сообщения
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дубна

E2-90-537

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ON THE SPIN EFFECTS AT HIGH ENERGIES. THE QUARK-LOOP CONTRIBUTION

1990

At present, great interest in high energy physics stems from the spin phenomena. The modern theory of strong interaction has many difficulties there [1]. They are very crucial for QCD too. Really, the perturbative QCD leads to small polarization at large momenta transfer [2] which decreases as a power of s. This result is in contradiction with the experiment.

Here we are interested in small momenta transfer. It is obvious that the perturbative theory cannot be used in this region. The high energy hadron interaction at small angles can be investigated on the basis of different model approaches [3]. Some of them lead to the spin effects which do not disappear in the $s \to \infty$ limit in different processes, including elastic scattering [4,5]. In the case of elastic reactions the t-channel exchange with the vacuum quantum numbers (pomeron) contributes. So investigation of the pomeron spin structure is very important.

I would like to note that some large-distance contribution from the hadron wave function (see for example [6,7]) can lead to the spin effects which do not vanish at high energies. But in what follows we shall investigate the elementary elastic quark-quark subprocess that plays an important role in different hadron reactions determined by the pomeron contribution.

The amplitude with the vacuum quantum numbers in the t-channel is due to the two-gluon exchange [8]. The nonperturbative properties of the theory [9] which are important in this case where taken into account in the model [10]. The analysis of full matrix structure of the gluon-gluon interaction amplitude in the t-channel exchange leads in this model to the spin-flip amplitude growing as s [10].

$$T_{flip}(t) \simeq i \frac{s\sqrt{|t|}}{m^3} \phi_f(t). \qquad (1)$$

In the leading logarithmic approximation the contribution

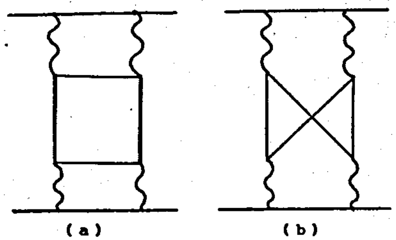

Fig.1 The quark-loop contribution to the imaginary part of the ladder diagrams in QED.

to the spin-non-flip amplitude from the diagrams (fig 1a,b) in QED (In QCD we have an additional graph /12/) is the following /11/

$$T_{\text{non-flip}}(t) \simeq i\, \frac{s \ln s/s_0}{m^2} \phi_{nf}(t). \qquad (2)$$

So the spin-flip amplitude (1) is suppressed with respect to the spin-non-flip amplitude (2) only logarithmically. This result was confirmed by calculation of contributions such as a gluon ladder, quark loops, $q\bar{q}$ sea /13,14/.

The result (1) is true for different individual ladder diagrams. But these growing as s terms of the spin-flip amplitude can compensate each other in the total sum of diagrams which obey the gauge invariance. To check the possibility of this compensation, one must to analyse a certain set of diagrams which can cancel only each other. An example of that set is the diagrams with quark loops in the t-channel exchange because they have the N_f factors in the leading terms.

In /12/ the spin effects were calculated in the diagrams with quark loops in QCD at large distances and it was shown that they are cancelled in the leading logarithmic approximation.

In this paper we shall calculate the nonleading terms of quark loops in the t-channel exchange in QED up to the end and show that the behaviour (1) really takes place in this case. Physical mechanisms leading to that behaviour of the spin-flip amplitude will be discussed too. Some long formulas which can be useful in future investigations are in the appendices.

Let us calculate the qq scattering

$$q(p_1)+q(p_2) \to q(p_3)+q(p_4)$$

at high energies and fixed momenta transfer. In calculations we shall use the symmetric coordinate system in which the sum of quark momenta before and after scattering is directed along the z-axis and the momenta transfer Δ-along x axis /9,10/

$$p=(p_1+p_3)/2=(p_0,0,0,p_z), \quad p'=(p_2+p_4)/2=(p_0,0,0,-p_z);$$

$$r=(p_1-p_3)/2=(p_2-p_4)/2=(0,r_x,0,0)=(0,-\Delta/2\ ,0,0);$$

$$(pr)=(p'r)=0, \quad p^2=p'^2=m^2+\Delta^2/4; \quad s\simeq 2(pp').$$

In what follows we shall investigate only the imaginary part of diagrams because the pomeron is approximately purely imaginary. In our model we suppose, following /9,10/, that nonperturbative effects are important in this region and use the following general representations for the quark and gluon propagators

$$G^{q}(p)=i(\hat{p}+m)D(-p^2), \qquad G^{g}_{\alpha\beta}(q)=-ig_{\alpha\beta}F(-q^2).$$

In the region we are interested in, the momenta p^2 and q^2 are small, and we do not use any concrete form for the functions $F(-q^2)$ and $G(-p^2)$. We assume only that these propagators become perturbative at large momenta. The connection between the nonperturbative part of the gluon propagator and gluon condensate is written in /9/. Some gluon and quark propagators in these calculations are on the mass shell. In this case we suppose for them the usual pole behaviour. As a result in the one-loop approximation only the diagram of fig.1 contributes. For the diagram, fig.1a, for a definite flavor in the quark loop we have

$$\mathrm{Im}T^a \simeq c^a \frac{g^8}{2(2\pi)^8} \int d^4k\, d^4q\, d^4l\ \delta[(p-k)^2-m^2]\delta[(p'+l)^2-m^2]$$
$$\delta[(k+q)^2-\sigma^2]\delta[(q+l)^2-\sigma^2]\ H^a\ S^a_{\lambda\mu,\nu\sigma}\ N^{\lambda\mu,\nu\sigma}, \qquad (3)$$

were m and σ are the masses of a constituent quark and a quark in the loop, respectively, c^a is a color factor,

$$H^a= F[-(k+r)^2]F[-(k-r)^2]F[-(l+r)^2]F[-(l-r)^2]$$
$$D[-(q+r)^2+\sigma^2]D[-(q-r)^2+\sigma^2],$$

$$S^a_{\lambda\mu,\nu\sigma}= \mathrm{Sp}\{\gamma_\lambda[\hat{q}+\hat{k}+\sigma]\gamma_\mu[\hat{q}-\hat{r}+\sigma]\gamma_\sigma[\hat{q}+\hat{l}+\sigma]\gamma_\nu[\hat{q}+\hat{r}+\sigma]\}, \qquad (4)$$

$$N^{\lambda\mu,\nu\sigma}=\bar{u}(p-r)\gamma^\lambda(\hat{p}-\hat{k}+m)\gamma^\mu u(p+r)\ \bar{u}(p'+r)\gamma^\nu(\hat{p}'+\hat{l}+m)\gamma^\sigma u(p'-r).$$

Both the Sudakov and the light-cone variables can be used for integrations in (3). We use the light-cone variables:

$$k=(xp_+,k_-,\vec{k}_\perp),\ l=(yp_+,l_-,\vec{l}_\perp),\ q=(-zp_+,q_-,\vec{q}_\perp),\ p_\pm=p_0\pm p_z.$$

The integration over k_-,l_-,q_-,y can be performed with the help of δ-functions. As a result we obtain the following solution for the pole quantities

$$k_-=[r_\perp^2-m^2x/(1-x)-k_\perp^2/(1-x)]/p_+;\ q_-=[(k_\perp+q_\perp)^2+\sigma^2]/(p_+(x-z))-k_-$$
$$l_-=[(l_\perp+q_\perp)^2+\sigma^2]/(p_+(y-z))-q_-,\ y\simeq(\Delta^2+m^2)/s. \qquad (5)$$

After integration in (3) we have

$$\mathrm{Im}T=\frac{c\ \alpha_s^4}{s(2\pi)^4}\int\frac{dxdz\ \theta(1-x)\theta(x-z)\theta(z-y)}{(1-x)(x-z)(z-y)}$$
$$\int d^2k_\perp d^2l_\perp d^2q_\perp H <S_{\lambda\mu,\nu\sigma}\ N^{\lambda\mu,\nu\sigma}>|_{k_-,l_-,q_-,y} \qquad (6)$$

where c is a colour factor of the diagram, when the momentum transfer is sufficiently small: $\Delta^2=4\ r^2<4\ m^2$, we find from (5) that $y\simeq m^2/s\to 0$. At the pole points (5) the quark and gluon propagators depend on the variables $\tilde{a}^2=a^2|_{k,l,q,y}$, which are the following for diagrams Fig 1a and Fig 1b:

$$-(k\tilde{\pm}r)^2=\{x^2m^2+[\vec{k}_\perp\pm(1-x)\vec{r}_\perp]^2\}/(1-x);\quad -(l\tilde{\pm}r)^2=(\vec{l}_\perp\pm\vec{r}_\perp)^2;$$
$$-(q\tilde{\pm}r)^2+\sigma^2=x/(x-z)[q_\perp^2+2\vec{q}_\perp(z/x\ \vec{k}_\perp\pm(x-z)/x\ \vec{r}_\perp)+h^a(x,z,k_\perp^2,r_\perp^2)].$$
$$-(k+q\tilde{+}l+r)^2+\sigma^2=x/z[q_\perp^2+2\vec{q}_\perp(z\vec{k}_\perp+x\vec{l}_\perp+z\vec{r}_\perp)/x+h^b(x,z,k_\perp^2,r_\perp^2)]. \qquad (7)$$

For simplicity we do not write the expressions for $h^{a,b}$ here.

In what follows we shall calculate the amplitude with the spin-flip in the upper quark line. The corresponding matrix element of the $<S\ N>$ product can be calculated with the help of the formulas from [12]. It can be shown that it has terms growing as s^2, which leads to the growing as s contribution in the amplitude (6).

It is easy to see that the integrals over $d^2k_\perp$, $d^2l_\perp$ are convergent in the upper limit. The main logarithmic asymptotics of (3,6) is connected with the integration over $d^2q_\perp$ near $q_\perp^2\simeq s$. In this region the momenta squared in the quark loop are large and we can use the asymptotically free quark propagators D.

The same form of propagators can be used in the intermediate momenta transfer $m^2<<\Delta^2<<s$ where the results obtained above are true. So in what follows we shall use the perturbative quark propagators in the loop. It is convenient to extract the term $x^2(1-x)$ in the denominator of the integral over dx. All other terms from the denominator of (6) and from propagators arguments in the loop (10) which depends on x and z are collected into the A coefficient (9). The matrix element of the SN product has the form

$$<S^a_{\lambda\mu,\nu\sigma}\ N^{\lambda\mu,\nu\sigma}>_{flip}=[\tilde{b}^a(x,z)\ q_\perp^2+\tilde{f}^a(x,z,q_\perp,k_\perp,\Delta)], \qquad (8)$$

Using (8) we can write the integral over $d^2q_\perp$ as follows

$$A\int_0^s\frac{d^2q_\perp(\tilde{b}q_\perp^2+\tilde{f}(\vec{q}_\perp))}{(q_\perp^2+2\vec{q}_\perp\vec{v}1+h1)(q_\perp^2+2\vec{q}_\perp\vec{v}2+h2)}=\pi\int_0^1 d\alpha\int_0^s\frac{dq'^2(bq'^2+f)}{(q'^2+h)^2}=$$
$$=\pi\int_0^1 d\alpha[b*(\ln(s/m^2)-1)-b\ln(h/m^2)+f/h]. \qquad (9)$$

Here we use the notation $f(\vec{q})=A\tilde{f}(\vec{q})$, $b=A\tilde{b}$ ($A^a=\frac{(x-z)}{z}$; $A^b=1$) and

$$f=f(\vec{q}_\perp=-\vec{v})+bv^2;\ h=\alpha h1+(1-\alpha)h2-v^2;\ \vec{v}=\alpha\vec{v}1+(1-\alpha)\vec{v}2.$$

The functions b,f,h can be found in Appendix A. With the help of (6,9) one can write the spin-flip amplitude as follows

$$T_{flip}(s,t)=im\Delta s[\ln s/s_0\ T_1(t)+T_0(t)+\ldots]. \qquad (10)$$

The logarithmic terms proportional to T_1 in (10) were calculated in [12]. They are not determined by the integration in the end-point region as in (2), but determined by the contribution from large transverse momenta on the corresponding loops. Similar terms can be seen in the spin-non- flip amplitudes but they are cancelled in the sum of diagrams [11]. This cancellation is true for the sum of the spin- flip contributions too [12] both in the case of QED and QCD. As a result the spin-flip amplitude is absent in the leading logarithmic approximation.

So one must calculate the nonlogarithmic term T_0 in (10) in order to check the existence of the growing as s contribution in the spin-flip amplitude. All these terms have a different structure and it is difficult to believe in full compensation of them. Now, accurate calculations of these contributions are impossible in QCD because the diagram propagators are in the nonperturbative region.

But in the case of quantum electrodynamics the calculation of the nonlogarithmic terms for the sum of diagrams (fig.1a,b) is possible. For simplicity we shall calculate the function $T_0(t)$ in (10) for zero momenta transfer. We may conclude that a term proportional to s exists in the spin-flip amplitude if $T_0(0)$ is not equal to zero.

Let us introduce the photon mass λ to avoid singularities at $t=0$. After some algebraic transformation one can write the functions $T_0^{a,b}(0)$ in the general form

$$T_0(0)= \frac{\alpha^4}{8\pi} \int_0^1 d\alpha \int_0^1 \frac{dx(1-x)}{x^2} \int_0^x dz \int_0^\infty \frac{dk^2}{(k^2+m_x^2)^2} \int_0^\infty \frac{dl^2}{(l^2+\lambda^2)^2}$$

$$\left[-B \ln(H_0/m^2) +F_0/H_0+F_1/H_0^2\right]. \qquad (11)$$

Here the functions B, F_0, F_1, H_0 for the diagrams Fig 1a,b are calculated in Appendix B, the quantity m_x^2 is the following

$$m_x^2=m^2x^2+\lambda^2(1-x). \qquad (12)$$

The integration over $d\alpha$, dl^2 is trivial for the diagram Fig 1a because the functions in (11) do not depend on α and l^2. As a result we have

$$T^a(0)=T_\lambda^a(0)/\lambda^2+T_0^a(0). \qquad (13)$$

The integrals over dk^2. dz was calculated with the help of the REDUCE program

$$T_0^a(0)= \frac{\alpha^4}{8\pi} \int_0^1 dx\phi^a(x) \qquad (14)$$

where

$$\phi^a(x)=-8[48\ln(1-x)x^6-114\ln(1-x)x^5+120\ln(1-x)x^4-90\ln(1-x)x^3$$
$$-60\ln(1-x)x^2+192\ln(1-x)x-96\ln(1-x)-48\ln(x)x^6+$$
$$54\ln(x)x^5+118x^5-275x^4+109x^3+144x^2-96x]/(45\, m^2x^4). \qquad (15)$$

Note that in calculations we use the $m=\sigma$ limit. Integration with another mass in the loop is possible but more complicated. In the integral over dk^2 we set $\lambda^2=0$. This can be done because the resulting integral (14) is convergent in the lower limit.

Investigation of the nonplanar integral is more

complicated because the integral functions depend on all variables. It is easy to see that in this case after integration over dl^2 we have not only $1/\lambda^2$ divergence but a $\ln(\lambda^2/m^2)$ term too. The simple procedure permits us to extract these terms. As a result we obtain for the diagram of fig. 1b[15]

$$T^b(0)=T_\lambda^b(0)/\lambda^2+T_{ln}^b(0)\ln\lambda^2/m^2+T_0^b(0). \qquad (16)$$

After integration, we find that $T_\lambda^b(0)=-T_\lambda^b(0)$ and the $1/\lambda^2$ terms compensate each other in the sum of contributions (fig.1a,b).

So we must compute the term $T_{ln}^b(0)$ in (16). After long but not complicated calculations of the integrals over $d\alpha$, dk^2, dz we find with the help of the REDUCE program

$$T_{ln}^b(0)= \frac{\alpha^4}{8\pi} \int_0^1 dx\phi_{ln}^b(x) \qquad (17)$$

where

$$\phi_{ln}^b(x)=-2[2\ln(1-x)x^8- 459\ln(1-x)x^7 +1435\ln(1-x)x^6- 245\ln(1-x)$$
$$x^5 -3885\ln(1-x)x^4+ 4088\ln(1-x)x^3 +2184\ln(1-x)x^2-$$
$$5280\ln(1-x)x +2160\ln(1-x)- 2\ln(x)x^8 +459\ln(x)x^7-$$
$$1435\ln(x)x^6 +1050\ln(x)x^5 +2x^7- 458x^6 +1282x^5- 1150x^4$$
$$+2364x^3- 4200x^2 +2160x]/ (315m_x{}^2m^2x^2(x- 2)). \qquad (18)$$

It is easy to see that the corresponding integral over dx has a divergence in the $\lambda=0$ limit. This leads to the additional $\ln(\lambda^2/m^2)$ term in it. To show this, we shall integrate (17) from ε to 1. After definition of the following integrals

$$C=\int_0^1 \frac{dx\ln(1-x)}{(x-2)}, \quad S=\int_0^1 \frac{dx\ln(1-x)}{x} \qquad (19)$$

we can express all integrals in (17) through (19) and analytical functions. For example

$$\int \frac{dx\ln(1-x)}{x^2(x-2)} =1/4\{C(x)-S(x)-2[x\ln(1-x)-\ln(1-x)-x\ln(x)]/x\},$$

where $C(x)$, $S(x)$ are the corresponding indefinite integrals in (19). As a result we obtain

$$T_{ln}^b(0)= \frac{\alpha^4}{144\pi m^4} (179 +24C+ 236S-120\ln(\varepsilon)).$$

The integrals (19) are connected with the Spence functions. Their magnitudes are well known

$$C=\pi^2/12, \quad S=-\pi^2/6,$$

It can be shown from the analysis of the integral (m_x^2 is determined in (12))

$$\int_{\varepsilon}^{1} \frac{dx\ x}{m_x^2}$$

that

$$ln(\varepsilon)=ln(\lambda^2/m^2)/2.$$

The resulting contribution to the total amplitude $T(0)$ in (16) from the $T_{1n}(0)$ is the following

$$T(0)= \frac{\alpha^4}{432\pi m^4}\ [537-112\pi^2-180ln(\lambda^2/m^2)]ln(\lambda^2/m^2). \qquad (20)$$

So the magnitude $T(0)$ is not equal to zero. The calculations of the constant terms in (13,16) can not change this result. Thus, the planar and nonplanar graphs have different dependences on momenta transfer. For the nonzero t, the logarithmic form $ln(\lambda^2/m^2)$ turns into $ln(\Delta^2/m^2)$. As a result there is no singularity at $\lambda=0$ in the spin-flip amplitude (10) at all $|t|$.

So we see that in quantum electrodynamics the contributions proportional to s are not compensated in the sum of diagrams (fig.1a,b) and the asymptotic form (1) is true for the spin- flip amplitude as $s\to\infty$, t-fixed. Thus we can conclude that the obtaned amplitude is suppressed only logarithmically with respect to the spin-non-flip amplitude (2). The behaviour like (1) must be expected in QCD too.

It seems that the heavy quarks in the loop can contribute in the diagrams Fig 1a,b. In this case we can use the perturbative quark propagators in the loop at all momenta transfer. It can be shown that after integration over d^2q in (9) only the terms from the ratio f/h in (9) which do not depend on σ^2 in the limit $\sigma^2\to\infty$ can contribute. As a result we obtain a heavy quark contribution to the spin-flip amplitude in the form

$$T^{heavy}_{flip}=2\frac{im\ \Delta\ s\ \alpha_s^4\ \pi}{(2\pi)^4} \int d^2l_\perp\ F[-(l+\tilde{r})^2]F[-(l-\tilde{r})^2]$$
$$\int_0^1 \frac{dx}{x^2(1-x)} \int d^2k_\perp\ F[-(k+\tilde{r})^2]F[-(k-\tilde{r})^2]I_{heavy}(x) \qquad (21)$$

From (A1,A2) we have

$$I^a_{heavy}=c^a\int_0^x dz16(x^3\ -2x^2\ +3xz\ -2z^2)=\ 8c^ax^3(6x-7)/3$$
$$I^b_{heavy}=c^b\int_0^x dz\ 8[-2x^3+\ 3x^2-4xz+4z^2+(2l_x x^2-4l_x xz)/\Delta]=$$
$$=-8c^bx^3(6x-7)/3 \qquad (22)$$

So we see that the terms (22) are compensated in the $\sigma^2\to\infty$ limit in QED. This permits us to conclude that the heavy quarks in the loops cannot contribute in QCD too.

Let us exemplify by the diagram (Fig.1a) the physical reason which leads to the growing as s contribution to the spin-flip amplitude. This diagram can be decomposed into two quark-quark subgraphs with the two-gluon exchange in the t-channel. The corresponding spin-flip amplitude in this case has the following energy dependence:

$$T^{qq}_{flip}(t)\simeq \frac{m\sqrt{|t|}}{s_{qq}}\ T^{qq}_{non-flip}(t)\ ,$$

where s_{qq} is the quark subprocess energy. It can be shown that in the integration region which contributes to the T_0 spin-flip amplitude the energies in the up and down subgraph are of the following order of magnitude:

$$s^{up}_{qq}\ \simeq\ m^2; \qquad s^{down}_{qq}\ \simeq\ s.$$

Thus the up quark subprocess is at low energies and the spin-flip amplitude has no energy suppression in it

$$T^{qq}_{flip}(t)\ \simeq\ \frac{\sqrt{|t|}}{m}\ T^{qq}_{non-flip}(t)\ .$$

The spin-non-flip amplitude growing as s contributes to the down quark subprocess. As a result, we obtain the behaviour (1,10) for the total diagram.

So, in QCD the terms $\approx s$ in the spin-flip amplitude are determined by the nonperturbative region in the up subprocess of the diagram. From our point of view the $q\bar{q}$ sea contribution can be very important here [12-15]. This can explain the success of the meson-cloud model [5] which takes into account similar effects phenomenologically.

Thus, the quark loop effects in gluon t-channel exchange and $q\bar{q}$ sea contributions lead to the spin-flip amplitude growing as s. A similar contribution can be obtained from the nonperturbative diquark state in the wave function for example[6]. In all cases that behaviour of the spin-flip amplitude is determined by the long-distance effects.

It is important to note that quark loops investigated here lead to the transverse polarization of the muon (electron) beam in the $\mu(e)+p\to\mu_{\uparrow}(e_{\uparrow})+X$ reactions which slowly changes with s growing. In this case the upper part of diagrams can be calculated by using QED, in the investigation of the lower part one can use some model for the nonperturbative gluon propagator (see [16] e.g.). This conclusion can be checked in the future $\mu(e)p$ experiments.

So it is shown that in QCD the spin effects which decrease very slowly (only logarithmically) with energy growth really can be obtained in the $s\to\infty$ limit. This means that the pomeron in QCD has a complicated spin structure. But it is necessary to use the properties of the theory at large distances to obtain some quantitative estimations.

The author expresses his deep gratitude to V.G.Kadyshevsky and V.A.Matveev for the interest in the work and support.

Appendix A

In this part of paper we shall write some formulas obtained for diagrams Fig 1a,b after integration over d^2q.

The REDUCE calculations lead to the following form for the functions b, f, h in (12) for the planar diagram Fig 1a

$$b^a=16m\Delta s^2(x^3 -2x^2z -x^2 +2xz^2 +2xz -2z^2); \qquad (A1)$$

$$\begin{aligned}
&f^a=\{16m\Delta s^2[4\alpha^2(rr)x^6 \quad -16\alpha^2(rr)x^5z \quad -8\alpha^2(rr)x^5 \quad +28\alpha^2(rr)x^4z^2\\
&+32\alpha^2(rr)x^4z \quad +4\alpha^2(rr)x^4 \quad -24\alpha^2(rr)x^3z^3 \quad -56\alpha^2(rr)x^3z^2\\
&-16\alpha^2(rr)x^3z \quad +8\alpha^2(rr)x^2z^4 \quad +48\alpha^2(rr)x^2z^3 \quad +28\alpha^2(rr)x^2z^2\\
&-16\alpha^2(rr)xz^4 \quad -24\alpha^2(rr)xz^3 \quad +8\alpha^2(rr)z^4 \quad +8\alpha(kr)x^4z \quad -24\alpha(kr)x^3z^2\\
&-8\alpha(kr)x^3z \quad +24\alpha(kr)x^2z^3 \quad +24\alpha(kr)x^2z^2 \quad -8\alpha(kr)xz^4 \quad -24\alpha(kr)xz^3\\
&+8\alpha(kr)z^4 \quad -4\alpha(rr)x^6 \quad +16\alpha(rr)x^5z \quad +8\alpha(rr)x^5 \quad -28\alpha(rr)x^4z^2\\
&-32\alpha(rr)x^4z \quad -4\alpha(rr)x^4 \quad +24\alpha(rr)x^3z^3 \quad +56\alpha(rr)x^3z^2 \quad +16\alpha(rr)x^3z\\
&-8\alpha(rr)x^2z^4 \quad -48\alpha(rr)x^2z^3 \quad -28\alpha(rr)x^2z^2 \quad +16\alpha(rr)xz^4\\
&+24\alpha(rr)xz^3 \quad -8\alpha(rr)z^4 \quad -(kk)x^3z \quad +4(kk)x^2z^2 \quad -5(kk)xz^3 \quad +2(kk)z^4\\
&-4(kr)x^4z \quad +12(kr)x^3z^2 \quad +4(kr)x^3z \quad -12(kr)x^2z^3 \quad -12(kr)x^2z^2\\
&+4(kr)xz^4 \quad +12(kr)xz^3 \quad -4(kr)z^4 \quad +\sigma^2x^6 \quad -3\sigma^2x^5 \quad +3\sigma^2x^4z \quad +2\sigma^2x^4\\
&-2\sigma^2x^3z^2 \quad -3\sigma^2x^3z \quad +2\sigma^2x^2z^2 \quad -m^2x^5z \quad +4m^2x^4z^2 \quad -5m^2x^3z^3 \quad +2m^2x^2z^4\\
&-(rr)x^5z \quad -(rr)x^5 \quad +4(rr)x^4z^2 \quad +5(rr)x^4z \quad +(rr)x^4 \quad -5(rr)x^3z^3
\end{aligned}$$

$$\begin{aligned}
&-11(rr)x^3z^2 \quad -4(rr)x^3z \quad +2(rr)x^2z^4 \quad +11(rr)x^2z^3 \quad +7(rr)x^2z^2\\
&-4(rr)xz^4 \quad -6(rr)xz^3 \quad +2(rr)z^4]\}/(x^2(x-1));
\end{aligned}$$

$$\begin{aligned}
&h^a=(-4\alpha^2(rr)x^3 \quad +8\alpha^2(rr)x^2z \quad +4\alpha^2(rr)x^2 \quad -4\alpha^2(rr)xz^2 \quad -8\alpha^2(rr)xz\\
&+4\alpha^2(rr)z^2 \quad +4\alpha(kr)x^2z \quad -4\alpha(kr)xz^2 \quad -4\alpha(kr)xz \quad +4\alpha(kr)z^2\\
&+4\alpha(rr)x^3 \quad -8\alpha(rr)x^2z \quad -4\alpha(rr)x^2 \quad +4\alpha(rr)xz^2 \quad +8\alpha(rr)xz \quad -4\alpha(rr)z^2\\
&-(kk)xz \quad +(kk)z^2 \quad -2(kr)x^2z \quad +2(kr)xz^2 \quad +2(kr)xz \quad -2(kr)z^2 \quad +\sigma^2x^3\\
&-\sigma^2x^2 \quad -m^2x^3z \quad +m^2x^2z^2 \quad -(rr)x^3z \quad +(rr)x^2z^2 \quad +2(rr)x^2z \quad -2(rr)xz^2\\
&-(rr)xz \quad +(rr)z^2)/(x^2(x-1)).
\end{aligned}$$

For the nonplanar diagram Fig 1b we have

$$\begin{aligned}
&b^b= 16s^2(l_xmx^2 \quad -2l_xmxz \quad -m\Delta x^3 \quad +2m\Delta x^2z \quad +m\Delta x^2 \quad -2m\Delta xz^2 \quad -2m\Delta xz \quad +\\
&\quad 2m\Delta z^2) \qquad (A2)
\end{aligned}$$

$$\begin{aligned}
&f^b=\{8s^2[-4\alpha^2(ll)l_xmx^5 \quad +8\alpha^2(ll)l_xmx^4z \quad +4\alpha^2(ll)l_xmx^4\\
&-8\alpha^2(ll)l_xmx^3z \quad +2\alpha^2(ll)l_xmx^5 \quad -4\alpha^2(ll)l_xmx^4z \quad -2\alpha^2(ll)l_xmx^4\\
&+4\alpha^2(ll)l_xmx^3z \quad -2\alpha^2(ll)m\Delta x^6 \quad +4\alpha^2(ll)m\Delta x^5z \quad +2\alpha^2(ll)m\Delta x^5\\
&-4\alpha^2(ll)m\Delta x^4z^2 \quad -4\alpha^2(ll)m\Delta x^3z \quad +4\alpha^2(ll)m\Delta x^2z^2 \quad +4\alpha^2m\Delta(lr)x^6\\
&-16\alpha^2m\Delta(lr)x^5z \quad -7\alpha^2m\Delta(lr)x^5 \quad +24\alpha^2m\Delta(lr)x^4z^2 \quad +26\alpha^2m\Delta(lr)x^4z\\
&+3\alpha^2m\Delta(lr)x^4 \quad -16\alpha^2m\Delta(lr)x^3z^3 \quad -36\alpha^2m\Delta(lr)x^3z^2 \quad -10\alpha^2m\Delta(lr)x^3z\\
&+24\alpha^2m\Delta(lr)x^2z^3 \quad +12\alpha^2m\Delta(lr)x^2z^2 \quad -8\alpha^2m\Delta(lr)xz^3 \quad -2\alpha^2m\Delta(rr)x^6\\
&+12\alpha^2m\Delta(rr)x^5z \quad +4\alpha^2m\Delta(rr)x^5 \quad -28\alpha^2m\Delta(rr)x^4z^2 \quad -24\alpha^2m\Delta(rr)x^4z\\
&-2\alpha^2m\Delta(rr)x^4 \quad +32\alpha^2m\Delta(rr)x^3z^2 \quad +56\alpha^2m\Delta(rr)x^3z^2 \quad +12\alpha^2m\Delta(rr)x^3z\\
&-16\alpha^2m\Delta(rr)x^2z^4 \quad -64\alpha^2m\Delta(rr)x^2z^3 \quad -28\alpha^2m\Delta(rr)x^2z^2\\
&+32\alpha^2m\Delta(rr)xz^4 \quad +32\alpha^2m\Delta(rr)xz^3 \quad -16\alpha^2m\Delta(rr)z^4 \quad +4\alpha(kl)l_xmx^5\\
&-4\alpha(kl)l_xmx^4z \quad -4\alpha(kl)l_xmx^4 \quad +4\alpha(kl)l_xmx^3z \quad -4\alpha(kl)l_xmx^5\\
&+8\alpha(kl)l_xmx^4z \quad +4\alpha(kl)l_xmx^4 \quad -8\alpha(kl)l_xmx^3z^2 \quad -8\alpha(kl)l_xmx^3z\\
&+8\alpha(kl)l_xmx^2z^2 \quad -2\alpha(kl)m\Delta x^4z \quad +2\alpha(kl)m\Delta x^3z \quad -4\alpha k_x(ll)mx^4z\\
&+8\alpha k_x(ll)mx^3z^2 \quad +4\alpha k_x(ll)mx^3z \quad -8\alpha k_x(ll)mx^2z^2 \quad +8\alpha k_xl_xm\Delta x^4z\\
&-12\alpha k_xl_xm\Delta x^3z^2 \quad -8\alpha k_xl_xm\Delta x^3z \quad +8\alpha k_xl_xm\Delta x^2z^3 \quad +12\alpha k_xl_xm\Delta x^2z^2\\
&-8\alpha k_xl_xm\Delta xz^3 \quad -6\alpha m\Delta(kr)x^4z \quad +24\alpha m\Delta(kr)x^3z^2 \quad +6\alpha m\Delta(kr)x^3z\\
&-32\alpha m\Delta(kr)x^2z^3 \quad -24\alpha m\Delta(kr)x^2z^2 \quad +16\alpha m\Delta(kr)xz^4 \quad +32\alpha m\Delta(kr)xz^3\\
&-16\alpha m\Delta(kr)z^4 \quad +2\alpha(ll)l_xmx^5 \quad -4\alpha(ll)l_xmx^4z \quad -2\alpha(ll)l_xmx^4\\
&+4\alpha(ll)l_xmx^3z \quad +2\alpha(ll)m\Delta x^6 \quad -4\alpha(ll)m\Delta x^5z \quad -2\alpha(ll)m\Delta x^5\\
&+4\alpha(ll)m\Delta x^4z^2 \quad +4\alpha(ll)m\Delta x^3z \quad -4\alpha(ll)m\Delta x^2z^2 \quad -4\alpha m\Delta(lr)x^6\\
&+16\alpha m\Delta(lr)x^5z \quad +7\alpha m\Delta(lr)x^5 \quad -24\alpha m\Delta(lr)x^4z^2 \quad -24\alpha m\Delta(lr)x^4z\\
&-3\alpha m\Delta(lr)x^4 \quad +16\alpha m\Delta(lr)x^3z^3 \quad +36\alpha m\Delta(lr)x^3z^2 \quad +8\alpha m\Delta(lr)x^3z\\
&-24\alpha m\Delta(lr)x^2z^3 \quad -12\alpha m\Delta(lr)x^2z^2 \quad +8\alpha m\Delta(lr)xz^3 \quad +2\alpha m\Delta(rr)x^6\\
&-12\alpha m\Delta(rr)x^5z \quad -5\alpha m\Delta(rr)x^5 \quad +28\alpha m\Delta(rr)x^4z^2 \quad +26\alpha m\Delta(rr)x^4z\\
&+3\alpha m\Delta(rr)x^4 \quad -32\alpha m\Delta(rr)x^3z^3 \quad -56\alpha m\Delta(rr)x^3z^2 \quad -14\alpha m\Delta(rr)x^3z\\
&+16\alpha m\Delta(rr)x^2z^4 \quad +64\alpha m\Delta(rr)x^2z^3 \quad +28\alpha m\Delta(rr)x^2z^2 \quad -32\alpha m\Delta(rr)xz^4
\end{aligned}$$

$$-32\alpha m\Delta(rr)xz^3 +16\alpha m\Delta(rr)z^4 +2(kk)l_x mx^3z -6(kk)l_x mx^2z^2 +4(kk)l_x mxz^3 +(kk)m\Delta x^3z -5(kk)m\Delta x^2z^2 +8(kk)m\Delta xz^3 -4(kk)m\Delta z^4 -2k_x l_x m\Delta x^4z +6k_x l_x m\Delta x^3z^2 +2k_x l_x m\Delta x^3z -4k_x l_x m\Delta x^2z^3 -6k_x l_x m\Delta x^2z^2 +4k_x l_x m\Delta xz^3 +4m\Delta(kr)x^4z -12m\Delta(kr)x^3z^2 -4m\Delta(kr)x^3z +16m\Delta(kr)x^2z^3 +12m\Delta(kr)x^2z^2 -8m\Delta(kr)xz^4 -16m\Delta(kr)xz^3 +8m\Delta(kr)z^4 -m\Delta(lr)x^5z +3m\Delta(lr)x^4z^2 +2m\Delta(lr)x^4z -2m\Delta(lr)x^3z^3 -6m\Delta(lr)x^3z^2 -m\Delta(lr)x^3z +4m\Delta(lr)x^2z^3 +3m\Delta(lr)x^2z^2 -2m\Delta(lr)xz^3 +2l_x\sigma^2mx^5 -4l_x\sigma^2mx^4z -2l_x\sigma^2mx^4 +4l_x\sigma^2mx^3z +2l_x m^3x^5z -6l_x m^3x^4z^2 +4l_x m^3x^3z^3 -2m\Delta\sigma^2x^6 +5m\Delta\sigma^2x^5 -4m\Delta\sigma^2x^4z -3m\Delta\sigma^2x^4 +4m\Delta\sigma^2x^3z^2 +4m\Delta\sigma^2x^3z -4m\Delta\sigma^2x^2z^2 +\Delta m^3x^5z -5\Delta m^3x^4z^2 +8\Delta m^3x^3z^3 -4\Delta m^3x^2z^4 +m\Delta(rr)x^5z +m\Delta(rr)x^5 -5m\Delta(rr)x^4z^2 -4m\Delta(rr)x^4z -m\Delta(rr)x^4 +8m\Delta(rr)x^3z^3 +11m\Delta(rr)x^3z^2 +3m\Delta(rr)x^3z -4m\Delta(rr)x^2z^4 -16m\Delta(rr)x^2z^3 -6m\Delta(rr)x^2z^2 +8m\Delta(rr)xz^4 +8m\Delta(rr)xz^3 -4m\Delta(rr)z^4]\}/(x^2(x-1))$$

$$h^b=(-\alpha^2(ll)x^3 +\alpha^2(ll)x^2 +2\alpha^2(lr)x^3 -4\alpha^2(lr)x^2z -2\alpha^2(lr)x^2 +4\alpha^2(lr)xz -\alpha^2(rr)x^3 +4\alpha^2(rr)x^2z +\alpha^2(rr)x^2 -4\alpha^2(rr)xz^2 -4\alpha^2(rr)xz +4\alpha^2(rr)z^2 +4\alpha(kr)x^2z -4\alpha(kr)xz^2 -4\alpha(kr)xz +4\alpha(kr)z^2 +\alpha(ll)x^3 -\alpha(ll)x^2 -2\alpha(lr)x^3 +4\alpha(lr)x^2z +2\alpha(lr)x^2 -4\alpha(lr)xz +\alpha(rr)x^3 -4\alpha(rr)x^2z -\alpha(rr)x^2 +4\alpha(rr)xz^2 +4\alpha(rr)xz -4\alpha(rr)z^2 -(kk)xz +(kk)z^2 -2(kr)x^2z +2(kr)xz^2 +2(kr)xz -2(kr)z^2 +\sigma^2x^3 -\sigma^2x^2 -m^2x^3z +m^2x^2z^2 -(rr)x^3z +(rr)x^2z^2 +2(rr)x^2z -2(rr)xz^2 -(rr)xz +(rr)z^2)/(x^2(x-1)).$$

Here we use the following notation: (ab) is a scalar production of $a_\perp$ and $b_\perp$. A similar representation was obtained for the diagram fig 1c, but it is more complicated. The formulas from this Appendix can be useful in future investigations.

Appendix B

In this appendix we shall obtain formulas for the integrals in zero momenta transfer limit. To do this, we must extract the kinematical factor Δ from all integrals and take the others functions at $r^2=0$ limit. This procedure is trivial for the diagram Fig 1a and we obtain for it representation (18) with the functions (Note that the factor $m\Delta s$ is picked out before the total integral (10)):

$$B^a=16(x^3 -2x^2z -x^2 +2xz^2 +2xz -2z^2); \qquad \text{(B1)}$$

$$F_0{}^a=\{16[-(kk)x^3z +4(kk)x^2z^2 -5(kk)xz^3 +2(kk)z^4 +\sigma^2x^6 -3\sigma^2x^5 +3\sigma^2x^4z +2\sigma^2x^4 -2\sigma^2x^3z^2 -3\sigma^2x^3z +2\sigma^2x^2z^2 -m^2x^5z +4m^2x^4z^2 -5m^2x^3z^3 +2m^2x^2z^4]\}/(x^2(x-1));$$

$$F_1 = 0;$$

$$H_0{}^a=[-(kk)xz +(kk)z^2 +\sigma^2x^3 -\sigma^2x^2 -m^2x^3z +m^2x^2z^2]/(x^2(x-1)).$$

To do this, in the case of diagram Fig 1b we must decompose the functions h^b, f^b (A3) which contain the terms proportional to r. After that the kinematical factor of the spin-flip amplitude is extracted. Note that from the symmetry point of view the terms linear in l, k in the numerators are falled out. As a result we obtain for the diagram Fig 1b representation (18) with the functions

$$B^b=16(-x^3 +2x^2z +x^2 -2xz^2 -2xz +2z^2); \qquad \text{(B2)}$$

$$F_0{}^b=\{8[-2\alpha^2(ll)x^6 +4\alpha^2(ll)x^5z +3\alpha^2(ll)x^5 -4\alpha^2(ll)x^4z^2 -4\alpha^2(ll)x^4z -\alpha^2(ll)x^4 +4\alpha^2(ll)x^3z^2 +2\alpha(ll)x^6 -4\alpha(ll)x^5z -3\alpha(ll)x^5 +4\alpha(ll)x^4z^2 +4\alpha(ll)x^4z +\alpha(ll)x^4 -4\alpha(ll)x^3z^2 +(kk)x^3z -5(kk)x^2z^2 +8(kk)xz^3 -4(kk)z^4 -2\sigma^2x^6 +5\sigma^2x^5 -4\sigma^2x^4z -3\sigma^2x^4 +4\sigma^2x^3z^2 +4\sigma^2x^3z -4\sigma^2x^2z^2 +m^2x^5z -5m^2x^4z^2 +8m^2x^3z^3 -4m^2x^2z^4]\}/(x^2(x-1));$$

$$F_1{}^b=\{8\alpha(ll)[-\alpha^3(ll)x^6 +4\alpha^3(ll)x^5z +2\alpha^3(ll)x^5 -4\alpha^3(ll)x^4z^2 -8\alpha^3(ll)x^4z -\alpha^3(ll)x^4 +8\alpha^3(ll)x^3z^2 +4\alpha^3(ll)x^3z -4\alpha^3(ll)x^2z^2 +2\alpha^2(ll)x^6 -8\alpha^2(ll)x^5z -4\alpha^2(ll)x^5 +8\alpha^2(ll)x^4z^2 +16\alpha^2(ll)x^4z +2\alpha^2(ll)x^4 -16\alpha^2(ll)x^3z^2 -8\alpha^2(ll)x^3z +8\alpha^2(ll)x^2z^2 -2\alpha(kk)x^4z^2 +\alpha(kk)x^4z +6\alpha(kk)x^3z^3 -\alpha(kk)x^3z^2 -\alpha(kk)x^3z -4\alpha(kk)x^2z^4 -4\alpha(kk)x^2z^3 +3\alpha(kk)x^2z^2 +4\alpha(kk)xz^4 -2\alpha(kk)xz^3 -\alpha(ll)x^6 +4\alpha(ll)x^5z +2\alpha(ll)x^5 -4\alpha(ll)x^4z^2 -8\alpha(ll)x^4z -\alpha(ll)x^4 +8\alpha(ll)x^3z^2 +4\alpha(ll)x^3z -4\alpha(ll)x^2z^2 +\alpha\sigma^2x^6 -4\alpha\sigma^2x^5z -2\alpha\sigma^2x^5 +4\alpha\sigma^2x^4z^2 +8\alpha\sigma^2x^4z +\alpha\sigma^2x^4 -8\alpha\sigma^2x^3z^2 -4\alpha\sigma^2x^3z +4\alpha\sigma^2x^2z^2 +\alpha m^2x^6z -5\alpha m^2x^5z^2 -\alpha m^2x^5z +8\alpha m^2x^4z^3 +5\alpha m^2x^4z^2 -4\alpha m^2x^3z^4 -8\alpha m^2x^3z^3 +4\alpha m^2x^2z^4 +(kk)x^4z^2 -(kk)x^4z -3(kk)x^3z^3 +3(kk)x^3z^2 +(kk)x^3z +2(kk)x^2z^4 -2(kk)x^2z^3 -4(kk)x^2z^2 +5(kk)xz^3 -2(kk)z^4 -\sigma^2x^6 +4\sigma^2x^5z +2\sigma^2x^5 -4\sigma^2x^4z^2 -8\sigma^2x^4z -\sigma^2x^4 +8\sigma^2x^3z^2 +4\sigma^2x^3z -4\sigma^2x^2z^2 -m^2x^6z +5m^2x^5z^2 +m^2x^5z -8m^2x^4z^3 -5m^2x^4z^2 +4m^2x^3z^4 +8m^2x^3z^3 -4m^2x^2z^4]\}/(x(x-1))^2;$$

$$H_0{}^b=[-\alpha^2(ll)x^3 +\alpha^2(ll)x^2 +\alpha(ll)x^3 -\alpha(ll)x^2 -(kk)xz +(kk)z^2 +\sigma^2x^3 -\sigma^2x^2 -m^2x^3z +m^2x^2z^2]/(x^2(x-1)).$$

A similar method can be used for obtaining derivatives of the amplitudes with respect to Δ^2 at zero momenta transfer.

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Received by Publishing Department
on November 30, 1990.