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H^+H^- INTERACTION UP TO HIGHER ORDERS OF PERTURBATION THEORY IN THE MODEL WITH TWO HIGGS DOUBLETS (SELF-ENERGY AND VERTEX DIAGRAMS)

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Introduction

This work continues the previous papers [1,2] devoted to the calculation of Higgs-Higgs interaction in the framework of the standard $SU(2)\otimes U(1)$ model with only one doublet of Higgs particles.

There exist a rather large number of different variants of the Standard Model (SM) extension. Among them there is the left-right symmetrical model [3,4], vector-like theories based on another type of gauge groups (see, for example, [5,6]) and the model with the horizonthal interaotion [7]. Most of these models are characterized by enlarging of the Higgs seotor, namely by introduotion of two and even more Higgs boson doublets [8-10]. The supersymmetric extension of the SM [11,12] requires also an introduction of at least two doublets of the scalars.

The interest in these models has grown up in connection with the problem of suppression of CP-violation in strong interactions [13] and the problem of $B^0\bar{B}^0$-mixing [14], because they give the alternative theoretical solution to the problem of electroweak violation of CP-invariance [10,15]. Let us mention in this connection that the measurements of the B-meson life-time [16], the ratio [17] $\bar{R} = \Gamma(b \to e\nu u) / \Gamma(b \to ce\nu)$ and of ε'/ε [18] indicate also possible difficulties in the Standard Model (the description of CP-violation). The theoretical value of the ratio ε'/ε is larger than that measured in these experiments [19]

$$\varepsilon'/\varepsilon = (-0.4 \pm 1.4 \pm 0.6)\cdot 10^{-3}.$$

The paper is organized as follows. In sect.2, the simplest extension of the Glashow - Weinberg - Salam model by introducing two doublets of scalar particles is considered. The renormalization scheme is discussed also here. In table 1 we present the values of the coupling constants (f^{HHV}, f^{HVV}, f^{3H} e.t.c.), derived within the Minimal (i.e. two-doublet) Supersymmetric extension of the Standard Model (MSSM). But in our calculations we try to avoid the concretization of the values of vertex constants. In sect.3, one-loop integrals that correspond to self-energy diagrams as well as $H^{\pm}W^{\pm}$-mixing are calculated. In Sect.4 the vertex diagrams are computed in a similar way.

The obtained results will be used as in [1,2], for calculating the H^+H^- interaction amplitude with one-loop corrections included (see, in this context, the papers [20-22])

2. Two Higgs doublet model

The simplest extension of the Standard Model consists in the introduction of the second doublet of Higgs bosons

$$\Phi_1 = \begin{pmatrix} \phi_1^+ \\ \frac{v_1 + \eta_1 + i\chi_1}{\sqrt{2}} \end{pmatrix}, \quad \Phi_2 = \begin{pmatrix} \phi_2^+ \\ \frac{v_2 + \eta_2 + i\chi_2}{\sqrt{2}} \end{pmatrix}. \qquad (2.1)$$

The Higgs sector of the Lagrangian has the following form [23]

$$\mathcal{L}_H = \mathcal{L}_{GH} + \mathcal{L}_{FH} - V(\Phi_1, \Phi_2). \qquad (2.2)$$

It contains:

1) the terms that define the interaction of gauge fields with the Higgs bosons

$$\mathcal{L}_{GH} = |D_\mu \Phi_1|^2 + |D_\mu \Phi_2|^2, \qquad (2.3)$$

where

$$D_\mu = \partial_\mu - \frac{ig}{2}\vec{\sigma}\vec{W}_\mu - \frac{ig'}{2}B_\mu, \qquad (2.4)$$

is the covariant derivative for the $SU(2)_L \otimes U(1)$ group;

2) The terms that describe the fermion-Higgs interaction [x)]

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x) The variant when the fermions get their masses due to the interaction with the only one Higgs doublet is often used. In this case g_{1f} is taken to be zero (see for details [24-26]).

$$\mathcal{L}_{FH} = -\sum_f g_{1f}\left[\bar{f}_R \Phi_1^+ f_L + \bar{f}_L \Phi_1 f_R\right] - \sum_f g_{2f}\left[\bar{f}_R \Phi_2^+ f_L + \bar{f}_L \Phi_2 f_R\right], \tag{2.5}$$

where f_L and f_R are left-handed fermion doublets and right-handed singlets;

3) Self-interaction potential

$$V(\Phi_1, \Phi_2) = -\mu_1^2 \Phi_1^+\Phi_1 - \mu_2^2 \Phi_2^+\Phi_2 + \lambda_1 (\Phi_1^+\Phi_1)^2 + \lambda_2 (\Phi_2^+\Phi_2)^2 + \lambda_3 (\Phi_1^+\Phi_1)(\Phi_2^+\Phi_2) + \lambda_4 (\Phi_1^+\Phi_2)(\Phi_2^+\Phi_1) + \frac{\lambda_5}{2}\left[(\Phi_1^+\Phi_2)^2 + (\Phi_2^+\Phi_1)^2\right]. \tag{2.6}$$

Eight Higgses are used to attach masses to gauge fields through the mechanism of spontaneous symmetry breaking. Three Goldstone fields

$$G^\pm = \phi_1^\pm \cos\beta + \phi_2^\pm \sin\beta\ , \tag{2.7}$$

$$G^0 = \sqrt{2}\left[\chi_1 \cos\beta + \chi_2 \sin\beta\right]$$

are absorbed to produce the longitudinal components and masses for $W^\pm$ and Z^0 bozons.

The remaining 5 Higgses

$$H^\pm = -\phi_1^\pm \sin\beta + \phi_2^\pm \cos\beta\ ;\ H_3^0 = \sqrt{2}\left[-\chi_1 \sin\beta + \chi_2 \cos\beta\right], \tag{2.8}$$

$$H_1^0 = \sqrt{2}\left[\eta_1 \cos\alpha + \eta_2 \sin\alpha\right];\ H_2^0 = \sqrt{2}\left[-\eta_1 \sin\alpha + \eta_2 \cos\alpha\right]$$

also get their masses.

The potential of the self-interaction (2.6) has the most general form of the renormalizable scalar potential that is consistent with the requirement of the discrete symmetry under the replacement $\Phi_1 \to \Phi_1$ and $\Phi_2 \to -\Phi_2$ (This requirement is imposed usually to suppress the flavour changing neutral currents, that are automatically suppressed in the SM).

The disadvantage of the model consists in the increase of the number of free parameters: the masses of 5 Higgs bosons, the angle

α of mixing in the sector of neutral scalars and the angle β that is connected with the ratio of two vacuum expectation values v_1 and v_2 as follows: $tg\ \beta = v_2/v_1$.

In contrast to this the minimal two-doublet supersymmetric model has only two additional free parameters ($tg\ \beta$ and $M_{H\pm}$, for example) /11,12/. Table 1 contains the vertex constants that follows from the supersymmetry model and are used in our paper to concretize the values of f^{HHV}, f^{3H} and other constants.

Table I

$f^{H^\pm H^\pm H_1^0}$	$-gM_W\left(\cos(\beta-\alpha) - \frac{\cos 2\beta \cos(\beta+\alpha)}{2\cos^2\theta_W}\right)$	$f^{H^\pm W^\pm H_1^0}$	$-\frac{g}{2}\sin(\alpha-\beta)$
$f^{H^\pm H^\pm H_2^0}$	$-gM_W\left(\sin(\beta-\alpha) + \frac{\cos 2\beta \sin(\beta+\alpha)}{2\cos^2\theta_W}\right)$	$f^{H^\pm W^\pm H_2^0}$	$-\frac{g}{2}\cos(\alpha-\beta)$
$f^{3H_1^0}$	$-\frac{3gM_Z}{2\cos\theta_W}\cos 2\alpha \cos(\beta+\alpha)$	$f^{H^\pm W^\pm H_3^0}$	$-i\frac{g}{2}$
$f^{2H_1^0 H_2^0}$	$\frac{gM_Z}{2\cos\theta_W}\left(\begin{array}{l}2\sin 2\alpha \cos(\beta+\alpha) + \\ + \sin(\beta+\alpha)\cos 2\alpha\end{array}\right)$	$f^{H^\pm H^\pm Z}$	$-\frac{g\cos 2\theta_W}{2\cos\theta_W}$
$f^{H_1^0 2H_2^0}$	$-\frac{gM_Z}{2\cos\theta_W}\left(\begin{array}{l}2\sin 2\alpha \sin(\beta+\alpha) - \\ - \cos(\beta+\alpha)\cos 2\alpha\end{array}\right)$	$f^{H^\pm H^\pm A}$	$-e$
$f^{3H_2^0}$	$-\frac{3gM_Z}{2\cos\theta_W}\cos 2\alpha \sin(\beta+\alpha)$	$f^{H_1^0 H_3^0 Z}$	$-\frac{ig\sin(\alpha-\beta)}{2\cos\theta_W}$
$f^{H_1^0 2H_3^0}$	$\frac{gM_Z}{2\cos\theta_W}\cos 2\beta \cos(\beta+\alpha)$	$f^{H_2^0 H_3^0 Z}$	$-\frac{ig\cos(\alpha-\beta)}{2\cos\theta_W}$
$f^{H_2^0 2H_3^0}$	$-\frac{gM_Z}{2\cos\theta_W}\cos 2\beta \sin(\beta+\alpha)$	$f^{H^\pm H^\pm 2H_1^0}$	$-\frac{g^2}{2}\left(\cos^2(\alpha-\beta) - \frac{\cos 2\alpha \cos 2\beta}{2\cos^2\theta_W}\right)$
$f^{H_1^0 2W}$	$-gM_W\cos(\beta-\alpha)$	$f^{H^\pm H^\pm H_1^0 H_2^0}$	$-\frac{g^2}{4}tg^2\theta_W \sin 2\alpha \cos 2\beta$
$f^{H_2^0 2W}$	$-gM_W\sin(\beta-\alpha)$	$f^{H^\pm H^\pm 2H_2^0}$	$-\frac{g^2}{2}\left(\sin^2(\alpha-\beta) + \frac{\cos 2\alpha \cos 2\beta}{2\cos^2\theta_W}\right)$
$f^{H_1^0 2Z}$	$-\frac{gM_Z}{\cos\theta_W}\cos(\beta-\alpha)$	$f^{4H^\pm}$	$-\frac{g^2\cos^2 2\beta}{2\cos^2\theta_W}$
$f^{H_2^0 2Z}$	$-\frac{gM_Z}{\cos\theta_W}\sin(\beta-\alpha)$	$f^{H^\pm H^\pm 2H_3^0}$	$+\frac{g^2\cos^2 2\beta}{4\cos^2\theta_W}$

$f^{H^\pm H^\pm 2W}$	$-g^2/2$	$f^{H^\pm H^\pm ZA}$	$-\dfrac{eg\cos 2\theta_W}{\cos\theta_W}$
$f^{H^\pm H^\pm 2Z}$	$-\dfrac{g^2\cos^2 2\theta_W}{2\cos^2\theta_W}$	$f^{H_j^0 H_j^0 2W}$	$-g^2/2$
$f^{H^\pm H^\pm 2A}$	$-2e^2$	$f^{H_j^0 H_j^0 2Z}$	$-\dfrac{g^2}{2\cos^2\theta_W}$
$f^{H_1^0 uu}$	$-\dfrac{gm_u\sin\alpha}{2M_W\sin\beta}$	$f^{H_1^0 dd}$	$-\dfrac{gm_d\cos\alpha}{2M_W\cos\beta}$
$f^{H_2^0 uu}$	$-\dfrac{gm_u\cos\alpha}{2M_W\sin\beta}$	$f^{H_2^0 dd}$	$\dfrac{gm_d\sin\alpha}{2M_W\cos\beta}$
$f^{H_3^0 uu}$	$\dfrac{igm_u ctg\beta}{2M_W}\gamma_5$	$f^{H_3^0 dd}$	$\dfrac{igm_d tg\beta}{2M_W}\gamma_5$
$f^{H^\pm ud}$	$\dfrac{g}{2\sqrt{2}M_W}\left[(m_d tg\beta + m_u ctg\beta)\pm(m_d tg\beta - m_u ctg\beta)\gamma_5\right]$		

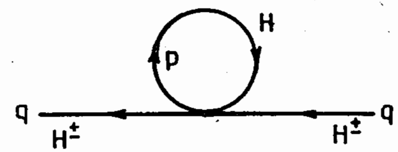
In [27,28] the general approach to the renormalization in the unitary gauge was developed. We shall apply this scheme (with the use of the standard parameters, recommended by the Trieste Conference [29] to the case of the two-doublet model of electroweak interactions

$$
\begin{aligned}
W_0 &= \sqrt{Z_W}\,W & e_0 &= \left(\sqrt{Z_A}\right)^{-1} e \\
Z_0 &= \sqrt{Z_Z}\,Z & M_{0W}^2 &= Z_{M_W} Z_W^{-1} M_W^2 \\
A_0 &= \sqrt{Z_A}\,A + \sqrt{Z_M}\,Z & M_{0Z}^2 &= Z_{M_Z} Z_Z^{-1} M_Z^2 \qquad (2.9)\\
H_0^\pm &= \sqrt{Z_{H^\pm}}\,H^\pm & M_{0H^\pm}^2 &= Z_{M_{H^\pm}} Z_{H^\pm}^{-1} M_{H^\pm}^2 \\
H_{10}^0 &= \sqrt{Z_{H_1^0}}\,H_1^0 + \sqrt{Z_{M1}}\,H_2^0 & H_{20}^0 &= \sqrt{Z_{M2}}\,H_1^0 + \sqrt{Z_{H_2^0}}\,H_2^0 .
\end{aligned}
$$

The renormalization of other parameters follows from these relations.

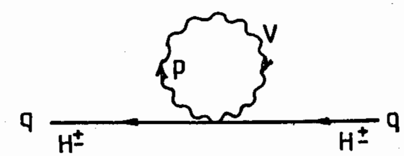
3. Self-energy diagrams

1.



$$\Pi = \frac{if^{4H}}{16\pi^2} M_H^2\left[2P - 1 + \ln\frac{M_H^2}{M_W^2}\right]. \qquad (3.1)$$

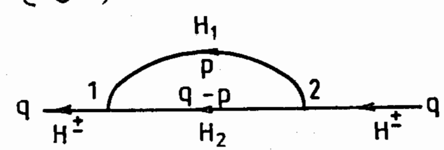
2.



$$\Pi = \frac{if^{2H2V}}{16\pi^2}\,\varepsilon M_V^2\left[6P - 1 + 3\ln\frac{\varepsilon M_V^2}{M_W^2}\right], \qquad (3.2)$$

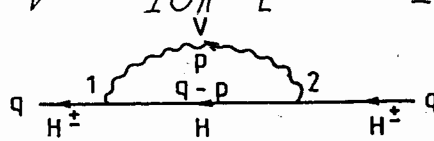
$$\varepsilon = \begin{cases} 1, & \text{if } V \text{ is a heavy vector boson .} \\ 0, & \text{if } V \text{ is a photon .} \end{cases}$$

3.



$$\Pi(q^2) = \frac{if_1^{3H} f_2^{3H}}{16\pi^2}\left[-2P - I_0(q^2, M_{H1}^2, M_{H2}^2)\right]. \qquad (3.3)$$

4.

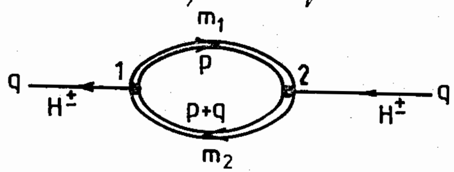


$$\Pi(q^2) = \pm\frac{if_1^{HHV} f_2^{HHV}}{16\pi^2}\left\{\left[-2q^2(2+\varepsilon) + 2M_H^2(1-\varepsilon) - \frac{2\varepsilon}{M_V^2}\times\right.\right.$$

$$\left.\times(q^2 + M_H^2)^2\right]P + \varepsilon q^2 - \varepsilon M_V^2 + M_H^2(1+\varepsilon) + \varepsilon\,(M_V^2 -$$

$$-q^2-M_H^2\Big)\ln\frac{\varepsilon M_V^2}{M_W^2}-M_H^2\ln\frac{M_H^2}{M_W^2}-\Big(2q^2+\varepsilon M_V^2-2M_H^2+\frac{\varepsilon}{M_V^2}\left(q^2+M_H^2\right)^2\Big)I_0\left(q^2,M_H^2,\varepsilon M_V^2\right)\Big\}. \qquad (3.4)$$

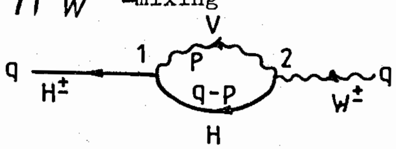
5.



$$\Pi(q^2)=\frac{i f_1^H f_2^H}{4\pi^2}\Big\{\Big[(1-b_1b_2)(q^2+2m_1^2+2m_2^2)+(1+b_1b_2)\cdot 2m_1m_2\Big]P+\Big[\frac{1}{2}(1-b_1b_2)(q^2+m_1^2+m_2^2)+(1+b_1b_2)m_1m_2\Big]\cdot I_0(q^2,m_1^2,m_2^2)+(1-b_1b_2)\Big(\frac{m_1^2}{2}\ln\frac{m_1^2}{M_W^2}+\frac{m_2^2}{2}\ln\frac{m_2^2}{M_W^2}+\frac{q^2}{4}\Big)+\frac{1}{2}(1+b_1b_2)m_1m_2\Big\}. \qquad (3.5)$$

6.

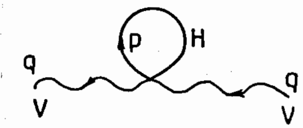
$H^\pm W^\pm$ -mixing



$$\Pi_\mu(q^2)=\frac{i f_1^{HHV} f_2^{HW}}{16\pi^2}q_\mu\Big\{\Big[-3-\frac{1}{M_V^2}(q^2+M_H^2)\Big]P-\frac{1}{2}\frac{1}{M_V^2}\frac{q^2+M_H^2-M_V^2}{q^2}\Big[M_H^2\Big(1+\ln\frac{M_H^2}{M_W^2}\Big)-M_V^2\Big(1+\ln\frac{M_V^2}{M_W^2}\Big)\Big]-\Big(\frac{1}{2}\frac{1}{M_V^2}\frac{(q^2+M_H^2-M_V^2)^2}{q^2}+2\Big)I_0(q^2,M_H^2,M_V^2)\Big\}. \qquad (3.6)$$

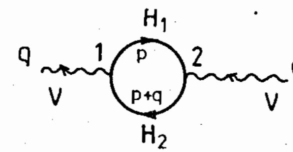
Let us still present the result of a calculation of the diagrams needed for finding of the counter terms and amplitude $H^+H^-\to H^+H^-$.

7.



$$\Pi_{\alpha\beta}=\frac{i f^{2H2V}}{16\pi^2}\delta_{\alpha\beta}M_H^2\Big[2P-1+\ln\frac{M_H^2}{M_W^2}\Big]. \qquad (3.7)$$

8.

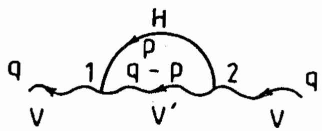


$$\Pi_{\alpha\beta}(q^2)=\frac{i f_1^{HHV} f_2^{HHV}}{16\pi^2}\Big\{q_\alpha q_\beta\Big[-\frac{2}{3}P+\frac{2}{9}-\frac{4}{3}\cdot\frac{(M_{H1}^2-M_{H2}^2)^2}{q^4}+\frac{M_{H1}^2}{q^2}\ln\frac{M_{H1}^2}{M_W^2}\Big(\frac{2}{3}+\frac{4}{3}\frac{M_{H1}^2-M_{H2}^2}{q^2}\Big)+\frac{M_{H2}^2}{q^2}\ln\frac{M_{H2}^2}{M_W^2}\Big(\frac{2}{3}+\frac{4}{3}\frac{M_{H2}^2-M_{H1}^2}{q^2}\Big)-\Big(\frac{1}{3}+\frac{2}{3}\frac{M_{H1}^2+M_{H2}^2}{q^2}+\frac{4}{3}\frac{(M_{H1}^2-M_{H2}^2)^2}{q^4}\Big)I_0(q^2,M_{H1}^2,M_{H2}^2)\Big]-q^2\delta_{\alpha\beta}\Big[-2\Big(\frac{1}{3}+\frac{M_{H1}^2+M_{H2}^2}{q^2}\Big)P+\frac{2}{9}+\frac{M_{H1}^2+M_{H2}^2}{q^2}-\frac{1}{3}\frac{(M_{H1}^2-M_{H2}^2)^2}{q^4}+$$

$$+\frac{M_{H1}^2}{q^2}\ln\frac{M_{H1}^2}{M_W^2}\left(-\frac{1}{3}+\frac{1}{3}\frac{M_{H1}^2-M_{H2}^2}{q^2}\right)+\frac{M_{H2}^2}{q^2}\ln\frac{M_{H2}^2}{M_W^2}\left(-\frac{1}{3}+\frac{1}{3}\frac{M_{H2}^2-M_{H1}^2}{q^2}\right)-\left(\frac{1}{3}+\frac{2}{3}\frac{M_{H1}^2+M_{H2}^2}{q^2}+\frac{1}{3}\frac{(M_{H1}^2-M_{H2}^2)^2}{q^4}\right)\times$$

$$\times I_0(q^2,M_{H1}^2,M_{H2}^2)\Big]\Big\}. \qquad (3.8)$$

9.



$$\Pi_{\alpha\beta}(q^2)=\frac{i f_1^{HVV} f_2^{HVV}}{16\pi^2}\frac{1}{M_V^2}\Big\{q_\alpha q_\beta\Big[-\frac{2}{3}P+\frac{1}{18}+\frac{M_V^2-M_H^2}{2q^2}-\frac{1}{3}\frac{(M_V^2-M_H^2)^2}{q^4}-\frac{M_V^2}{q^2}\ln\frac{M_V^2}{M_W^2}\left(\frac{1}{3}+\frac{M_H^2-M_V^2}{3q^2}\right)+\frac{M_H^2}{q^2}\ln\frac{M_H^2}{M_W^2}\times$$

$$\times\left(\frac{2}{3}+\frac{M_H^2-M_V^2}{3q^2}\right)-\left(\frac{1}{3}+\frac{2}{3}\frac{M_H^2-M_V^2/2}{q^2}+\frac{1}{3}\frac{(M_H^2-M_V^2)^2}{q^4}\right)\times$$

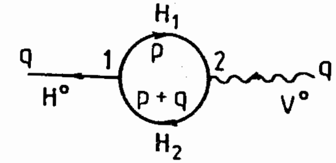
$$\times I_0(q^2,M_V^2,M_H^2)\Big]-q^2\delta_{\alpha\beta}\Big[\left(-\frac{1}{6}-\frac{1}{2}\frac{M_H^2}{q^2}+\frac{3}{2}\frac{M_V^2}{q^2}\right)\times$$

$$P+\frac{1}{18}+\frac{1}{4}\frac{M_H^2}{q^2}+\frac{1}{4}\frac{M_V^2}{q^2}-\frac{1}{12}\frac{(M_H^2-M_V^2)^2}{q^4}-\frac{M_V^2}{q^2}\ln\frac{M_V^2}{M_W^2}\times$$

$$\times\left(\frac{1}{12}+\frac{M_H^2-M_V^2}{12q^2}\right)+\frac{M_H^2}{q^2}\ln\frac{M_H^2}{M_W^2}\left(-\frac{1}{12}+\frac{M_H^2-M_V^2}{12q^2}\right)+\Big(\frac{5M_V^2}{6q^2}-$$

$$-\frac{1}{6}\frac{M_H^2}{q^2}-\frac{1}{12}-\frac{1}{12}\frac{(M_H^2-M_V^2)^2}{q^4}\Big)I_0(q^2,M_V^2,M_H^2)\Big]\Big\}. \qquad (3.9)$$

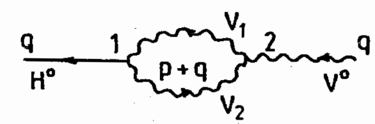
10. H^0Z^0 - mixing



$$\Pi_\mu(q^2)=\frac{i f_1^{3H} f_2^{HHV}}{16\pi^2}q_\mu\Big\{\frac{M_{H2}^2-M_{H1}^2}{q^2}+\frac{M_{H1}^2}{q^2}\ln\frac{M_{H1}^2}{M_W^2}-\frac{M_{H2}^2}{q^2}\times$$

$$\times\ln\frac{M_{H2}^2}{M_W^2}+\frac{M_{H2}^2-M_{H1}^2}{q^2}I_0(q^2,M_{H1}^2,M_{H2}^2)\Big\}. \qquad (3.10)$$

11. H^0Z^0 - mixing



$$\Pi_\mu(q^2)=\frac{i f_1^{HVV} f_2^{3V}}{16\pi^2}q_\mu\Big\{\left[\frac{q^2}{2M_{V1}^2}-\frac{q^2}{2M_{V2}^2}+\frac{3}{2}\frac{M_{V2}^2}{M_{V1}^2}-\frac{3}{2}\frac{M_{V1}^2}{M_{V2}^2}\right]\times$$

$$P+\frac{M_{V1}^2}{q^2}\left(\ln\frac{M_{V1}^2}{M_W^2}-1\right)\left[3+\frac{1}{4}\frac{q^2(M_{V2}^2-M_{V1}^2)}{M_{V1}^2M_{V2}^2}+\frac{1}{4}\times\right.$$

$$\left.\times\frac{(M_{V2}^2-M_{V1}^2)^2}{M_{V1}^2M_{V2}^2}\right]+\frac{M_{V2}^2}{q^2}\left(\ln\frac{M_{V2}^2}{M_W^2}-1\right)\left[-3+\frac{1}{4}\frac{q^2(M_{V1}^2-M_{V2}^2)}{M_{V1}^2M_{V2}^2}\right.$$

$$\left.+\frac{1}{4}\frac{(M_{V1}^2-M_{V2}^2)^2}{M_{V1}^2M_{V2}^2}\right]+\left[\frac{1}{4}\frac{q^2(M_{V2}^2-M_{V1}^2)}{M_{V2}^2M_{V1}^2}+\frac{1}{2}\frac{M_{V2}^4-M_{V1}^4}{M_{V1}^2M_{V2}^2}+\right.$$

$$\left.+\frac{9}{4}\frac{M_{V2}^2-M_{V1}^2}{q^2}+\frac{1}{4}\frac{M_{V2}^6-M_{V1}^6}{M_{V1}^2M_{V2}^2q^2}\right]I_0(q^2,M_{V1}^2,M_{V2}^2)\Big\}. \qquad (3.11)$$

Obviously, when the masses of the virtual particles are equated the $H^0 Z^0$-mixing diagrams don't contribute.
Since

$$\Pi^{ren}_{H^\pm}(q^2) = \Pi_{H^\pm}(q^2) - \delta M^2_{H^\pm} - (Z_{H^\pm}-1)(q^2+M^2_{H^\pm}), \quad (3.12)$$

it is necessary to find the counter terms $\delta M^2_{H^\pm}$ and $Z_{H^\pm}-1$.

$$\frac{\delta M^2_{H^\pm}}{M^2_{H^\pm}} = \frac{\Pi_{H^\pm}(-M^2_{H^\pm})}{M^2_{H^\pm}} = \frac{ig^2}{16\pi^2}\Big\{\Big[9 - 6R - \frac{1}{R}\Big(\frac{3}{2} + \cos^2 2\beta\Big) - 5r_\pm^{-1} - \frac{1}{R} r_\pm^{-1}\big(3 - 2\cos^2 2\beta\big) + \frac{1}{R^2} r_\pm^{-1}\Big(\frac{3}{2} - \frac{1}{2}\cos^2 2\beta\Big) -$$

$$- \big(r_1 \sin^2(\alpha-\beta) + r_2\cos^2(\alpha-\beta) - r_3\big) - \frac{1}{2} r_\pm^{-1}\big(r_1\cos^2(\alpha-\beta) + r_2 \sin^2(\alpha-\beta)\big) + \frac{1}{2} r_\pm^{-1}\big(r_1^2 \sin^2(\alpha-\beta) + r_2^2\cos^2(\alpha-\beta) - r_3^2\big) +$$

$$+ \frac{1}{4R} r_\pm^{-1}\cos 2\beta\big(r_1\cos 2\alpha - r_2 \cos 2\alpha + r_3 \cos 2\beta\big) - \sum_{i,j}^{1/2 N_f}\Big(\frac{m_i^2 \, ctg^2\beta + m_j^2 \, tg^2\beta}{M_W^2} - 2\,\frac{m_i^2 m_j^2 (tg^2\beta + ctg^2\beta + 1)}{M_W^2 M_{H^\pm}^2} -$$

$$-2\,\frac{m_i^4 \, ctg^2\beta + m_j^4\, tg^2\beta}{M_W^2 M_{H^\pm}^2}\Big)\Big] P + \text{finite part}\Big\} \quad (3.13)$$

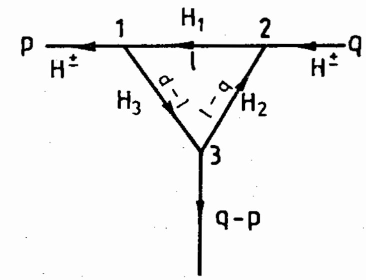
and $\left(r_\pm = M^2_{H^\pm}/M^2_W,\ r_i = M^2_{H_i}/M^2_W\right)$

$$Z_{H^\pm} - 1 = \frac{ig^2}{16\pi^2}\Big\{\Big[-7 + 4R + \frac{3}{2}\,\frac{1}{R} + \big(r_1 \sin^2(\alpha-\beta) + r_2\cos^2(\alpha-\beta) - r_3\big) + \sum_{i,j}^{1/2 N_f}\frac{m_i^2\, ctg^2\beta + m_j^2\, tg^2\beta}{M_W^2}\Big] P +$$

$$+ 4(1-R) P_{IR} + \text{finite part}\Big\}. \quad (3.14)$$

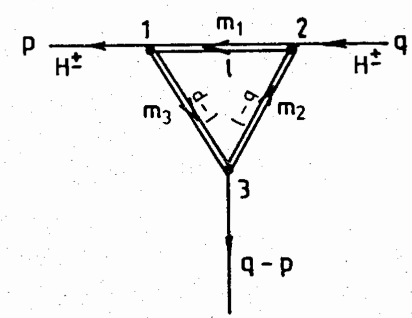
4. Vertex diagrams

1.



$$\Gamma(p^2, q^2, (p-q)^2) = \frac{i f_1^{3H} f_2^{3H} f_3^{3H}}{16\pi^2} I_1\big(q^2, (p-q)^2, p^2, M^2_{H1}, M^2_{H2}, M^2_{H3}\big). \quad (4.1)$$

2.



and a similar diagram with the opposite lepton current direction.

$$\Gamma(p^2, q^2, (p-q)^2) = \frac{i f_1^H f_2^H f_3^H}{4\pi^2}\Big\{-\big[2m_1 B_1 + 2m_2 B_2 + 2m_3 B_3\big] P - \frac{m_1 B_1 + m_2 B_2 + m_3 B_3}{2} - \frac{m_1 B_1 + m_2 B_2}{2} \times$$

$$\times I_0(q^2, m_1^2, m_2^2) - \frac{m_1 B_1 + m_3 B_3}{2} I_0(p^2, m_1^2, m_3^2) - \quad (4.2)$$

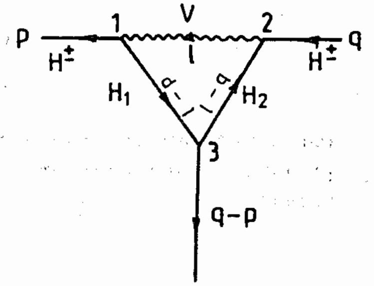
$$- \frac{m_2 B_2 + m_3 B_3}{2} I_0\big((p-q)^2, m_2^2, m_3^2\big) - \frac{1}{2}\Big[m_1 B_1\big((p-q)^2 + m_2^2 + m_3^2\big) + m_2 B_2\big(p^2 + m_1^2 + m_3^2\big) + m_3 B_3\big(q^2 + m_1^2 + m_2^2\big) +$$

$$+2m_1m_2m_3B_4\Big]I_1\left(q^2,(p-q)^2,p^2,m_1^2,m_2^2,m_3^2\right),$$

where

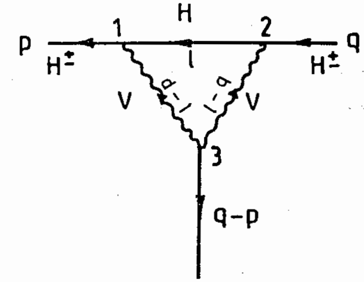
$$B_1=1+b_1b_2-b_1b_3-b_2b_3\ ,$$
$$B_2=1-b_1b_2-b_1b_3+b_2b_3\ ,$$
$$B_3=1-b_1b_2+b_1b_3-b_2b_3\ ,\qquad (4.3)$$
$$B_4=1+b_1b_2+b_1b_3+b_2b_3\ .$$

3.



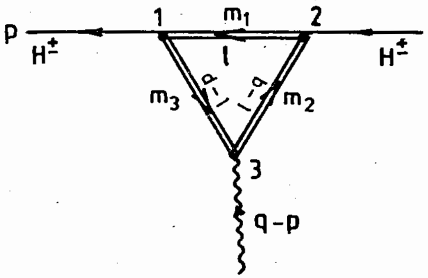
$$\Gamma\left(p^2,q^2,(p-q)^2\right)=\pm\frac{i f_1^{HHV}f_2^{HHV}f_3^{3H}}{16\pi^2}\Big\{-2\Big[1-\varepsilon-\frac{\varepsilon}{M_V^2}\big(p^2+q^2+$$
$$+M_{H1}^2+M_{H2}^2\big)\Big]P-\varepsilon+\varepsilon\ln\frac{\varepsilon M_V^2}{M_W^2}+I_0\left((p-q)^2,M_{H1}^2,M_{H2}^2\right)+$$
$$+I_0\left(q^2,\varepsilon M_V^2,M_{H2}^2\right)\left[-1+\frac{\varepsilon}{M_V^2}\left(q^2+M_{H2}^2\right)\right]+I_0\left(p^2,\varepsilon M_V^2,M_{H1}^2\right)\times$$
$$\times\left[-1+\frac{\varepsilon}{M_V^2}\left(p^2+M_{H1}^2\right)\right]-I_1\Big(p^2,(p-q)^2,q^2,\varepsilon M_V^2,M_{H1}^2,$$
$$M_{H2}^2\Big)\Big[-p^2-q^2+2(p-q)^2-\varepsilon M_V^2+M_{H1}^2+M_{H2}^2-\frac{\varepsilon}{M_V^2}\left(p^2+M_{H1}^2\right)\times$$
$$\times\left(q^2+M_{H2}^2\right)\Big]\Big\}.\qquad (4.4)$$

4.



$$\Gamma\left(p^2,q^2,(p-q)^2\right)=\pm\frac{i f_1^{HHV}f_2^{HHV}f_3^{HVV}}{16\pi^2}\Big\{-\Big[3\frac{1}{M_V^2}(p-q)^2+\frac{1}{M_V^4}\times$$
$$\times\left(\frac{(p-q)^2\left(p^2+q^2+(p-q)^2+2M_H^2\right)}{2}+2\left(p^2+M_H^2\right)\left(q^2+M_H^2\right)\right)\Big]P+$$
$$+\frac{1}{M_V^2}\,\frac{p^2+q^2+(p-q)^2+2M_H^2}{2}\left(1-\ln\frac{M_V^2}{M_W^2}\right)-\Big[-\frac{1}{2}+\frac{1}{2}\,\frac{p^2-q^2}{M_V^2}+$$
$$+\frac{1}{2}\,\frac{\left(p^2+M_H^2\right)\left(q^2+M_H^2\right)}{M_V^4}\Big]I_0\left(p^2,M_H^2,M_V^2\right)-\Big[-\frac{1}{2}+\frac{1}{2}\,\frac{q^2-p^2}{M_V^2}+$$
$$+\frac{1}{2}\,\frac{\left(p^2+M_H^2\right)\left(q^2+M_H^2\right)}{M_V^4}\Big]I_0\left(q^2,M_H^2,M_V^2\right)-\Big[1+\frac{1}{2}\,\frac{1}{M_V^2}\big(-p^2-$$
$$-q^2+2(p-q)^2-2M_H^2\big)+\frac{1}{4}(p-q)^2\,\frac{p^2+q^2+(p-q)^2+2M_H^2}{M_V^4}\Big]\times$$
$$\times I_0\left((p-q)^2,M_V^2,M_V^2\right)+\Big[p^2+q^2-\frac{1}{2}(p-q)^2+M_V^2-2M_H^2+$$
$$+\frac{1}{M_V^2}\left(p^4+q^4-p^2q^2-\frac{1}{2}\left(p^2+q^2\right)(p-q)^2+M_H^2\left(p^2+q^2-(p-q)^2\right)+M_H^4\right)-$$
$$-\frac{1}{2}\,\frac{1}{M_V^4}(p-q)^2\left(p^2+M_H^2\right)\left(q^2+M_H^2\right)\Big]I_1\Big(q^2,(p-q)^2,p^2,M_H^2,$$
$$M_V^2,M_V^2\Big)\Big\}.\qquad (4.5)$$

5.



and a similar diagram with the opposite lepton current direction

$$\Gamma_\mu(p^2,q^2,(p-q)^2) = \frac{i f_1^H f_2^H f_3^V}{4\pi^2}\left\{p_\mu \Gamma^{(1)} + q_\mu \Gamma^{(2)}\right\}, \tag{4.6}$$

$$\Gamma^{(1)}(p^2,q^2,(p-q)^2) = B_1' P + \frac{1}{4}B_1' - \frac{1}{2}\frac{m_2^2}{(p-q)^2}B_1' \times \left(\ln\frac{m_2^2}{M_W^2} - 1\right) + \frac{1}{2}\frac{m_3^2}{(p-q)^2}B_1'\left(\ln\frac{m_3^2}{M_W^2} - 1\right) + \frac{1}{2}B_1' \times \bar{I}_0(p^2,m_1^2,m_3^2) + \frac{1}{2}\frac{m_2^2-m_3^2}{(p-q)^2}B_1'\bar{I}_0((p-q)^2,m_2^2,m_3^2) + \frac{1}{2}\left[(q^2+m_1^2+m_2^2)B_1' + 2m_1m_2B_2'\right]\bar{I}_1(q^2,(p-q)^2,p^2,m_1^2,m_2^2,m_3^2) + \left[\frac{p^2+q^2-(p-q)^2+2m_1^2}{2}B_1' + m_1m_2B_2' + m_1m_3B_3' + m_2m_3B_4'\right]\frac{1}{p^4+q^4+(p-q)^4-2p^2q^2-2p^2(p-q)^2-2q^2(p-q)^2} \times \left[2q^2\bar{I}_0(q^2,m_1^2,m_2^2) + ((p-q)^2-p^2-q^2)\bar{I}_0(p^2,m_1^2,m_3^2) + (p^2-q^2-(p-q)^2)\bar{I}_0((p-q)^2,m_2^2,m_3^2) + \left((p^2+q^2-(p-q)^2)(m_1^2-m_2^2-q^2) - 2q^2(m_1^2-m_3^2-p^2)\right)\bar{I}_1(q^2,(p-q)^2,p^2,m_1^2,m_2^2,m_3^2)\right] \tag{4.7}$$

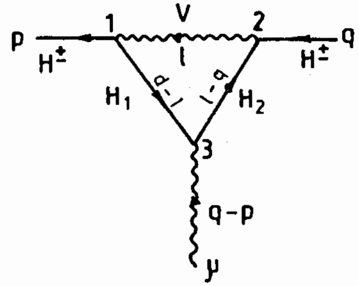
and

$$\Gamma^{(2)}(p^2,q^2,(p-q)^2) = B_1' P + \frac{1}{4}B_1' + \frac{1}{2}\frac{m_2^2}{(p-q)^2}B_1' \times \left(\ln\frac{m_2^2}{M_W^2} - 1\right) - \frac{1}{2}\frac{m_3^2}{(p-q)^2}B_1'\left(\ln\frac{m_3^2}{M_W^2} - 1\right) + \frac{1}{2}B_1' \times \bar{I}_0(q^2,m_1^2,m_2^2) + \frac{1}{2}\frac{m_3^2-m_2^2}{(p-q)^2}B_1'\bar{I}_0((p-q)^2,m_2^2,m_3^2) + \frac{1}{2}\left[(p^2+m_1^2+m_3^2)B_1' + 2m_1m_3B_3'\right]\bar{I}_1(q^2,(p-q)^2,p^2,m_1^2,m_2^2,m_3^2) + \left[\frac{p^2+q^2-(p-q)^2+2m_1^2}{2}B_1' + m_1m_2B_2' + m_1m_3B_3' + m_2m_3B_4'\right]\frac{1}{p^4+q^4+(p-q)^4-2p^2q^2-2p^2(p-q)^2-2q^2(p-q)^2} \times \left[2p^2\bar{I}_0(p^2,m_1^2,m_3^2) + ((p-q)^2-p^2-q^2)\bar{I}_0(q^2,m_1^2,m_2^2) + (q^2-p^2-(p-q)^2)\bar{I}_0((p-q)^2,m_2^2,m_3^2) + \left((p^2+q^2-(p-q)^2)(m_1^2-m_3^2-p^2) - 2p^2(m_1^2-m_2^2-q^2)\right)\bar{I}_1(q^2,(p-q)^2,p^2,m_1^2,m_2^2,m_3^2)\right], \tag{4.8}$$

with

$$\begin{aligned} B_1' &= a_3(1-b_1b_2) - b_1b_3 + b_2b_3, \\ B_2' &= a_3(1+b_1b_2) - b_1b_3 - b_2b_3, \\ B_3' &= a_3(1+b_1b_2) + b_1b_3 + b_2b_3, \\ B_4' &= a_3(1-b_1b_2) + b_1b_3 - b_2b_3. \end{aligned} \tag{4.9}$$

6.



$$\Gamma_\mu(p^2,q^2,(p-q)^2) = \frac{i f_1^{HHV} f_2^{HHV} f_3^{HHV}}{16\pi^2}\left\{p_\mu \Gamma^{(1)} + q_\mu \Gamma^{(2)}\right\}, \qquad (4.10)$$

$$\Gamma^{(1)}(p^2,q^2,(p-q)^2) = 2\left[-1+\varepsilon+\frac{\varepsilon}{M_V^2}(q^2+M_{H2}^2)\right]P + \varepsilon \times \frac{M_V^2-M_{H1}^2}{p^2}\left(\ln\frac{\varepsilon M_V^2}{M_W^2}-1\right) + \left(\frac{M_{H1}^2}{(p-q)^2} - \frac{M_{H1}^2}{p^2} + \frac{\varepsilon}{M_V^2}\frac{M_{H1}^2(p^2+M_{H1}^2)}{p^2}\right)\times$$

$$\times\left(\ln\frac{M_{H1}^2}{M_W^2}-1\right) - \frac{M_{H2}^2}{(p-q)^2}\left(\ln\frac{M_{H2}^2}{M_W^2}-1\right) - \left[\frac{\varepsilon M_V^2-M_{H1}^2}{p^2} - \frac{\varepsilon}{M_V^2}\times\frac{(p^2+M_{H1}^2)(M_V^2-M_{H1}^2)}{p^2}\right] I_0(p^2,\varepsilon M_V^2,M_{H1}^2) - \left[1-\frac{\varepsilon}{M_V^2}(q^2+M_{H2}^2)\right]\times$$

$$\times I_0(q^2,\varepsilon M_V^2,M_{H2}^2) - \frac{M_{H1}^2-M_{H2}^2}{(p-q)^2} I_0((p-q)^2,M_{H1}^2,M_{H2}^2) + 2\left[-p^2-q^2+2(p-q)^2+M_{H1}^2+M_{H2}^2-\varepsilon M_V^2-\frac{\varepsilon}{M_V^2}(p^2+M_{H1}^2)(q^2+M_{H2}^2)\right]\times$$

$$\times\frac{1}{p^4+q^4+(p-q)^4-2p^2(p-q)^2-2q^2(p-q^2)-2p^2q^2}\left[(p^2+q^2-(p-q)^2) I_0(p^2,\varepsilon M_V^2,M_{H1}^2) - 2q^2 I_0(q^2,\varepsilon M_V^2,M_{H2}^2) - (p^2-q^2-(p-q)^2) I_0((p-q)^2,M_{H1}^2,M_{H2}^2) + \left((p^2+q^2-(p-q)^2)(q^2+M_{H2}^2-\varepsilon M_V^2) + 2q^2(\varepsilon M_V^2-M_{H1}^2-\right.\right.$$

$$\left.\left.-p^2) - \frac{1}{2}\left(p^4+q^4+(p-q)^4-2p^2q^2-2p^2(p-q)^2-2q^2(p-q)^2\right)\right)\times I_1(p^2,(p-q)^2,q^2,\varepsilon M_V^2,M_{H1}^2,M_{H2}^2)\right], \qquad (4.11)$$

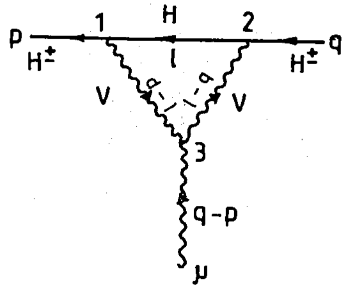
and

$$\Gamma^{(2)}(p^2,q^2,(p-q)^2) = 2\left[-1+\varepsilon+\frac{\varepsilon}{M_V^2}(p^2+M_{H1}^2)\right]P + \varepsilon\frac{M_V^2-M_{H2}^2}{q^2}\left(\ln\frac{\varepsilon M_V^2}{M_W^2}-1\right) - \frac{M_{H1}^2}{(p-q)^2}\left(\ln\frac{M_{H1}^2}{M_W^2}-1\right) + \left(\frac{M_{H2}^2}{(p-q)^2} - \frac{M_{H2}^2}{q^2} + \frac{\varepsilon}{M_V^2}\frac{M_{H2}^2(q^2+M_{H2}^2)}{q^2}\right)\left(\ln\frac{M_{H2}^2}{M_W^2}-1\right) - \left[1-\frac{\varepsilon}{M_V^2}(p^2+M_{H1}^2)\right]\times$$

$$\times I_0(p^2,\varepsilon M_V^2,M_{H1}^2) - \left[\frac{\varepsilon M_V^2-M_{H2}^2}{q^2} - \frac{\varepsilon}{M_V^2}\frac{(q^2+M_{H2}^2)(M_V^2-M_{H2}^2)}{q^2}\right]\times$$

$$\times I_0(q^2,\varepsilon M_V^2,M_{H2}^2) - \frac{M_{H2}^2-M_{H1}^2}{(p-q)^2} I_0((p-q)^2,M_{H1}^2,M_{H2}^2) + 2\left[-p^2-q^2+2(p-q)^2+M_{H1}^2+M_{H2}^2-\varepsilon M_V^2-\frac{\varepsilon}{M_V^2}(p^2+M_{H1}^2)(q^2+M_{H2}^2)\right]\times$$

$$\times\frac{1}{p^4+q^4+(p-q)^4-2p^2q^2-2p^2(p-q)^2-2q^2(p-q)^2}\left[-2p^2 I_0(p^2,\varepsilon M_V^2,M_{H1}^2) + (p^2+q^2-(p-q)^2) I_0(q^2,\varepsilon M_V^2,M_{H2}^2) - (q^2-p^2-(p-q)^2) I_0((p-q)^2,M_{H1}^2,M_{H2}^2) + \left((p^2+q^2-(p-q)^2)(p^2+M_{H1}^2-\varepsilon M_V^2) + 2p^2(\varepsilon M_V^2-M_{H2}^2-q^2) - \frac{1}{2}\left(p^4+q^4+(p-q)^4-2p^2q^2-2p^2(p-q)^2-2q^2(p-q)^2\right)\right) I_1(p^2,(p-q)^2,q^2,\varepsilon M_V^2,M_{H1}^2,M_{H2}^2)\right]. \qquad (4.12)$$

7. 

$$\Gamma_\mu(p^2,q^2,(p-q)^2) = \pm \frac{i f_1^{HHV} f_2^{HHV} f_3^{3V}}{16\pi^2}\left\{p_\mu \Gamma^{(1)} + q_\mu \Gamma^{(2)}\right\}, \tag{4.13}$$

$$\Gamma^{(1)}(p^2,q^2,(p-q)^2) = \Big[3 + \frac{1}{M_V^2}\Big(\frac{14}{3}q^2 - \frac{5}{3}p^2 + \frac{8}{3}(p-q)^2 + 3M_H^2\Big) - \frac16 \frac{(p-q)^2}{M_V^4}\big((p-q)^2 - p^2 + q^2\big)\Big] P +$$

$$+ M_V^2\Big[\frac12\frac1{p^2} + \frac1{M_V^2}\Big(\frac76 - \frac{M_H^2}{2p^2} - \frac23\frac{p^2-q^2}{(p-q)^2}\Big) + \frac16\frac1{M_V^4}\big(p^2-q^2-(p-q)^2\big)\Big]\Big(\ln\frac{M_V^2}{M_W^2}-1\Big) + M_H^2\Big[-\frac12\frac1{p^2} + \frac12\frac{p^2+M_H^2}{M_V^2 p^2}\Big]\times$$

$$\times\Big(\ln\frac{M_H^2}{M_W^2}-1\Big) + \Big[\frac32 + \frac{M_H^2}{p^2} - \frac12\frac{M_V^2}{p^2} + \frac1{M_V^2}\Big(p^2 + \frac{q^2}{2} + M_H^2 - \frac12\frac{M_H^4}{p^2}\Big) - \frac12\frac1{M_V^4}(p^2+M_H^2)(q^2+M_H^2)\Big] I_0(p^2, M_H^2, M_V^2) +$$

$$+ \Big[-\frac12 + \frac12\frac1{M_V^2}(q^2-p^2) + \frac12\frac1{M_V^4}(p^2+M_H^2)(q^2+M_H^2)\Big] I_0(q^2, M_H^2, M_V^2) + \Big[-\frac23 + \frac23\frac{p^2-q^2}{(p-q)^2} + \frac32\frac1{M_V^2}\big(q^2-p^2+(p-q)^2\big) +$$

$$+ \frac1{12}\frac1{M_V^4}(p-q)^2\big(p^2-q^2-(p-q)^2\big)\Big] I_0((p-q)^2, M_V^2, M_V^2) +$$

$$+ \Big[M_V^2 - 2M_H^2 + 3q^2 - p^2 + \frac12(p-q)^2 + \frac1{M_V^2}\Big((q^2+M_H^2)^2 + (q^2+M_H^2)(p^2+M_H^2) - (p^2+M_H^2)^2 + \frac12(p-q)^2(p^2+q^2+2M_H^2)\Big) + \frac12\frac1{M_V^4}(p-q)^2(p^2+M_H^2)(q^2+M_H^2)\Big] I_1(q^2, (p-q)^2, p^2, M_H^2, M_V^2, M_V^2) +$$

$$+ 2\Big[\frac{p^2+q^2}{2} - 3pq + 2M_H^2 - M_V^2 + \frac12\frac{(p-q)^2}{M_V^2}(p^2+q^2+2M_H^2) - \frac1{M_V^2}(p^2+M_H^2)(q^2+M_H^2) - \frac12\frac{(p-q)^2}{M_V^4}(p^2+M_H^2)(q^2+M_H^2)\Big]\frac{1}{p^4+q^4+(p-q)^4-2p^2q^2-2p^2(p-q)^2-2q^2(p-q)^2}\times$$

$$\times\Big[-2q^2 I_0(q^2,M_H^2,M_V^2) + 2pq\, I_0(p^2,M_H^2,M_V^2) + 2(q^2-pq)\, I_0((p-q)^2,M_V^2,M_V^2) + 2\big((q^2-pq)(M_H^2-M_V^2) + q^2(pq-p^2)\big)\times$$

$$\times I_1(q^2,(p-q),p^2,M_H^2,M_V^2,M_V^2)\Big] + \frac{q^2-p^2+(p-q)^2}{(p-q)^2}\times$$

$$\times\Big(\frac23 + \frac49\frac{(p-q)^2}{M_V^2} + \frac1{18}\frac{(p-q)^4}{M_V^4}\Big), \tag{4.14}$$

and

$$\Gamma^{(2)}(p^2,q^2,(p-q)^2) = \Big[3 + \frac1{M_V^2}\Big(\frac{14}{3}p^2 - \frac53 q^2 +$$

$$+\frac{8}{3}(p-q)^2+3M_H^2\Big)+\frac{1}{6M_V^4}(p-q)^2\big(q^2-p^2-(p-q)^2\big)\Big]P+$$

$$+M_V^2\Big[\frac{1}{2}\frac{1}{q^2}+\frac{1}{M_V^2}\Big(\frac{7}{6}-\frac{1}{2}\frac{M_H^2}{q^2}-\frac{2}{3}\frac{q^2-p^2}{(p-q)^2}\Big)+\frac{1}{6}\frac{1}{M_V^4}\times$$

$$\times\big(q^2-p^2-(p-q)^2\big)\Big]\Big(\ln\frac{M_V^2}{M_W^2}-1\Big)+M_H^2\Big[-\frac{1}{2}\frac{1}{q^2}+\frac{1}{2}\frac{1}{M_V^2}\times$$

$$\times\frac{q^2+M_H^2}{q^2}\Big]\Big(\ln\frac{M_H^2}{M_W^2}-1\Big)+\Big[-\frac{1}{2}+\frac{1}{2}\frac{1}{M_V^2}(p^2-q^2)+$$

$$+\frac{1}{2}\frac{1}{M_V^4}(p^2+M_H^2)(q^2+M_H^2)\Big]I_0(p^2,M_H^2,M_V^2)+\Big[\frac{3}{2}+\frac{M_H^2}{q^2}-$$

$$-\frac{1}{2}\frac{M_V^2}{q^2}+\frac{1}{M_V^2}\Big(q^2+\frac{p^2}{2}+M_H^2-\frac{M_H^4}{2q^2}\Big)-\frac{1}{2}\frac{1}{M_V^4}(p^2+M_H^2)\times$$

$$\times(q^2+M_H^2)\Big]I_0(q^2,M_H^2,M_V^2)+\Big[-\frac{2}{3}+\frac{2}{3}\frac{q^2-p^2}{(p-q)^2}+$$

$$+\frac{3}{2}\frac{1}{M_V^2}\big(p^2-q^2+(p-q)^2\big)+\frac{1}{12}\frac{1}{M_V^4}(p-q)^2\big(q^2-p^2-$$

$$-(p-q)^2\big)\Big]I_0\big((p-q)^2,M_V^2,M_V^2\big)+\Big[M_V^2-2M_H^2+3p^2-q^2+$$

$$+\frac{1}{2}(p-q)^2+\frac{1}{M_V^2}\Big((p^2+M_H^2)^2+(p^2+M_H^2)(q^2+M_H^2)-(q^2+M_H^2)^2$$

$$+\frac{1}{2}(p-q)^2(p^2+q^2+2M_H^2)\Big)+\frac{1}{2}\frac{1}{M_V^4}(p-q)^2(p^2+M_H^2)(q^2+M_H^2)\Big]\times$$

$$\times I_1\big(q^2,(p-q)^2,p^2,M_H^2,M_V^2,M_V^2\big)+2\Big[\frac{p^2+q^2}{2}-3pq+2M_H^2-$$

$$-M_V^2+\frac{1}{2}\frac{1}{M_V^2}(p-q)^2(p^2+q^2+2M_H^2)-\frac{1}{M_V^2}(p^2+M_H^2)(q^2+M_H^2)-$$

$$-\frac{1}{2}\frac{(p-q)^2}{M_V^4}(p^2+M_H^2)(q^2+M_H^2)\Big]\frac{1}{p^4+q^4+(p-q)^4-2p^2q^2-2p^2(p-q)^2-2q^2(p-q)^2}\times$$

$$\times\Big[-2p^2I_0(p^2,M_H^2,M_V^2)+2pq\,I_0(q^2,M_H^2,M_V^2)+2(p^2-$$

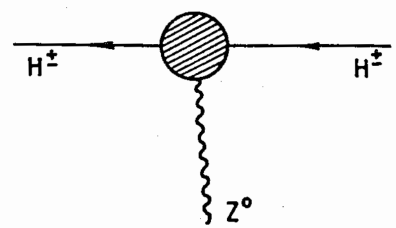
$$-pq)\,I_0\big((p-q)^2,M_V^2,M_V^2\big)+2\Big((p^2-pq)(M_H^2-M_V^2)+p^2\times$$

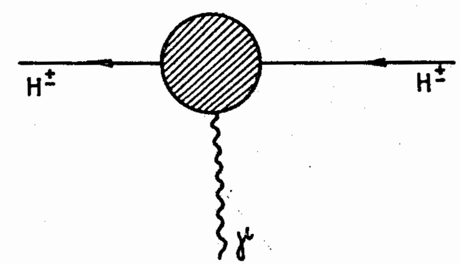
$$\times(pq-q^2)\Big)I_1\big(q^2,(p-q)^2,p^2,M_H^2,M_V^2,M_V^2\big)\Big)+\Big(\frac{2}{3}+$$

$$+\frac{4}{9}\frac{(p-q)^2}{M_V^2}+\frac{1}{18}\frac{(p-q)^4}{M_V^4}\Big)\frac{p^2-q^2+(p-q)^2}{(p-q)^2}\qquad(4.15)$$

In the diagrams (3.4),(4.3),(4.4) and (4.7) the sign "+" corresponds to the case when V is a gauge Z^0 boson or a photon and the sign "−" to the case when V is a gauge $W^\pm$ boson.

The counter terms need for the renormalization of the vertex function in the framework of the supersymmetric (SUSY) model are



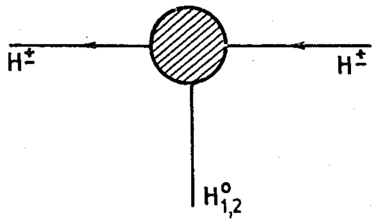


$$\Gamma^{c.t}_{H\bar{H}A}(p^2,q^2,t)=-ie\,(Z_{H^\pm}-1)\,,\qquad(4.16)$$

$$\Gamma^{c.t}_{H\bar{H}Z}(p^2,q^2,t)=-\frac{ig(2R-1)}{2\sqrt{R}}\Big[(Z_{H^\pm}-1)+\frac{1}{2}(Z_Z-1)+$$

$$+\frac{\delta g}{g}+\frac{2R+1}{2(2R-1)}\,\frac{\delta R}{R}\Big]-ie\Big[(Z_{H^\pm}-1)+\frac{1}{2}(Z_M-1)-\frac{1}{2}(Z_A-1)\Big]. \qquad (4.17)$$

And next,



$$\Gamma^{c.t}_{H^\pm H^\pm H^0_{1,2}}(p^2,q^2,t)=-igM_W\Big[\begin{Bmatrix}\cos(\beta-\alpha)\\ \sin(\beta-\alpha)\end{Bmatrix}\mp\frac{1}{2R}\cos 2\beta\times\begin{Bmatrix}\cos(\beta+\alpha)\\ \sin(\beta+\alpha)\end{Bmatrix}\Big]\times\Big[(Z_{H^\pm}-1)+\frac{1}{2}(Z_{H^0_{1,2}}-1)+\frac{\delta g}{g}+\frac{1}{2}\frac{\delta M_W^2}{M_W^2}\Big]-igM_W\Big[\pm\frac{1}{2R}\cos 2\beta\begin{Bmatrix}\cos(\beta+\alpha)\\ \sin(\beta+\alpha)\end{Bmatrix}\Big]\times\frac{\delta R}{R}-igM_W\Big[\begin{Bmatrix}\sin(\beta-\alpha)\\ \cos(\beta-\alpha)\end{Bmatrix}\pm\frac{1}{2R}\cos 2\beta\begin{Bmatrix}\sin(\beta+\alpha)\\ \cos(\beta+\alpha)\end{Bmatrix}\Big]\times\Big[(Z_{H^\pm}-1)+\frac{1}{2}(Z_{M2,M1}-1)+\frac{\delta g}{g}+\frac{1}{2}\frac{\delta M_W^2}{M_W^2}\Big]-igM_W\times\Big[\mp\frac{1}{2R}\cos 2\beta\begin{Bmatrix}\sin(\beta+\alpha)\\ \cos(\beta+\alpha)\end{Bmatrix}\Big]\frac{\delta R}{R}. \qquad (4.18)$$

5. Summary

The self-energy and vertex diagrams calculated here will be used in subsequent papers to find out a renormalized expression for the H^+H^- amplitude up to the fourth order of perturbation theory. We have not here written the final general expressions for Π^{REN}

and Γ^{REN} because they would take too much space. But they can be quite easily obtained in the framework of the SUSY model by summing up the expressions (3.1)–(3.5) and (4.1)–(4.15) with the concrete values of the coupling constants f^{3H}, f^{HHV}, f^{H} etc.

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H^+H^--взаимодействие с учетом высших порядков теории возмущений в модели с двумя хиггсовскими дублетами (Собственно-энергетические и вершинные диаграммы)

В рамках модели электрослабого взаимодействия, содержащей два дублета хиггсовских бозонов и произвольное число фермионов, вычислены собственно-энергетические и вершинные блоки, входящие в амплитуду хиггс-хиггс взаимодействия в четвертом порядке теории возмущений. Перенормировки осуществляются на массовой поверхности физических полей после спонтанного нарушения симметрии. Значения констант, как правило, в работе не конкретизируются. Однако в случае такой необходимости использовались значения, полученные для них в рамках модели, являющейся минимальным суперсимметричным расширением стандартной модели.

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Dvoeglazov V.V., Skachkov N.B. E2-91-114

H^+H^- Interaction up to Higher Orders of Perturbation Theory in the Model with Two Higgs Doublets (Self-Energy and Vertex Diagrams)

Self-energy and vertex blocks that enter into the amplitude of Higgs-Higgs-interaction are calculated up to the fourth order of perturbation theory in the framework of the model of electroweak interaction with two Higgs doublets and an arbitrary number of fermions. The renormalization is performed on the mass shell of the physical fields after a spontaneous symmetry breaking. The values of the coupling constants are, as a rule, not concretized in the paper. In the cases where we need to use them, we take their values obtained in the model with the minimal supersymmetric extension of the standard model (MSSM).

The investigation has been performed at the Laboratory of Theoretical Physics, JINR.

Communication of