92-91

сообщения
объединенного
института
ядерных
исследований
дубна

E2-91-11

A.A.Bel'kov, D.Ebert, A.V.Lanyov*

STATUS OF CP VIOLATION IN K DECAYS WITHIN CHIRAL PERTURBATION THEORY

(Talk presented at the International Triangle Seminar: Standard Model and Beyond: From LEP to UNK and LHC, Dubna, October 1-5, 1990)

* Institute for High Energy Physics, Moscow Region, 142284, Protvino, USSR

1991

1. Introduction

CP violation has been observed for the first time in the decay of long-living neutral kaons into two pions. From the phenomenological point of view, there are two possible mechanisms of CP violation in K^0 decays. The first one is connected with mixing in the mass matrix of neutral kaons, resulting in the fact that the short- and long-living $K^0_{S,L}$ -mesons (mass eigenstates) became a mixture of CP eigenstates K^0_1 $(CP = +1)$ and K^0_2 $(CP = -1)$ characterized by the phenomenological parameter ϵ, $K^0_S \approx K^0_1 + \epsilon K^0_2$, $K^0_L \approx K^0_2 + \epsilon K^0_1$. The parameter ϵ is well known experimentally, $\epsilon = 2.27 \cdot 10^{-3}$. In the standard six-quark model the $(K^0 - \bar{K}^0)$ -mixing has been related to the box diagram of Fig.1a in the second order of weak interactions. The CP violation originates from the phase appearing in the Kobayashi-Maskawa mixing matrix of heavy quarks [1] . The diagram of Fig.1a can be associated with the Wolfenstein superweak interaction [2] with strangeness change $|\Delta S| = 2$.

The second mechanism of CP violation is related to the possibility of direct deday $K^0_2 \to 2\pi$ (direct CP violation). It is characterized by the phenomenological parameter ϵ' :

$$\eta_{00} = \frac{T(K^0_L \to \pi^0\pi^0)}{T(K^0_S \to \pi^0\pi^0)} = \epsilon - 2\epsilon'.$$

In the standard model the main contribution to direct CP violation comes from the penguin diagram of Fig.1b (first order of weak interactions).

The first experimental confirmation of the existence of direct CP violation in $K^0 \to 2\pi$ decays has been given in [3] , where a significant nonzero result on the ratio of the parameters ϵ' and ϵ has been found:

$$Re(\epsilon'/\epsilon) = (3.3 \pm 1.5) \cdot 10^{-3} \quad (CERN). \tag{1}$$

On the other hand, the very recent data of [4] show no evidence for direct CP violation:

$$Re(\epsilon'/\epsilon) = (-0.5 \pm 1.5) \cdot 10^{-3} \quad (FNAL). \tag{2}$$

Recall that the enhancement of transitions with isospin change $|\Delta I| = 1/2$

$$A^{exp}(K^0 \to \pi^+\pi^-) \;/\; A^{exp}(K^+ \to \pi^+\pi^0) \approx 15$$
$$(\Delta I = 1/2,\ 3/2) \qquad (\Delta I = 3/2)$$

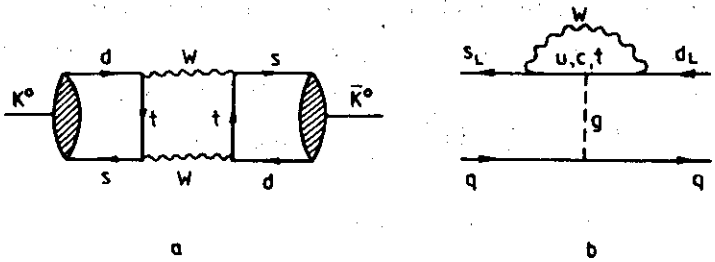

Figure 1: a) Box diagram contributing to $K^0 - \bar{K}^0$ mixing ; b) Penguin diagram

is a problem closely connected with direct CP violation since penguin diagrams are the dominant mechanism of this enhancement.

There are two main reasons for searching new sources of experimental information on direct CP violation in K decays:

-until now this effect has been observed only in $K^0 \to 2\pi$ decays;

-the ambiguity of the experimental status of direct CP violation (see Eqs.1,2) needs the measuring of the parameter ϵ' in any alternative possible processes.

We can obtain new information of CP violation by measuring the charge asymmetry of differential distributions in decays $K^\pm \to 3\pi$, $K^\pm \to 2\pi\gamma$. In particular, the latter decay was regarded in [5] as the most suitable process. However, as experiments show, neither a charge asymmetry nor interference between direct emission and inner bremsstrahlung amplitudes in $K^\pm \to 2\pi\gamma$ decay could be observed.

The effects of direct CP violation in neutral $K^0(\bar{K}^0) \to 3\pi$ decays can be extracted in an analysis of the time-dependent asymmetry of the decay probability dominated by $(K^0 - \bar{K}^0)$-mixing. Here possible modifications of the Li-Wolfenstein relation, connecting the direct CP violation parameter ϵ'_{+-0} of $K^0_S \to \pi^+\pi^-\pi^0$ decay with ϵ' (Born approximation)

$$\epsilon'_{+-0} = -2\epsilon' \qquad (3)$$

need a detailed investigation.

Recall that the calculation of hadronic matrix elements of weak quark operators contained in $\mathcal{L}_W^{\Delta S=1,2}$,

$$\langle h_1 h_2 | \mathcal{L}_W^{\Delta S=1} | h \rangle, \quad \langle h_1 h_2 h_3 | \mathcal{L}_W^{\Delta S=1} | h \rangle \quad (\text{ for decays }),$$
$$\langle K^0 | \mathcal{L}_W^{\Delta S=2} | \bar{K}^0 \rangle \quad (\, B \text{ parameter of } K^0 - \bar{K}^0 \text{ mixing }\,)$$

is the fundamental theoretical problem in nonleptonic weak interactions. The main difficulty encountered here is that the energy region $E < 1\,GeV$, characteristic for kaon physics, belongs to the nonperturbative regime of QCD. The various nonperturbative methods, developed for the description of low energy processes, can be classified into three groups:

- Effective chiral Lagrangians and $1/N_C$ expansion
- Lattice QCD
- QCD Sum Rules.

In this talk we shall briefly review the effective Lagrangian approach where effects of long distances are characterized by the expansion parameter $1/(4\pi F_0)^{-2} \sim 1/N_C$ (F_0 is the bare constant of $\pi \to \mu\nu$ decay, N_C is the number of colours), and then apply it to nonleptonic kaon decays. Notice that effective chiral Lagrangians have first been introduced by accounting for group theoretical properties of strong and weak interactions [6]. Later on, they have been derived in a more fundamental way from bosonization of QCD based on the path integral approach. In this approach mesons are treated as bound $q\bar{q}$-states[7–10, 12, 13].

2. Group theoretical approach to effective Lagrangians

The $SU(N_C)$ gauge-invariant part of the QCD Lagrangian for strong interactions of quarks reads

$$\mathcal{L}_{QCD}^{(quark)} = \bar{q}_L^a i\not{D} q_L^a + \bar{q}_R^a i\not{D} q_R^a - [\bar{q}_L^a m_{ab}^0 q_R^b + h.c.], \tag{4}$$

where $q_{L,R} = P_{L,R}q$ is the left/right-handed projection of the quark field q with $P_{L,R} = (1 \mp \gamma_5)/2$, D_μ is the covariant derivative and m^0 is the current quark mass matrix. In the limit $m^0 = 0$, the Lagrangian Eq.4 becomes also invariant under transformations with respect to the chiral flavour group $G = SU(3)_L \times SU(3)_R$,

$$q_L \to g_L q_L, \quad q_R \to g_R q_R; \quad g_{L,R} \subset SU(3)_{L,R}. \tag{5}$$

Next, consider left-handed transformations g_L which generate a left-handed $(V - A)$ quark current $J^{ab}_{(V-A)\mu}$ transforming like

$$J^{ab}_{(V-A)\mu} \equiv 2\bar{q}_{La}\gamma_\mu q_L^b \sim (8_L, 1_R)$$

under G. Consequently, the (symmetrized) effective weak current×current Lagrangian transforms like

$$\mathcal{L}_W \sim J^{ab}_\mu J^{cd\mu} \sim (8_L, 1_R) + (27_L, 1_R).$$

Effective chiral meson Lagrangians are now obtained by using a realization of the chiral group by meson fields

$$U = \exp\left(i\sqrt{2}\frac{\Phi}{F_0}\right).$$

Here $F_0 = 93\ MeV$ and

$$\Phi = \begin{pmatrix} \pi_0/\sqrt{2} + \eta/\sqrt{6} & \pi^+ & K^+ \\ \pi^- & -\pi_0/\sqrt{2} + \eta/\sqrt{6} & K^0 \\ K^- & \bar{K}^0 & -\sqrt{\frac{2}{3}}\eta \end{pmatrix},$$

is the pseudoscalar meson field matrix. The matrix U transforms like $U \to g_L U g_R^+$ under G.

The effective meson Lagrangians corresponding to the chiral group $SU(3)_L \times SU(3)_R$ take the following general form

i) Strong interactions:

$$\begin{aligned} \mathcal{L}_S &= \mathcal{L}_S^{(2)} + \mathcal{L}_S^{(4)} + \ldots, \\ \mathcal{L}_S^{(2)} &= \frac{F_0^2}{4}\mathrm{Tr}\,\partial_\mu U \partial^\mu U^+ + \frac{F_0^2}{4}\mathrm{Tr}\,(MU + h.c.), \qquad (6) \\ \mathcal{L}_S^{(4)} &= a_1 \mathrm{Tr}\,(\partial_\mu U^+ \partial_\nu U)^2 + a_2 \mathrm{Tr}\,(\partial^2 U^+ \partial^2 U) + \ldots \qquad (7) \end{aligned}$$

ii) Weak interactions:

$$\begin{aligned} \mathcal{L}_W &= \mathcal{L}_W^{(2)} + \mathcal{L}_W^{(4)} + \ldots, \\ \mathcal{L}_W^{(2)} &= g_8\,(\partial_\mu U \partial^\mu U^+)^a{}_b + g_{27}\,(\partial^\mu U \cdot U^+)^a{}_c\,(\partial_\mu U \cdot U^+)^d{}_b, \qquad (8) \\ &\qquad \text{(symmetrized)} \\ \mathcal{L}_W^{(4)} &= h_1 \mathrm{Tr}\,(\lambda_6 L_\mu L^\mu L_\nu L^\nu) + h_2 \mathrm{Tr}\,(\lambda_6 L_\mu L_\nu L^\mu L^\nu) + \ldots \qquad (9) \end{aligned}$$

Here $L_\mu = (\partial_\mu U)U^+$ is the simplest object transforming like a left-handed weak current. The superscripts (2), (4) denote Lagrangians containing two and four meson field derivatives, respectively. Notice that the coefficients a_i, g_8, g_{27}, h_i are not predicted by symmetry. We can eventually fix them at least partly from bosonization of QCD using the path integral approach.

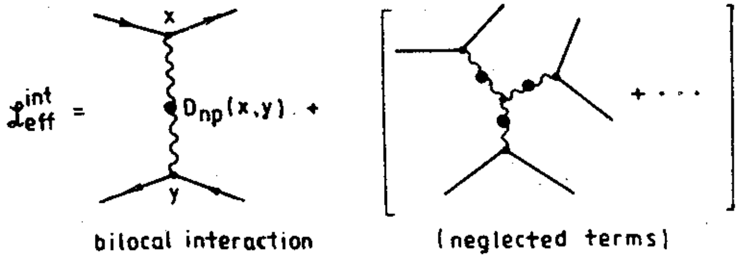

Figure 2: Effective quark interactions generated by (nonperturbative) gluon exchange

3. Bosonization approach: From QCD to the Nambu-Jona-Lasinio model

Let us consider the bosonization of QCD in general outline. The QCD generating functional is

$$Z = \int \mathcal{D}q\,\mathcal{D}\bar{q}\,\mathcal{D}A\ \exp\{i \int d^4x [\bar{q}(i\partial\!\!\!/ - m_0)q + g\ \bar{q}A\!\!\!/^a \frac{\tilde{\lambda}^a}{2} q - \frac{1}{4} F^a_{\mu\nu} F^a_{\mu\nu}]\}, \quad (10)$$

where $F^a_{\mu\nu} = \partial_\mu A^a_\nu - \partial_\nu A^a_\mu + g\ f^{abc} A^b_\mu A^c_\nu$ is the gluon field strength tensor; m_0 is the diagonal bare quark mass matrix. After integrating over gluon fields the generating functional Eq.10 takes the form [12, 13]

$$Z = \int \mathcal{D}q\,\mathcal{D}\bar{q}\ \exp[i \int d^4x \bar{q}(i\partial\!\!\!/ - m_0)q] \exp(iW[j]), \quad (11)$$

where $W[j]$ is given as an expansion in terms of flavour singlet quark currents

$$\begin{aligned} j^a_\mu(x) &= \bar{q}(x)\,\gamma_\mu\,\tilde{\lambda}^a\,q(x), \\ W[j] &= \sum_{n=2}^{\infty} \frac{1}{n!} \int d^4x_1 \ldots d^4x_n\, D^{a_1 \ldots a_n}_{\mu_1 \ldots \mu_n}(x_1, \ldots, x_n) \prod_{i=1}^{n} j^{a_i}_{\mu_i}(x_i). \end{aligned} \quad (12)$$

The coefficient functions $D^{a_1 \ldots a_n}_{\mu_1 \ldots \mu_n}$ are the connected n-point gluon Green's functions containing all the information on gluon dynamics. The expansion in terms of currents $j^a_\mu(x)$ is represented by the diagrams shown in Fig. 2. As a matter of fact, one usually takes into account only the first bilocal interaction term arising in the expansion Eq.12.

Performing a suitable Fierz transformation this procedure leads, in local approximation, to the well-known Nambu-Jona-Lasinio (NJL) model [13] :

$$\begin{aligned} Z = \int \mathcal{D}q\,\mathcal{D}\bar{q}\ \exp\{i \int d^4x\ [\bar{q}(x)(i\partial\!\!\!/ - m_0)q(x) + \frac{G_1}{2}\left((\bar{q}\lambda^a q)^2 + (\bar{q}i\gamma_5\lambda^a q)^2\right) - \\ - \frac{G_2}{2}\left((\bar{q}\gamma_\mu\lambda^a q)^2 + (\bar{q}\gamma_\mu\gamma_5\lambda^a q)^2\right)]\}, \end{aligned} \quad (13)$$

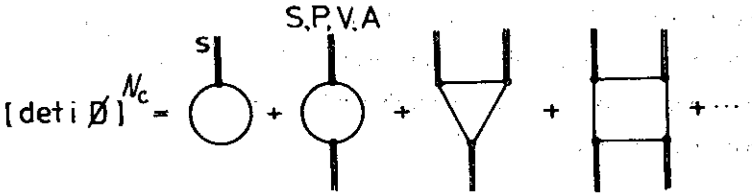

Figure 3: Loop expansion of the quark determinant

where G_1 and G_2 are universal quark coupling constants with dimensions $(\text{length})^2$, and λ^a are the generators of the flavour group $U(3)$. The NJL-Lagrangian Eq.13 accumulates all the general properties of the low-energy meson physics [7, 8] : the spontaneous breaking of chiral symmetry by a nonvanishing quark condensate, the existence of Goldstone bosons and PCAC, the KSFR and Weinberg relations, vector meson dominance etc. The inclusion of the 't Hooft determinantal interaction even allows one to solve the $U(1)$-problem [14] .

4. Effective meson Lagrangians from the NJL-model [7, 8]

By introducing collective meson fields S, P, V and A with quantum numbers $J^{PC} = (0^{++}, 0^{-+}, 1^{--}, 1^{++})$, i.e.

$$\begin{aligned} S &\sim G_1\lambda^a(\bar{q}\lambda^a q), \quad P \sim G_1\lambda^a(\bar{q}\gamma_5\lambda^a q), \\ V_\mu &\sim G_2\lambda^a(\bar{q}\gamma_\mu\lambda^a q), \quad A_\mu \sim G_2\lambda^a(\bar{q}\gamma_\mu\gamma_5\lambda^a q), \end{aligned}$$

into Eq.13, the action becomes bilinear in the quark fields. After this, the quark integration is Gaussian and can easily be performed. Thus one obtains ($\Phi \equiv S + iP$)

$$Z = \int \mathcal{D}\Phi\,\mathcal{D}\Phi^+\,\mathcal{D}V\,\mathcal{D}A\ \exp\{i\int dx^4\,[-\frac{1}{4G_1}\mathrm{Tr}\,(\Phi - m_0)^+(\Phi - m_0) - \\ -\frac{1}{4G_2}\mathrm{Tr}\,(V_\mu^2 + A_\mu^2)]\}\ Z_f(\Phi, V, A), \qquad (14)$$

$$Z_f(\Phi, V, A) = \int \mathcal{D}q\,\mathcal{D}\bar{q}\ \exp\{i\int dx^4\ \bar{q}\,i\,\not{D}q\} \approx (\det\, i\,\not{D})^{N_C}, \qquad (15)$$

where $i\,\not{D} = i\,(\not{\partial} + \not{V} + \not{A}\gamma_5) - (P_R\Phi + P_L\Phi^+)$ is the Dirac operator containing collective fields. The quark determinant Eq.15 can be evaluated either by a loop expansion over quark loops emitting collective fields [7] (Fig. 3) or by using recursion relations of the heat kernel technique [8]. It is worth mentioning that there arises a quark condensate $\langle \bar{q}q \rangle \neq 0$ due to the nonvanishing vacuum expectation value of the scalar meson field. The current quark mass m_i^0 and the quark condensate then combine to yield the mass m_i of constituent quarks

$$m_i = m_i^0 - 2\,G_1 \langle \bar{q}_i q_i \rangle = m_i^0 + i\,8\,G_1 N_C \int^{\Lambda} \frac{d^4k}{(2\pi)^4} \frac{m_i}{k^2 - m_i^2}. \qquad (16)$$

The evaluation of the quark determinant leads to the following effective Lagrangian, describing strong interactions of mesons in the low-energy limit of QCD*) [8]

$$\mathcal{L}_{mes}^{QCD} = \mathcal{L}_{mes}^{(2)} + \mathcal{L}_{mes}^{(4)} + \mathcal{L}_{SB} \qquad (17)$$

$$\mathcal{L}_{mes}^{(2)} = -\frac{F_0^2}{4} \mathrm{Tr}\,(L_\mu L^\mu) \qquad \text{(kinetic term)}, \qquad (18)$$

$$\mathcal{L}_{mes}^{(4)} = \frac{1}{64\,\pi^2} \mathrm{Tr}\,(\frac{1}{2}[L_\mu, L_\nu]^2 + (L_\mu L^\mu)^2) \qquad (p^4\text{–interaction)}, \qquad (19)$$

$$\mathcal{L}_{SB} = \frac{F_0^2}{4} \mathrm{Tr}\,(MU + h.c.) \qquad \text{(chiral symmetry breaking)}. \qquad (20)$$

The experimental status of the Lagrangian $\mathcal{L}_{mes}^4$ was discussed in ref.[15]. The inclusion of an additional interaction term $\mathcal{L}_G$ taking into account the $U(1)$ anomaly

$$\mathcal{L}_G = \frac{aF_0^2}{16N_C} [\mathrm{Tr}\,(\ln U - \ln U^+)]^2 \qquad (21)$$

(or by using the equivalent 't Hooft interaction term) allows one to give a correct description of the masses of the nonet of pseudoscalar mesons. The values of the parameters μ_i^2 in $M =$ diag(μ_u^2, μ_d^2, μ_s^2) and the parameter a in Eq.21 are fixed by the masses m_π, m_K, m_η, $m_{\eta'}$: $a = 0.729\ GeV^2$, $\mu_u^2 = 0.0114\ GeV^2$, $\mu_d^2 = 0.025\ GeV^2$, $\mu_s^2 = 0.47 GeV^2$ with the angle of (η, η')-mixing $\varphi = -19^o$. Finally, an additional chiral symmetry breaking term

$$\mathcal{L}'_{SB} = -\frac{F_0^2}{4\Lambda_\xi^2} \mathrm{Tr}\,(M\partial^2 U + h.c.), \;\; \Lambda_\xi^2 = 0.78\ GeV^2 \qquad (22)$$

allows one to describe the splitting of the constants F_π and F_K .

*)Higher order derivative terms as well as a tachyonic term in Eq.19 are disregarded.

5. Bosonization of weak interactions

The weak nonleptonic $|\Delta S| = 1$ Lagrangian taking into account hard gluon corrections has been given on the quark-level as follows [16]

$$\mathcal{L}_W^{(|\Delta S|=1)} = \sqrt{2}\, G_F \sin\theta_C \cos\theta_C \sum_{i=1}^{6} c_i(\mu) \mathcal{O}_i^{(q)}. \tag{23}$$

Here, $\mathcal{O}_i^{(q)}$ are four-quark operators consisting of products of left- and/or right-handed quark currents ; $c_i(\mu)$ are Wilson coefficient functions calculated in the QCD leading-log approximation which depend on the renormalization scale μ . The operators $\mathcal{O}_5^{(q)}$ and $\mathcal{O}_6^{(q)}$ containing right-handed currents are generated by penguin diagrams (see Fig. 1b).

The transition from quarks to mesons in the Lagrangian can be realized within the bosonization approach. Let us consider the total effective quark Lagrangian containing strong and weak interactions

$$\mathcal{L}_{tot} = \bar{q}(i\partial\!\!\!/ - m_0)q + \sum_i G_i(\bar{q}O_i q)(\bar{q}O_i q) + G_F \sum_{ij} c_{ij}\,(\bar{q}\tilde{O}_i q)(\bar{q}\tilde{O}_j q). \tag{24}$$

Here, the first sum, diagonal over operators $O_i = \{\lambda^a,\, \gamma_5\lambda^a,\, \gamma^\mu\lambda^a,\, \gamma^\mu\gamma_5\lambda^a\}$, corresponds to strong interactions of the NJL-type and the second sum, nondiagonal over operators $\tilde{O}_i = \{P_{R,L}\gamma_\mu\lambda^a,\, P_{R,L}\lambda^a\}$ corresponds to the effective weak interaction . Obviously, the bosonization procedure of weak interactions should transform averages of quark currents

$$\langle\bar{q}\tilde{O}_i q\rangle = \int \mathcal{D}q\,\mathcal{D}\bar{q}\; e^{i\mathcal{L}_{NJL}}\bar{q}\tilde{O}_i q \;/\; \int Dq\, D\bar{q}\; e^{i\mathcal{L}_{NJL}} \equiv Z_q[0]^{-1}\frac{\delta}{\delta\bar{\eta}_i} Z_q[\bar{\eta}_i]|_{\bar{\eta}_i=0} \tag{25}$$

into corresponding averages of meson currents $\langle J_i\rangle$. Here the generating functional $Z_q[\bar{\eta}_i]$ in Eq.25 is defined by

$$\begin{aligned} Z_q[\bar{\eta}_i] = \int \mathcal{D}q\,\mathcal{D}\bar{q}\; \exp\{\, i \int dx\, [\,\bar{q}(i\partial\!\!\!/ - m_0)q \;&+\; \sum_i G_i(\bar{q}O_i q)(\bar{q}O_i q) + \\ &+\; \sum_i \bar{\eta}_i\, \bar{q}\tilde{O}_i q\,]\,\}, \end{aligned} \tag{26}$$

where $\bar{\eta}_i$ are external sources coupling to quark currents $\bar{q}\tilde{O}_i q$. The above bosonization approach allows one then to formulate the following bosonization prescription which implies a replacement of quark currents by their equivalent hadronic currents

$$\mathcal{O}_i^{(q)}[:J_\mu^+J^\mu:] \longrightarrow \mathcal{O}_i^{(m)}[:J_\mu^+J^\mu:]\,, \qquad i=1,\ldots,4\,;$$

$$:(\bar q_R q_L)(\bar q_L q_R): \sim :(\partial_\mu A^\mu)(\partial_\nu V^\nu): \qquad i=5. \qquad (27)$$

$$(\text{ quarks }) \qquad\qquad (\text{ mesons })$$

The exact expressions for weak nonleptonic meson Lagrangians describing $\Delta I = 1/2,\ 3/2$ transitions can be found in ref. [17] . Here we quote only the meson $(V-A)$-currents which, in accordance with Noether's theorem, correspond to the Lagrangians Eqs.18,19

$$\mathcal{L}_{mes}^{(2)} \rightarrow J_\mu^a = i\,\frac{F_0^2}{4}\mathrm{Tr}\,(\,\lambda^a L_\mu\,)\,,$$
$$\mathcal{L}_{mes}^{(4)} \rightarrow J_\mu^a = \frac{i}{64\,\pi^2}\mathrm{Tr}\,(\,[L^\nu,\lambda^a][L_\mu,L_\nu] - 2\,\{\,\lambda^a, L_\mu\}L^\nu L_\nu\,)\,. \qquad (28)$$

Let us also give the expression for the electroweak current responsible for structure photon emission in electroweak meson decays

$$\begin{aligned} J_\mu^{a(\gamma)} &= -e\,\frac{N_C}{32\,\pi^2}\mathrm{Tr}\,\lambda^a\{\,\frac{2}{3}\,\mathcal{F}_{\mu\nu}(\,U[Q,\partial^\nu U^+] + h.c.\,) + \\ &+\frac{1}{\mu^2}\,[\frac{1}{6}\,(\,U\,(\,\mathcal{F}_{\mu\nu}[Q,\partial^\nu\partial^2U^+] + \partial_\alpha\mathcal{F}_{\mu\nu}[Q,\partial^\alpha\partial^\nu U^+]\,) - \\ &- \partial_\mu\mathcal{F}_{\nu\alpha}\partial^\alpha U\,[Q,\partial^\nu U^+] - \mathcal{F}_{\nu\alpha}\,\partial^\nu U\,[Q,\partial_\mu\partial^\alpha U^+]\,) \\ &- \frac{1}{90}\mathcal{F}_{\mu\nu}(\partial^\nu\partial_\alpha U^+\cdot Q\cdot\partial^\alpha U - 2\{\,Q,\partial^\nu\partial_\alpha U^+\cdot\partial^\alpha U\,\}\,) + h.c.\,]\,\}, \end{aligned} \qquad (29)$$

where Q is the operator of electric charge and $\mathcal{F}_{\mu\nu} = \partial_\mu A_\nu - \partial_\nu A_\mu$, A_μ being the photon field; $\mu = \frac{1}{3}\sum_1^3 m_i \approx 340\ MeV$ is the average constituent quark mass.

6. Analysis of nonleptonic K decays

Our analysis which is based on the effective meson Lagrangian $\mathcal{L}_W^m$, obtained from Eq.23 by applying the bosonization prescription Eq.27, provides an explicit realization of the $1/N_C$ expansion. In this approach [17] , four-quark operators $\mathcal{O}_i^{(q)}$ are replaced by mesonic operators $\mathcal{O}_i^{(m)}$. Here the mesonic currents in the operators $\mathcal{O}_i^{(m)}$ and their respective coefficients are renorm-invariant. In fact, for non-penguin diagrams, all dependencies on the anomalous dimensions vanish in the limit $N_C \rightarrow \infty$ (the ratio $\gamma/b \rightarrow 0$ in this limit). For penguin diagrams, it is true because their contributions are $O(1/N_C)$. Note that the coefficients c_i' are considered here as phenomenological parameters to be determined from experiment. They differ from the Wilson coefficients $c_i(\mu)$ by corrections $O(1/N_C,\mu)$ which cannot be controlled theoretically.

The corresponding bosonized chiral Lagrangian allows one to reproduce in three approximation (see Fig. 4a) the results obtained for $K \rightarrow 2\pi$ decay amplitudes in

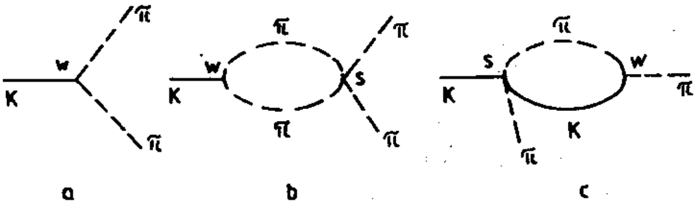

Figure 4: Diagrams for $K \to 2\pi$ decay in one-loop approximation; W denotes weak vertices, S denotes strong vertices

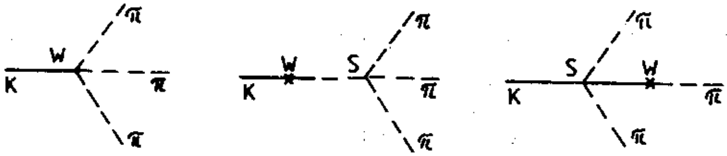

Figure 5: Three diagrams for $K \to 3\pi$ decays.

the usual factorization method. However, in case of more complicated processes, the factorization method becomes questionable due to arising ambiguities. Analogously, the isotopically invariant part of $K \to 3\pi$ decay amplitudes is described by the Born diagrams of Fig. 5.

Besides the Born diagrams it is necessary to take into account one-loop contributions (unitary corrections) connected with meson rescattering. For the regularization of the UV divergences, appearing in the one-loop approximation of chiral perturbation theory we will use the results of the superpropagator (SP) method proposed for the first time in [18]. It is possible to show that to lowest order in the $1/F_0$ expansion, the SP regularization gives the same results for one-loop integrals as dimensional regularization if one makes the following substitutions:

$$\mu^2 \to (4\pi F_0)^2, \quad (C - \frac{1}{\epsilon}) \to C_{SP},$$

where

$$C_{SP} = 2C + 1 + \frac{1}{2}\left[\frac{d}{dz}(\ln \Gamma^{-2}(2z+2))\right]_{z=0} = -1 + 4C \approx 1.309$$

has a finite value; $C = 0.577$ is the Euler constant and $\epsilon = (4 - D)/2$.

Meson rescattering processes play an essential role in the description of $K \to 2\pi$, $K \to 3\pi$ decays. In ref. [19, 20] it was shown that their contribution (Fig.4 b,c) results in an additional enhancement of the $|\Delta I| = 1/2$ transition by about 1.5 times and a similar suppression of the $|\Delta I| = 3/2$ transition. The meson rescattering plays an even more important role in the description of the $K \to 3\pi$ Dalitz-plot slope parameters. The rescattering is of particular importance in estimating observable direct CP violation effects which are determined by interference of $\pi\pi$ scattering phases in the final states of $|\Delta I| = 1/2, 3/2$ transitions in $K \to 2\pi$ and $K \to 3\pi$ decays.

In order to determine the parameters c_i' in the bosonized nonleptonic weak Lagrangian, we used the experimental data on decay parameters of $K \to 2\pi$, $K \to 3\pi$ decays—partial decay width's B_i and expansion coefficients g_i, h_i of the matrix element squared with respect to the Dalitz variables X, Y, i.e.

$$|T_{K\to 3\pi}|^2 \propto 1 + gY + hY^2 + \ldots$$

One obtains the following values of the parameters searched for (primes are omitted).

$$\begin{aligned} (-c_1 + c_2 + c_3)^{exp.} &= 5.32 \pm 0.08\,, \qquad c_5^{exp.} = -0.60 \pm 0.006\,, \\ c_4^{exp.} &= 0.327 \pm 0.006\,. \end{aligned} \tag{30}$$

7. Direct CP violation in $K \to 3\pi$ decays

The coefficients c_i $(i \neq 4)$ in the operator expansion Eq.23 for the six-quark Kobayashi-Maskawa model are complex quantities [21]. Their imaginary parts determine the contributions of the operators $\mathcal{O}_i$ with $|\Delta I| = 1/2$ to direct CP violation. The main contribution to this effect comes from the penguin diagrams. Therefore the parameter ϵ' characterizing the direct CP violation in $K^0 \to 2\pi$ decays is practically predicted by the imaginary part of the coefficient c_5. Using the experimental value Eq.1 we estimate

$$|\,\mathrm{Im}\ c_5\,|^{exp.} = (7.9 \pm 3.1) \cdot 10^{-5}. \tag{31}$$

Direct CP violation results in charge asymmetries of probabilities and slope parameters of $K^\pm \to 3\pi$ decays:

$$\Delta\Gamma = \frac{\Gamma_{K^-\to 3\pi} - \Gamma_{K^+\to 3\pi}}{\Gamma_{K^-\to 3\pi} + \Gamma_{K^+\to 3\pi}}, \qquad \Delta g = \frac{g_{K^-\to 3\pi} - g_{K^+\to 3\pi}}{g_{K^-\to 3\pi} + g_{K^+\to 3\pi}}.$$

The parameter values Eqs.30,31 lead to the following estimates for CP asymmetries in $K^\pm \to \pi^\pm\pi^\pm\pi^\mp$ and $K^\pm \to \pi^0\pi^0\pi^\pm$ decays [17]

$$|\Delta\Gamma(K^\pm \to \pi^\pm\pi^\pm\pi^\mp)| \;=\; \frac{\Gamma(K^+ \to \pi^+\pi^0\pi^0)}{\Gamma(K^+ \to \pi^+\pi^+\pi^-)}\,|\Delta\Gamma(K^\pm \to \pi^0\pi^0\pi^\pm)| \approx 0.31 \cdot 10^{-4},$$

$$| \Delta g(K^\pm \to \pi^\pm\pi^\pm\pi^\mp) | \approx | \Delta g(K^\pm \to \pi^0\pi^0\pi^\pm) | \approx 0.11 \cdot 10^{-2}. \quad (32)$$

Note that our predictions on $\Delta\Gamma$ are about two orders of magnitude larger than the results of [29]. Previous attempts to observe charge CP asymmetries in $K \to 3\pi$ decays are still not successful [22] :

$$\begin{aligned} \Delta\Gamma(K^\pm \to \pi^\pm\pi^\pm\pi^\mp)^{exp.} &= (0.35 \pm 0.60) \cdot 10^{-3}, \\ \Delta g(K^\pm \to \pi^\pm\pi^\pm\pi^\mp)^{exp.} &= (0.7 \pm 0.5) \cdot 10^{-2}, \\ \Delta\Gamma(K^\pm \to \pi^0\pi^0\pi^\pm)^{exp.} &= (-0.15 \pm 2.75) \cdot 10^{-3}. \end{aligned}$$

The effects of CP violation in neutral $K^0(\bar{K}^0) \to 3\pi$ decays are characterized by a time-dependent asymmetry of the decay probability

$$\Delta(t) = \frac{I(K^0(t) \to 3\pi) - I(\bar{K}^0(t) \to 3\pi)}{I(K^0(t) \to 3\pi) + I(\bar{K}^0(t) \to 3\pi)} = \frac{-Z + F(t)}{1 - ZF(t)}, \quad (33)$$

where $Z = \langle K_S^0 | K_L^0 \rangle$ is the parameter of $(K^0 - \bar{K}^0)$-mixing;

$$F(t) \approx \frac{2\exp[-\frac{1}{2}(\Gamma_S + \Gamma_L)t]}{\exp(-\Gamma_S t)R + \exp(-\Gamma_L t)} \mathrm{Re}\{ e^{i\Delta m \cdot t} [(1 + R)\epsilon + \epsilon'_{+-0}] \}. \quad (34)$$

Here $\Delta m = m_L - m_S$, $R = | T_S |^2 / | T_L |^2$ and the parameter ϵ'_{+-0} characterizes direct CP violation in $K^0(\bar{K}^0) \to \pi^+\pi^-\pi^0$ decays. Its value is connected with ϵ' by the relation Eq.3.

The Li-Wolfenstein relation Eq.3 can be derived from chiral QCD in the Born approximation neglecting the p^4-corrections from the interaction $\mathcal{L}^{(4)}_{mes}$ Eq.19. However, p^4 -corrections, (π^0, η, η')-mixing (isospin-violating diagrams of Fig.6) and meson rescattering have an essential influence on relation Eq.3. For example, the authors of [24] argued that the p^4-contributions essentially enhance the direct CP violation effects in the decays $K^0(\bar{K}^0) \to \pi^+\pi^-\pi^0$ as compared with $K \to 2\pi$ decays. Thus, their value $\epsilon'_{+-0} \sim 10\,\epsilon'$ significantly differs from the Li-Wolfenstein prediction.

The results of our calculations show that the p^4- contributions and (π^0, η, η')-mixing lead to a value of

$$\epsilon'_{+-0} = +5.1\ \epsilon'. \quad (35)$$

This result deviates from that of [25]

$$\epsilon'_{+-0} = -(1.3 \pm 0.7)\epsilon'. \quad (36)$$

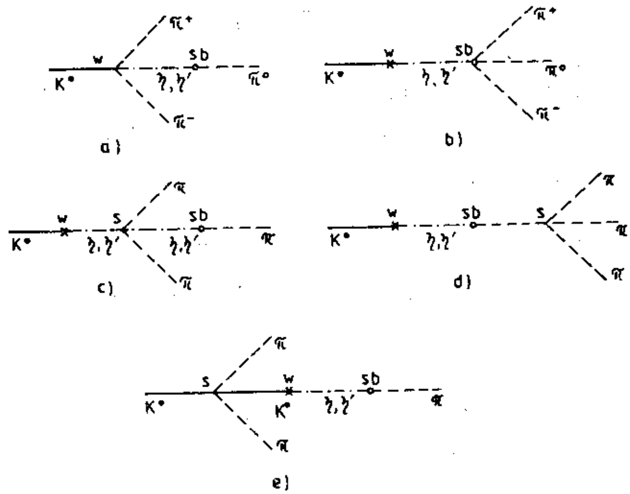

Figure 6: Diagrams of isospin-violating contributions to $K^0(\bar{K}^0) \to 3\pi$ decays

The essential difference between the results Eq.35 and Eq.36 is due to the fact that in our analysis penguin diagrams, providing the main mechanism of direct CP violation, are explicitly taken into account in the $K \to 2\pi$, $K \to 3\pi$ transition amplitudes by the imaginary part of c_5.

If one takes into account additional corrections corresponding to meson rescattering effects one obtains

$$\epsilon'_{+-0} = i\,(\,6.4 + 2.4\,i\,)\mid \epsilon' \mid .$$

In spite of an enhancement of direct CP violation in the decays $K^0(\bar{K}^0) \to \pi^+\pi^-\pi^0$ the value of this enhancement ($\mid \epsilon'_{-+0} \mid \approx 6.8 \mid \epsilon' \mid$) seems nevertheless to be insufficient for the experimental detection of a direct CP violation contribution to the asymmetry Eq.33.

8. Direct CP violation in $K^\pm \to \pi^\pm\pi^0\gamma$ decay

The process $K^\pm \to \pi^\pm\pi^0\gamma$ gets contributions from inner bremsstrahlung (IB) and direct emission (DE) of photons. The IB contribution is strongly suppressed due to the $\mid \Delta I \mid = 1/2$ rule, but there is no such suppression for the DE amplitude. The latter is the sum of direct magnetic (M1) and electric (E1) dipole transitions

$$\begin{aligned} T^{DE} &= T_{M1} + T_{E1}, \\ T_{M1} &= i\, e\, \epsilon_\mu\, h_{M1}\, \epsilon_{\mu\nu\alpha\beta}\, k_\nu\, p_{+\alpha}\, q_\beta, \\ T_{E1} &= -e\, \epsilon_\mu\, h_{E1} [(q \cdot k) p_{+\mu} - (q \cdot p_+) k_\mu], \end{aligned} \tag{37}$$

where h_{M1}, h_{E1} are real form factors; ϵ_μ is the polarization vector of the photon; k, p_+, q are four-momenta of the kaon, pion and photon, respectively. The DE amplitudes contain $|\Delta I| = 1/2$ and $3/2$ transitions.

The experimental values for the DE branching ratio (a cut off 55 $MeV < T_{\pi^+} <$ 90 MeV is used)

$$B_{DE}^{exp.} = \begin{cases} (1.56 \pm 0.35 \pm 0.50) \cdot 10^{-5} & \text{Ref.[26]} \\ (2.05 \pm 0.45) \cdot 10^{-5} & \text{Ref.[27]} \end{cases} \tag{38}$$

are in agreement with the estimate of the M1-transition branching ratio [28]

$$B_{M1} = 1.8 \cdot 10^{-5}.$$

Thus, the DE amplitude is dominated by M1 transitions arising from diagrams with anomalous vertices. The E1 transition is determined by the electroweak current Eq.29 yielding a very small probability $B_{E1} = 2.4 \cdot 10^{-7}$. It is this reason why neither interference between E1 and IB amplitudes nor a charge CP-asymmetry were observed in the experiments.

9. Summary

In this talk we have briefly reviewed the recent progress in the application of bosonization methods to derive effective Lagrangians for strong and electroweak interactions of mesons including higher order derivative terms, anomalies, γ-structure currents etc. On this basis, using chiral perturbation theory with superpropagator regularization to include final state interactions, a simultaneous analysis of $K \to 2\pi$ and $K \to 3\pi$ data was performed and predictions on charge CP-asymmetries in $K^\pm \to 3\pi$ decays were discussed. The possibility of the experimental observation of the charge CP-asymmetry of the slope parameter Δg was examined in the recent proposal of the Neutrino Tagging Collaboration [30]. For neutral $K^0(\bar{K}^0) \to 3\pi$ decays various possibilities of modifications of the Li-Wolfenstein relation $\epsilon'_{+-0} = -2\epsilon'$, relating direct CP violation parameters, have been discussed. Finally, the origin for the suppression of direct CP violation effects in the $K^{\pm} \to \pi^{\pm}\pi^{0}\gamma$ decay has been clarified.

References

1. Kobayashi M., Maskawa K. *Progr. Theor. Phys.* **49** (1973) 652.

2. Wolfenstein L. *Phys. Rev. Lett.* **13** (1964) 562.

3. Burkhardt H. et.al. *Phys. Lett.* **206B** (1988) 169.

4. Winstein B. *Talk at the XIV Internat. Symp. on Lepton and Photon interactions,* Stanford, August 1989.

5. Burshay S. *Phys. Rev. Lett.,* **18** (1967) 515.
   Costa G., Kabir P.K. *Phys. Rev. Lett.,* **18** (1967) 429.
   Christ N. *Phys. Rev.,* **159** (1967) 1292.

6. Weinberg S. *Phys. Rev. Lett.* **18** (1967) 188; *Phys. Rev.* **166** (1968) 1568.
   Coleman S., Wess J., Zumino B. *Phys. Rev.* **177** (1969) 2239.
   Gasiorowicz S., Geffen D.A. *Rev. Mod. Phys.* **41** (1969) 531.

7. Ebert D., Volkov M.K. *Yad. Fiz.* **36** (1982) 1265; *Z. Phys.* **C16** (1983) 205.

8. Ebert D., Reinhardt H. *Nucl. Phys.,* **B271** (1986) 188; *Phys. Lett.*, **B173** (1986) 453.

9. Karchev N.I., Slavnov A.A. *Teor. Mat. Fiz.* **65** (1985) 192.
   Simic P. *Phys. Rev. Lett.* **55** (1985) 40.
   D'yakonov D., Eides M. *Pis'ma v ZheTF* **38** (1983) 358 .
   Andrianov A.A. et. al. *Teor. Mat. Fiz.* **70** (1987) 63; *Phys. Lett.,* **B186** (1987) 401.

10. Dhar. A., Wadia S.R. *Phys. Rev. Lett.* **52** (1984) 959.
    Dhar A., Shankar R., Wadia S.R. *Phys. Rev.* **D31** (1985) 3256.

11. Gerard J.-M. *Preprint MPI-PAE/PTh 32/89*, Munich, 1989.
Cheng H.-Y. *Preprint IUHET-132*, IP-ASTP-02-87, 1987.
Cheng H.-Y, Cheung C.Y., Yeung W,B, *Preprint IP-ASTP-13*, 1988.

12. Praschifka J., Cahill R.T., Roberts C.D. *Int. Journ. Mod. Phys.* **A4** (1989) 4929; *Phys. Rev.* **D36** (1987) 209.

13. Ball R. *Proc. of the Workshop on "Skyrmions and Anomalies"*, Mogilany, Poland, 1987.

14. Alkofer R., Reinhardt H. *Phys. Lett.* **B207** (1988) 482.
Takizawa M. et al. *Nucl. Phys.* **A507** (1990) 611.

15. Bel'kov A.A., Lanyov A.V. *Yad. Fiz.,* **45** (1987) 496.
Bel'kov A.A., Ebert D., Pervushin V.N. *Phys. Lett.* **193B** (1987) 315.

16. Gaillard M.K., Lee B.W. *Phys. Rev. Lett.,* **33** (1974) 108.
Altarelli G., Maiani L. *Phys. Lett.,* **52B**(1974) 351.
Vainshtein A.I., Zakharov V.I., Shifman M.A. *Nucl. Phys.* **B120** (1977), 316.

17. Bel'kov A.A., Lanyov A.V. *Yad. Fiz.,* **47** (1988) 516 .
Bel'kov A.A., Lanyov A.V., Ebert D. *Yad. Fiz.,* **50** (1989) 1093 .
Bel'kov A.A., Bohm G., Ebert D., Lanyov A.V. *Phys. Lett.,* **232B** (1989) 118.

18. Volkov M.K. *Yad. Fiz.,* **6** (1967) 1100; **7** (1967) 445.
Volkov M.K. *Ann. Phys.,* **49** (1968) 202; *Fortsch. Phys.*, **28** (1974) 499.

19. Bel'kov A.A., Bohm G., Ebert D., Lanyov A.V. *Phys. Lett.,* **B220** (1989) 459.

20. Bardeen W.A., Buras A.J., Gerard J.-M. *Phys. Lett.,* **B192** (1987) 138.

21. Gilman F.J., Wise M.B. *Phys. Lett.,* **83B** (1979) 83; *Phys. Rev.*, **D20** (1979) 2392.
Guberina B., Peccei R.D. *Nucl. Phys.,* **B163** (1980) 289.

22. Particle Data Group *Phys. Lett.,* **170B** (1986) 1.

23. Bolotov V.N. et al. *Yad. Fiz.,* **44** (1986) 117.

24. Donoghue J.F., Holstein B.R., Valencia G. *Phys. Rev.* **D36** (1987) 789.

25. Fajfer S., Gerard J.-M. *Z. Phys.,* **C42** (1989) 425.

26. Abrams R.J. et al. *Phys. Rev. Lett.,* **29** (1972) 1118.

27. Bolotov V.N. et al. *Yad. Fiz.,* **45** (1987) 1652.

28. Bel'kov A.A. et al. *Yad. Fiz.,* **44** (1968) 690.

29. Avilers C. *Phys. Rev.* **D23** (1981) 1124;
Grinstein B., Rey S.-J., Wise M.B. *Phys. Rev.* **D33** (1986) 1495.

30. Collaboration "Neutrino Tagging" *Preprint PHE 90-07*, Berlin-Zeuthen, 1990.

Received by Publishing Department
on January 14, 1991.