$$+ g_{12} H^+H^-(H_3^0)^2 + g_{13} H^+H^- H_1^0 H_2^0 + g_{14}(H_1^0)^4 +$$

$$+ g_{15}(H_1^0)^3 H_2^0 + g_{16}(H_1^0)^2(H_2^0)^2 + g_{17} H_1^0 (H_2^0)^3 +$$

$$+ g_{18}(H_2^0)^4 + g_{19}(H_1^0)^2(H_3^0)^2 + g_{20}(H_2^0)^2(H_3^0)^2 +$$

$$+ g_{21} H_1^0 H_2^0 (H_3^0)^2 + g_{22}(H_3^0)^4,$$

where g_i are the constants expressed through the Higgs masses and the trigonometric functions of the angles α and β. They are given in Appendix A (compare with /5/).

The Lagrangian for the MSSM (that we use to concretize the vertex constants in the diagrams for the amplitude of H^+H^- interaction) can be found in /3,4/ . As it follows from these papers, the Lagrangian of interaction of Higgs particles with the gauge bosons is identical with the Lagrangian of the ordinary two-doublet model. The expression for the potential of the self-interaction due to the presence in SUSY of definite sum rules for the masses in the Higgs sector becomes much more simple. The part of the supersymmetric Lagrangian responsible for self-interaction $\mathcal{L}_{4H}$, and not written in /3,4/ is given in Appendix B (compare with /5/). Still, in the two-doublet SUSY model a number of useful relations exist:

$$M^2_{H^0_{1,2}} = \frac{1}{2}\left[M^2_{H^0_3} + M^2_Z \pm \sqrt{(M^2_{H^0_3}+M^2_Z)^2 - 4M^2_{H^0_3}M^2_Z\cos^2 2\beta}\right], \tag{2.17}$$

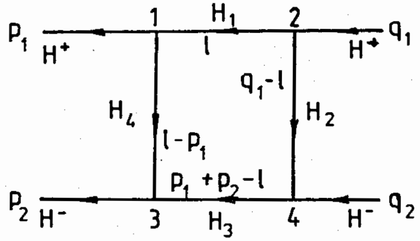
$$M^2_{H^\pm} = M^2_{H^0_3} + M^2_W \quad , \tag{2.18}$$

$$tg\, 2\alpha = tg\, 2\beta \,\frac{M^2_{H^0_1}+M^2_{H^0_2}}{M^2_{H^0_3}-M^2_Z} . \tag{2.19}$$

3. One-loop amplitudes

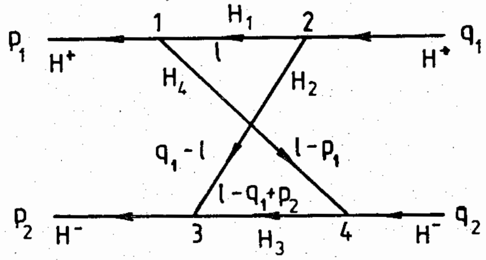
Let us present the diagrams with two-particle exchange.

1\.



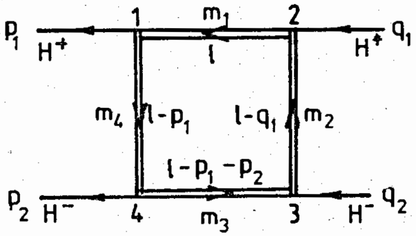
$$i\mathcal{M} = \frac{i f_1^{4H} f_2^{4H} f_3^{4H} f_4^{4H}}{16\pi^2}\, I_2\left(q_1^2, q_2^2, p_2^2, p_1^2, s, t, M^2_{H1}, M^2_{H2},\right.$$
$$\left. M^2_{H3}, M^2_{H4}\right). \tag{3.1}$$

2\.



$$i\mathcal{M} = \frac{i f_1^{4H} f_2^{4H} f_3^{4H} f_4^{4H}}{16\pi^2}\, I_2\left(q_1^2, p_2^2, q_2^2, p_1^2, u, t, M^2_{H1}, M^2_{H2},\right.$$
$$\left. M^2_{H3}, M^2_{H4}\right). \tag{3.2}$$

3\.



and a similar diagrams with the opposite lepton current direction

$$i\mathcal{M} = \frac{i f_1^H f_2^H f_3^H f_4^H}{4\pi^2}\Big\{ 2B_5 P + \frac{1}{2}B_5 + \frac{1}{2}B_5 I_0(s,$$
$$m_1^2, m_3^2) + \frac{1}{2}B_5 I_0(t, m_2^2, m_4^2) - \frac{1}{2}B_5(q_1q_2 - m_2^2) -$$

$$-B_6 m_1 m_2 - B_7 m_1 m_3 - B_9 m_2 m_3\Big]\, I_1\left(q_1^2, q_2^2, s, m_1^2, m_2^2, m_3^2\right) + \frac{1}{2}\Big[B_5(q_1 p_1 + m_1^2) + B_6 m_1 m_2 + B_8 m_1 m_4 + B_{10} m_2 m_4\Big]\times$$

$$\times I_1\left(q_1^2, t, p_1^2, m_1^2, m_2^2, m_4^2\right) - \frac{1}{2}\Big[B_5(p_1 p_2 - m_4^2) - B_7 m_1 m_3 - B_8 m_1 m_4 - B_{11} m_3 m_4\Big]\, I_1\left(s, p_2^2, p_1^2, m_1^2, m_3^2, m_4^2\right) -$$

$$-\frac{1}{2}\Big[B_5(p_2 q_2 - m_3^2) - B_9 m_2 m_3 - B_{10} m_2 m_4 - B_{11} m_3 m_4\Big]\times$$

$$\times I_1\left(q_2^2, p_2^2, t, m_2^2, m_3^2, m_4^2\right) - \frac{1}{2}\Big[\frac{1}{2} B_5\Big(\left(p_1^2 + m_1^2 + m_4^2\right)\left(q_2^2 + m_2^2 + m_3^2\right) + \left(p_2^2 + m_3^2 + m_4^2\right)\left(q_1^2 + m_1^2 + m_2^2\right) - \left(s + m_1^2 + m_3^2\right)\left(t + m_2^2 + m_4^2\right)\Big) + B_6 m_1 m_2 \left(p_2^2 + m_3^2 + m_4^2\right) + B_7 m_1 m_3 \left(t + m_2^2 + m_4^2\right) + B_8 m_1 m_4 \left(q_2^2 + m_2^2 + m_3^2\right) + B_9 m_2 m_3 \left(p_1^2 + m_1^2 + m_4^2\right) +$$

$$+ B_{10} m_2 m_4 \left(s + m_1^2 + m_3^2\right) + B_{11} m_3 m_4 \left(q_1^2 + m_1^2 + m_2^2\right) + 2 B_{12} \times m_1 m_2 m_3 m_4\Big]\, I_2\left(q_1^2, q_2^2, p_2^2, p_1^2, s, t, m_1^2, m_2^2, m_3^2, m_4^2\right)\Big\} \qquad (3.3)$$

where

$$B_5 = 1 - b_1 b_2 + b_1 b_3 - b_1 b_4 - b_2 b_3 + b_2 b_4 - b_3 b_4 + b_1 b_2 b_3 b_4,$$

$$B_6 = 1 + b_1 b_2 + b_1 b_3 - b_1 b_4 + b_2 b_3 - b_2 b_4 - b_3 b_4 - b_1 b_2 b_3 b_4,$$

$$B_7 = 1 + b_1 b_2 - b_1 b_3 - b_1 b_4 - b_2 b_3 - b_2 b_4 + b_3 b_4 + b_1 b_2 b_3 b_4,$$

$$B_8 = 1 + b_1 b_2 - b_1 b_3 + b_1 b_4 - b_2 b_3 + b_2 b_4 - b_3 b_4 - b_1 b_2 b_3 b_4,$$

$$B_9 = 1 - b_1 b_2 - b_1 b_3 - b_1 b_4 + b_2 b_3 + b_2 b_4 + b_3 b_4 - b_1 b_2 b_3 b_4,$$

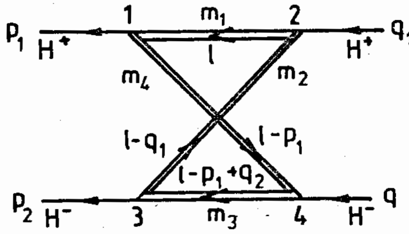
$$B_{10} = 1 - b_1 b_2 - b_1 b_3 + b_1 b_4 + b_2 b_3 - b_2 b_4 - b_3 b_4 + b_1 b_2 b_3 b_4,$$

$$B_{11} = 1 - b_1 b_2 + b_1 b_3 + b_1 b_4 - b_2 b_3 - b_2 b_4 + b_3 b_4 - b_1 b_2 b_3 b_4, \qquad (3.4)$$

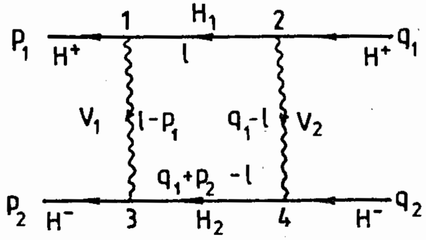
$$B_{12} = 1 + b_1 b_2 + b_1 b_3 + b_1 b_4 + b_2 b_3 + b_2 b_4 + b_3 b_4 + b_1 b_2 b_3 b_4 .$$

The constants b_i define the strength of the Higgs–fermion pseudoscalar interaction (see formula (1.2) in [7a]).

The expressions for the amplitude corresponding to the diagram

4. 

and a similar diagram with the opposite lepton current direction are described by the above expression (3.3) but with the substitution $p_2 \Leftrightarrow -q_2$.

5. 

$$
i\mathcal{M} = \frac{i f_1^{HHV} f_2^{HHV} f_3^{HHV} f_4^{HHV}}{16\pi^2} \Big\{ \Big[-2 + 2(\varepsilon_1+\varepsilon_2-\varepsilon_1\varepsilon_2) + \frac{\varepsilon_1}{M_{V1}^2} \times
$$

$$
\Big(8q_1q_2 - \varepsilon_2(p_1p_2+q_1q_2+M_{V2}^2) + 2(1-\varepsilon_2)t + \left(1-\frac{3}{2}\varepsilon_2\right) \times
$$

$$
\times (C+D) + \left(1-\frac{1}{2}\varepsilon_2\right)(E+F)\Big) + \frac{\varepsilon_2}{M_{V2}^2}\times\Big(8p_1p_2 - \varepsilon_1(p_1p_2+
$$

$$
+q_1q_2+M_{V1}^2) + 2(1-\varepsilon_1)t + \left(1-\frac{1}{2}\varepsilon_1\right)(C+D) + \Big(1-
$$

$$
-\frac{3}{2}\varepsilon_1\Big)(E+F)\Big) + \frac{\varepsilon_1\varepsilon_2}{M_{V1}^2M_{V2}^2}\Big(-\frac{t}{2}(C+D+E+F) - 2CE -
$$

$$
-2DF - \frac{2CD+2EF+CF+ED}{3}\Big)\Big]P + \frac{\varepsilon_1\varepsilon_2}{M_{V1}^2M_{V2}^2}\Big(\frac{t}{3} +
$$

$$
+M_{V1}^2+M_{V2}^2\Big)\left(\frac{t}{6} + \frac{p_1p_2+q_1q_2}{3} + \frac{(C-E)(D-F)}{6t}\right) +
$$

$$
+\left(\ln\frac{\varepsilon_1 M_{V1}^2}{M_W^2} - 1\right)\Big[\frac{C+D-E-F}{2t}\varepsilon_1\left(1-\frac{\varepsilon_2}{2}\right) - \frac{1}{3}\varepsilon_1\varepsilon_2 -
$$

$$
-\frac{1}{6}\varepsilon_1\varepsilon_2\frac{p_1p_2+q_1q_2}{t} - \frac{1}{3}\varepsilon_1\varepsilon_2\frac{(C-E)(D-F)}{t^2} + \varepsilon_1\varepsilon_2\frac{M_{V1}^2}{M_{V2}^2}\times
$$

$$
\times\left(\frac{1}{3} + \frac{1}{4}\frac{C+D-E-F}{t} + \frac{1}{6}\frac{p_1p_2+q_1q_2}{t} + \frac{1}{3}\frac{(C-E)(D-F)}{t^2}\right)
$$

$$
+\varepsilon_1\varepsilon_2\frac{t}{M_{V2}^2}\Big(-\frac{1}{3} - \frac{1}{4}\frac{C+D+E+F}{t} - \frac{1}{6}\frac{p_1p_2+q_1q_2}{t} +
$$

$$
+\frac{1}{2}\frac{EF-CD}{t^2} + \frac{1}{6}\frac{(C-E)(D-F)}{t^2}\Big)\Big] + \left(\ln\frac{\varepsilon_2 M_{V2}^2}{M_W^2} - 1\right)\times
$$

$$
\times\Big[-\frac{C+D-E-F}{2t}\varepsilon_2\left(1-\frac{\varepsilon_1}{2}\right) - \frac{1}{3}\varepsilon_1\varepsilon_2 - \frac{1}{6}\varepsilon_1\varepsilon_2\frac{p_1p_2+q_1q_2}{t} -
$$

$$
-\frac{1}{3}\varepsilon_1\varepsilon_2\frac{(C-E)(D-F)}{t^2} + \varepsilon_1\varepsilon_2\frac{M_{V2}^2}{M_{V1}^2}\Big(\frac{1}{3}+\frac{1}{4}\frac{E+F-C-D}{t} +
$$

$$
+\frac{1}{6}\frac{p_1p_2+q_1q_2}{t} + \frac{1}{3}\frac{(C-E)(D-F)}{t^2}\Big) + \varepsilon_1\varepsilon_2\frac{t}{M_{V1}^2}\Big(-\frac{1}{3} -
$$

$$
-\frac{1}{4}\frac{C+D+E+F}{t} - \frac{1}{6}\frac{p_1p_2+q_1q_2}{t} + \frac{1}{2}\frac{CD-EF}{t^2} +
$$

$$
+\frac{1}{6}\frac{(C-E)(D-F)}{t^2}\Big)\Big] + \bar{I}_0(p_1^2, M_{H1}^2, \varepsilon_1 M_{V1}^2)\left[1 - \frac{\varepsilon_1 C}{M_{V1}^2}\right] +
$$

$$
+\bar{I}_0(p_2^2, M_{H2}^2, \varepsilon_1 M_{V1}^2)\left[1-\frac{\varepsilon_1 D}{M_{V1}^2}\right] - \bar{I}_0(s, M_{H1}^2, M_{H2}^2) +
$$

$$
+\bar{I}_0(q_1^2, M_{H1}^2, \varepsilon_2 M_{V2}^2)\left[\frac{\varepsilon_1 C}{M_{V1}^2} - \frac{\varepsilon_1\varepsilon_2}{M_{V1}^2M_{V2}^2}CE\right] + \bar{I}_0(q_2^2, M_{H2}^2, \varepsilon_2 M_{V2}^2)\times
$$

$$
\times\left[\frac{\varepsilon_1 D}{M_{V1}^2} - \frac{\varepsilon_1\varepsilon_2}{M_{V1}^2M_{V2}^2}DF\right] + \bar{I}_0(t, \varepsilon_1 M_{V1}^2, \varepsilon_2 M_{V2}^2)\Big[+\varepsilon_1+\varepsilon_2 - 2 -
$$

$$
-\frac{1}{3}\varepsilon_1\varepsilon_2 + \varepsilon_1\varepsilon_2\frac{1}{3}\frac{p_1p_2+q_1q_2}{t} + \varepsilon_1\varepsilon_2\frac{2}{3}\frac{(C-E)(D-F)}{t^2} +
$$

$$
+\varepsilon_1\frac{t}{M_{V1}^2}\Big(\frac{4q_1q_2}{t} - \frac{1}{3}\varepsilon_2\frac{p_1p_2+q_1q_2}{t} + \frac{1-\varepsilon_2}{2}\frac{C+D}{t} +
$$

$$+\frac{1}{2}\frac{E+F}{t}+1-\frac{2}{3}\varepsilon_2-\frac{1}{2}\varepsilon_2\frac{CD-EF}{t^2}-\frac{1}{6}\varepsilon_2\times$$

$$\times\frac{(C-E)(D-F)}{t^2}) +\varepsilon_1\varepsilon_2\frac{M_{V2}^2}{M_{V1}^2}\left(-\frac{1}{2}+\frac{1}{4}\frac{C+D-E-F}{t}\right)+$$

$$+\varepsilon_2\frac{t}{M_{V2}^2}(\frac{4p_1p_2}{t}-\frac{1}{3}\varepsilon_1\frac{p_1p_2+q_1q_2}{t}+\frac{1}{2}\frac{C+D}{t}+\frac{1-\varepsilon_1}{2}\times$$

$$\times\frac{E+F}{t}+1-\frac{2}{3}\varepsilon_1-\frac{1}{2}\varepsilon_1\frac{EF-CD}{t^2}-\frac{1}{6}\varepsilon_1\times$$

$$\times\frac{(C-E)(D-F)}{t^2}) +\varepsilon_1\varepsilon_2\frac{M_{V1}^2}{M_{V2}^2}\left(-\frac{1}{2}+\frac{1}{4}\frac{E+F-C-D}{t}\right)+$$

$$+\frac{\varepsilon_1\varepsilon_2}{M_{V1}^2M_{V2}^2}(\frac{t^2}{3}-\frac{1}{4}t(C+D+E+F)-\frac{1}{6}t(p_1p_2+q_1q_2)$$

$$-\frac{1}{3}(C-E)(D-F)-\frac{1}{2}(CF+DE))]+I_1(p_1^2,p_2^2,s,$$

$$M_{H1}^2,\varepsilon_1M_{V1}^2,M_{H2}^2)\left[4(p_1p_2)-\varepsilon_1M_{V1}^2+C+D-\frac{\varepsilon_1}{M_{V1}^2}CD\right]+$$

$$+I_1(q_1^2,q_2^2,s,M_{H1}^2,\varepsilon_2M_{V2}^2,M_{H2}^2)[4(q_1q_2)-\varepsilon_2M_{V2}^2+E+$$

$$+F-\frac{\varepsilon_2}{M_{V2}^2}EF]+I_1(p_1^2,q_1^2,t,\varepsilon_1M_{V1}^2,M_{H1}^2,\varepsilon_2M_{V2}^2)[-4(q_1q_2)-$$

$$-4(p_1p_2)+(\varepsilon_1-1)C-E-F+\varepsilon_2M_{V2}^2+\frac{\varepsilon_1}{M_{V1}^2}C(4q_1q_2+E+$$

$$+F-\varepsilon_2M_{V2}^2)+\frac{\varepsilon_2}{M_{V2}^2}E(4p_1p_2+C(1-\varepsilon_1)+F)-\frac{\varepsilon_1\varepsilon_2}{M_{V1}^2M_{V2}^2}CEF]+$$

$$+I_1(p_2^2,q_2^2,t,\varepsilon_1M_{V1}^2,M_{H2}^2,\varepsilon_2M_{V2}^2)[-4(q_1q_2)-4(p_1p_2)+(\varepsilon_1-1)\times$$

$$\times D-E-F+\varepsilon_2M_{V2}^2+\frac{\varepsilon_1}{M_{V1}^2}D(4q_1q_2+E+F-\varepsilon_2M_{V2}^2)+$$

$$+\frac{\varepsilon_2}{M_{V2}^2}F(4p_1p_2+D(1-\varepsilon_1)+E)-\frac{\varepsilon_1\varepsilon_2}{M_{V1}^2M_{V2}^2}DEF]+[1-$$

$$-\frac{\varepsilon_1}{M_{V1}^2}C-\frac{\varepsilon_2}{M_{V2}^2}E+\frac{\varepsilon_1\varepsilon_2}{M_{V1}^2M_{V2}^2}CE]\frac{4}{p_1^4+q_1^4+t^2-2p_1^2q_1^2-2p_1^2t-2q_1^2t}\times$$

$$\times[((p_2p_1)(p_1q_1)-p_1^2(p_2q_1))I_0(p_1^2,\varepsilon_1M_{V1}^2,M_{H1}^2)+((p_1q_1)(q_1p_2)-$$

$$-q_1^2(p_1p_2))I_0(q_1^2,M_{H1}^2,\varepsilon_2M_{V2}^2)-((p_2p_1)(p_1q_1)+(p_2q_1)(q_1p_1)-$$

$$-p_1^2(p_2q_1)-q_1^2(p_2p_1))I_0(t,\varepsilon_1M_{V1}^2,\varepsilon_2M_{V2}^2)+[((p_2p_1)(p_1q_1)-$$

$$-p_1^2(p_2q_1))(C-t-\varepsilon_2M_{V2}^2)+((p_2q_1)(q_1p_1)-q_1^2(p_1p_2))(C-\varepsilon_1M_{V1}^2)]\times$$

$$\times I_1(p_1^2,q_1^2,t,\varepsilon_1M_{V1}^2,M_{H1}^2,\varepsilon_2M_{V2}^2)]-[1-\frac{\varepsilon_1}{M_{V1}^2}D-\frac{\varepsilon_2}{M_{V2}^2}F+$$

$$+\frac{\varepsilon_1\varepsilon_2}{M_{V1}^2M_{V2}^2}DF]\frac{4}{p_2^4+q_2^4+t-2p_2^2q_2^2-2p_2^2t-2q_2^2t}\times$$

$$\times[((p_1p_2)(p_2q_2)-p_2^2(p_1q_2))I_0(p_2^2,\varepsilon_1M_{V1}^2,M_{H2}^2)+((p_2q_2)(q_2p_1)-$$

$$-q_2^2(p_1p_2))I_0(q_2^2,M_{H2}^2,\varepsilon_2M_{V2}^2)-((p_1p_2)(p_2q_2)+(p_1q_2)(q_2p_2)-$$

$$-p_2^2(p_1q_2)-q_2^2(p_1p_2))I_0(t,\varepsilon_1M_{V1}^2,\varepsilon_2M_{V2}^2)-[((p_1p_2)(p_2q_2)-$$

$$-p_2^2(p_1q_2)\Big)\left(\mathcal{D}-t-\varepsilon_2M_{V2}^2\right)+\Big((p_1q_2)(q_2p_2)-q_2^2(p_1p_2)\Big)\left(\mathcal{D}-\varepsilon_1M_{V1}^2\right)\Big]$$

$$\times\bar{I}_1\left(p_2^2,q_2^2,t,\varepsilon_1M_{V1}^2,M_{H2}^2,\varepsilon_2M_{V2}^2\right)\Big]+\bar{I}_2\Big(p_1^2,p_2^2,q_2^2,q_1^2,s,t,$$

$$M_{H1}^2,\varepsilon_1M_{V1}^2,M_{H2}^2,\varepsilon_2M_{V2}^2\Big)\Big[\left(4(q_1q_2)+E+F-\varepsilon_2M_{V2}^2\right)\Big(4(p_1p_2)+$$

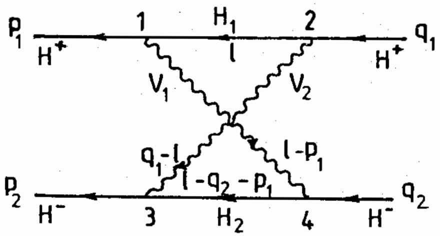
$$+C+\mathcal{D}-\varepsilon_1M_{V1}^2\Big)-\frac{\varepsilon_1}{M_{V1}^2}C\mathcal{D}\left(4(q_1q_2)+E+F-\varepsilon_2M_{V2}^2\right)-\frac{\varepsilon_2}{M_{V2}^2}EF\times$$

$$\left(4(p_1p_2)+C+\mathcal{D}-\varepsilon_1M_{V1}^2\right)+\frac{\varepsilon_1\varepsilon_2}{M_{V1}^2M_{V2}^2}C\mathcal{D}EF\Big]\Bigg\}. \qquad (3.5)$$

where

$$C=p_1^2+M_{H1}^2\ ,\quad \mathcal{D}=p_2^2+M_{H2}^2\ ,\qquad (3.6)$$
$$E=q_1^2+M_{H1}^2\ ,\quad F=q_2^2+M_{H2}^2\ .$$

The diagram

6. 

is obtained from the (3.5) but with the substitution $p_2 \Leftrightarrow -q_2$. Besides these diagrams the diagrams shown in Fig.1. contribute to the H^+H^--amplitude (up to the 4-th order of perturbation theory).

$$i\mathcal{M}_{a)}=ig^2\left[\begin{Bmatrix}\cos(\beta-\alpha)\\ \sin(\beta-\alpha)\end{Bmatrix}\mp\frac{1}{2R}\begin{Bmatrix}\cos(\beta+\alpha)\\ \sin(\beta+\alpha)\end{Bmatrix}\cos 2\beta\right]^2\frac{M_W^2}{t+M_{H^0_{1,2}}^2}, \qquad (3.7)$$

$$i\mathcal{M}_{b)}^{Z}=\frac{ig^2\cos^2 2\theta_W}{4\cos^2\theta_W}\,\frac{2s+t-p_1^2-p_2^2-q_1^2-q_2^2+\frac{1}{M_Z^2}(p_1^2-q_1^2)(q_2^2-p_2^2)}{t+M_Z^2}, \qquad (3.8)$$

$$i\mathcal{M}_{B)}^{A}=ie^2\,\frac{2s+t-p_1^2-p_2^2-q_1^2-q_2^2}{t}\ , \qquad (3.9)$$

$$i\mathcal{M}_{c)}=-g\left[\begin{Bmatrix}\cos(\beta-\alpha)\\ \sin(\beta-\alpha)\end{Bmatrix}\mp\frac{1}{2R}\cos 2\beta\begin{Bmatrix}\cos(\beta+\alpha)\\ \sin(\beta+\alpha)\end{Bmatrix}\right]\frac{M_W\,\Gamma^{Ren.}_{H^\pm H^\pm H^0_{1,2}}(p_1^2,q_1^2,t)}{t+M_{H^0_{1,2}}^2}+\begin{Bmatrix}p_1\leftrightarrow p_2\\ q_1\leftrightarrow q_2\end{Bmatrix} \qquad (3.10)$$

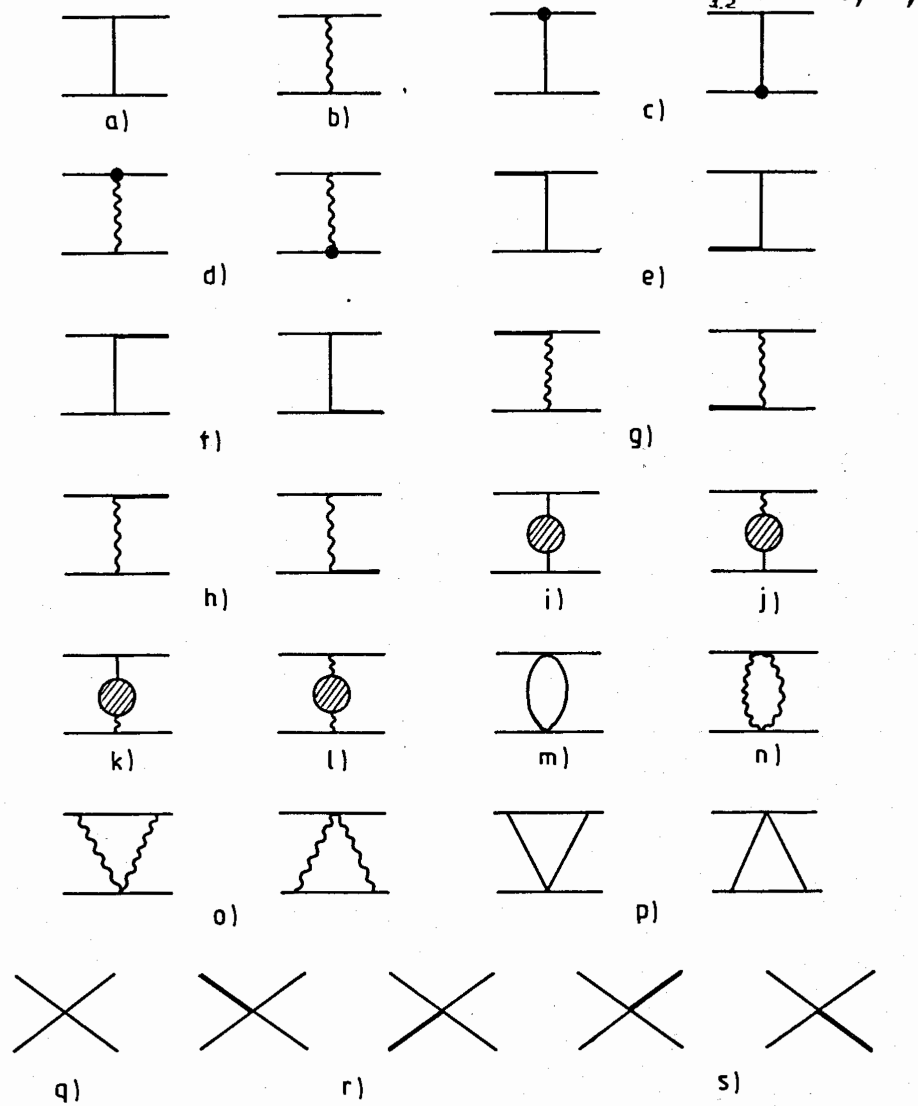


Fig.1. The diagrams describing the interaction of two Higgs particle (S -channel); a solid line corresponds to the renormalized propagator and a black circle corresponds to the renormalized vertex.

$$i\mathcal{M}^{Z}_{d)} = -\frac{g\cos 2\theta_W}{2\cos\theta_W}\left\{\left[s-p_2^2+\frac{t-p_1^2-q_1^2}{2}+\frac{1}{2M_Z^2}(t+p_1^2-q_1^2)(q_2^2-p_2^2)\right]\cdot\frac{\Gamma^{Ren.(1)}_{H^\pm H^\mp Z}(p_1^2,q_1^2,t)}{t+M_Z^2}+\left[s-q_2^2+\frac{t-p_1^2-q_1^2}{2}+\frac{1}{2M_Z^2}(t+q_1^2-p_1^2)(p_2^2-q_2^2)\right]\cdot\frac{\Gamma^{Ren(2)}_{H^\pm H^\mp Z}(p_1^2,q_1^2,t)}{t+M_Z^2}\right\}+\left\{\begin{matrix}p_1\leftrightarrow p_2\\ q_1\leftrightarrow q_2\end{matrix}\right\}, \quad (3.11)$$

$$i\mathcal{M}^{A}_{d)} = -e\left\{\left[s-p_2^2+\frac{t-p_1^2-q_1^2}{2}\right]\frac{\Gamma^{Ren.(1)}_{H^\pm H^\mp A}(p_1^2,q_1^2,t)}{t}+\left[s-q_2^2+\frac{t-p_1^2-q_1^2}{2}\right]\frac{\Gamma^{Ren.(2)}_{H^\pm H^\mp A}(p_1^2,q_1^2,t)}{t}\right\}+\left\{\begin{matrix}p_1\leftrightarrow p_2\\ q_1\leftrightarrow q_2\end{matrix}\right\}, \quad (3.12)$$

$$i\mathcal{M}_{e)} = g^2\left[\begin{Bmatrix}\cos(\beta-\alpha)\\ \sin(\beta-\alpha)\end{Bmatrix}\mp\frac{1}{2R}\cos 2\beta\begin{Bmatrix}\cos(\beta+\alpha)\\ \sin(\beta+\alpha)\end{Bmatrix}\right]^2\cdot \quad (3.13)$$
$$\cdot\frac{M_W^2\,\Pi^{Ren}(p_1^2)}{(p_1^2+M_{H^\pm}^2)(t+M^2_{H^0_{1,2}})}-g^2\begin{Bmatrix}\sin(\alpha-\beta)\\ \cos(\alpha-\beta)\end{Bmatrix}\left[\begin{Bmatrix}\cos(\beta-\alpha)\\ \sin(\beta-\alpha)\end{Bmatrix}\mp\frac{1}{2R}\cdot\cos 2\beta\begin{Bmatrix}\cos(\beta+\alpha)\\ \sin(\beta+\alpha)\end{Bmatrix}\right]\frac{(q_1^2-t)\,\Pi_{H^\pm W^\mp}(p_1^2)}{2M_W(t+M^2_{H^0_{1,2}})}+\{p_1\to p_2\},$$

$$i\mathcal{M}_{f)} = g^2\left[\begin{Bmatrix}\cos(\beta-\alpha)\\ \sin(\beta-\alpha)\end{Bmatrix}\mp\frac{1}{2R}\cos 2\beta\begin{Bmatrix}\cos(\beta+\alpha)\\ \sin(\beta+\alpha)\end{Bmatrix}\right]^2\cdot \quad (3.14)$$
$$\cdot\frac{M_W^2\,\Pi^{Ren}(q_1^2)}{(q_1^2+M_{H^\pm}^2)(t+M^2_{H^0_{1,2}})}+g^2\begin{Bmatrix}\sin(\alpha-\beta)\\ \cos(\alpha-\beta)\end{Bmatrix}\left[\begin{Bmatrix}\cos(\beta-\alpha)\\ \sin(\beta-\alpha)\end{Bmatrix}\mp\frac{1}{2R}\cdot\cos 2\beta\begin{Bmatrix}\cos(\beta+\alpha)\\ \sin(\beta+\alpha)\end{Bmatrix}\right]\frac{(p_1^2-t)\,\Pi_{H^\pm W^\mp}(q_1^2)}{2M_W(t+M^2_{H^0_{1,2}})}+\{q_1\to q_2\},$$

$$i\mathcal{M}^{Z}_{g)} = \frac{g^2\cos^2 2\theta_W}{4\cos^2\theta_W}\left[2s+t-p_1^2-p_2^2-q_1^2-q_2^2+\frac{1}{M_Z^2}(p_1^2-q_1^2)(q_2^2-p_2^2)\right]\frac{\Pi^{Ren}(p_1^2)}{(p_1^2+M_{H^\pm}^2)(t+M_Z^2)}+\{p_1\to p_2\}, \quad (3.15)$$

$$i\mathcal{M}^{A}_{g)} = e^2\left[2s+t-p_1^2-p_2^2-q_1^2-q_2^2\right]\frac{\Pi^{Ren}(p_1^2)}{(p_1^2+M_{H^\pm}^2)\,t}+\{p_1\to p_2\}, \quad (3.16)$$

$$i\mathcal{M}^{Z}_{h)} = \frac{g^2\cos^2 2\theta_W}{4\cos^2\theta_W}\left[2s+t-p_1^2-p_2^2-q_1^2-q_2^2+\frac{1}{M_Z^2}(p_1^2-q_1^2)\cdot(q_2^2-p_2^2)\right]\frac{\Pi^{Ren}(q_1^2)}{(q_1^2+M_{H^\pm}^2)(t+M_Z^2)}+\{q_1\to q_2\}, \quad (3.17)$$

$$i\mathcal{M}^{A}_{h)} = e^2\left[2s+t-p_1^2-p_2^2-q_1^2-q_2^2\right]\frac{\Pi^{Ren}(q_1^2)}{(q_1^2+M_{H^\pm}^2)\,t}+\{q_1\to q_2\}, \quad (3.18)$$

$$i\mathcal{M}_{i)} = g^2\left[\begin{Bmatrix}\cos(\beta-\alpha)\\ \sin(\beta-\alpha)\end{Bmatrix}\mp\frac{1}{2R}\begin{Bmatrix}\cos(\beta+\alpha)\\ \sin(\beta+\alpha)\end{Bmatrix}\cos 2\beta\right]^2\frac{M_W^2\,\Pi^{Ren}_{H^0_{1,2}}(t)}{(t+M^2_{H^0_{1,2}})}+$$
$$+g^2\left[\frac{1}{2}\sin 2(\beta-\alpha)+\frac{1}{2R}\sin 2\alpha\cos 2\beta+\frac{1}{8R^2}\sin 2(\beta+\alpha)\cos^2 2\beta\right]\cdot\frac{2M_W^2\,\Pi_{H_1^0H_2^0}(t)}{(t+M^2_{H_1^0})(t+M^2_{H_2^0})}, \quad (3.19)$$

$$i\mathcal{M}_{j,k)} = 0, \quad i\mathcal{M}^{AA}_{\ell)} = e^2(p_1+q_1)_\mu(p_2+q_2)_\nu\frac{\Pi^{A}_{\mu\nu}(t)}{t^2}, \quad (3.20)$$

$$i\mathcal{M}^{ZZ}_{\ell)} = \frac{g^2\cos^2 2\theta_W}{4\cos^2\theta_W}\left[p_{1\mu}\frac{p_1^2-q_1^2+M_Z^2}{M_Z^2}+q_{1\mu}\frac{q_1^2-p_1^2+M_Z^2}{M_Z^2}\right]\cdot\left[p_{2\nu}\frac{p_2^2-q_2^2+M_Z^2}{M_Z^2}+q_{2\nu}\frac{q_2^2-p_2^2+M_Z^2}{M_Z^2}\right]\frac{\Pi^{Z}_{\mu\nu}(t)}{(t+M_Z^2)^2}, \quad (3.21)$$

$$i\mathcal{M}^{ZA}_{l)} = \frac{eg\cos 2\theta_W}{2R}\Big\{\Big[p_{1\mu}\frac{p_1^2-q_1^2+M_Z^2}{M_Z^2} + q_{1\mu}\frac{q_1^2-p_1^2+M_Z^2}{M_Z^2}\Big]\cdot (p_{2\nu}+q_{2\nu})\frac{\Pi^{ZA}_{\mu\nu}(t)}{t\,(t+M_Z^2)} + (p_{1\mu}+q_{1\mu})\Big[p_{2\nu}\frac{p_2^2-q_2^2+M_Z^2}{M_Z^2} + q_{2\nu}\frac{q_2^2-p_2^2+M_Z^2}{M_Z^2}\Big]\frac{\Pi^{AZ}_{\mu\nu}(t)}{t\,(t+M_Z^2)}\,, \tag{3.22}$$

$$i\mathcal{M}_{m)} = \frac{g^4}{4}\frac{\cos^4 2\beta}{R^2}\Pi^{(3)}(t,M^2_{H^\pm},M^2_{H^\pm}) + \frac{g^4}{16}\frac{\cos^4 2\beta}{R^2}\cdot\Pi^{(3)}(t,M^2_{H_3^0},M^2_{H_3^0}) + \frac{g^4}{4}\Big[\left\{\begin{matrix}\cos^2(\alpha-\beta)\\ \sin^2(\alpha-\beta)\end{matrix}\right\}\mp\frac{\cos 2\alpha\cos 2\beta}{2R}\Big]^2\cdot\Pi^{(3)}(t,M^2_{H^0_{1,2}},M^2_{H^0_{1,2}}) + \frac{g^4}{16}tg^4\theta_W\sin^2 2\alpha\cos^2 2\beta\,\Pi^{(3)}(t,M^2_{H_1^0},M^2_{H_2^0}), \tag{3.23}$$

$$i\mathcal{M}_{n)} = \frac{g^4}{4}\Pi^{(I,5)}(t,M^2_{W^\pm},M^2_{W^\pm}) + \frac{g^4}{4}\frac{\cos^4 2\theta_W}{R^2}\Pi^{(I,5)}(t,M_Z^2,M_Z^2) + 4e^4\Pi^{(I,5)}(t,0,0) + e^2g^2\frac{\cos^2 2\theta_W}{R}\Pi^{(I,5)}(t,M_Z^2,0), \tag{3.24}$$

$$i\mathcal{M}_{o)} = -\frac{ig^4}{8}\left\{\begin{matrix}\sin^2(\alpha-\beta)\\ \cos^2(\alpha-\beta)\end{matrix}\right\}\Gamma^{(4)}(p_1^2,q_1^2,t,M^2_{H^0_{1,2}},M^2_{W^\pm},M^2_{W^\pm}) + \frac{ig^4}{8}\Gamma^{(4)}(p_1^2,q_1^2,t,M^2_{H_3^0},M^2_{W^\pm},M^2_{W^\pm}) + \frac{ig^4}{8}\frac{\cos^4 2\theta_W}{R^2}\Gamma^{(4)}(p_1^2,q_1^2,t,M^2_{H^\pm},M_Z^2,M_Z^2) + 2ie^4\Gamma^{(4)}(p_1^2,q_1^2,t,M^2_{H^\pm},0,0) + \frac{ie^2g^2}{2}\frac{\cos^2 2\theta_W}{R}\Gamma^{(4)}(p_1^2,q_1^2,t,M^2_{H^\pm},M_Z^2,0) + \left\{\begin{matrix}p_1\to p_2\\ q_1\to q_2\end{matrix}\right\}, \tag{3.25}$$

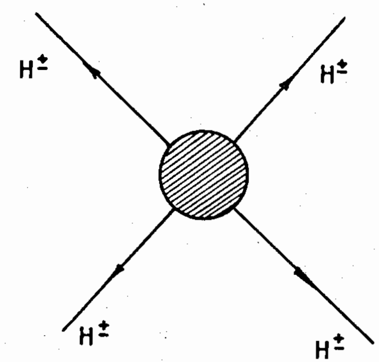
See the diagramm No.5 of the work /1/ for $\Pi^{(I,5)}$.

$$i\mathcal{M}_{p)} = \frac{ig^4}{2}M_W^2\frac{\cos^2 2\beta}{R}\Big[\left\{\begin{matrix}\cos(\beta-\alpha)\\ \sin(\beta-\alpha)\end{matrix}\right\}\mp\frac{1}{2R}\cos 2\beta\left\{\begin{matrix}\cos(\beta+\alpha)\\ \sin(\beta+\alpha)\end{matrix}\right\}\Big]^2\cdot\Gamma^{(1)}(p_1^2,q_1^2,t,M^2_{H^0_{1,2}},M^2_{H^\pm},M^2_{H^\pm}) + \frac{ig^4}{2}M_W^2\Big[\left\{\begin{matrix}\cos^2(\alpha-\beta)\\ \sin^2(\alpha-\beta)\end{matrix}\right\}\mp\frac{\cos 2\alpha\cos 2\beta}{2R}\Big]\cdot\Big[\left\{\begin{matrix}\cos(\beta-\alpha)\\ \sin(\beta-\alpha)\end{matrix}\right\}\mp\frac{1}{2R}\cos 2\beta\left\{\begin{matrix}\cos(\beta+\alpha)\\ \sin(\beta+\alpha)\end{matrix}\right\}\Big]^2\cdot\Gamma^{(1)}(p_1^2,q_1^2,t,M^2_{H^\pm},M^2_{H^0_{1,2}},M^2_{H^0_{1,2}}) + \frac{ig^4}{4}M_W^2\,tg^2\theta_W\sin 2\alpha\cdot\cos 2\beta\Big[\frac{1}{2}\sin 2(\beta-\alpha) + \frac{1}{2R}\sin 2\alpha\cos 2\beta + \frac{1}{8R^2}\cos^2 2\beta\cdot\sin 2(\beta+\alpha)\Big]\Gamma^{(1)}(p_1^2,q_1^2,t,M^2_{H^\pm},M^2_{H^0_{1,2}},M^2_{H^0_{2,1}}) + \left\{\begin{matrix}p_1\to p_2\\ q_1\to q_2\end{matrix}\right\}, \tag{3.26}$$

$$i\mathcal{M}_{q)} = -\frac{ig^2}{2}\frac{\cos^2 2\beta}{R}\,, \tag{3.27}$$

$$i\mathcal{M}_{r)} = -\frac{g^2}{2}\frac{\cos^2 2\beta}{R}\frac{\Pi^{Ren}(p_1^2)}{p_1^2+M^2_{H^\pm}} + \{p_1\to p_2\}\,, \tag{3.28}$$

$$i\mathcal{M}_{s)} = -\frac{g^2}{2}\frac{\cos^2 2\beta}{R}\frac{\Pi^{Ren}(q_1^2)}{q_1^2+M^2_{H^\pm}} + \{q_1\to q_2\}\,. \tag{3.29}$$



The counter-term for the renormalization of H^+H^- amplitude is read

$$i\mathcal{M}^{c.t} = -\frac{g^2}{2\cos^2\theta_W}\cos^2 2\beta\left[2\frac{\delta g}{g} + 2(Z_{H^\pm}-1) - \frac{\delta R}{R}\right] \tag{3.30}$$

4. Summary

The Higgs part of the Lagrangian for a two-doublet model and for a SUSY extension of the Standard Model is presented here without any approximation. On the basis of the above Lagrangian the H^+H^- amplitude up to the fourth order of perturbation theory has been calculated. The one-loop expressions presented in this paper and in [1] would be used for a calculation of the other electroweak processes **with a part of the Higgs bosons**(see the phenomenological review [4]).

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Appendix A.

The constants entering in the self-interaction potential are read

$$g_1 = \frac{2M_{H^\pm}^2}{v}\cos(\beta-\alpha) + \frac{M_{H2}^2}{2v}\left(\frac{3\sin(\beta+\alpha)+\sin(\alpha-3\beta)}{\sin 2\beta}\right), \tag{A.1}$$

$$g_2 = \frac{2M_{H^\pm}^2}{v}\sin(\beta-\alpha) + \frac{M_{H1}^2}{2v}\left(\frac{3\cos(\beta+\alpha)+\cos(\alpha-3\beta)}{\sin 2\beta}\right), \tag{A.2}$$

$$g_3 = \frac{M_{H2}^2}{4v}\left(\frac{3\sin(\beta+\alpha)+\sin(\beta-3\alpha)}{\sin 2\beta}\right), \tag{A.3}$$

$$g_4 = \frac{M_{H1}^2}{4v}\left(\frac{\cos(\beta+\alpha)-\cos(\beta-3\alpha)}{\sin 2\beta}\right) + \frac{M_{H2}^2}{2v}\left(\frac{\cos(\beta+\alpha)-\cos(\beta-3\alpha)}{\sin 2\beta}\right), \tag{A.4}$$

$$g_5 = \frac{M_{H1}^2}{2v}\left(\frac{\sin(\beta+\alpha)+\sin(\beta-3\alpha)}{\sin 2\beta}\right) + \frac{M_{H2}^2}{4v}\left(\frac{\sin(\beta+\alpha)+\sin(\beta-3\alpha)}{\sin 2\beta}\right), \tag{A.5}$$

$$g_6 = \frac{M_{H1}^2}{4v}\left(\frac{3\cos(\beta+\alpha)+\cos(\beta-3\alpha)}{\sin 2\beta}\right), \tag{A.6}$$

$$g_7 = \frac{M_{H3}^2}{v}\cos(\beta-\alpha) + \frac{M_{H2}^2}{4v}\left(\frac{3\sin(\beta+\alpha)+\sin(\alpha-3\beta)}{\sin 2\beta}\right), \tag{A.7}$$

$$g_8 = \frac{M_{H3}^2}{v}\sin(\beta-\alpha) + \frac{M_{H1}^2}{4v}\left(\frac{3\cos(\beta+\alpha)+\cos(\alpha-3\beta)}{\sin 2\beta}\right), \tag{A.8}$$

$$g_9 = \frac{M_{H1}^2}{8v^2}\left(\frac{3\cos(\beta+\alpha)+\cos(\alpha-3\beta)}{\sin 2\beta}\right)^2 + \frac{M_{H2}^2}{8v^2}\left(\frac{3\sin(\beta+\alpha)+\sin(\alpha-3\beta)}{\sin 2\beta}\right)^2, \tag{A.9}$$

$$g_{10} = \frac{M_{H^\pm}^2}{v^2}\cos^2(\beta-\alpha) + \frac{M_{H1}^2}{8v^2}\frac{(\cos(\alpha+\beta)-\cos(3\alpha-\beta))(3\cos(\beta+\alpha)+\cos(3\beta-\alpha))}{\sin^2 2\beta} + \frac{M_{H2}^2}{8v^2}\frac{(3\sin(\beta+\alpha)-\sin(3\beta-\alpha))(3\sin(\beta+\alpha)-\sin(3\alpha-\beta))}{\sin^2 2\beta}, \tag{A.10}$$

$$g_{11} = \frac{M_{H^\pm}^2}{v^2}\sin^2(\beta-\alpha) + \frac{M_{H1}^2}{8v^2}\frac{(3\cos(\beta+\alpha)+\cos(\alpha-3\beta))(3\cos(\beta+\alpha)+\cos(\beta-3\alpha))}{\sin^2 2\beta} + \frac{M_{H2}^2}{8v^2}\frac{(\sin(\beta+\alpha)-\sin(\beta-3\alpha))(3\sin(\beta+\alpha)-\sin(3\beta-\alpha))}{\sin^2 2\beta}, \tag{A.11}$$

$$g_{12} = g_9, \tag{A.12}$$

$$g_{13} = \frac{M_{H^\pm}^2}{v^2}\sin 2(\beta-\alpha) + \frac{M_{H1}^2}{4v^2}\frac{(\sin(\beta+\alpha)-\sin(\beta-3\alpha))(3\cos(\beta+\alpha)+\cos(3\beta-\alpha))}{\sin^2 2\beta} + \frac{M_{H2}^2}{4v^2}\frac{(\cos(\beta+\alpha)-\cos(\beta-3\alpha))(3\sin(\beta+\alpha)-\sin(3\beta-\alpha))}{\sin^2 2\beta}, \tag{A.13}$$

$$g_{14}=\frac{M_{H1}^2}{8v^2}\,\frac{\sin^2 2\alpha\,\sin^2(\beta-\alpha)}{\sin^2 2\beta}+\frac{M_{H2}^2}{32v^2}\,\frac{\left(3\sin(\beta+\alpha)+\sin(\beta-3\alpha)\right)^2}{\sin^2 2\beta}, \qquad (A.14)$$

$$g_{15}=\frac{M_{H1}^2}{4v^2}\,\frac{\sin^2 2\alpha\,\sin 2(\alpha-\beta)}{\sin^2 2\beta}+\frac{M_{H2}^2}{8v^2}\,\frac{\left(\cos(\beta+\alpha)-\cos(\beta-3\alpha)\right)\left(3\sin(\beta+\alpha)+\sin(\beta-3\alpha)\right)}{\sin^2 2\beta}, \qquad (A.15)$$

$$g_{16}=\frac{M_{H1}^2}{16v^2}\,\frac{8\sin^2 2\alpha\cos^2(\beta-\alpha)+\cos^2(\beta+\alpha)-\cos^2(\beta-3\alpha)}{\sin^2 2\beta}+\frac{M_{H2}^2}{16v^2}\,\frac{8\sin^2 2\alpha\sin^2(\beta-\alpha)+\sin^2(\beta+\alpha)-\sin^2(\beta-3\alpha)}{\sin^2 2\beta}, \qquad (A.16)$$

$$g_{17}=\frac{M_{H1}^2}{8v^2}\,\frac{\left(\sin(\beta+\alpha)-\sin(\beta-3\alpha)\right)\left(3\cos(\beta+\alpha)+\cos(\beta-3\alpha)\right)}{\sin^2 2\beta}+\frac{M_{H2}^2}{4v^2}\,\frac{\sin^2 2\alpha\,\sin 2(\alpha-\beta)}{\sin^2 2\beta}, \qquad (A.17)$$

$$g_{18}=\frac{M_{H1}^2}{32v^2}\,\frac{\left(3\cos(\beta+\alpha)-\cos(\beta-3\alpha)\right)^2}{\sin^2 2\beta}+\frac{M_{H2}^2}{8v^2}\,\frac{\sin^2 2\alpha\cos^2(\beta-\alpha)}{\sin^2 2\beta}, \qquad (A.18)$$

$$g_{19}=\frac{M_{H3}^2}{2v^2}\cos^2(\beta-\alpha)-\frac{M_{Ht}^2}{2v^2}\cos^2(\beta-\alpha)+\frac{1}{2}g_{10}, \qquad (A.19)$$

$$g_{20}=\frac{M_{H3}^2}{2v^2}\sin^2(\beta-\alpha)-\frac{M_{Ht}^2}{2v^2}\sin^2(\beta-\alpha)+\frac{1}{2}g_{11}, \qquad (A.20)$$

$$g_{21}=\frac{M_{H3}^2}{2v^2}\sin 2(\beta-\alpha)-\frac{M_{Ht}^2}{2v^2}\sin 2(\beta-\alpha)+\frac{1}{2}g_{13}, \qquad (A.21)$$

$$g_{22}=\frac{1}{4}g_9 . \qquad (A.22)$$

Appendix B

The Lagrangian of self-interaction $\mathcal{L}_{4H}$ in the supersymmetric model without singlet Higgs fields has the form [x)]

$$\begin{aligned}\mathcal{L}_{4H}=&-\frac{g^2}{4}H^+H^-\Big[(H_1^0)^2\Big(\cos^2(\alpha-\beta)-\frac{\cos 2\alpha\cos 2\beta}{2\cos^2\theta_W}\Big)\\&+H_1^0H_2^0\left(tg^2\theta_W\sin 2\alpha\cos 2\beta\right)+(H_2^0)^2\Big(\sin^2(\alpha-\beta)+\\&+\frac{\cos 2\alpha\cos 2\beta}{2\cos^2\theta_W}\Big)\Big]-\frac{g^2}{8\cos^2\theta_W}\Big[(H^+H^-)^2\cos^2 2\beta-\\&-H^+H^-(H_3^0)^2\cos^2 2\beta+\frac{1}{4}(H_1^0)^4\cos^2 2\alpha-\frac{1}{2}(H_1^0)^3H_2^0\cdot\\&\cdot\sin 4\alpha+(H_1^0H_2^0)^2\Big(\sin^2 2\alpha-\frac{1}{2}\cos^2 2\alpha\Big)+\frac{1}{2}H_1^0(H_2^0)^3\cdot\\&\cdot\sin 4\alpha+\frac{1}{4}(H_2^0)^4\cos^2 2\alpha+\frac{1}{4}(H_3^0)^4\cos^2 2\beta-\\&-\frac{1}{2}(H_3^0)^2(H_1^0)^2\cos 2\beta\cos 2\alpha+\frac{1}{2}(H_3^0)^2(H_2^0)^2\cos 2\beta\cos 2\alpha+\\&+(H_3^0)^2H_1^0H_2^0\cos 2\beta\sin 2\alpha\Big]. \qquad (B.1)\end{aligned}$$

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x) The Lagrangians that correspond to other forms of interaction in the models that are supersymmetric extensions of the Standard Model can be found in /3,4/. The general receipt for constructing such Lagrangians can be found there.

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Двоеглазов В.В., Скачков Н.Б. E2-91-115
H^+H^--взаимодействие с учетом высших порядков теории возмущений в модели с двумя хиггсовскими дублетами
/Лагранжиан и однопетлевые амплитуды/

Однопетлевая амплитуда H^+H^--взаимодействия в четвертом порядке теории возмущений вычислена в рамках модели с двумя хиггсовскими дублетами с произвольным числом фермионов. Использованный нами лагранжиан не содержит каких-либо приближений. Для конкретизации констант связи f^{3H}, f^{H}, f^{HHV} и др. использовались значения, следующие из модели с минимальным суперсимметричным расширением стандартной модели.

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Dvoeglazov V.V., Skachkov N.B. E2-91-115
H^+H^- Interaction up to Higher Orders of Perturbation Theory in the Model with Two Higgs Doublets
(Lagrangian and One-Loop Amplitudes)

The one-loop amplitude of the Higgs-Higgs interaction is calculated in the fourth order of perturbation theory in the framework of the model with two Higgs doublets and an arbitrary number of fermions. The Lagrangian of this interaction is taken without any approximations. The vertex coupling constants like f^{3H}, f^{HHV}, etc., are taken as given by the minimal SUSY extension of the Standard Model.

The investigation has been performed at the Laboratory of Theoretical Physics, JINR.

Communication of the Joint Institute for Nuclear Research. Dubna 1991