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HIGGS-HIGGS INTERACTION UP TO HIGHER ORDERS OF PERTURBATION THEORY IN THE STANDARD MODEL

I. Self-Energy and Vertex Diagrams

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I. Introduction

The Higgs sector of the electroweak theory attracts much attention of theorists because of its connection with the cornerstones of the theory. The search for Higgs scalars is included in most of the experimental programs of the newcoming and acting accelerators [1].

The Higgs particles are suggested to be found in the decays of different particles (Z^0 -bosons, heavy quarkoniums, etc.) as well as in photon-photon interactions and gluon-gluon fusion. In this connection, let us mention not so long ago attempts to explain anomalous events seen at SpS as the manifestation of the bound state of two Higgs bosons, i.e. of higgsonium [2,3] or of the bound state of vector bosons [4] in accordance with the Veltman paper [5]. Nowadays, even after clarifying the experimental situation with these anomalous events, the interest in Higgsonium still has the right for existence at least from the viewpoint of prepredness to new unexpected news from the experiment. This is the reason why we start a more complete study of this problem.

Let us mention that the previous investigation of the problem of existence of a two-Higgs bound state was based on the Born approximation of their interaction amplitude [2,3]. In the present paper we shall present the results of our calculation of the amplitude of Higgs-Higgs interaction up to fourth order of perturbation theory in the framework of the Standard Model (SM) of Weinberg-Salam-Glashow. We are going to consider the same problem in the framework of the extended variants of the SM. To this end, we shall use the notation that would allow us to perform easily the generalization of the obtained results to other models.

The results, presented here, have rather technical (but original) nature. So to reduce the volume of the articles we shall use the standard notation used in [6] and the renormalization scheme analogous to that suggested by Sirlin [7]. We choose also the unitary gauge and the parameters, recommended by the Trieste conference [8].

The paper is organized as follows. In sect.2, we present the main notation - the Lagrangian corresponding to the Higgs sector of the theory and the Table containing the values of the coupling constants that correspond to the variant of the SM. The results of the calculation of the corresponding self-energy diagrams will be presented in Sect.3, while those for the vertex diagrams in Sect.4.

The Appendix contains the definition of some integrals met in calculations, and their connection with the integrals calculated in /9,10/ is given. The results obtained in this paper will be used in the following paper for calculation of the matrix elements of the amplitude of Higgs-Higgs interaction.

2. The Lagrangian and the coupling constants for the Higgs sector of the Standard Model

This Lagrangian has the following form /11/

$$\mathcal{L} = -\frac{1}{2}(\partial_\mu \chi)^2 - \frac{1}{2}M_\chi^2\chi^2 - \frac{eM_W}{(1-R)^{1/2}}W_\mu^+W_\mu^-\chi - \tag{I}$$

$$-\frac{eM_Z}{2R^{1/2}(1-R)^{1/2}}Z_\mu^2\chi - \frac{e}{2(1-R)^{1/2}}\frac{1}{M_W}\sum_f m_f\bar{f}f\chi -$$

$$-\frac{e^2}{4(1-R)}W_\mu^+W_\mu^-\chi^2 - \frac{e^2}{8R(1-R)}Z_\mu^2\chi^2 - \frac{eM_\chi^2}{4(1-R)^{1/2}}\frac{1}{M_W}\chi^3 - \frac{e^2}{32(1-R)}\frac{M_\chi^2}{M_W^2}\chi^4,$$

where e is the electron charge, M_χ is the Higgs mass, M_W and M_Z are masses of the vector bosons, m_f are the fermion masses, $R = M_W^2/M_Z^2$.

We, as it was mentioned in the Introduction, are going to use the results obtained here for calculations in the framework of the models that generalize the SM to the case of the models with two Higgs doublets. Thus, we shall leave some room, as the authors of /6/, for the notation of the vertex coupling constants. The exact form of the constants f^χ, $f^{V\chi}$, $f^{3\chi}$, $f^{4\chi}$, etc. (for the case of the SM) is given in Table I.

Here $g = e/\sin\theta_W$ (θ_W is the Weinberg angle, v is the vacuum expectation value, $|Q_f|$ is the modulus of the fermion (quark) electric charge, N_f is the number of fermions ($Tr|Q_f| = 1/2\, N_f$), K_{ij} is, in a general case, a non-diagonal matrix that coincides in the quark sector with the Kobayashi-Maskawa matrix.

Table I. Coupling constants for the case of the SM

f^W	$-\frac{1}{2\sqrt{2}}g$	f^{WW2A}	$ig^2\sqrt{R(1-R)}$
f^Z	$-\frac{1}{4}g\, M_Z/M_W$	$f^{W\chi}$	$-ig\,M_W$
f^A	eQ_i	$f^{Z\chi}$	$-ig\,M_Z^2/M_W$
f^{WWZ}	$-g\,M_W/M_Z$	$f^{2W2\chi}$	$-\frac{1}{2}ig^2$
f^{WWA}	e	$f^{2Z2\chi}$	$-\frac{1}{2}ig^2M_Z^2/M_W^2$
f^{2W2Z}	$-ig^2M_W^2/M_Z^2$	f^χ	$-\frac{1}{2}ig\, m_i/M_W$
f^{4W}	ig^2	$f^{3\chi}$	$-\frac{3}{2}ig\,M_\chi^2/M_W$
f^{2W2A}	$-ie^2$	$f^{4\chi}$	$-\frac{3}{4}ig^2M_\chi^2/M_W^2$

Vertices except 4χ, 3χ and $\bar{f}f\chi$ have the index content, of course (see e.g. /10/).

The form of the propagators is chosen as in /6/. Each loop integr is multiplied by the factor $i^k(2\pi)^{-4}$ where k is the number that corresponds to the perturbation theory order. The value of P is defined by the following formula

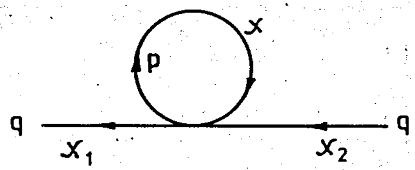
$$2P = -\frac{1}{\varepsilon} + \gamma + \ln\frac{M_W^2}{4\pi\mu^2}, \tag{2.2}$$

where γ is the Euler constant and $2\varepsilon = 4-d$, where d is the space dimension.

The renormalization constants are fixed in a way analogous to that used in QED. The condition of the nonrenormalizability of the external lines (i.e. the lines that correspond to the particles on the mass shell $p_i^2 = m_i^2$) as well as the condition of the electric charge e nonrenormalizability (as defined from γe-scattering in the limit of the zero energy) allow us to define (as in /6,7,11/) all the renormalization constants.

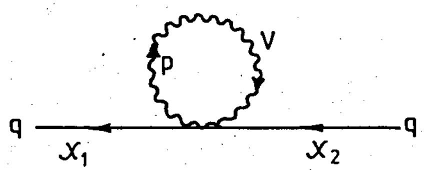
3. Self-energy diagrams for scalar bosons

1.



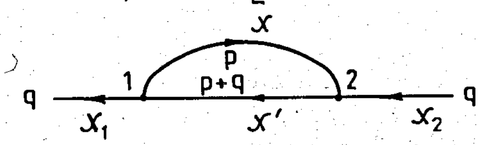
$$\Pi = \frac{i f^{4\chi}}{16\pi^2} M_\chi^2 \left[2P - 1 + \ln \frac{M_\chi^2}{M_W^2} \right] . \qquad (3.1)$$

2.



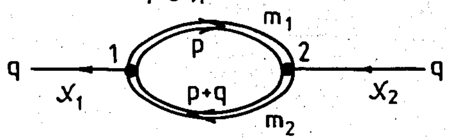
$$\Pi = \frac{i f^{2V2\chi}}{16\pi^2} M_V^2 \left[6P - 1 + 3 \ln \frac{M_V^2}{M_W^2} \right] . \qquad (3.2)$$

3.



$$\Pi(q^2) = \frac{i f_1^{3\chi} f_2^{3\chi}}{16\pi^2} \left[-2P - I_0(q^2, M_\chi^2, M_{\chi'}^2) \right] . \qquad (3.3)$$

4.



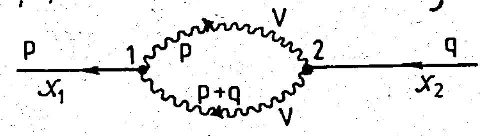
$$\Pi(q^2) = \frac{i f_1^{\chi} f_2^{\chi}}{4\pi^2} \Big\{ \Big[(1 - b_1 b_2)(q^2 + 2m_1^2 + 2m_2^2) + (1 + b_1 b_2) \cdot$$

$$\cdot 2 m_1 m_2 \Big] P + \Big[\frac{1}{2}(1 - b_1 b_2)(q^2 + m_1^2 + m_2^2) + (1 + b_1 b_2) m_1 m_2 \Big] \cdot$$

$$\cdot I_0(q^2, m_1^2, m_2^2) + (1 - b_1 b_2)\Big(\frac{m_1^2}{2} \ln \frac{m_1^2}{M_W^2} + \frac{m_2^2}{2} \ln \frac{m_2^2}{M_W^2} +$$

$$+ \frac{q^2}{4} \Big) + \frac{1}{2}(1 + b_1 b_2) m_1 m_2 \Big\} . \qquad (3.4)$$

5.



$$\Pi(q^2) = \frac{i f_1^{VV\chi} f_2^{VV\chi}}{16\pi^2} \Big\{ \Big[-\frac{q^4}{2M_V^4} - 3\frac{q^2}{M_V^2} - 6 \Big] P - \Big(\frac{q^4}{4M_V^4} +$$

$$+ \frac{q^2}{M_V^2} + 3 \Big) I_0(q^2, M_V^2, M_V^2) - \frac{q^2}{2M_V^2} \ln \frac{M_V^2}{M_W^2} + \frac{q^2}{2M_V^2} - 2 \Big\} \qquad (3.5)$$

In the framework of the SM with only one Higgs doublet we get

$$\Pi^{\chi}(q^2) = \frac{i g^2}{16\pi^2} M_\chi^2 \Big\{ \Big[-\frac{3}{4} \frac{q^4}{M_W^2 M_\chi^2} - 3\frac{q^2}{M_\chi^2} - \frac{3}{2}\frac{1}{R}\frac{q^2}{M_\chi^2} +$$

$$+ \frac{q^2}{M_W^2 M_\chi^2} \mathrm{Tr}\, m_i^2 - 3 r_W - 9 r_W^{-1} - \frac{9}{2}\frac{1}{R} r_Z^{-1} +$$

$$+ 6 \frac{1}{M_W^2 M_\chi^2} \mathrm{Tr}\, m_i^4 \Big] P + t + \frac{3}{4}\frac{q^4}{M_W^2 M_\chi^2} + \Big(\frac{5}{2} +$$

$$+ \frac{5}{4}\frac{1}{R} \Big) \frac{q^2}{M_\chi^2} + \frac{21}{8} r_W + \frac{9}{2} r_W^{-1} + \frac{9}{4}\frac{1}{R} r_Z^{-1}$$

$$- \frac{3}{2} r_W \ln r_W + \Big(\frac{1}{8}\frac{q^4}{M_W^4} + \frac{3}{4}\frac{1}{R}\frac{q^2}{M_W^2} + \frac{9}{4}\frac{1}{R^2} \Big)$$

$$\cdot r_W^{-1} \ln R - \Big(\frac{1}{8}\frac{q^2}{M_W^2} + \frac{1}{2} + \frac{3}{2}\frac{M_W^2}{q^2} \Big) \frac{1}{M_\chi^2} \cdot$$

$$\times L(q^2,M_W^2,M_W^2) - \left(\frac{1}{16}\frac{q^2}{M_Z^2} + \frac{1}{4} + \frac{3}{4}\frac{M_Z^2}{q^2}\right)\frac{1}{R}\frac{1}{M_\chi^2}\times$$

$$\times L(q^2,M_Z^2,M_Z^2) - \frac{9}{16} r_W \frac{1}{q^2} L(q^2,M_\chi^2,M_\chi^2) - \qquad (3.6)$$

$$- \frac{3}{4}\frac{q^2}{M_W^2 M_\chi^2} Tr\, m_i^2 + \frac{1}{2}\frac{q^2}{M_W^2 M_\chi^2} Tr\, m_i^2 \ln\frac{m_i^2}{M_W^2} -$$

$$- \frac{7}{2}\frac{1}{M_W^2 M_\chi^2} Tr\, m_i^4 + \frac{3}{M_W^2 M_\chi^2} Tr\, m_i^4 \ln\frac{m_i^2}{M_W^2} + \frac{1}{4}\frac{1}{M_W^2 M_\chi^2}\times$$

$$\times Tr\, m_i^2 L(q^2,m_i^2,m_i^2) + \frac{1}{M_W^2 M_\chi^2}\frac{1}{q^2} Tr\, m_i^4 L(q^2,m_i^2,m_i^2)\Big\}.$$

The corresponding counter-terms are

$$\frac{\delta M_\chi^2}{M_\chi^2} = Z_{M_\chi} - Z_\chi = \frac{ig^2}{16\pi^2}\Big\{\Big[3 + \frac{3}{2}\frac{1}{R} - \frac{15}{4} r_W -$$

$$- 9 r_W^{-1} - \frac{9}{2}\frac{1}{R} r_Z^{-1} - \frac{1}{M_W^2} Tr\, m_i^2 + \frac{6}{M_W^2 M_\chi^2} Tr\, m_i^4\Big]$$

$$\times P + t - \frac{5}{2} - \frac{5}{4}\frac{1}{R} + \frac{27}{8} r_W + \frac{9}{2} r_W^{-1} +$$

$$+ \frac{9}{4}\frac{1}{R} r_Z^{-1} - \frac{3}{2} r_W \ln r_W + \left(\frac{1}{8} r_W - \frac{3}{4}\frac{1}{R} +\right.$$

$$\left.+ \frac{9}{4}\frac{1}{R} r_Z^{-1}\right)\ln R + \frac{3}{4}\frac{1}{M_W^2} Tr\, m_i^2 - \frac{1}{2}\frac{1}{M_W^2}\times$$

$$\times Tr\, m_i^2 \ln\frac{m_i^2}{M_W^2} - \frac{7}{2}\frac{1}{M_W^2 M_\chi^2} Tr\, m_i^4 + \frac{3}{M_W^2 M_\chi^2} Tr\, m_i^4 \times$$

$$\times \ln\frac{m_i^2}{M_W^2} + \left(\frac{1}{8} - \frac{1}{2} r_W^{-1} + \frac{3}{2} r_W^{-2}\right)\frac{1}{M_W^2} L(-M_\chi^2,M_W^2,M_W^2) +$$

$$+ \left(\frac{1}{16} - \frac{1}{4} r_Z^{-1} + \frac{3}{4} r_Z^{-2}\right)\frac{1}{M_W^2} L(-M_\chi^2,M_Z^2,M_Z^2) +$$

$$+ \frac{9}{16}\frac{1}{M_W^2} L(-M_\chi^2,M_\chi^2,M_\chi^2) + \frac{1}{4}\frac{1}{M_W^2 M_\chi^2} Tr\, m_i^2 L(-M_\chi^2,$$

$$m_i^2,m_i^2) - \frac{1}{M_W^2 M_\chi^4} Tr\, m_i^4 L(-M_\chi^2,m_i^2,m_i^2)\Big\} \qquad (3.7)$$

and

$$Z_\chi - 1 = \frac{ig^2}{16\pi^2}\Big\{\Big[-3 - \frac{3}{2}\frac{1}{R} + \frac{3}{2} r_W + \frac{1}{M_W^2} Tr\, m_i^2\Big]\times$$

$$\times P + \frac{3}{2} + \frac{3}{4}\frac{1}{R} + 3 r_W^{-1} + \frac{3}{2}\frac{1}{R} r_Z^{-1} + \left(\frac{3}{4}\frac{1}{R} -\right.$$

$$\left.- \frac{1}{4} r_W\right)\ln R - \frac{1}{4}\frac{1}{M_W^2} Tr\, m_i^2 + \frac{1}{2}\frac{1}{M_W^2} Tr\, m_i^2 \ln\frac{m_i^2}{M_W^2} -$$

$$- \frac{2}{M_W^2 M_\chi^2} Tr\, m_i^4 + \left(\frac{1}{4} - \frac{1}{4} r_W - \frac{3}{r_W(r_W-4)}\right)\frac{1}{M_\chi^2} L(-M_\chi^2,$$

$$M_W^2,M_W^2) + \left(\frac{1}{8} - \frac{1}{8} r_Z - \frac{3}{2 r_Z(r_Z-4)}\right)\frac{1}{R}\frac{1}{M_\chi^2} L(-M_\chi^2,$$

$$M_Z^2,M_Z^2) + \frac{3}{8}\frac{1}{M_W^2} L(-M_\chi^2,M_\chi^2,M_\chi^2) - \frac{1}{4}\frac{1}{M_W^2 M_\chi^2}\times \qquad (3.8)$$

$$\times Tr\, m_i^2 L(-M_\chi^2,m_i^2,m_i^2) - \frac{1}{2}\frac{1}{M_W^2 M_\chi^4} Tr\, m_i^4 L(-M_\chi^2,$$

$$m_i^2,m_i^2)\Big\}.$$

Consequently,

$$\Pi^{Ren}(q^2) = \Pi^{\chi}(q^2) - \delta M_\chi^2 - (Z_\chi - 1)(q^2 + M_\chi^2) =$$

$$= \frac{ig^2}{16\pi^2} M_\chi^2 \Big\{ \Big[-\frac{3}{4}\frac{q^4}{M_W^2 M_\chi^2} - \frac{3}{4} r_W - \frac{3}{2}\frac{q^2}{M_W^2} \Big] P +$$

$$+ \frac{3}{4}\frac{q^4}{M_W^2 M_\chi^2} + \frac{1}{8}\frac{q^4}{M_W^2 M_\chi^2} \ln R + \frac{q^2}{M_\chi^2}\Big(1 + \frac{2}{R} - 3 r_W^{-1} -$$

$$- \frac{3}{2}\frac{1}{R} r_Z^{-1}\Big) + \frac{1}{4}\frac{q^2}{M_W^2} \ln R + \frac{1}{8} r_W \ln R + 1 +$$

$$+ \frac{1}{2R} - \frac{3}{4} r_W - 3 r_W^{-1} - \frac{3}{2}\frac{1}{R} r_Z^{-1} - \Big(\frac{q^2}{M_\chi^2} + 1\Big)\frac{1}{2M_W^2} \times$$

$$\times \operatorname{Tr} m_i^2 + \Big(\frac{q^2}{M_\chi^2} + 1\Big)\frac{2}{M_W^2 M_\chi^2} \operatorname{Tr} m_i^4 - \Big(\frac{1}{8}\frac{q^2}{M_W^2} + \frac{1}{2} +$$

$$+ \frac{3}{2}\frac{M_W^2}{q^2}\Big)\frac{1}{M_\chi^2} L(q^2, M_W^2, M_W^2) - \Big(\frac{1}{16}\frac{q^2}{M_Z^2} + \frac{1}{4} + \frac{3}{4}\frac{M_Z^2}{q^2}\Big) \times$$

$$\times \frac{1}{R}\frac{1}{M_\chi^2} L(q^2, M_Z^2, M_Z^2) - \frac{9}{16}\frac{1}{q^2} r_W L(q^2, M_\chi^2, M_\chi^2) +$$

$$+ \frac{1}{4}\frac{1}{M_W^2 M_\chi^2} \operatorname{Tr} m_i^2 L(q^2, m_i^2, m_i^2) + \frac{1}{M_W^2 M_\chi^2}\frac{1}{q^2} \operatorname{Tr} m_i^4 \cdot$$

$$\cdot L(q^2, m_i^2, m_i^2) + \Big(-\frac{q^2}{4M_\chi^2} + \frac{q^2}{4M_W^2} + \frac{3q^2}{M_\chi^2 r_W (r_W - 4)} + \frac{1}{4} +$$

$$+ \frac{1}{8} r_W - \frac{3}{2} r_W^{-1} + \frac{3}{r_W(r_W - 4)}\Big)\frac{1}{M_\chi^2} L(-M_\chi^2, M_W^2, M_W^2) +$$

$$+ \Big(-\frac{q^2}{8M_\chi^2} + \frac{q^2}{8M_Z^2} + \frac{3q^2}{M_\chi^2 r_Z (r_Z - 4)} + \frac{1}{8} + \frac{1}{16} r_Z - \frac{3}{4} r_Z^{-1} +$$

$$+ \frac{3}{2 r_Z (r_Z - 4)}\Big)\frac{1}{R}\frac{1}{M_\chi^2} L(-M_\chi^2, M_Z^2, M_Z^2) - \Big(\frac{3q^2}{8M_W^2} + \frac{15}{16} r_W\Big) \times$$

$$\times \frac{1}{M_\chi^2} L(-M_\chi^2, M_\chi^2, M_\chi^2) + \frac{q^2}{4M_W^2 M_\chi^4} \operatorname{Tr} m_i^2 L(-M_\chi^2, m_i^2, m_i^2) +$$

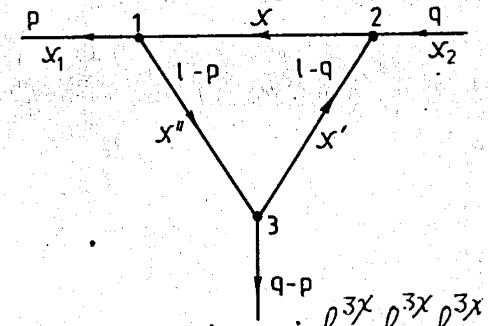
$$+ \frac{1}{2}\Big(1 - \frac{q^2}{M_\chi^2}\Big)\frac{1}{M_W^2 M_\chi^4} \operatorname{Tr} m_i^4 L(-M_\chi^2, m_i^2, m_i^2)\Big\}. \qquad (3.9)$$

In this Section and in what follows $r_W = M_\chi^2/M_W^2$, $r_Z = M_\chi^2/M_Z^2$. $b_{1,2}$ are constants defined by the strength of the Higgs-fermion pseudoscalar interaction. The form of the integral $I_0(q^2, M_1^2, M_2^2)$ is given in Appendix A.

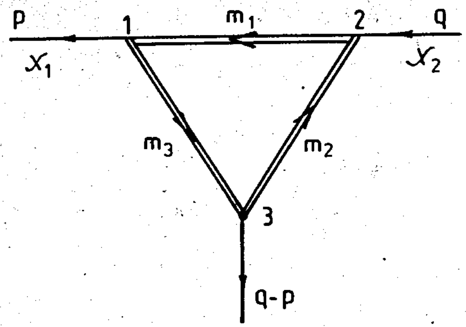
4. Vertex diagrams

The technique of the calculation of the diagrams shown below is very much alike to that suggested in paper /10/.

1.



$$\Gamma(p^2, q^2, (p-q)^2) = \frac{i f_1^{3\chi} f_2^{3\chi} f_3^{3\chi}}{16\pi^2} I_1(q^2, (p-q)^2, p^2, M_\chi^2, M_{\chi'}^2, M_{\chi''}^2). \qquad (4.1)$$

2. 

and a similar diagram with the opposite lepton current direction.

$$\Gamma\left(p^2,q^2,(p-q)^2\right)=\frac{i f_1^{\chi} f_2^{\chi} f_3^{\chi}}{4\pi^2}\Big\{\left[-2m_1B_1-2m_2B_2-2m_3B_3\right]\cdot P-\frac{m_1B_1+m_2B_2+m_3B_3}{2}-\frac{m_1B_1+m_2B_2}{2}\bar{I}_0\left(q^2,m_1^2,m_2^2\right)-$$

$$-\frac{m_1B_1+m_3B_3}{2}\bar{I}_0\left(p^2,m_1^2,m_3^2\right)-\frac{m_2B_2+m_3B_3}{2}\bar{I}_0\left((p-q)^2,m_2^2,m_3^2\right)-\frac{1}{2}\Big[\left(m_1(p-q)^2+m_1m_2^2+m_1m_3^2\right)B_1+\left(m_2p^2+\right.$$

$$\left.+m_2m_1^2+m_2m_3^2\right)B_2+\left(m_3q^2+m_3m_1^2+m_3m_2^2\right)B_3+ \tag{4.2}$$

$$+2m_1m_2m_3B_4\Big]\bar{I}_1\left(q^2,(p-q)^2,p^2,m_1^2,m_2^2,m_3^2\right),$$

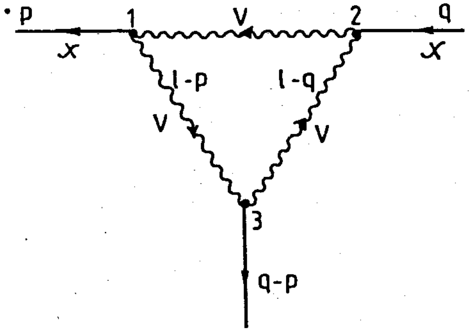
where

$$B_1=1+b_1b_2-b_1b_3-b_2b_3,$$

$$B_2=1-b_1b_2-b_1b_3+b_2b_3, \tag{4.3}$$

$$B_3=1-b_1b_2+b_1b_3-b_2b_3,$$

$$B_4=1+b_1b_2+b_1b_3+b_2b_3.$$

3. 

and a similar diagram with the opposite loop current direction.

$$\Gamma\left(p^2,q^2,(p-q)^2\right)=\frac{i f_1^{V\chi} f_2^{V\chi} f_3^{V\chi}}{16\pi^2}\Big\{\Big[-\frac{3}{4}\frac{p^2+q^2+(p-q)^2}{M_V^4}-$$

$$-\frac{1}{8}\frac{\left(p^2+q^2+(p-q)^2\right)^2}{M_V^6}\Big]P+\frac{1}{4}\frac{p^2+q^2+(p-q)^2}{M_V^4}\Big(1-$$

$$-\ln\frac{M_V^2}{M_W^2}\Big)-\Big[\frac{1}{4M_V^4}\Big(\frac{p^2+q^2-(p-q)^2}{2}+p^2-q^2\Big)+\frac{p^2}{8M_V^6}\cdot$$

$$\cdot\frac{p^2+q^2+(p-q)^2}{2}\Big]\bar{I}_0\left(p^2,M_V^2,M_V^2\right)-\Big[\frac{1}{4M_V^4}\Big(\frac{p^2+q^2-(p-q)^2}{2}+$$

$$+q^2-p^2\Big)+\frac{q^2}{8M_V^6}\frac{p^2+q^2+(p-q)^2}{2}\Big]\bar{I}_0\left(q^2,M_V^2,M_V^2\right)+\Big[\frac{1}{4M_V^4}\cdot$$

$$\Big(\frac{p^2+q^2-(p-q)^2}{2}-(p-q)^2\Big)-\frac{(p-q)^2}{8M_V^6}\frac{p^2+q^2+(p-q)^2}{2}\Big]\bar{I}_0\big((p-q)^2,$$

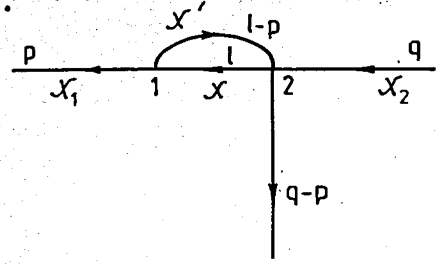
$$M_V^2,M_V^2\big)+\Big[3+\frac{p^2+q^2+(p-q)^2}{2M_V^2}+\frac{p^4+q^4-p^2q^2+(p-q)^4-p^2(p-q)^2-q^2(p-q)^2}{4M_V^4}-$$

$$-\frac{p^2q^2(p-q)^2}{8M_V^6}\Big]\bar{I}_1\left(q^2,(p-q)^2,p^2,M_V^2,M_V^2,M_V^2\right). \tag{4.4}$$

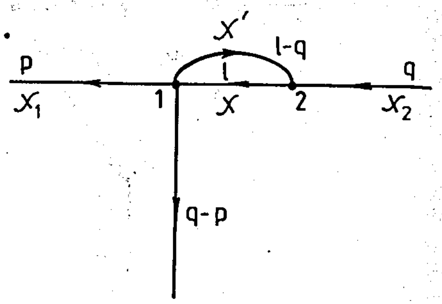
The form of $\bar{I}_1$ is given in Appendix A.

The diagrams

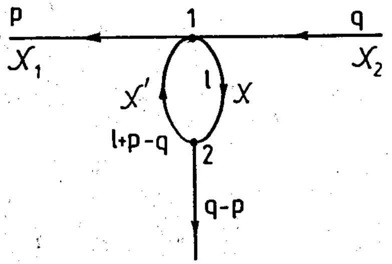
4.



5.

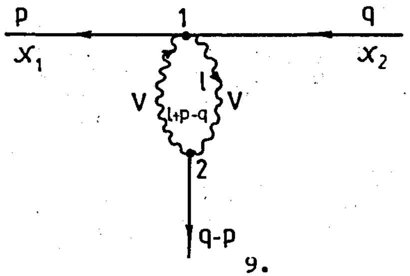


6.

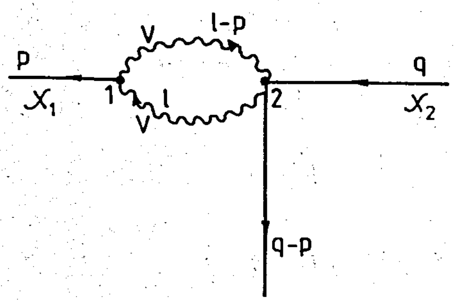


and

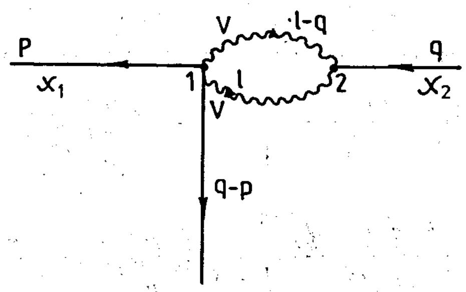
7.



8.



9.



can be derived from the self-energy diagrams easily (see Sect.3).

As a result, we get

$$
\begin{aligned}
\Gamma(p^2,q^2,(p-q)^2) &= \frac{ig^3}{16\pi^2}\frac{M_\chi^2}{M_W}\Big\{\Big[\frac{3}{4}\,\frac{p^2q^2+p^2(p-q)^2+q^2(p-q)^2}{M_W^2M_\chi^2} - \\
&- \frac{27}{8}r_W - 9r_W^{-1} - \frac{9}{2}\frac{1}{R}r_Z^{-1} + 6\frac{1}{M_W^2M_\chi^2}\,Tr\, m_i^2\Big]P + t - \\
&- \frac{p^2+q^2+(p-q)^2}{4M_\chi^2} - \frac{p^2+q^2+(p-q)^2}{8M_\chi^2}\frac{1}{R}(1+\ln R) - \\
&- 3r_W^{-1} - \frac{3}{2}\frac{1}{R}r_Z^{-1} + \frac{3}{2}\frac{1}{M_W^2M_\chi^2}\,Tr\, m_i^4 + \Big[-\frac{3}{2}r_W^{-1} - \\
&- \frac{q^2+(p-q)^2-p^2}{4M_\chi^2} + \frac{p^2(q^2+(p-q)^2)}{8M_W^2M_\chi^2}\Big]\bar{I}_0(p^2,M_W^2,M_W^2) + \\
&+ \Big[-\frac{3}{2}r_W^{-1} - \frac{p^2+(p-q)^2-q^2}{4M_\chi^2} + \frac{q^2(p^2+(p-q)^2)}{8M_W^2M_\chi^2}\Big]\bar{I}_0(q^2, \\
&M_W^2,M_W^2) + \Big[-\frac{3}{2}r_W^{-1} - \frac{p^2+q^2-(p-q)^2}{4M_\chi^2} + \frac{(p-q)^2(p^2+q^2)}{8M_W^2M_\chi^2}\Big]\times \\
&\times\bar{I}_0((p-q)^2,M_W^2,M_W^2) + \Big[-\frac{3}{4}\frac{1}{R}r_Z^{-1} - \frac{q^2+(p-q)^2-p^2}{8M_\chi^2}\frac{1}{R} + \\
&+ \frac{p^2(q^2+(p-q)^2)}{16M_W^2M_\chi^2}\Big]\bar{I}_0(p^2,M_Z^2,M_Z^2) + \Big[-\frac{3}{4}\frac{1}{R}r_Z^{-1} - \\
&- \frac{p^2+(p-q)^2-q^2}{8M_\chi^2}\frac{1}{R} + \frac{q^2(p^2+(p-q)^2)}{16M_W^2M_\chi^2}\Big]\bar{I}_0(q^2,M_Z^2,M_Z^2) + \\
&+ \Big[-\frac{3}{4}\frac{1}{R}r_Z^{-1} - \frac{p^2+q^2-(p-q)^2}{8M_\chi^2}\frac{1}{R} + \frac{(p-q)^2(p^2+q^2)}{16M_W^2M_\chi^2}\Big]\times
\end{aligned}
$$

$$\cdot I_0\left((p-q)^2, M_Z^2, M_Z^2\right) - \frac{9}{16} r_W \left[I_0(p^2, M_\chi^2, M_\chi^2) + I_0(q^2, M_\chi^2, M_\chi^2) + I_0\left((p-q)^2, M_\chi^2, M_\chi^2\right)\right] + \frac{1}{M_W^2 M_\chi^2} \operatorname{Tr} m_i^4 \left[I_0(p^2, m_i^2, m_i^2) + I_0(q^2, m_i^2, m_i^2) + I_0\left((p-q)^2, m_i^2, m_i^2\right)\right] - \frac{27}{8} r_W M_\chi^2 \cdot$$

$$\cdot I_1\left(q^2, (p-q)^2, p^2, M_\chi^2, M_\chi^2, M_\chi^2\right) - \left[6 r_W^{-1} M_W^2 + \left(p^2+q^2+(p-q)^2\right) r_W^{-1} + \frac{p^4+q^4+(p-q)^4 - p^2q^2 - p^2(p-q)^2 - q^2(p-q)^2}{2M_\chi^2} - \frac{p^2q^2(p-q)^2}{4M_W^2M_\chi^2}\right] I_1\left(q^2, (p-q)^2, p^2, M_W^2, M_W^2, M_W^2\right) -$$

$$- \left[3\frac{1}{R} r_Z^{-1} M_Z^2 + \frac{p^2+q^2+(p-q)^2}{2}\frac{1}{R} r_Z^{-1} + \frac{p^4+q^4+(p-q)^4}{4M_\chi^2} \cdot \frac{1}{R} - \frac{p^2q^2+p^2(p-q)^2+q^2(p-q)^2}{4M_\chi^2}\frac{1}{R} - \frac{p^2q^2(p-q)^2}{8M_W^2M_\chi^2}\right] \cdot$$

$$\cdot I_1\left(q^2, (p-q)^2, p^2, M_Z^2, M_Z^2, M_Z^2\right) + \frac{p^2+q^2+(p-q)^2}{2} \cdot \frac{1}{M_W^2M_\chi^2} \operatorname{Tr} m_i^4 \, I_1\left(q^2, (p-q)^2, p^2, m_i^2, m_i^2, m_i^2\right) + \frac{4}{M_W^2M_\chi^2} \cdot \operatorname{Tr} m_i^6 \, I_1\left(q^2, (p-q)^2, p^2, m_i^2, m_i^2, m_i^2\right)\Bigg\}. \tag{4.5}$$

The appropriate counter-term is

$$\Gamma^{c.t.} = -\frac{3}{2} g \frac{M_\chi^2}{M_W}\left[\frac{3}{2}(Z_\chi - 1) + \frac{\delta g}{g} + \frac{\delta M_\chi^2}{M_\chi^2} - \frac{1}{2}\frac{\delta M_W^2}{M_W^2}\right]. \tag{4.6}$$

As seen from /6/

$$\frac{\delta g}{g} = Z_A^{-1/2}\left(1 - \frac{\delta R}{1-R}\right)^{-1/2} - 1 \simeq \frac{1}{2}\left[\frac{\delta R}{1-R} - (Z_A - 1)\right], \tag{4.7}$$

$$\frac{\delta R}{R} = \frac{Z_{M_W} Z_W^{-1}}{Z_{M_Z} Z_Z^{-1}} - 1 \simeq \frac{\delta M_W^2}{M_W^2} - \frac{\delta M_Z^2}{M_Z^2} \tag{4.8}$$

and

$$\frac{\delta M_W^2}{M_W^2} = Z_{M_W} - Z_W = \frac{ig^2}{16\pi^2}\Bigg\{\left[\frac{34}{3} - \frac{3}{2}\frac{1}{R} - \frac{1}{3}N_f + \frac{1}{M_W^2}\operatorname{Tr} m_i^2\right] P - 19 + \frac{14}{9} + \frac{23}{12}\frac{1}{R} + \frac{1}{12}\frac{1}{R^2} - \frac{1}{2} r_W +$$

$$+ \frac{1}{12} r_W^2 + \left(-\frac{7}{2} + \frac{7}{12}\frac{1}{R} + \frac{1}{24}\frac{1}{R^2}\right)\frac{1}{R}\ln R + \left(-\frac{3}{4} + \frac{1}{4} r_W - \frac{1}{24} r_W^2\right) r_W \ln r_W + \frac{7}{36} N_f - \frac{1}{12}\frac{1}{M_W^2}\operatorname{Tr} m_i^2 -$$

$$- \frac{1}{6}\frac{1}{M_W^4}\operatorname{Tr} m_i^4 + \sum_{i,j}^{\frac12 N_f}\left[\frac{1}{3}\frac{m_i^2 m_j^2}{M_W^4} - \left(\frac{1}{3} - \frac{m_i^2 + m_j^2}{2M_W^2}\right) \cdot \ln\frac{m_i m_j}{M_W^2} + \frac{(m_i^2 - m_j^2)^3}{12 M_W^6}\ln\frac{m_i^2}{m_j^2} + \left(\frac{1}{6} - \frac{m_i^2 + m_j^2}{12 M_W^2} - \right.\right.$$

$$\left.\left. - \frac{(m_i^2 - m_j^2)^2}{12 M_W^4}\right)\frac{1}{M_W^2} L(-M_W^2, m_i^2, m_j^2)\right] K_{ij} K_{ij}^+ + \left(\frac{1}{2} - \frac{r_W}{6} + \frac{r_W^2}{24}\right)\frac{1}{M_W^2} L(-M_W^2, M_W^2, M_\chi^2) + \left(-\frac{17}{6} - 2R + \frac{2}{3}\frac{1}{R} + \right.$$

$$+\frac{1}{24}\frac{1}{R^2}\Big)\frac{1}{M_W^2}L(-M_W^2,M_W^2,M_Z^2)\Big\}\ , \qquad (4.9)$$

$$\frac{\delta M_Z^2}{M_Z^2} = Z_{M_Z} - Z_Z = \frac{ig^2}{16\pi^2}\Big\{\Big[-\frac{7}{3}+14R-\frac{11}{6}\frac{1}{R}-\frac{1}{3}\Big(2-\frac{1}{R}\Big)\times$$

$$\times N_f - \frac{8}{3}\frac{(1-R)^2}{R}Tr\, Q_i^2 + \frac{1}{M_W^2}Tr\, m_i^2\Big]P + \frac{35}{18}-\frac{34}{3}R -$$

$$- 8R^2 + \frac{35}{18}\frac{1}{R} - \frac{1}{2} r_W + \frac{1}{12}r_Z r_W + \frac{5}{6}\frac{1}{R}\ln R +$$

$$+ \Big(-\frac{3}{4}+\frac{1}{4} r_Z - \frac{1}{24}r_Z^2\Big) r_W \ln r_Z + \Big(\frac{7}{36}N_f - \frac{14}{9}\times$$

$$\times Tr\, Q_i^2\Big)\Big(2-\frac{1}{R}\Big) + \frac{14}{9}R\,Tr\, Q_i^2 + \frac{1}{M_W^2}Tr\Big[\Big(-\frac{1}{12} -$$

$$- \frac{8}{3}|Q_i| + \frac{16}{3}Q_i^2\Big)m_i^2 + \Big(\frac{8}{3}|Q_i| - \frac{32}{3}Q_i^2\Big)R m_i^2 +$$

$$+ \frac{16}{3}Q_i^2R^2m_i^2\Big] + \frac{1}{M_W^2}Tr\Big[\frac{1}{2} m_i^2 - \frac{1}{3} M_Z^2\Big(\frac{1}{2} - 2|Q_i| +$$

$$+ 4Q_i^2\Big) - \frac{1}{3} M_W^2\big(2|Q_i|-8Q_i^2\big) - \frac{4}{3}\frac{M_W^4}{M_Z^2}Q_i^2\Big]\ln\frac{m_i^2}{M_W^2} +$$

$$+ \Big(\frac{1}{24}+\frac{2}{3}R-\frac{17}{6}R^2-2R^3\Big)\frac{1}{M_W^2}L(-M_Z^2,M_W^2,M_W^2) +$$

$$+ \Big(\frac{1}{2}-\frac{1}{6} r_Z+\frac{1}{24}r_Z^2\Big)\frac{1}{M_W^2}L(-M_Z^2,M_Z^2,M_\chi^2) + Tr\Big[\Big(\frac{1}{12} -$$

$$- \frac{1}{3}|Q_i| + \frac{2}{3}Q_i^2\Big) + \Big(\frac{1}{3}|Q_i|-\frac{4}{3}Q_i^2\Big)R + \frac{2}{3} Q_i^2R^2 +$$

$$+ \Big(\frac{1}{12}-\frac{4}{3}|Q_i|+\frac{8}{3}Q_i^2\Big)\frac{m_i^2}{M_Z^2} + \Big(\frac{4}{3}|Q_i|-\frac{16}{3}Q_i^2\Big)R\frac{m_i^2}{M_Z^2} +$$

$$+ \frac{8}{3}Q_i^2R^2\frac{m_i^2}{M_Z^2}\Big]\frac{1}{M_W^2}L(-M_Z^2,m_i^2,m_i^2)\Big\} \qquad (4.10)$$

and, finally,

$$Z_A - 1 = \frac{ie^2}{16\pi^2}\Big\{\Big[-14 + \frac{8}{3}Tr\, Q_i^2\Big]P + \frac{2}{3}\big(1+Tr\, Q_i^2\big) +$$

$$+ \frac{4}{3} Tr\, Q_i^2 \ln\frac{m_i^2}{M_W^2}\Big\}^{(*)}\ , \qquad (4.11)$$

thus

$$\frac{\delta g}{g} = \frac{ig^2}{16\pi^2}\Big\{\Big[\frac{43}{6} - \frac{1}{6}N_f\Big]P - \frac{1}{3}(1-R)\Big(1+Tr\, Q_i^2 +$$

$$+ 2\,Tr\, Q_i^2 \ln\frac{m_i^2}{M_W^2}\Big) + \frac{1}{2}\frac{R}{1-R}\big[W(-1) - Z(-1)\big]\Big\}, \qquad (4.12)$$

where $W(-1)$ and $Z(-1)$ are finite parts of the counterterms $\delta M_W^2/M_W^2$ and $\delta M_Z^2/M_Z^2$, respectively.

Then

$$\Gamma^{c.t}\big(p^2,q^2,(p-q)^2\big) = -\frac{ig^3}{16\pi^2}\,\frac{3}{2}\,\frac{M_\chi^2}{M_W}\Big\{\Big[-\frac{3}{2}r_W - 9r_W^{-1} -$$

---

x) Some difference of the renormalization constants from those given in [6] appears firstly due to the approximation $s,t,u, M_V^2, M_\chi^2 \gg m_f^2$ done there (and not used in our consideration) and secondly due to the different definition of the $f(d)$ –function (used in the trace calculations, see Appeneix A), that, as it is known [12], does not influence the physical quantities.

$$-\frac{9}{2}\frac{1}{R}r_z^{-1}+6\frac{1}{M_W^2M_\chi^2}\mathrm{Tr}\,m_i^4\Big]P+\frac{1}{2}\Big(\frac{R}{1-R}-1\Big)W(-1)-$$

$$-\frac{1}{2}\frac{R}{1-R}Z(-1)+\chi(-1)+\frac{3}{2}\chi^F(-1)-\frac{1}{3}(1-R)\times$$

$$\times\Big(1+\mathrm{Tr}\,Q_i^2+2\,\mathrm{Tr}\,Q_i^2\ln\frac{m_i^2}{M_W^2}\Big)\Big\}=\frac{ig^3}{16\pi^2}\frac{3}{2}\frac{M_\chi^2}{M_W}\Big\{\Big[-\frac{3}{2}\times$$

$$r_W-9r_W^{-1}-\frac{9}{2}\frac{1}{R}r_z^{-1}+6\frac{1}{M_W^2M_\chi^2}\mathrm{Tr}\,m_i^4\Big]P+\frac{49}{6}-$$

$$-\frac{28}{3}R-4R^2-\frac{25}{24}\frac{1}{R}-\frac{1}{24}\frac{1}{R^2}-\Big(\frac{10}{9}-\frac{10}{9}R\Big)\times$$

$$\times\mathrm{Tr}\,Q_i^2+\frac{29}{8}r_W-\frac{1}{4}r_W^2+\frac{1}{4}r_Wr_z+9r_W^{-1}+\frac{9}{2}\frac{1}{R}r_z^{-1}+$$

$$+\Big(-\frac{9}{8}+\frac{1}{8}r_z-\frac{1}{8}r_W+\frac{1}{48}r_W^2-\frac{1}{48}r_Wr_z-\frac{1}{48}r_z^2\Big)\times$$

$$\times r_W\ln r_W+\Big(-\frac{47}{12}+\frac{7}{3}\frac{1}{R}-\frac{1}{4}\frac{1}{R^2}-\frac{1}{48}\frac{1}{R^3}+\frac{3}{8}\times$$

$$\times r_z-\frac{1}{8}r_z^2+\frac{1}{48}r_z^3\Big)\frac{1}{1-R}\ln R+\Big(-\frac{1}{4}r_W+\frac{3}{8}\frac{1}{R}+$$

$$+\frac{9}{4}\frac{1}{R}r_z^{-1}\Big)\ln R+\frac{1}{M_W^2}\mathrm{Tr}\Big[\frac{5}{12}m_i^2+\Big(\frac{4}{3}|Q_i|-$$

$$-\frac{8}{3}Q_i^2\Big)Rm_i^2+\frac{8}{3}Q_i^2R^2m_i^2\Big]+\frac{1}{M_W^4}\mathrm{Tr}\Big[\frac{1}{12}m_i^4-$$

$$-\frac{1}{12}\frac{R}{1-R}m_i^4-\frac{13}{4}r_W^{-1}m_i^4\Big]+\mathrm{Tr}\Big[\frac{1}{4}\frac{m_i^2}{M_W^2}\Big(1-\frac{R}{1-R}\Big)+$$

$$+\frac{1}{12}\frac{1}{1-R}-\frac{1}{3}|Q_i|+3\frac{m_i^4}{M_W^2M_\chi^2}\Big]\ln\frac{m_i^2}{M_W^2}+\frac{R}{1-R}\Big(-\frac{1}{48}-$$

$$-\frac{1}{9}R+\frac{17}{12}R^2+R^3\Big)\frac{1}{M_W^2}L(-M_z^2,M_W^2,M_W^2)+\frac{R}{1-R}\times$$

$$\times\frac{1}{M_W^2}\Big(-\frac{1}{4}+\frac{1}{12}r_z-\frac{1}{48}r_z^2\Big)L(-M_z^2,M_z^2,M_\chi^2)+\frac{1}{1-R}\times$$

$$\times\Big(-\frac{1}{4}+\frac{1}{2}R+\frac{1}{12}r_W-\frac{1}{6}r_z+\frac{1}{24}r_Wr_z-\frac{1}{48}r_W^2\Big)\frac{1}{M_W^2}\times$$

$$\times L(-M_W^2,M_W^2,M_\chi^2)+\frac{1}{1-R}\Big(\frac{25}{12}-\frac{11}{6}R-2R^2-\frac{7}{24}\frac{1}{R}-$$

$$-\frac{1}{48}\frac{1}{R^2}\Big)\frac{1}{M_W^2}L(-M_W^2,M_W^2,M_z^2)+\Big(-\frac{1}{4}-\frac{1}{8}r_W^{-1}+\frac{3}{2}r_W^{-2}-$$

$$-\frac{9}{2}\frac{1}{r_W^2(r_W-4)}\Big)\frac{1}{M_W^2}L(-M_\chi^2,M_W^2,M_W^2)+\Big(-\frac{1}{8}-\frac{1}{16}r_z^{-1}+$$

$$+\frac{3}{4}r_z^{-2}-\frac{9}{4}\frac{1}{r_z^2(r_z-4)}\Big)\frac{1}{M_W^2}L(-M_\chi^2,M_z^2,M_z^2)+\frac{9}{8}\frac{1}{M_W^2}\times$$

$$\times L(-M_\chi^2,M_\chi^2,M_\chi^2)-\frac{1}{M_W^2}\mathrm{Tr}\Big[\frac{1}{24}\frac{R}{1-R}-\Big(\frac{1}{6}|Q_i|-\frac{1}{3}Q_i^2\Big)R$$

$$-\frac{1}{3}Q_i^2R^2+\frac{1}{24}\frac{R}{1-R}\frac{m_i^2}{M_z^2}-\Big(\frac{2}{3}|Q_i|-\frac{4}{3}Q_i^2\Big)\frac{m_i^2}{M_z^2}R-$$

$$-\frac{4}{3}Q_i^2R^2\frac{m_i^2}{M_z^2}\Big]L(-M_z^2,m_i^2,m_i^2)-\frac{1}{M_W^2}\mathrm{Tr}\Big[\frac{m_i^2}{8M_\chi^2}+\frac{7}{8}\frac{m_i^4}{M_\chi^4}\Big]\times$$

$$\times L(-M_\chi^2,m_i^2,m_i^2)+\frac{1}{2}\Big(\frac{R}{1-R}-1\Big)\sum_{i,j}^{1/2N_f}\Big[\frac{1}{3}\frac{m_i^2m_j^2}{M_W^4}-$$

$$-\Big(\frac{1}{3}-\frac{m_i^2+m_j^2}{2M_W^2}\Big)\ln\frac{m_i m_j}{M_W^2}+\frac{(m_i^2-m_j^2)^3}{12M_W^6}\ln\frac{m_i^2}{m_j^2}+\Big(\frac{1}{6}-\frac{1}{12}\frac{m_i^2+m_j^2}{M_W^2}-$$

$$-\frac{1}{12}\frac{(m_i^2-m_j^2)^2}{M_W^4}\Big)\frac{1}{M_W^2}L(-M_W^2,m_i^2,m_j^2)\Big]K_{ij}K_{ij}^+\Big\}. \qquad (4.13)$$

As a result, the renormalized vertex is

$$\Gamma^{Ren}(p^2,q^2,(p-q)^2)=\frac{ig^3}{16\pi^2}\frac{M_\chi^2}{M_W}\Big\{\Big[\frac{3}{4}\frac{p^2q^2+p^2(p-q)^2+q^2(p-q)^2}{M_W^2M_\chi^2}-$$

$$-\frac{9}{8}r_W+\frac{9}{2}r_W^{-1}+\frac{9}{4}\frac{1}{R}r_Z^{-1}-3\frac{1}{M_W^2M_\chi^2}\mathrm{Tr}\,m_i^4\Big]P-$$

$$-\frac{1}{4}\frac{p^2+q^2+(p-q)^2}{M_\chi^2}\Big(1+\frac{3}{2}\frac{1}{R}\Big)-\frac{1}{4}\frac{p^2q^2+p^2(p-q)^2+q^2(p-q)^2}{M_W^2M_\chi^2}\times$$

$$\times\Big(3+\frac{1}{2}\ln R\Big)-\frac{49}{4}+14R+6R^2+\frac{25}{16}\frac{1}{R}+$$

$$+\frac{1}{16}\frac{1}{R^2}-\frac{33}{16}r_W-\frac{15}{2}r_W^{-1}-\frac{15}{4}\frac{1}{R}r_Z^{-1}+\frac{3}{8}r_W^2$$

$$-\frac{3}{8}r_Wr_Z+\frac{5}{3}(1-R)\,\mathrm{Tr}\,Q_i^2-\frac{1}{M_W^2}\mathrm{Tr}\Big[\frac{5}{12}m_i^2+\Big(\frac{4}{3}|Q_i|-$$

$$-\frac{8}{3}Q_i^2\Big)Rm_i^2+\frac{8}{3}Q_i^2R^2m_i^2\Big]+\frac{1}{M_W^4}\mathrm{Tr}\Big[\frac{1}{12}m_i^4-$$

$$-\frac{1}{12}\frac{R}{1-R}m_i^4-\frac{3}{8}r_W^{-1}m_i^4\Big]+\Big(\frac{3}{8}r_W-\frac{9}{16}\frac{1}{R}-\frac{9}{8}\times$$

$$\times\frac{1}{R}r_Z^{-1}\Big)\ln R-\frac{1}{1-R}\Big(-\frac{47}{8}+\frac{7}{2}\frac{1}{R}-\frac{3}{8}\frac{1}{R^2}-$$

$$-\frac{1}{32}\frac{1}{R^3}+\frac{9}{16}r_Z-\frac{3}{16}r_Z^2+\frac{1}{32}r_Z^3\Big)\ln R+\Big(\frac{3}{16}r_W-$$

$$-\frac{3}{16}r_Z-\frac{1}{32}r_W^2+\frac{1}{32}r_Wr_Z+\frac{1}{32}r_Z^2\Big)r_W\ln r_W-$$

$$-\mathrm{Tr}\Big[\frac{1}{4}\Big(1-\frac{R}{1-R}\Big)\frac{m_i^2}{M_W^2}+\frac{1}{12}\frac{1}{1-R}-\frac{1}{3}|Q_i|+\frac{3}{2}\times$$

$$\times\frac{m_i^4}{M_W^2M_\chi^2}\Big]\ln\frac{m_i^2}{M_W^2}+\frac{1}{2}\frac{1}{p^2}L(p^2,M_W^2,M_W^2)\Big[-\frac{3}{2}r_W^{-1}-$$

$$-\frac{q^2+(p-q)^2-p^2}{4M_\chi^2}+\frac{p^2(q^2+(p-q)^2)}{8M_W^2M_\chi^2}\Big]+\frac{1}{2}\frac{1}{q^2}L(q^2,M_W^2,M_W^2)\times$$

$$\times\Big[-\frac{3}{2}r_W^{-1}-\frac{p^2+(p-q)^2-q^2}{4M_\chi^2}+\frac{q^2(p^2+(p-q)^2)}{8M_W^2M_\chi^2}\Big]+\frac{1}{2}\frac{1}{(p-q)^2}\times$$

$$\times L((p-q)^2,M_W^2,M_W^2)\Big[-\frac{3}{2}r_W^{-1}-\frac{p^2+q^2-(p-q)^2}{4M_\chi^2}+\frac{(p-q)^2(p^2+q^2)}{8M_W^2M_\chi^2}\Big]$$

$$+\frac{1}{2}\frac{1}{p^2}L(p^2,M_Z^2,M_Z^2)\Big[-\frac{3}{4}\frac{1}{R}r_Z^{-1}-\frac{q^2+(p-q)^2-p^2}{8M_\chi^2}\frac{1}{R}+$$

$$+\frac{p^2(q^2+(p-q)^2)}{16M_W^2M_\chi^2}\Big]+\frac{1}{2}\frac{1}{q^2}L(q^2,M_Z^2,M_Z^2)\Big[-\frac{3}{4}\frac{1}{R}r_Z^{-1}-\frac{1}{R}\times$$

$$\times\frac{p^2+(p-q)^2-q^2}{8M_\chi^2}+\frac{q^2(p^2+(p-q)^2)}{16M_W^2M_\chi^2}\Big]+\frac{1}{2}\frac{1}{(p-q)^2}L((p-q)^2,M_Z^2,$$

$$M_Z^2)\Big[-\frac{3}{4}\frac{1}{R}r_Z^{-1}-\frac{p^2+q^2-(p-q)^2}{8M_\chi^2}\frac{1}{R}+\frac{(p-q)^2(p^2+q^2)}{16M_W^2M_\chi^2}\Big]-$$

$$-\frac{9}{16} r_W \left[\frac{1}{2}\frac{1}{p^2} L(p^2, M_\chi^2, M_\chi^2) + \frac{1}{2}\frac{1}{q^2} L(q^2, M_\chi^2, M_\chi^2) + \frac{1}{2}\frac{1}{(p-q)^2} \times L((p-q)^2, M_\chi^2, M_\chi^2)\right] + \frac{1}{M_W^2} Tr\left[\frac{1}{16}\frac{R}{1-R} - \left(\frac14|Q_i| - \frac12 Q_i^2\right)R - \frac12 Q_i^2 R^2 + \frac{1}{16}\frac{R}{1-R}\frac{m_i^2}{M_Z^2} + \left(|Q_i| - 2Q_i^2\right)\frac{m_i^2}{M_Z^2} R - 2Q_i^2 R^2 \frac{m_i^2}{M_Z^2}\right] L(-M_Z^2, m_i^2, m_i^2) + \frac{1}{M_W^2} Tr\left[\frac{3}{16}\frac{m_i^2}{M_\chi^2} + \frac{21}{16}\frac{m_i^4}{M_\chi^4}\right] L(-M_\chi^2, m_i^2, m_i^2) + \frac{1}{M_W^2 M_\chi^2} Tr\, m_i^4 \left[\frac12 \frac{1}{p^2} \times L(p^2, m_i^2, m_i^2) + \frac12\frac{1}{q^2}L(q^2,m_i^2,m_i^2) + \frac12\frac{1}{(p-q)^2}L((p-q)^2, m_i^2, m_i^2)\right] - \frac34\left(\frac{R}{1-R} - 1\right)\sum_{i,j}^{1/2 N_f}\left[\frac13 \frac{m_i m_j}{M_W^4} - \left(\frac13 - \frac{m_i^2+m_j^2}{2M_W^2}\right)\ln\frac{m_i m_j}{M_W^2} + \frac{(m_i^2-m_j^2)^3}{12 M_W^6}\ln\frac{m_i^2}{m_j^2} + \left(\frac16 - \frac{m_i^2+m_j^2}{12M_W^2} - \frac{(m_i^2-m_j^2)^2}{12 M_W^4}\right)\frac{1}{M_W^2} L(-M_W^2, m_i^2, m_j^2)\right] K_{ij}K_{ij}^+ - \frac{27}{8} r_W M_\chi^2 \times I_1(q^2, (p-q)^2, p^2, M_\chi^2, M_\chi^2, M_\chi^2) - \left[6 r_W^{-1} M_W^2 + \left(p^2+q^2+(p-q)^2\right) r_W^{-1} + \frac12 \frac{p^4+q^4+(p-q)^4 - p^2q^2 - p^2(p-q)^2 - q^2(p-q)^2}{M_\chi^2} - \frac14 \frac{p^2q^2(p-q)^2}{M_W^2 M_\chi^2}\right] I_1(q^2,(p-q)^2,p^2,M_W^2,M_W^2,M_W^2) -$$

$$-\left[3\frac1R r_Z^{-1} M_Z^2 + \frac{p^2+q^2+(p-q)^2}{2}\frac1R r_Z^{-1} + \frac14 \frac{p^4+q^4+(p-q)^4}{M_\chi^2}\frac1R - \frac14 \frac{p^2q^2+p^2(p-q)^2+q^2(p-q)^2}{M_\chi^2}\frac1R - \frac18 \frac{p^2q^2(p-q)^2}{M_W^2M_\chi^2}\right] I_1\left(q^2, (p-q)^2, p^2, M_Z^2, M_Z^2, M_Z^2\right) + \frac{1}{M_W^2M_\chi^2} Tr\, m_i^4\left(4m_i^2 + \frac{p^2+q^2+(p-q)^2}{2}\right) I_1\left(q^2,(p-q)^2,p^2,m_i^2,m_i^2,m_i^2\right)\Big\}. \quad (4.14)$$

5. Summary

The expressions obtained here for the self-energy and vertex diagrams would allow us to derive (in a subsequent paper) renormalized expressions for the Higgs-Higgs interaction amplitudes in the next-to-leading order of perturbation theory.

Let us mention that the obtained formulae contain the matrix elements of the Kobayashi-Maskawa matrix K_{ij} . They appeared due to the renormalization after account was taken of vertex counter-terms.

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Appendix A. Integrals and Traces

The integrals I_0, I_1 and I_2 are taken from the t'Hooft-Veltman paper /9/

$$I_0(q^2, M_1^2, M_2^2) = \int_0^1 dx \ln\frac{q^2x(1-x)+M_1^2x+M_2^2(1-x)}{M_W^2} \quad \text{(A.1)}$$

$$= -2 + \ln\frac{M_1M_2}{M_W^2} - \frac12\frac{M_1^2-M_2^2}{q^2}\ln\frac{M_1^2}{M_2^2} + \frac{1}{2q^2}L(q^2,M_1^2,M_2^2),$$

where

$$L(q^2, M_1^2, M_2^2) = \left[(q^2+M_1^2+M_2^2) - 4M_1^2M_2^2\right] \int_0^1 \frac{dx}{q^2x(1-x)+M_1^2x+M_2^2(1-x)} . \qquad \text{(A.2)}$$

$$I_1\left(q^2, (p-q)^2, p^2, M_1^2, M_2^2, M_3^2\right) = \int_0^1 dx \int_0^x dy \left[ax^2+by^2+cxy+dx+ey+f\right]^{-1}, \qquad \text{(A.3)}$$

with

$$\begin{aligned} a &= -(p-q)^2 , & d &= M_2^2 - M_3^2 + (p-q)^2 \\ b &= -q^2 , & e &= M_1^2 - M_2^2 + 2pq - q^2 \\ c &= -2(pq-q^2) , & f &= M_3^2 - i\varepsilon . \end{aligned} \qquad \text{(A.4)}$$

$$I_2\left(q_1^2, q_2^2, p_2^2, p_1^2, s, t, M_1^2, M_2^2, M_3^2, M_4^2\right) = \int_0^1 dx \int_0^x dy \int_0^y dz \left[ax^2 + by^2 + gz^2 + cxy + hxz + jyz + dx + ey + kz + f\right]^{-2}, \qquad \text{(A.5)}$$

$$\begin{aligned} a &= -p_2^2 , & f &= M_4^2 - i\varepsilon \\ b &= -q_2^2 , & g &= -q_1^2 \\ c &= 2p_2q_2 , & h &= 2p_2q_1 \\ d &= M_3^2 - M_4^2 + p_2^2 , & j &= -2q_1q_2 \\ e &= M_2^2 - M_3^2 + q_2^2 - 2q_2p_2 , & k &= M_1^2 - M_2^2 + q_1^2 + 2q_1q_2 - 2q_1p_2 \end{aligned} \qquad \text{(A.6)}$$

The traces of γ -matrices in an d-dimensional space are

$$Tr\,\gamma_\alpha = Tr\,\gamma_5 = Tr\,\gamma_\alpha\gamma_\beta\gamma_5 = 0 , \qquad \text{(A.7)}$$

$$Tr\,\gamma_\alpha\gamma_\beta = f(d)\,\delta_{\alpha\beta} , \qquad \text{(A.8)}$$

$$Tr\,\gamma_\alpha\gamma_\beta\gamma_\rho\gamma_\sigma = f(d)\,d_{\alpha\beta\rho\sigma} , \qquad \text{(A.9)}$$

$$Tr\,\gamma_\alpha\gamma_\beta\gamma_\rho\gamma_\sigma\gamma_5 = f(d)\,\varepsilon_{\alpha\beta\rho\sigma} \qquad \text{(A.10)}$$

where

$$d_{\alpha\beta\rho\sigma} = \delta_{\alpha\beta}\delta_{\rho\sigma} - \delta_{\alpha\rho}\delta_{\beta\sigma} + \delta_{\alpha\sigma}\delta_{\beta\rho} \qquad \text{(A.11)}$$

In the present work we use $f(d) = 2\omega = 4 - 2\varepsilon$ (which is not crucial).

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