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RELATIVISTIC PARTIAL WAVE INTEGRAL EQUATIONS FOR THE TWO-FERMION WAVE FUNCTION

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1. Introduction

The 3-dimensional equations derived on the basis of quantum field theory [1-9], as well as the well-known 4-dimensional Bethe-Salpeter equation [10], are widely used for the relativistic description of the particle bound states. The formalism of these equations is more close to the formalism of the nonrelativistic theory and the wave function has a clear probabilistic interpretation.

In most of the 3-dimensional approaches the wave function is defined by projecting the Bethe-Salpeter wave function onto a certain space-like surface [6-8]. The same projection procedure is used to define the corresponding free and complete Green functions that are employed further to construct the interaction operator $\hat{V} = G_0^{-1} - G^{-1}$. The simplest choice of the space-like surface as a plane perpendicular to the time axis is equivalent to the operation of equating individual particle times, i.e. to the description of a particle system in terms of a single time [1,6].

As a consequence of the transition to a single-time formalism (i.e. to its different covariant modifications) there appears the parametric dependence of the operator of interaction (that serve as a kernel in the integral equation) on the total energy of the composite system. This dependence of the interaction kernel was explicitly calculated for the case of scalar particles in [9] to the second order in coupling constant. The numerical calculation of the energy spectrum of a system of two scalar particles interacting via the scalar photon exchange was performed in [11]. The results obtained in [11] allow us to explain the mass spectrum of the recently observed resonance states in the e^+e^--system [12] as well as the spectrum of the diproton resonance [13].

The aim of the present article is to prepare an analogous apparatus of integral equations for the calculations in the spin case. We shall use the covariant three-dimensional equation for a two-fermion system derived previously in [14,15]. In the next section we shall perform the invariant separation of states of the system with the total spins S=0 and S=1 and study the spin structure of the interaction kernels of the corresponding equations. In sect.3 the system of partial wave equations for the wave functions describing the states with the total angular momentum of the system J and spin S will be derived as well as the explicit form of matrix elements of the quasipotential $V^{J}_{ee'}$.

2. Spin structure of the 3-dimensional covariant equation for a two-fermion system

A covariant single-time wave function (WF) that describes the relative motion of two fermions (with momenta p_1 and p_2) is defined through the Bethe-Salpeter WF by the following integral [15]:

$$\widetilde{\Psi}^{\sigma_1\sigma_2}_{MK}(p_1,p_2) = -\frac{(2\pi)^4}{4m^2}\,\bar{u}^{\sigma_1}_{\alpha}(\Lambda^{-1}_{\lambda_{\mathcal{P}}}p_1)\,u^{\sigma_2}_{\beta}(\Lambda^{-1}_{\lambda_{\mathcal{P}}}p_2)\times \tag{2.1}$$
$$\times\int d^4x\, e^{\frac{i(p_1-p_2)x}{2}}\,\delta[\lambda_{\mathcal{P}}x]\langle 0|T\{\Psi_\alpha(x)\bar{\Psi}_\beta(-x)\}|M\vec{K};S\sigma\rangle$$

Here $\Psi(x)$ and $\bar{\Psi}(x)$ are the field operators of the fermion and antifermion in the Heisenberg representation, $\mathcal{P}=p_1+p_2$, $x=x_1-x_2$, $\lambda^{\mu}_{\mathcal{P}}=\mathcal{P}^{\mu}/\sqrt{\mathcal{P}^2}$ is the 4-vector of the system velocity and the vector $|M\vec{K};S\sigma\rangle$ characterizes the bound state as a whole, moving with the momentum $\vec{K}$ and having the total mass M, spin S and its projection on the z-axis σ ; $\Lambda^{-1}_{\lambda_{\mathcal{P}}}$ is the Lorentz boost operator into the rest system of the composite particle moving with the 4-velocity $\lambda^{\mu}_{\mathcal{P}}=\mathcal{P}^{\mu}/\sqrt{\mathcal{P}^2}$, i.e. $\Lambda_{\lambda_{\mathcal{P}}}(M,\vec{0})=(\mathcal{P}^0,\vec{\mathcal{P}})$ where $\mathcal{P}^0=\sqrt{\vec{\mathcal{P}}^2+M^2}$.

It should be underlined that the presence of the $\delta(\lambda_{\mathcal{P}}x)$-function $(x=x_1-x_2)$ in the integrand of (2.1) ensures the coincidence of individual times of fermions $x_1^0=x_2^0$ in the c.m.s. $\vec{\mathcal{P}}=0$ x).

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x) Let us mention that this way is not the only possible . In the Logunov-Tavkhelidze approach the elimination of the relative time out of the Green function is equivalent to the integration

$$\widetilde{G}(\vec{p},\vec{q};E)=(2\pi)^{-2}\int\!\!\int_{-\infty}^{\infty}dp_0\,dq_0\;G(p_0,q_0,\vec{p},\vec{q};E) \tag{2.2}$$

which allows us to obtain in the c.m.s. the quasipotential of the same form as in [8,9,11,23]. It was used in [16,17] to calculate the hyperfine splitting of the ground state of the muonium and positronium.

In the present paper we use the relativistic equation for the spin WF [14,15,18,19]

$$2\Delta^0_{p,m\lambda_{\mathcal{P}}}\left[M-2\Delta^0_{p,m\lambda_{\mathcal{P}}}\right]\widetilde{\Psi}^{\sigma_1\sigma_2}_{MK}(\vec{\Delta}_{p,m\lambda_{\mathcal{P}}})= \tag{2.3}$$
$$=\frac{1}{(2\pi)^3}\sum_{\nu_1\nu_2}\int\frac{d^3\vec{\Delta}_{k,m\lambda_{\mathcal{P}}}}{2\Delta^0_{k,m\lambda_{\mathcal{P}}}}\,V^{\sigma_1\sigma_2}_{\nu_1\nu_2}(\vec{\Delta}_{p,m\lambda_{\mathcal{P}}},\vec{\Delta}_{k,m\lambda_{\mathcal{P}}};M)\,\widetilde{\Psi}^{\nu_1\nu_2}_{MK}(\vec{\Delta}_{k,m\lambda_{\mathcal{P}}})$$

Here the vector

$$\vec{\Delta}_{p,m\lambda_{\mathcal{P}}}\equiv\overset{\circ}{\vec{p}}=\left(\overrightarrow{\Lambda^{-1}_{\lambda_{\mathcal{P}}}p_1}\right)=-\left(\overrightarrow{\Lambda^{-1}_{\lambda_{\mathcal{P}}}p_2}\right)=-\vec{\Delta}_{p_2,m\lambda_{\mathcal{P}}}= \tag{2.4}$$
$$=\vec{p}_1-\frac{\vec{\mathcal{P}}}{\sqrt{\mathcal{P}^2}}\left(p_1^0-\frac{\vec{p}_1\vec{\mathcal{P}}}{\mathcal{P}_0+\sqrt{\mathcal{P}^2}}\right)$$

represents a covariant generalization of the vector of the first particle momentum in the c.m.s. $(\vec{p}_1=-\vec{p}_2=\vec{p})$ before the interaction [20,21,18] . The covariant generalization of the particle momentum after the interaction ($\vec{k}_1=-\vec{k}_2=\vec{k}$) is defined analogously . The time components of 4-vectors $\Delta^{\mu}_{p,m\lambda_{\mathcal{P}}}$ and $\Delta^{\mu}_{k,m\lambda_{\mathcal{P}}}$ are defined through the equation of the mass-shell hyperboloid:

$$\left(\Delta^0_{p,m\lambda_{\mathcal{P}}}\right)^2-\vec{\Delta}^2_{p,m\lambda_{\mathcal{P}}}=m^2,\quad\left(\Delta^0_{k,m\lambda_{\mathcal{P}}}\right)^2-\vec{\Delta}^2_{k,m\lambda_{\mathcal{P}}}=m^2. \tag{2.5}$$

Usually in the first approximation the relativistic scattering amplitude T, is used to construct the interaction kernel. The scattering amplitude T is connected with the Bethe-Salpeter scatterin amplitude M defined through the Feynman diagrams by the relation

$$T(nM|\vec{p}_1\sigma_1\vec{p}_2\sigma_1|\vec{k}_1\gamma_1\vec{k}_2\gamma_2)=\frac{1}{i}(2\pi)^3\delta(\overset{\circ}{\vec{P}}-\overset{\circ}{\vec{K}})\times \tag{2.6}$$
$$\times\bar{u}_1(\vec{p}_1\sigma_1)\bar{u}_2(\vec{p}_2\sigma_2)(2\pi)^{-8}\int d\alpha\,d\beta\left(\frac{1}{\Delta_p/2+\alpha-i\varepsilon}+\right.$$
$$\left.+\frac{1}{\Delta_p/2-\alpha-i\varepsilon}\right)\left(\frac{1}{\Delta_k/2+\beta-i\varepsilon}+\frac{1}{\Delta_k/2-\beta-i\varepsilon}\right)\times$$
$$\times\mathcal{M}(p-\alpha n|k-\beta n|K)\;u_1(\vec{k}_1\gamma_1)\,u_2(\vec{k}_2\gamma_2)\;.$$

In the approximation of the one-boson exchange (with the mass μ) the quasipotential has in ξ - gauge the following form (compare with [8,23])

$$V^{(2)\sigma_1\sigma_2}_{\nu_1\nu_2}(p_1,p_2,\kappa_1,\kappa_2;M) = \left\{\bar u(\vec p_1\sigma_1)\gamma^\mu u(\vec\kappa_1\nu_1)\right\} g_{\mu\nu}\cdot$$
$$\cdot\left\{\bar u(\vec p_2\sigma_2)\gamma^\nu u(\vec\kappa_2\nu_2)\right\} V_0^F(\mathring{\vec p},\mathring{\vec \kappa};M) + (\xi-1)\cdot \quad (2.7)$$
$$\cdot\ \bar u(\vec p_1\sigma_1)\hat n\, u(\vec\kappa_1\nu_1)\,\bar u(\vec p_2\sigma_2)\hat n\, u(\vec\kappa_2\nu_2)\, V_0^{G}(\mathring{\vec p},\mathring{\vec \kappa};M),$$

where

$$V_0^F(\mathring{\vec p},\mathring{\vec \kappa};M) = -\frac{g_v^2}{\mathcal E_q(\mathcal E_q+\mathcal E_p+\mathcal E_\kappa - M)}, \quad (2.8)$$

$$V_0^G(\mathring{\vec p},\mathring{\vec \kappa};M) = -\frac{g_v^2}{2\mathcal E_q}\,\frac{\mathcal E_p+\mathcal E_\kappa-M}{(\mathcal E_q+\mathcal E_p+\mathcal E_\kappa-M)^2} \quad (2.9)$$

and

$$\mathcal E_q=\sqrt{\mathring{\vec q}^{\,2}+\mu^2}, \quad \mathring q_\mu=(\mathring p-\mathring k)_\mu,$$
$$\mathcal E_p=\sqrt{\mathring{\vec p}^{\,2}+m^2},$$
$$\mathcal E_k=\sqrt{\mathring{\vec k}^{\,2}+m^2}.$$

It is convenient to "remove" following [20,22] with the help of the Wigner D - functions the spin indices σ_i and ν_i each of which were "sitting" on its own momentum (the terminology of the authors of paper [20]) onto one and the same momentum, $\mathring{\vec p}\cdot(\equiv \vec\Delta_{p,m\lambda_{\mathcal P}})$ for example. We shall use the expression for the current found in the same paper

$$j^\mu_{\sigma_{\mathring p}\nu_{\mathring p}}(\mathring{\vec p},\mathring{\vec k}) = \bar u_{\sigma_{\mathring p}}(\mathring{\vec p})\gamma^\mu u_{\nu_{\mathring p}}(\mathring{\vec k}) = \quad (2.10)$$
$$= \frac{2}{\sqrt{2m(\Delta_0+m)}}\overset{*}{\chi}_{\sigma_{\mathring p}}\left\{\mathring p^\mu(\Delta_0+m)+2W^\mu(\mathring{\vec p})(\vec\sigma\vec\Delta)\right\}\chi_{\nu_{\mathring p}},$$

where $W^\mu(\mathring{\vec p})$ is the 4-vector of the relativistic spin (the Pauli-Lubansky-Shirokov vector [20,21] .) and the 4-vector Δ^μ is defined in the following way [4,22,18] :

$$\Delta_\mu = \left(\Lambda^{-1}_{\mathring p}\mathring k\right)_\mu, \quad (2.11a)$$

$$\Delta_0 = \left(\Lambda^{-1}_{\mathring p}\mathring k\right)_0 = \left(\mathring k_0\mathring p_0 - \mathring{\vec k}\mathring{\vec p}\right)/m, \quad (2.11b)$$

$$\vec\Delta = \left(\overrightarrow{\Lambda^{-1}_{\mathring p}\mathring k}\right) = \mathring{\vec k} - \frac{\mathring{\vec p}}{m}\left(\mathring k_0 - \frac{\mathring{\vec k}\mathring{\vec p}}{\mathring p_0+m}\right). \quad (2.11c)$$

The space component (2.11c) has the meaning of the difference of two vectors $\mathring{\vec k}$ and $\mathring{\vec p}$ (belonging to the mass-shell hyperboloid (2.5)) in the Lobachevsky momentum space realized on the upper sheet of this hyperboloid (see for details [4,22]). After the use of (2.10) we get the expression for the quasipotential in ξ - gauge

$$V^{(2)\sigma_{1\mathring p}\sigma_{2\mathring p}}_{\nu_{1\mathring p}\nu_{2\mathring p}}(\mathring{\vec p},\mathring{\vec k};M) = \overset{*}{\chi}{}^{\sigma_{1\mathring p}}\overset{*}{\chi}{}^{\sigma_{2\mathring p}}\, V^{(2)}(\mathring{\vec p},\mathring{\vec k};M)\,\chi_{\nu_{1\mathring p}}\chi_{\nu_{2\mathring p}}. \quad (2.12)$$

Here $\chi^{\sigma_{i\mathring p}}$ are the 2-component Pauli spinors and

$$\begin{aligned}
V^{(2)}(\mathring{\vec p},\mathring{\vec k};M) = -g_v^2\Bigg\{&\frac{1}{\mathcal E_q(\mathcal E_q+\mathcal E_p+\mathcal E_\kappa-M)}\Bigg[-4m^2+\\
&+\frac{\left(\mathcal E_p(\Delta_0+m)+\mathring{\vec p}\vec\Delta\right)^2}{m(\Delta_0+m)}\left[4+(\xi-1)\frac{\mathcal E_p+\mathcal E_\kappa-M}{\mathcal E_q+\mathcal E_p+\mathcal E_\kappa-M}\right]\Bigg]+ \qquad (2.13)\\
&+\frac{(\vec\sigma_1\vec\Delta)(\vec\sigma_2\vec\Delta)-(\vec\sigma_1\vec\sigma_2)\vec\Delta^2}{\mathcal E_q(\mathcal E_q+\mathcal E_p+\mathcal E_\kappa-M)}\,\frac{2m}{\Delta_0+m}+\frac{i(\vec\sigma_1+\vec\sigma_2)[\mathring{\vec p}\vec\Delta]}{\mathcal E_q(\mathcal E_q+\mathcal E_p+\mathcal E_\kappa-M)}\cdot\\
&\cdot\frac{\mathcal E_p(\Delta_0+m)+\mathring{\vec p}\vec\Delta}{m(\Delta_0+m)}\left[4+(\xi-1)\frac{\mathcal E_p+\mathcal E_\kappa-M}{\mathcal E_q+\mathcal E_p+\mathcal E_\kappa-M}\right]-\\
&-\frac{\vec\sigma_1[\mathring{\vec p}\vec\Delta]\,\vec\sigma_2[\mathring{\vec p}\vec\Delta]}{\mathcal E_q(\mathcal E_q+\mathcal E_p+\mathcal E_\kappa-M)}\,\frac{1}{m(\Delta_0+m)}\left[4+(\xi-1)\frac{\mathcal E_p+\mathcal E_\kappa-M}{\mathcal E_q+\mathcal E_p+\mathcal E_\kappa-M}\right]\Bigg\},
\end{aligned}$$

(ξ is the parameter that fixes the gauge). The vector n^μ from [8,23] is taken to be equal to the 4-velocity of the bound state as a whole $n^\mu = \lambda^\mu_{\mathcal P} = \mathcal P^\mu/\sqrt{\mathcal P^2}$.

After "removing" all the polarization indices of the WF onto one and the same momentum $\overset{\circ}{\vec{p}}$ ($\sigma \to \sigma_{\overset{\circ}{p}}$) which corresponds to the quantization of all spins of the particles onto one and the same direction along the vector $\overset{\circ}{\vec{p}}$ we can perform a covariant summation of the spins [20]. Thus, we arrive at the WF and the quasipotential that are characterized by the total spin S and its projection $\sigma_{\overset{\circ}{p}}$ [18,19]:

$$\tilde{\Psi}_{S\sigma_{\overset{\circ}{p}}}(\overset{\circ}{\vec{p}}) = \sum_{\sigma_{1\overset{\circ}{p}},\sigma_{2\overset{\circ}{p}}=\pm 1/2} \langle \tfrac{1}{2}\ \tfrac{1}{2}\ \sigma_{1\overset{\circ}{p}}\sigma_{2\overset{\circ}{p}} \mid S\,\sigma_{\overset{\circ}{p}} \rangle \tilde{\Psi}_{\sigma_{1\overset{\circ}{p}}\sigma_{2\overset{\circ}{p}}}(\overset{\circ}{\vec{p}}), \tag{2.14}$$

$$V^{S\sigma_{\overset{\circ}{p}}}_{S'\nu_{\overset{\circ}{p}}}(\overset{\circ}{\vec{p}},\overset{\circ}{\vec{k}};M) = \sum_{\sigma_{1\overset{\circ}{p}},\sigma_{2\overset{\circ}{p}}=\pm 1/2}\ \sum_{\nu_{1\overset{\circ}{p}},\nu_{2\overset{\circ}{p}}=\pm 1/2} \langle \tfrac{1}{2}\ \tfrac{1}{2}\ \sigma_{1\overset{\circ}{p}}\sigma_{2\overset{\circ}{p}} \mid S\,\sigma_{\overset{\circ}{p}} \rangle \times \tag{2.15}$$

$$\times V^{\sigma_{1\overset{\circ}{p}}\sigma_{2\overset{\circ}{p}}}_{\nu_{1\overset{\circ}{p}}\nu_{2\overset{\circ}{p}}}(\overset{\circ}{\vec{p}},\overset{\circ}{\vec{k}};M) \langle \tfrac{1}{2}\ \tfrac{1}{2}\ \nu_{1\overset{\circ}{p}}\nu_{2\overset{\circ}{p}} \mid S'\,\nu_{\overset{\circ}{p}} \rangle .$$

For composite particles with the total spin S=0 (mesons) the corresponding WF is antisymmetric

$$\tilde{\Psi}_0(\overset{\circ}{\vec{p}}) = \frac{1}{\sqrt{2}}\left\{ \tilde{\Psi}_{\frac{1}{2}\overset{\circ}{p},-\frac{1}{2}\overset{\circ}{p}}(\overset{\circ}{\vec{p}}) - \tilde{\Psi}_{-\frac{1}{2}\overset{\circ}{p},\frac{1}{2}\overset{\circ}{p}}(\overset{\circ}{\vec{p}}) \right\}. \tag{2.16}$$

For the state with the total spin S=1 the WF's are

$$\tilde{\Psi}_{10}(\overset{\circ}{\vec{p}}) = \frac{1}{\sqrt{2}}\left\{ \tilde{\Psi}_{\frac{1}{2}\overset{\circ}{p},-\frac{1}{2}\overset{\circ}{p}}(\overset{\circ}{\vec{p}}) + \Psi_{-\frac{1}{2}\overset{\circ}{p},\frac{1}{2}\overset{\circ}{p}}(\overset{\circ}{\vec{p}}) \right\}, \tag{2.17}$$

$$\tilde{\Psi}_{11}(\overset{\circ}{\vec{p}}) = \tilde{\Psi}_{\frac{1}{2}\overset{\circ}{p},\frac{1}{2}\overset{\circ}{p}}(\overset{\circ}{\vec{p}}), \tag{2.18}$$

$$\tilde{\Psi}_{1,-1}(\overset{\circ}{\vec{p}}) = \tilde{\Psi}_{-\frac{1}{2}\overset{\circ}{p},-\frac{1}{2}\overset{\circ}{p}}(\overset{\circ}{\vec{p}}) . \tag{2.19}$$

After this summation of the spins the equation (2.3) takes the form:

$$2\Delta^{0}_{p,m\lambda_{\mathcal{P}}}\left(M - 2\Delta^{0}_{p,m\lambda_{\mathcal{P}}}\right)\tilde{\Psi}^{S\sigma_{\overset{\circ}{p}}}(\overset{\circ}{\vec{p}}) = \tag{2.20}$$

$$= \frac{1}{(2\pi)^3}\sum_{S',\nu_{\overset{\circ}{p}}}\int \frac{d^3\vec{\Delta}_{k,m\lambda_{\mathcal{P}}}}{2\Delta^{0}_{k,m\lambda_{\mathcal{P}}}} V^{S\sigma_{\overset{\circ}{p}}}_{S'\nu_{\overset{\circ}{p}}}(\overset{\circ}{\vec{p}},\overset{\circ}{\vec{k}};M)\tilde{\Psi}^{S'\nu_{\overset{\circ}{p}}}(\overset{\circ}{\vec{k}}) .$$

Simple calculations lead us to the following results for different components of the quasipotential[x)] :

$$V^{00}_{00}(\vec{p},\vec{k};M) = -\frac{g_V^2}{\mathcal{E}_q(\mathcal{E}_q+\mathcal{E}_p+\mathcal{E}_k-M)}\left\{-4m^2 + 2\mathcal{E}_p\mathcal{E}_k\left[4+(\xi-1)\frac{\mathcal{E}_p+\mathcal{E}_k-M}{\mathcal{E}_q+\mathcal{E}_p+\mathcal{E}_k-M}\right]\right\}, \tag{2.21}$$

$$V^{10}_{10}(\vec{p},\vec{k};M) = -\frac{g_V^2}{\mathcal{E}_q(\mathcal{E}_q+\mathcal{E}_p+\mathcal{E}_k-M)}\left\{-4m^2 + (\mathcal{E}_p\mathcal{E}_k+\vec{p}\vec{k}+m^2)\left[4+(\xi-1)\frac{\mathcal{E}_p+\mathcal{E}_k-M}{\mathcal{E}_q+\mathcal{E}_p+\mathcal{E}_k-M}\right] - \frac{4m\Delta_3^2}{\Delta_0+m} - \frac{2(\eta_1^2+\eta_2^2)}{m(\Delta_0+m)}\left[4+(\xi-1)\frac{\mathcal{E}_p+\mathcal{E}_k-M}{\mathcal{E}_q+\mathcal{E}_p+\mathcal{E}_k-M}\right]\right\}, \tag{2.22}$$

$$V^{10}_{11}(\vec{p},\vec{k};M) = -\frac{g_V^2}{\mathcal{E}_q(\mathcal{E}_q+\mathcal{E}_p+\mathcal{E}_k-M)}\left\{\frac{2\sqrt{2}\,m\Delta_3(\Delta_1+i\Delta_2)}{\Delta_0+m} + \frac{i\sqrt{2}(\mathcal{E}_p+\mathcal{E}_k)(\eta_1+i\eta_2)}{\Delta_0+m}\left[4+(\xi-1)\frac{\mathcal{E}_p+\mathcal{E}_k-M}{\mathcal{E}_q+\mathcal{E}_p+\mathcal{E}_k-M}\right] - \frac{\sqrt{2}\,\eta_3(\eta_1+i\eta_2)}{m(\Delta_0+m)}\left[4+(\xi-1)\frac{\mathcal{E}_p+\mathcal{E}_k-M}{\mathcal{E}_q+\mathcal{E}_p+\mathcal{E}_k-M}\right]\right\}, \tag{2.23}$$

$$V^{10}_{1,-1}(\vec{p},\vec{k};M) = -\frac{g_V^2}{\mathcal{E}_q(\mathcal{E}_q+\mathcal{E}_p+\mathcal{E}_k-M)}\left\{-\frac{2\sqrt{2}\,m\Delta_3(\Delta_1-i\Delta_2)}{\Delta_0+m} + \frac{i\sqrt{2}(\mathcal{E}_p+\mathcal{E}_k)(\eta_1-i\eta_2)}{\Delta_0+m}\left[4+(\xi-1)\frac{\mathcal{E}_p+\mathcal{E}_k-M}{\mathcal{E}_q+\mathcal{E}_p+\mathcal{E}_k-M}\right] + \right.$$

x) In what follows we shall omit the circles at the top of the momenta: $\overset{\circ}{\vec{p}} \equiv \vec{p}$ and $\overset{\circ}{\vec{k}} \equiv \vec{k}$.

$$+\frac{\sqrt{2}\,\eta_3(\eta_1-i\eta_2)}{\Delta_0+m}\left[4+(\xi-1)\frac{\mathcal{E}_p+\mathcal{E}_k-\mathcal{M}}{\mathcal{E}_q+\mathcal{E}_p+\mathcal{E}_k-\mathcal{M}}\right]\Bigg\}, \qquad (2.24)$$

$$V_{10}^{11}(\vec{p},\vec{k};\mathcal{M}) = -\frac{g_V^2}{\mathcal{E}_q(\mathcal{E}_q+\mathcal{E}_p+\mathcal{E}_k-\mathcal{M})}\Bigg\{\frac{2\sqrt{2}\,m\Delta_3(\Delta_1-i\Delta_2)}{\Delta_0+m}+$$

$$+\frac{i\sqrt{2}(\mathcal{E}_p+\mathcal{E}_k)(\eta_1-i\eta_2)}{\Delta_0+m}\left[4+(\xi-1)\frac{\mathcal{E}_p+\mathcal{E}_k-\mathcal{M}}{\mathcal{E}_q+\mathcal{E}_p+\mathcal{E}_k-\mathcal{M}}\right]-$$

$$-\frac{\sqrt{2}\,\eta_3(\eta_1-i\eta_2)}{m(\Delta_0+m)}\left[4+(\xi-1)\frac{\mathcal{E}_p+\mathcal{E}_k-\mathcal{M}}{\mathcal{E}_q+\mathcal{E}_p+\mathcal{E}_k-\mathcal{M}}\right]\Bigg\}, \qquad (2.25)$$

$$V_{11}^{11}(\vec{p},\vec{k};\mathcal{M}) = -\frac{g_V^2}{\mathcal{E}_q(\mathcal{E}_q+\mathcal{E}_p+\mathcal{E}_k-\mathcal{M})}\Bigg\{-4m^2+$$

$$+(\mathcal{E}_p\mathcal{E}_k+\vec{p}\vec{k}+m^2)\left[4+(\xi-1)\frac{\mathcal{E}_p+\mathcal{E}_k-\mathcal{M}}{\mathcal{E}_q+\mathcal{E}_p+\mathcal{E}_k-\mathcal{M}}\right]-\frac{2m(\Delta_1^2+\Delta_2^2)}{\Delta_0+m}+$$

$$+\frac{2i(\mathcal{E}_p+\mathcal{E}_k)\eta_3}{\Delta_0+m}\left[4+(\xi-1)\frac{\mathcal{E}_p+\mathcal{E}_k-\mathcal{M}}{\mathcal{E}_q+\mathcal{E}_p+\mathcal{E}_k-\mathcal{M}}\right]-$$

$$-\frac{\eta_1^2+\eta_2^2+2\eta_3^2}{m(\Delta_0+m)}\left[4+(\xi-1)\frac{\mathcal{E}_p+\mathcal{E}_k-\mathcal{M}}{\mathcal{E}_q+\mathcal{E}_p+\mathcal{E}_k-\mathcal{M}}\right]\Bigg\}, \qquad (2.26)$$

$$V_{1,-1}^{11}(\vec{p},\vec{k};\mathcal{M}) = -\frac{g_V^2}{\mathcal{E}_q(\mathcal{E}_q+\mathcal{E}_p+\mathcal{E}_k-\mathcal{M})}\Bigg\{\frac{2m(\Delta_1-i\Delta_2)^2}{\Delta_0+m}-$$

$$-\frac{(\eta_1-i\eta_2)^2}{m(\Delta_0+m)}\left[4+(\xi-1)\frac{\mathcal{E}_p+\mathcal{E}_k-\mathcal{M}}{\mathcal{E}_q+\mathcal{E}_p+\mathcal{E}_k-\mathcal{M}}\right]\Bigg\}, \qquad (2.27)$$

$$V_{10}^{1,-1}(\vec{p},\vec{k};\mathcal{M}) = -\frac{g_V^2}{\mathcal{E}_q(\mathcal{E}_q+\mathcal{E}_p+\mathcal{E}_k-\mathcal{M})}\Bigg\{-\frac{2\sqrt{2}\,m\Delta_3(\Delta_1+i\Delta_2)}{\Delta_0+m}+$$

$$+\frac{i\sqrt{2}(\mathcal{E}_p+\mathcal{E}_k)(\eta_1+i\eta_2)}{\Delta_0+m}\left[4+(\xi-1)\frac{\mathcal{E}_p+\mathcal{E}_k-\mathcal{M}}{\mathcal{E}_q+\mathcal{E}_p+\mathcal{E}_k-\mathcal{M}}\right]+ \qquad (2.28)$$

$$+\frac{\sqrt{2}\,\eta_3(\eta_1+i\eta_2)}{m(\Delta_0+m)}\left[4+(\xi-1)\frac{\mathcal{E}_p+\mathcal{E}_k-\mathcal{M}}{\mathcal{E}_q+\mathcal{E}_p+\mathcal{E}_k-\mathcal{M}}\right]\Bigg\},$$

$$V_{11}^{1,-1}(\vec{p},\vec{k};\mathcal{M}) = -\frac{g_V^2}{\mathcal{E}_q(\mathcal{E}_q+\mathcal{E}_p+\mathcal{E}_k-\mathcal{M})}\Bigg\{\frac{2m(\Delta_1+i\Delta_2)^2}{\Delta_0+m}- \qquad (2.29)$$

$$-\frac{(\eta_1+i\eta_2)^2}{m(\Delta_0+m)}\left[4+(\xi-1)\frac{\mathcal{E}_p+\mathcal{E}_k-\mathcal{M}}{\mathcal{E}_q+\mathcal{E}_p+\mathcal{E}_k-\mathcal{M}}\right]\Bigg\},$$

$$V_{1,-1}^{1,-1}(\vec{p},\vec{k};\mathcal{M}) = -\frac{g_V^2}{\mathcal{E}_q(\mathcal{E}_q+\mathcal{E}_p+\mathcal{E}_k-\mathcal{M})}\Bigg\{-4m^2+(\mathcal{E}_p\mathcal{E}_k+\vec{p}\vec{k}+m^2)\times \qquad (2.30)$$

$$\times\left[4+(\xi-1)\frac{\mathcal{E}_p+\mathcal{E}_k-\mathcal{M}}{\mathcal{E}_q+\mathcal{E}_p+\mathcal{E}_k-\mathcal{M}}\right]-\frac{2m(\Delta_1^2+\Delta_2^2)}{\Delta_0+m}-\frac{2i(\mathcal{E}_p+\mathcal{E}_k)\eta_3}{\Delta_0+m}\times$$

$$\times\left[4+(\xi-1)\frac{\mathcal{E}_p+\mathcal{E}_k-\mathcal{M}}{\mathcal{E}_q+\mathcal{E}_p+\mathcal{E}_k-\mathcal{M}}\right]-\frac{\eta_1^2+\eta_2^2+2\eta_3^2}{m(\Delta_0+m)}\left[4+(\xi-1)\frac{\mathcal{E}_p+\mathcal{E}_k-\mathcal{M}}{\mathcal{E}_q+\mathcal{E}_p+\mathcal{E}_k-\mathcal{M}}\right]\Bigg\}.$$

To simplify the form of the quasipotential, let us choose the coordinate system so that the vector $\vec{k}$ be taken along the Z -axis, whereas the vector $\vec{p}$ belonging the XZ flat.

Let us expand the WF and quasipotential in spherical tensors $\{\Omega^{S}_{J\ell M}(\vec{n})\}_{\sigma}$:

$$\tilde{\Psi}(\vec{p})_{s\sigma} = \frac{1}{p}\sum_{JM}\tilde{\Psi}_{J\ell}(p)\{\Omega^{s}_{J\ell M}(\vec{n}_{\vec{p}})\}_{\sigma_p}, \qquad (2.31)$$

$$V^{s\sigma_p}_{s'\nu_p}(\vec{p},\vec{k};\mathcal{M}) = \sum_{J\ell\ell'M}\{\Omega^{s}_{J\ell M}(\vec{n}_{\vec{p}})\}^{\sigma_p}V^{J}_{\ell\ell'}(p,k;\mathcal{M})\{\overset{*}{\Omega}{}^{s'}_{J\ell' M}(\vec{n}_{\vec{k}})\}_{\nu_p}, \qquad (2.32)$$

$$\{\Omega^{S}_{J\ell M}(\vec{n})\} = \sum_{m\sigma}\langle \ell\, S\,;\, m\,\sigma\,|\,J\,M\rangle Y_{\ell m}(\theta,\phi)\chi_{s\sigma}, \qquad (2.33)$$

where $Y_{\ell m}(\theta,\phi)$ are spherical functions.

After the substitution of (2.32) and (2.33) into (2.20) we get the following equation

$$2\mathring{p}_0(\mathcal{M}-2\mathring{p}_0)\frac{1}{p}\tilde{\Psi}_{J\ell}(p) = \frac{1}{(2\pi)^3}\int_0^{\infty}\frac{k\,dk}{2\mathring{k}_0}\sum_{\ell'}V^{J}_{\ell\ell'}(p,k;\mathcal{M})\tilde{\Psi}_{J\ell'}(k). \qquad (2.34)$$

In the case when the total spin of the system is equal to zero (S=0) we have the following equation for the WF $\tilde{\Psi}_\ell(p)$

$$2\overset{\circ}{p}_0(M-2\overset{\circ}{p}_0)\frac{1}{p}\tilde{\Psi}_\ell(p)=\frac{1}{(2\pi)^3}\int\frac{k\,dk}{2\overset{\circ}{k}_0}V_\ell(p,k;M)\tilde{\Psi}_\ell(k), \quad (2.35)$$

where the $V_\ell(p,k;M)$ are the matrix elements of the quasipotential for the case of S=0, $J=\ell$. At l=0,1,2, the corresponding expressions take the form (the gauge parameter is chosen to be $\xi=1$, i.e. Feynman gauge)

$$V_0(p,k;M)=8\pi g_v^2\,\frac{m^2-2\varepsilon_p\varepsilon_k}{kp}\times \ln\left|\frac{\sqrt{(p+k)^2+\mu^2}+\varepsilon_p+\varepsilon_k-M}{\sqrt{(p-k)^2+\mu^2}+\varepsilon_p+\varepsilon_k-M}\right|, \quad (2.36)$$

$$V_1(p,k;M)=24\pi g_v^2\,\frac{m^2-2\varepsilon_p\varepsilon_k}{kp}\left\{-1+\frac{\varepsilon_p+\varepsilon_k-M}{2kp}\left[\sqrt{(p+k)^2+\mu^2}-\sqrt{(p-k)^2+\mu^2}\right]+\frac{2m^2+M^2-\mu^2-2M(\varepsilon_p+\varepsilon_k)+2\varepsilon_p\varepsilon_k}{2kp}\ln\left|\frac{\sqrt{(p+k)^2+\mu^2}+\varepsilon_p+\varepsilon_k-M}{\sqrt{(p-k)^2+\mu^2}+\varepsilon_p+\varepsilon_k-M}\right|\right\}, \quad (2.37)$$

$$V_2(p,k;M)=20\pi g_v^2\,\frac{m^2-2\varepsilon_p\varepsilon_k}{kp}\left\{-\frac{\varepsilon_p+\varepsilon_k-M}{4\vec{k}^2\vec{p}^2}\left[\left((p+k)^2+\mu^2\right)^{3/2}-\left((p-k)^2+\mu^2\right)^{3/2}\right]-\frac{3(\varepsilon_p+\varepsilon_k-M)}{4\vec{k}^2\vec{p}^2}\left(2m^2+M^2-2\mu^2-2M(\varepsilon_p+\varepsilon_k)+2\varepsilon_p\varepsilon_k-\vec{p}^2-\vec{k}^2\right)\left[\sqrt{(p+k)^2+\mu^2}-\sqrt{(p-k)^2+\mu^2}\right]+\frac{3(2m^2+M^2-\mu^2)}{2kp}+\frac{3(\varepsilon_p\varepsilon_k-M(\varepsilon_p+\varepsilon_k))}{kp}+\left[\frac{3\left(2m^2+M^2-\mu^2-2M(\varepsilon_p+\varepsilon_k)+2\varepsilon_p\varepsilon_k\right)^2}{4\vec{k}^2\vec{p}^2}-1\right]\times\ln\left|\frac{\sqrt{(p+k)^2+\mu^2}+\varepsilon_p+\varepsilon_k-M}{\sqrt{(p-k)^2+\mu^2}+\varepsilon_p+\varepsilon_k-M}\right|\right\}. \quad (2.38)$$

For the total spin S=1 the equation (2.34) gives the system of three partial wave equations for three different values of

$$\ell=J+1:\quad 2p_0(M-2p_0)\frac{1}{p}\tilde{\Psi}_{J,J+1}(p)=\frac{1}{(2\pi)^3}\int_0^\infty\frac{k\,dk}{2k_0}\times\left(V^J_{J+1,J+1}\tilde{\Psi}_{J,J+1}(k)+V^J_{J+1,J}\tilde{\Psi}_{J,J}(k)+V^J_{J+1,J-1}\tilde{\Psi}_{J,J-1}(k)\right); \quad (2.39)$$

$$\ell=J:\quad 2p_0(M-2p_0)\frac{1}{p}\tilde{\Psi}_{J,J}(p)=\frac{1}{(2\pi)^3}\int_0^\infty\frac{k\,dk}{2k_0}\times\left(V^J_{J,J+1}\tilde{\Psi}_{J,J+1}(k)+V^J_{J,J}\tilde{\Psi}_{J,J}(k)+V^J_{J,J-1}\tilde{\Psi}_{J,J-1}(k)\right); \quad (2.40)$$

$$\ell=J-1:\quad 2p_0(M-2p_0)\frac{1}{p}\tilde{\Psi}_{J,J-1}(p)=\frac{1}{(2\pi)^3}\int_0^\infty\frac{k\,dk}{2k_0}\times\left(V^J_{J-1,J+1}\tilde{\Psi}_{J,J+1}(k)+V^J_{J-1,J}\tilde{\Psi}_{J,J}(k)+V^J_{J-1,J-1}\tilde{\Psi}_{J,J-1}(k)\right) \quad (2.41)$$

It will become clear from the following that the system (2.39) -(2.41) for the state with S=1 would separate into the system of two coupled and one separate equation due to the fact that

$$V^J_{\ell\ell'}(p,k;M)=0 \qquad \text{at} \qquad \Delta\ell=\ell'-\ell=\pm1. \quad (2.42)$$

Our next aim is to calculate the quasipotential in the $J\ell$ -representation. For further practical applications it will be sufficient to restrict our consideration to $\ell,\ell'\leqslant 2$.

3. The matrix elements of the quasipotential

Here we shall be interested only in the states with J=1 and J=0 (with the total spin of the composite system of two fermions S=1).

Let us first consider the state with J=1. Using the general formula for the expansion coefficient $V^J_{\ell\ell'}(p,k;M)$ of the quasipotential that is diagonal in the total spin (S=S'=1) :

$$V^J_{ee'}(p,k;M) = \sum_{M,\sigma,\sigma'} \int_0^{\pi} \sin\theta_p d\theta_p \int_0^{2\pi} d\phi_p \int_0^{\pi} \sin\theta_k d\theta_k \int_0^{2\pi} d\phi_k \times \tag{3.1}$$
$$\times \left\{ \overset{*}{\Omega}{}^{s}_{JeM}(\vec n_{\vec p}) \right\}_\sigma V^{s\sigma}_{sy}(\vec p,\vec k;M) \left\{ \Omega^{s}_{Je'M}(\vec n_{\vec k}) \right\}^{\nu},$$

we get for coefficients of interest the following expressions (in the coordinate system with $\vec k \parallel OZ$ and $\vec p \in (XZ)$)

$$V^1_{00}(p,k;M) = -4\pi g_v^2 \left\{ \left[8A + 8\gamma + 3\gamma' + \gamma'' - 8\delta^2 \right] \times \ln\left| \frac{\sqrt{\gamma+1}+\delta}{\sqrt{\gamma-1}+\delta} \right| + 4B \ln\left| \frac{\gamma'+1}{\gamma'-1} \right| + \frac{8\delta}{\sqrt{\gamma'-\gamma}} B \times \right.$$
$$\times \left(\operatorname{arctg}\sqrt{\frac{\gamma'-\gamma}{\gamma+1}} - \operatorname{arctg}\sqrt{\frac{\gamma'-\gamma}{\gamma-1}} \right), \tag{3.2}$$

$$V^1_{01}(p,k;M) = V^1_{10}(p,k;M) = 0, \tag{3.3}$$

$$V^1_{02}(p,k;M) = 2\pi\sqrt{2}\, g_v^2 \left\{ \left[c_1 A + 4\gamma - 4\gamma'' - 4\delta^2 \right] \times \ln\left| \frac{\sqrt{\gamma+1}+\delta}{\sqrt{\gamma-1}+\delta} \right| + c_2 B \ln\left| \frac{\gamma'+1}{\gamma'-1} \right| + \frac{2\delta}{\sqrt{\gamma'-\gamma}} c_2 B \times \right.$$
$$\left. \times \left(\operatorname{arctg}\sqrt{\frac{\gamma'-\gamma}{\gamma+1}} - \operatorname{arctg}\sqrt{\frac{\gamma'-\gamma}{\gamma-1}} \right) + c_3 \right\}, \tag{3.4}$$

$$V^1_{11}(p,k;M) = 6\pi g_v^2 \left\{ \left[c_4 A - (\gamma-\delta^2)(4\gamma + 2\gamma' + 2\gamma'' - 4\delta^2) \right] \times \ln\left| \frac{\sqrt{\gamma+1}+\delta}{\sqrt{\gamma-1}+\delta} \right| + c_5 B \ln\left| \frac{\gamma'+1}{\gamma'-1} \right| + \frac{2\delta}{\sqrt{\gamma'-\gamma}} c_5 B \times \right.$$
$$\left. \times \left(\operatorname{arctg}\sqrt{\frac{\gamma'-\gamma}{\gamma+1}} - \operatorname{arctg}\sqrt{\frac{\gamma'-\gamma}{\gamma-1}} \right) + c_6 \right\}, \tag{3.5}$$

$$V^1_{12}(p,k;M) = V^1_{21}(p,k;M) = 0, \tag{3.6}$$

$$V^1_{20}(p,k;M) = 2\pi\sqrt{2}\, g_v^2 \left\{ \left[c_7 A + 4\gamma - 4\gamma'' - 4\delta^2 \right] \times \ln\left| \frac{\sqrt{\gamma+1}+\delta}{\sqrt{\gamma-1}+\delta} \right| + c_8 B \ln\left| \frac{\gamma'+1}{\gamma'-1} \right| + \frac{2\delta}{\sqrt{\gamma'-\gamma}} c_8 B \times \right.$$
$$\left. \times \left(\operatorname{arctg}\sqrt{\frac{\gamma'-\gamma}{\gamma+1}} - \operatorname{arctg}\sqrt{\frac{\gamma'-\gamma}{\gamma-1}} \right) + c_9 \right\}, \tag{3.7}$$

$$V^1_{22}(p,k;M) = 2\pi g_v^2 \left\{ \left[c_{10} A + (1 - 3(\gamma-\delta^2))(10\gamma + 3\gamma' - \gamma'' - 10\delta^2) \right] \times \ln\left| \frac{\sqrt{\gamma+1}+\delta}{\sqrt{\gamma-1}+\delta} \right| + c_{11} B \ln\left| \frac{\gamma'+1}{\gamma'-1} \right| + \frac{2\delta}{\sqrt{\gamma'-\gamma}} c_{11} B \times \right.$$
$$\left. \times \left(\operatorname{arctg}\sqrt{\frac{\gamma'-\gamma}{\gamma+1}} - \operatorname{arctg}\sqrt{\frac{\gamma'-\gamma}{\gamma-1}} \right) + c_{12} \right\}. \tag{3.8}$$

Here the following notation is introduced:

$$A = \frac{1-(\gamma-\delta^2)^2}{\gamma-\gamma'-\delta^2}, \quad B = \frac{1-\gamma'^2}{\gamma'-\gamma+\delta^2}, \tag{3.9}$$

$$\gamma = \frac{\vec p^{\,2} + \vec k^{\,2} + M^2}{2kp}, \quad \delta = \frac{\varepsilon_p + \varepsilon_k - M}{\sqrt{2kp}}, \tag{3.10}$$
$$\gamma' = \frac{\varepsilon_p \varepsilon_k + m^2}{kp}, \quad \gamma'' = \frac{\varepsilon_p \varepsilon_k - m^2}{kp}.$$

Note that the coefficients $V^J_{J,J+1}$, $V^J_{J+1,J}$, $V^J_{J,J-1}$, $V^J_{J-1,J}$ are equal to zero for all values of J. The constants C_i (i=1,2,...,12) in (3.2)-(3.8) have the form

$$C_1 = 4 - 6\frac{\varepsilon_\kappa^2}{\vec{K}^2} + 12\frac{p\varepsilon_\kappa}{(\varepsilon_p+m)\kappa}(\gamma-\delta^2) - 6\frac{\varepsilon_p-m}{\varepsilon_p+m}(\gamma-\delta^2)^2, \tag{3.11}$$

$$C_2 = 2 - 3\frac{\varepsilon_\kappa^2}{\vec{K}^2} + 6\frac{p\varepsilon_\kappa}{(\varepsilon_p+m)\kappa}\gamma' - 3\frac{\varepsilon_p-m}{\varepsilon_p+m}\gamma'^2, \tag{3.12}$$

$$\begin{aligned} C_3 = {} & \left[(\gamma+1)^{5/2} - (\gamma-1)^{5/2}\right]\frac{6}{5}\frac{\varepsilon_p-m}{\varepsilon_p+m}\delta + \left[(\gamma+1)^{3/2} - \right. \\ & \left. -(\gamma-1)^{3/2}\right] \times \left[4\frac{p\varepsilon_\kappa}{(\varepsilon_p+m)\kappa}\delta - 2\frac{\varepsilon_p-m}{\varepsilon_p+m}\delta\left(3\gamma+\gamma'-\delta^2\right)\right] + \\ & + \left[\sqrt{\gamma+1} - \sqrt{\gamma-1}\right] \times \left[6\frac{\varepsilon_\kappa^2}{\vec{K}^2}\delta - 12\frac{p\varepsilon_\kappa}{(\varepsilon_p+m)\kappa}\delta \times \right. \\ & \times \left(2\gamma+\gamma'-\delta^2\right) + 6\frac{\varepsilon_p-m}{\varepsilon_p+m}\delta\left((\gamma-\delta^2)^2 + (\gamma+\gamma')(\gamma-\delta^2) + \right. \\ & \left.\left. + \gamma^2 + \gamma\gamma' + \gamma'^2 - 1\right)\right] - 6\frac{\varepsilon_\kappa^2}{\vec{K}^2} + 12\frac{p\varepsilon_\kappa}{(\varepsilon_p+m)\kappa}(\gamma+\gamma'-\delta^2) - \\ & - 2\frac{\varepsilon_p-m}{\varepsilon_p+m}\left[1 + 3(\gamma^2+\gamma\gamma'+\gamma'^2-1) - 3\delta^2(2\gamma+\gamma'-\delta^2)\right], \end{aligned} \tag{3.13}$$

$$C_4 = 2\frac{m(2\varepsilon_p+\varepsilon_\kappa)}{pk} + 2\left(\frac{m}{\varepsilon_p+m} - 2\right)(\gamma-\delta^2), \tag{3.14}$$

$$C_5 = \frac{m(2\varepsilon_p+\varepsilon_\kappa)}{pk} + \left(\frac{m}{\varepsilon_p+m} - 2\right)\gamma', \tag{3.15}$$

$$\begin{aligned} C_6 = {} & \left[(\gamma+1)^{3/2} - (\gamma-1)^{3/2}\right]\frac{2}{3}\delta\frac{m}{\varepsilon_p+m} + \left[\sqrt{\gamma+1} - \right. \\ & \left. - \sqrt{\gamma-1}\right] \times \left[-4\frac{\varepsilon_p\varepsilon_\kappa}{pk} + 4(\delta^2-2\gamma) - 2\frac{m(2\varepsilon_p+\varepsilon_\kappa)}{pk} - \right. \\ & \left. - 2\left(\frac{m}{\varepsilon_p+m} - 2\right) \times \left(2\gamma+\gamma'-\delta^2\right)\right] + 4\frac{\varepsilon_p\varepsilon_\kappa}{pk} + \\ & + 4(\gamma-\delta^2) + 2\frac{m(2\varepsilon_p+\varepsilon_\kappa)}{pk} + 2\left(\frac{m}{\varepsilon_p+m} - 2\right) \times \left(\gamma+\gamma'-\delta^2\right), \end{aligned} \tag{3.16}$$

$$C_7 = 4 - 6\frac{m^2}{\vec{p}^2}, \tag{3.17}$$

$$C_8 = 2 - 3\frac{m^2}{\vec{p}^2}, \tag{3.18}$$

$$C_9 = 6\frac{m^2}{\vec{p}^2}\delta\left[\sqrt{\gamma+1} - \sqrt{\gamma-1}\right] - 6\frac{m^2}{\vec{p}^2}, \tag{3.19}$$

$$C_{10} = 16 - 6\frac{m(6\varepsilon_p+5\varepsilon_\kappa)}{pk}(\gamma-\delta^2) - 6\left(\frac{m}{\varepsilon_p+m} + 6\right) \times (\gamma-\delta^2)^2, \tag{3.20}$$

$$C_{11} = 8 - 3\frac{m(6\varepsilon_p+5\varepsilon_\kappa)}{pk}\gamma' - 3\left(\frac{m}{\varepsilon_p+m} + 6\right)\gamma'^2, \tag{3.21}$$

$$C_{12} = \left[(\gamma+1)^{5/2} - (\gamma-1)^{5/2}\right]\cdot\left[-6\delta + \frac{2}{5}\left(\frac{3m}{\varepsilon_p+m} + 18\right)\delta\right] + \left[(\gamma+1)^{3/2} - (\gamma-1)^{3/2}\right]\cdot\left[2\,\frac{\varepsilon_p\varepsilon_k + 2m^2}{pk}\,\delta - 2\,\frac{6m\varepsilon_p + 5m\varepsilon_k}{pk}\,\delta + 10\delta(3\gamma-\delta^2) - 2\left(\frac{m}{\varepsilon_p+m} + 6\right)\delta\left(3\gamma+\gamma'-\delta^2\right)\right] + \left[\sqrt{\gamma+1} - \sqrt{\gamma-1}\right]\cdot\left[-6\delta + 6\,\frac{\varepsilon_p\varepsilon_k + 2m^2}{pk}\,\delta(\delta^2-2\gamma) + 6\,\frac{6m\varepsilon_p+5m\varepsilon_k}{pk}\,\delta\left(2\gamma+\gamma'-\delta^2\right) - 30\,\delta\left(3\gamma^2 - 3\gamma\delta^2 + \delta^4\right) + 6\left(\frac{m}{\varepsilon_p+m} + 6\right)\cdot\left((\gamma-\delta^2)^2 + (\gamma+\gamma')\cdot(\gamma-\delta^2) + (\gamma^2+\gamma\gamma'+\gamma'^2-1)\right)\delta\right] \cancel{16} + 6\,\frac{\varepsilon_p\varepsilon_k + 2m^2}{pk}(\gamma-\delta^2) - 6(\gamma+\gamma'-\delta^2)\,\frac{6m\varepsilon_p+5m\varepsilon_k}{pk} + 30(\gamma-\delta^2)^2 - 2\left(\frac{m}{\varepsilon_p+m} + 6\right)\cdot\left(1 + 3(\gamma^2+\gamma\gamma'+\gamma'^2-1) - 3\delta^2(2\gamma+\gamma'-\delta^2)\right). \quad (3.22)$$

Now let us consider the coefficient $V^{J=0}_{\ell\ell'}(p,k;M) = V^0_{\ell\ell'}(p,k;M)$. There is only one nonzero coefficient

$$V^0_{11}(p,k;M) = \sqrt{\frac{3}{4\pi}}\sum_{\sigma,\nu}\int_{-1}^{1} d\cos\theta_p \int_0^{2\pi} d\phi_p \int_{-1}^{1} d\cos\theta_k \int_0^{2\pi} d\phi_k \times \langle 1\ 1\ 0\ \nu|0\ \nu\rangle\, V^{1\sigma}_{1\nu}(\vec p,\vec k;M)\,\langle 1\ 1\ \nu-\sigma\ \sigma|0\ \nu\rangle \times Y_{1,\nu-\sigma}(\theta_p,0). \quad (3.23)$$

All other coefficients $V^0_{\ell\ell'}(p,k;M)$ with $\ell,\ell'\neq 1$ are equal to zero. Formula (3.23) reduces to a one-dimensional integral

$$V^0_{11}(p,k;M) = 2\pi\int_{-1}^{1} d\cos\theta\left\{\left[4\varepsilon_p\varepsilon_k + 4\vec p\vec k - \frac{4m\Delta_3^2}{\Delta_0+m} - \frac{8\eta_2^2}{M(\Delta_0+m)}\right]\times V_0(\vec p,\vec k,M)\cos\theta - \left[\frac{4m\Delta_1\Delta_3}{\Delta_0+m} + \frac{8(\varepsilon_p+\varepsilon_k)\eta_2}{\Delta_0+m}\right]V_0(\vec p,\vec k;M)\sin\theta\right\}, \quad (3.24)$$

whose calculation gives the result

$$V^0_{11}(p,k;M) = 2\pi g_v^2\left\{\left[C_{13}A - (\gamma-\delta^2)(8\gamma+2\gamma'-2\gamma''-8\delta^2)\right]\times \ln\left|\frac{\sqrt{\gamma+1}+\delta}{\sqrt{\gamma-1}+\delta}\right| + C_{14}B\ln\left|\frac{\gamma'+1}{\gamma'-1}\right| + \frac{2\delta}{\sqrt{\gamma'-\gamma}}C_{14}B\times\left(\operatorname{arctg}\sqrt{\frac{\gamma'-\gamma}{\gamma+1}} - \operatorname{arctg}\sqrt{\frac{\gamma'-\gamma}{\gamma-1}}\right) + C_{15}\right\}, \quad (3.25)$$

where

$$C_{13} = -4\,\frac{m(2\varepsilon_p+\varepsilon_k)}{pk} - 4\left(\frac{m}{\varepsilon_p+m} + 2\right)(\gamma-\delta^2), \quad (3.26)$$

$$C_{14} = -2\,\frac{m(2\varepsilon_p+\varepsilon_k)}{pk} - 2\left(\frac{m}{\varepsilon_p+m} + 2\right)\gamma', \quad (3.27)$$

$$C_{15} = -\left[(\gamma+1)^{3/2} - (\gamma-1)^{3/2}\right]\frac{4}{3}\delta\frac{m}{\varepsilon_p+m} + \left[\sqrt{\gamma+1} - \sqrt{\gamma-1}\right]\delta\times\left[-\frac{4m^2}{pk} + 8(\delta^2-2\gamma) + \frac{4m(2\varepsilon_p+\varepsilon_k)}{pk} + 4\left(\frac{m}{\varepsilon_p+m} + 2\right)(2\gamma+\gamma'-\delta^2)\right] + 4\,\frac{m^2}{pk} + 8(\gamma-\delta^2) - 4\,\frac{m(2\varepsilon_p+\varepsilon_k)}{pk} - 4\left(\frac{m}{\varepsilon_p+m} + 2\right)(\gamma+\gamma'-\delta^2). \quad (3.28)$$

4. Summary

In the present paper we have prepared, in the framework of the three-dimensional approach to the relativistic description of bound states, the mathematical tool for describing the energy spectrum of a bound two-fermion system. As a result, we have obtained the system of one-dimensional partial-wave integral equations (2.39)-(2.41) for the wave function of the state with the total spin S=1 as well as the explicit form of the kernels of these equations. These kernels representing the interaction potentials are constructed from the Feynman matrix elements of the relativistic scattering amplitude according to the procedure developed in [1,2,8,9] in the framework of the single-time formalism (and its covariant generalizations) and are depended on the system total energy (the mass M of the bound state). Our further aim is to apply the developed formalism for calculating the mass spectrum of bound states, compound of two fermions.

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