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A.E.Dorokhov, N.I.Kochelev*

THE RELATION BETWEEN THE PROTON QUARK SPIN AND THE η'-MESON COUPLING

* High Energy Physics Institute, Academy of Sciencies of Kazakh SSR, SU-480082 Alma-Ata, USSR

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Recently, the work[1] has appeared devoted to the problem of quark mass corrections in the relation between the quark spin and the η'-meson coupling to the nucleon. However, in our opinion this work contains some inaccuracies. So, the resulting expression (21)[1] when the quark masses are put to zero ($m_u = m_d = m_s = 0$) is not transformed to its chiral limit eq. (13)[1].

The aim of this work is to consider the $SU(3)_f$ violation in this relation and to estimate the spin carried by proton valence quarks.

Papers[2,3] expounds the massless case. There, we have the Adler-Bardeen anomaly:

$$\partial_\mu j^5_\mu = \partial_\mu K_\mu, \tag{1}$$

where $j^\mu_5 = \sum_{i=1}^{N_f} \bar{q}_i \gamma_\mu \gamma_5 q_i$ and the current

$$K_\mu = N_f \frac{\alpha_s}{2\pi} \epsilon_{\nu\mu\sigma\rho} A^a_\mu (\partial_\sigma A^a_\rho - \frac{g}{3} f_{abc} A^b_\sigma A^c_\rho).$$

is connected with the topological charge by

$$\partial_\mu K_\mu = N_F \frac{\alpha_s}{8\pi} G^a_{\mu\nu} \tilde{G}^a_{\mu\nu} \equiv Q.$$

The matrix element of eq. (1) over the proton is expressed as[2]

$$\bar{\psi}(p')\gamma_5\psi(p)\{2M_p G_1(q^2) + q^2 G_2(q^2)\} =$$
$$= \bar{\psi}(p')\gamma_5\psi(p)\{2M_p \tilde{G}_1(q^2) + q^2 \tilde{G}_2(q^2)\}, \tag{2}$$

where $\lim_{q^2\to 0} q^2 G_2(q^2) = 0$ and $\tilde{G}_2(q^2)$ has a pole when $q^2 \to 0$ due to the mixing of η'-meson and ghost state[3]. Thus, the relation between the spin carried by quarks and the η'-meson coupling is as follows[2,3]:

$$\Delta\Sigma = G_1(0) - \tilde{G}_1(0) = \frac{\sqrt{N_f} f_{\eta_0}}{2M_p} g_{\eta_0 NN}. \tag{3}$$

However, we think that this statement is not quite correct. This expression relates only the spin of valence quarks to the anomaly. Namely, the proton axial form factor is defind by two diagrams of Fig. 1. The l. h. s. of the relation eq. (3) is the contribution of interaction with valence quarks, while the r. h. s. of this relation (with the minus sign) describes the anomaly-induced contribution of the sea quarks.

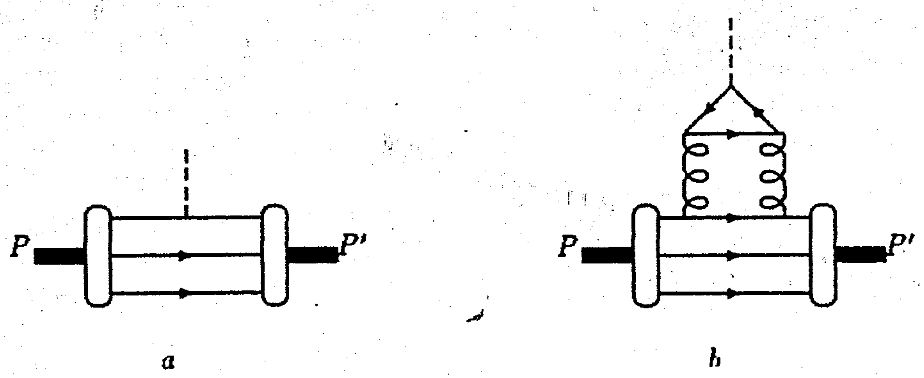

Figure 1: Interaction of the axial singlet current with a) valence quarks and b) sea quarks through the anomaly.

Now, let's consider the variation of eq. (3) under finite quark masses. To this end we should examine two effects: first, to add the mass terms to the r. h. s. of eq. (1):

$$\partial_\mu j^5_\mu = \partial_\mu K^\mu + \sum_f 2m_f \bar{q}_f i\gamma_5 q_f, \qquad (4)$$

and, second, to take into account the mixing in the coupling $f_{\eta'} g_{\eta' NN}$:

$$\frac{f_{\eta'} g_{\eta' NN}}{m^2_{\eta'}} \rightarrow \frac{f_{\eta_0} g_{\eta_0 NN}}{m^2_{\eta_0}} \cos\Theta_p - \frac{f_{\eta_8} g_{\eta_8 NN}}{m^2_{\eta_8}} \sin\Theta_p - \frac{f_\pi g_{\pi NN}}{m^2_{\pi_0}} \Theta_0,$$

where Θ_p, Θ_0 are the mixing angles of the η'-meson to η and π_0, respectively, and in addition the pole approximation is used.

So, at $m_u \neq m_d \neq m_s$ eq. (3) becomes

$$2 \quad M_p \Delta\Sigma - \sqrt{N_f} m^2_{\eta'} [\frac{f_{\eta_0} g_{\eta_0 NN}}{m^2_{\eta_0}} \cos\Theta_p - \frac{f_{\eta_8} g_{\eta_8 NN}}{m^2_{\eta_8}} \sin\Theta_p - \qquad (5)$$
$$- \frac{f_\pi g_{\pi NN}}{m^2_{\pi_0}} \Theta_0] = 2(m_u v_u + m_d v_d + m_s v_s),$$

where the definition

$$< p \mid \bar{q}_i i\gamma_5 q_i \mid p >= v_i \bar{\psi}(p) i\gamma_5 \psi(p)$$

is used.

Expressing the r. h. s. of eq. (5) through the combinations: $v_\pi = \frac{1}{\sqrt{2}}(v_u - v_d)$, $v_{\eta_0} = \frac{1}{\sqrt{3}}(v_u + v_d + v_s)$, $v_{\eta_8} = \frac{1}{\sqrt{6}}(v_u + v_d - 2v_s)$, we obtain:

$$m_u \quad v_u + m_d v_d + m_s v_s =$$
$$\frac{1}{\sqrt{3}} \quad (m_u + m_d + m_s) + \frac{1}{\sqrt{2}}(m_u - m_d) + \frac{1}{\sqrt{6}}(m_u + m_d - 2m_s). \quad (6)$$

By using the PCAC relations:

$$\sqrt{2} f_\pi g_{\pi NN} = 2(m_u - m_d), \sqrt{6} f_\eta g_{\eta NN} = 2(m_u + m_d - 2m_s) \qquad (7a)$$

and

$$v_{\eta_0} = \frac{1}{\sqrt{3}}(v_u + v_d + v_s) = 0, \qquad (7b)$$

we find in the first order in differences $m_u - m_d, m_d - m_s, m_u - m_s$:

$$2 \quad (m_u v_u + m_d v_d + m_s v_s) =$$
$$= \frac{\sqrt{2} f_\pi g_{\pi NN}(m_u - m_d)}{(m_u + m_d)} + \frac{\sqrt{6} f_{\eta_8} g_{\eta_8 NN}(m_u + m_d - 2m_s)}{(m_u + m_d + 4m_s)}. \qquad (8)$$

We should note here,that eq. (6b) has been used in some works[4] as a consequence of the EMC effect, the disappearence of the total helicity carried by quarks.

It is obvious that eq. (7) manifests the correct chiral limit behaviour while the r. h. s. of eq. (18) from ref. [1] does not.

Then, by using the current algebra relations:

$$m^2_\pi \quad = (m_u + m_d)K, m^2_{K+} = (m_u + m_s)K, \qquad (9)$$
$$m^2_{K^0} \quad = (m_d + m_s)K, m^2_{\eta_8} = \frac{1}{6}(m_u + m_d + 4m_s)K,$$

where K is a constant propotional to the quark condensate, eq. (7) can be expressed as:

$$2 \quad (m_u v_u + m_d v_d + m_s v_s) = \frac{\sqrt{2} f_\pi g_{\pi NN}(m^2_{K+} - m^2_{K^0} - m^2_{\pi+} + m^2_{\pi^0})}{m^2_\pi} -$$
$$- \frac{2\sqrt{2} f_{\eta_8} g_{\eta_8 NN}(m^2_K - m^2_\pi)}{\sqrt{3} m^2_{\eta_8}}. \qquad (10)$$

The mixing angles are defined by diagonalization of the mass matrix with the anomaly taken into account[6]:

$$M = \begin{pmatrix} m_s^2 & m_{18} & m_{13} \\ m_{81} & m_8^2 & m_{83} \\ m_{31} & m_{38} & m_\pi^2 \end{pmatrix}, \tag{11}$$

where $m_s^2 = m_{\eta'}^2 + m_\eta^2 - m_8^2$, $m_8^2 = \frac{1}{3}(4m_K^2 - m_\pi^2)$, $m_{81} = m_{18} = -\frac{2\sqrt{2}}{3}(m_K^2 - m_\pi^2)$, $m_{31} = m_{13} = \frac{2}{3}\delta m^2$, $m_{38} = m_{83} = \frac{1}{3}\delta m^2$, $\delta m^2 = (m_{K+}^2 - m_{K^0}^2 - m_{\pi+}^2 + m_{\pi^0}^2)$.

Thus, we obtain in the first order in mass differnces:

$$\Theta_p = -\frac{2\sqrt{2}(m_K^2 - m_\pi^2)}{3\Delta m_0^2}; \quad \Theta_0 = \sqrt{\frac{2}{3}}\frac{\delta m^2}{\Delta m_0^2}, \tag{12}$$

where Δm_0^2 is the mass of the η_0'-meson squared in the chiral limit. The result eq. (12) for the $\pi^0\eta'$ mixing angle Θ_0 doesn't also agree with the result of ref[1].

Upon substituting eq. (12) into eq. (5) the l. h. s. of eq. (5) becomes as follows:

$$2M_p\Delta\Sigma - \sqrt{N_f} f_{\eta_0} g_{\eta_0 NN} - \frac{2\sqrt{2}(m_K^2 - m_\pi^2)}{\sqrt{3}m_{\eta_8}^2} f_{\eta_8} g_{\eta_8 NN} + \sqrt{2}\delta m^2 \frac{f_\pi g_{\pi NN}}{m_\pi^2}. \tag{13}$$

By comparing eq. (9) and eq. (13) we see that all mass corrections are completely compensated! So, at least in the first order in $SU(3)_f$ violation eq. (3) does not changes.

Now, let's estimate the spin carried by valence quarks. More accurately, equation eq. (3) should be written as

$$2M\Delta\Sigma = \sqrt{3} f_{\eta_0}(0) g_{\eta_0 NN}(0), \tag{14}$$

where all constants are taken at $m_u = m_d = m_s = 0$.

In ref. [1,2] for estimating $\Delta\Sigma$ validity of the relation

$$f_{\eta_0} = f_\pi \tag{15}$$

has been assumed. However, this equality is correct only as $N_c \to \infty$[7], when the nonet symmetry is reconstructed, while in reality eq. (15) is violated due to the gluon anomaly which is of $1/N_c$ order.

Within the composite quark model with the interquark interaction through instantons proposed by us some years ago[8] the violation of eq. (15) can easily be understood. Really, the decay couplings f_i are inverse-propotional to the meson size[9]. Instantons provide strong repulsion for the η'-meson, which leads to the growth of its mass and, at the same time, attraction for the octet, which makes its massless. So, $f_{\eta'}(0) < f_\pi$. The model estimation provides the substantial suppression of $f_{\eta_0}(0) \approx 0.5 f_\pi$[8].

We think that one can obtain a more accurate estimation from the calculation of the width $\Gamma(\eta' \to 2\gamma)$ as it has been done in ref. [10]. There, it has been shown that if one assumes that

$$f_{\eta_0}(0) = f_\pi(1 - C_1/N_c), \tag{16}$$

then

$$\Gamma(\eta' \to 2\gamma) \approx \frac{1}{48\pi^3}[N_c N_f < Q_f^2 >]^2 \alpha^2 \frac{m_{\eta'}^3}{f_\pi^2}(1 - \frac{2C_1}{N_c}). \tag{17}$$

By comparing eq. (16) with experiment[11] [1] we have:

$$f_{\eta_0} \approx 0.8 f_\pi. \tag{18}$$

Substituting eq. (18) into eq. (14) and using $g_{\eta_0 NN} = 7.2$[12] we go to $\Delta\Sigma = 0.93$, that should compared with the result of ref. [2], $\Delta\Sigma = 1.14$.

Thus, we see that $1/N_c$ corrections substantially change the estimation of the spin carried by valence quarks in the proton. In our opinion, a more correct answer requires a more accurate determination of the variation of $f_{\eta'}$ outside the chiral limit.

So, $SU(3)_f$ violation does not change the relation between the spin of the proton valence quarks and the η'-meson coupling. At the same time, $1/N_c$ effects should be examined in this relation.

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[1] There arise questions concerning the $SU(3)_f$ violation in eq. (16), eq. (17). In ref. [10] it has been assumed that they compensate each other.

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