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HADRON-QUARK VERTEX FUNCTION. INTERCONNECTION BETWEEN 3D AND 4D WAVE FUNCTION

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I. Introduction

Two basic issues, Lorentz and gauge invariance, are of paramount importance to any realistic approach to strong interaction dynamics of which QCD is the leading contender to-day. This phenomenon is being witnessed in almost all QCD-motivated models, especially in lattice gauge theories where gauge invariance is meticulously observed (through Blaquette techniques), though a corresponding degree of confidence in the implementation of Lorentz invariance would probably remain in doubt until the limit $a \to 0$ is rigorously found. A related question concerns the desirability of a common underlying dynamical link all the way from low energy spectroscopy to high energy parton distributions, but it is not easy to achieve in practice. Spectroscopy (mostly $Q\bar{Q}$ systems) has been almost the exclusive preserve of N.R. potential models which have been sharpened over the years through built-in techniques of gauge invariance /1/, but they lack the Lorentz-invariant dynamics which is essential for carrying the extrapolation to the high energy domain. The latter is dominated by a different scenario - perturbative QCD, operator product expansions, empirical parton distribution, etc. QCD sum rules /2/ were no doubt a major step for downward extrapolation from the high energy end, but the actual (Borelization) techniques employed to determine the wave function somehow tended to "erase" the low energy link, so that the method stopped short of the prediction of L-excited hadron spectra, and had to rest content with that of some ground state masses. A dynamical equation based approach, on the other hand, does this job more naturally, precisely because of its built-in microcausal structure /3/. The links it provides between "low" and "high" energy physics are therefore more powerful in principle, and hence more reliable than in less microscopic methods, provided the conditions of Lorentz and gauge invariance are satisfied.

The need for a relativistic dynamical equation based approach is particularly acute in light-quark physics where the standard available tools are the Schwinger-Dyson (SDE) and/or the Bethe-Salpeter (BSE) equations /4/. However the very complexities of these equations have tended to encourage different versions mostly based on 3-D reduction techniques. A partial review of this history, including some classic references /5/, was given recently /6/. In particular the BSE has come to play an increasingly active role in the context of resurgence of the Nambu-Jonalasino (NJL) model /7/ of dynamical chiral symmetry breaking (DCSB) for generating constituent quark masses on the one hand, and almost massless pions on the other. Our preference for the BSE (with an effective kernel $\underline{K}$) stems basically from the ease with which it is able to keep the simultaneous link between spectroscopy and transition amplitudes through a vital interconnection /8/ between the wave function ϕ which satisfies a 3D form of the BSE (to determine the spectra) and the hadron quark vertex $\Gamma = D\phi$ which controls the structure of transition amplitudes on a direct 4D basis. ($D(\hat{q})$ is a universal denominator function in 3D form to be defined further below). This maintains Lorentz invariance, while gauge invariance can in principle be monitored through the interconnection that exists between the BS kernel $\underline{K}$ on the one hand and the Schwinger-Dyson vertex function Γ_μ on the other, via Ward-Takahashi identities. (Such possibilities have also been indicated by Miransky and co-workers /9/).

In this paper we do not address the issue of gauge invariance, but limit our study to the general structure of the BSE with an arbitrary Lorentz-invariant kernel, with a view to providing simultaneous access to spectroscopy and transition amplitudes via the interconnection noted above. This objective is certainly not new, having been addressed by Feynman et al. /10/ many years ago, and had also figured in our (two-tier) BS programme since the early Eighties /11/. In this paper we shall however refrain from making any explicit appeal to this model as such, but leave the structure general enough to bring out the basic connection between spectroscopy ("on-shell") and transition amplitudes ("off-shell"). In this respeot we shall find that a crucial role is played by the component $\hat{q}_\mu = q_\mu - P.q\, P_\mu / P^2$ of the internal 4-momentum q_μ which is always orthogonal to P_μ ($\hat{q}\cdot P = 0$) , irrespective of whether q_μ is on-shell ($q\cdot P = 0$) or off-shell ($q\cdot P \neq 0$). In view of this remarkable property of $\hat{q}_\mu$ which makes it an effectively 3D vector, our twin objective of (i) a 3D structure of the BSE /5/ as the controlling equation for the spectra, and (ii) a general enough (off-shell) structure of the BS vertex function $\Gamma(\hat{q})$ to facilitate applications to transition amplitudes in 4D form, is largely met if the BS kernel $\underline{K}$ depends on $\hat{q}_\mu$ rather than on q_μ. This formalism is briefly outlined in Seo.2, with emphasis on the generality of the vertex function $\Gamma(\hat{q}) = D(\hat{q}) \otimes \phi(\hat{q})$, where the denominator function $D(\hat{q})$ is universal, while only the 3D wave function $\phi(\hat{q})$ is kernel-dependent. The generality of this framework makes it amenable to the evaluation of matrix elements without requiring an explicit ansatz for $\phi(\hat{q})$, and is easily adaptable to the null-plane language /6/. <u>In Sec.3</u> we quickly trace the past history of previous approaches since FKR /10/, including our own, and draw attention to the new element provided by the role of the denominator function $D(\hat{q})$ as the connecting link between the 3D and 4D formulations. Among other things it reduces exactly to our earlier derivation of the 3D form of BSE in the on-shell ($q\cdot P = 0$) limit /12/, so as to be consistent (except for small off-shell corrections) with our previous results /13-15/ on the mass spectra for the special choice (Vector-Vector) for the kernel employed therein /6/, but the basic structure equally well accommodates other choices (e.g., Scalar-Scalar) for $\underline{K}$. The important gain is that the off-shell form of the Vertex function is now unambiguous, unlike our previous attempts /16/ which had left some ambiguishes on this score. <u>Sec.4</u> gives two simple applications, f_P values for $P \to \ell\bar{\ell}$ and F_π value for $\pi^0 \to 2\gamma$, as illustrations of the technology with the new vertex function $\Gamma(\hat{q})$ which expresses these amplitudes as 3D integrations over $d^3\hat{q}$. The difficulties with arbitrary extensions to bigger quark loops with more such vertex functions are traced to the appearance of unitarity cuts in the internal propagators. A possible cure to this basically infrared problem of confinement is suggested via a self-energy correotion to these propagators using the input kernel $K(\hat{q},\hat{q}')$ of this very formalism. <u>Seo.5</u> summarises our conclusions.

2. Interconnection between 4D and 3D Wave Funotions

Before addressing the dynamics of a $q\bar{q}$ hadron, it is convenient to collect some kinematical ingredients first for a composite of two spinless constituents /6/ since the inclusion of spin is a straightforward matter.

Kinematical preliminaries:

Let the constituents of masses m_1, m_2 and 4-momenta $p^\mu_{1,2}$ interacting through a suitable mechanism produoe a composite of 4-momentum P_μ and mass M . The internal 4-momentum q_μ is related to the individual ones by

$$p^\mu_{1,2} = \hat{m}_{1,2} \cdot P_\mu \pm q_\mu , \qquad (2.1)$$

where we have employed the Wightman-Garding (WG) definitions /17/ of the fractional momenta $\hat{m}_{1,2}$ as

$$\hat{m}_{1,2} = \frac{1}{2}\left[1 \pm \frac{m_1^2 - m_2^2}{M^2}\right]; \quad P^2 = -M^2 \tag{2.2}$$

At this stage we also introduce the inverse propagators $\Delta_{1,2}$ defined as

$$\Delta_{1,2} = m_{1,2}^2 + p_{1,2}^2 \tag{2.3}$$

associated with the two constituents. In accordance with our earlier approach /11-16/, we continue to use the constituent masses in (2.3) so that these inverse propagators are essentially non-perturbative in character. This is not inconsistent with the NJL picture /7/ which is already supposed to incorporate the effect of dynamical symmetry breaking (DCSB) in the definition of these masses. On the other hand, according to QCD perceptions /18/, these masses are supposed to "run" with their momenta, an effect we do not believe to be important for low and medium energy applications (e.g., the mass spectra), though it may well be so for high energy applications. For the moment however we shall consider these masses as constants in (2.3).

The WG definitions (2.2) ensure that on the mass shells of the respective constituents, the orthogonality condition

$$q \cdot P = 0 \tag{2.4}$$

is exactly satisfied even when $m_1 \neq m_2$. On the other hand it is possible to define a 4-vector $\hat{q}_\mu$ as

$$\hat{q}_\mu \equiv q_\mu - q \cdot P P_\mu / P^2 \tag{2.5}$$

such that

$$\hat{q} \cdot P \equiv 0 \tag{2.6}$$

irrespective of whether the individual (i=1,2) constituents are on shell ($\Delta_i = 0$) or off-shell ($\Delta_i \neq 0$).

Dynamical considerations

With the kinematical background, we formally define the 4D wave function $\Phi(P,q)$ of this system to the governed by a BSE with an effective kernel K /8/ :

$$i(2\pi)^4 \Delta_1 \Delta_2 \Phi(P,q) = \int d^4q' \, K(q,q') \, \Phi(P,q'), \tag{2.7}$$

where we now put an additional ansatz

$$K(q,q') \Rightarrow K(\hat{q},\hat{q}') \tag{2.8}$$

to emphasize the dependence of K on the quantities $\hat{q}, \hat{q}'$ each of which identically satisfies (2.6), in preparation for the objective stated in Sec.1. This equation does not of course represent the complete dynamics of the system in a field-theoretical sense, but may be regarded as merely a ladder approximation to the fuller Schwinger-Dyson equation, in the sense of an effective 4-fermion interaction of the NJL type /7/, though with an extended kernel. When the structure of the two-body kernel K is (more microscopically) defined, say in terms of the Schwinger-Dyson vertex function Γ_μ in QCD /18/, it should be possible to address the formal issues of gauge invariance within the BSE language, but this aspect will not be considered in this paper. On the other hand, the Lorentz invariance of (2.7) is explicit, even with the form (2.8). We now do a 3D reduction of (2.7), a task which is facilitated by noting that the longitudinal component $M\sigma$ of q_μ defined by

$$M\sigma \equiv M\, q \cdot P / P^2 \tag{2.9}$$

does not appear in the definition (2.8) of the kernel. In terms of $\hat{q}_\mu$ and σ we now have the following results:

$$\Delta_1 = m_1^2 + p_1^2 = \omega_1^2 - M^2(\hat{m}_1 + \sigma)^2$$
$$\Delta_2 = m_2^2 + p_2^2 = \omega_2^2 - M^2(\hat{m}_2 - \sigma)^2 \tag{2.10}$$

$$\omega_{1,2}^2 = m_{1,2}^2 + \hat{q}^2 \tag{2.11}$$

Now define a 3D wave function $\phi(\hat{q})$ as

$$\phi(\hat{q}) = \int_{-\infty}^{\infty} M d\sigma \, \Phi(P,q) \tag{2.12}$$

This definition can be directly incorporated on the RHS of Eq.(2.7) under the ansatz (2.8), since the variable σ' is not involved in K, and

$$d^4q' = d^3\hat{q}' \, M d\sigma' \tag{2.13}$$

so that Eq. (2.7) may be recast as:

$$(2\pi)^4 i\, \Delta_1 \Delta_2\, \Phi(P,q) = \int d^3\hat{q}'\, K(\hat{q};\hat{q}')\, \phi(\hat{q}') . \tag{2.14}$$

Dividing out by $\Delta_1\Delta_2$ in (2.14) and using once again the formula (2.12), gives an <u>exact</u> 3D reduction of the 4D form (2.7) as:

$$(2\pi)^3 D(\hat{q})\, \phi(\hat{q}) = \int d^3\hat{q}'\, K(\hat{q},\hat{q}')\, \phi(\hat{q}') . \tag{2.15}$$

where the 3D denominator function $D(\hat{q})$ is defined as

$$[D(\hat{q})]^{-1} \equiv \frac{1}{2\pi i} \int_{-\infty}^{\infty} d\sigma\, \frac{M}{\Delta_1 \Delta_2} \tag{2.16}$$

Integration over $d\sigma$ in (2.16) may be carried out by noting the following pole positions of $\Delta_{1,2}$ in the σ-plane in accordance with their representations (2.10):

$$\begin{aligned} \Delta_1^{\pm} &: \quad M(\sigma + \hat{m}_1) = \pm\, \omega_1 \mp i\varepsilon \\ \Delta_2^{\pm} &: \quad M(\sigma - \hat{m}_2) = \pm\, \omega_2 \mp i\varepsilon . \end{aligned} \tag{2.17}$$

The final result for D which is obtained by considering either the two poles $\Delta^{+}_{1,2}$, or the two poles $\Delta^{-}_{1,2}$ may be expressed <u>symmetrically</u> as follows:

$$D(\hat{q}) = D_0(\hat{q}) \Big/ \left[\frac{\hat{m}_1}{2\omega_1} + \frac{\hat{m}_2}{2\omega_2} \right] \tag{2.18}$$

$$\tfrac{1}{2} D_0(\hat{q}) = \omega_1^2 - M^2 \hat{m}_1^2 = \omega_2^2 - M^2 \hat{m}_2^2 \tag{2.19}$$

$$= \hat{q}^2 - \lambda(m_1^2, m_2^2, M^2)/4M^2 \tag{2.20}$$

$$\lambda = M^4 - 2M^2(m_1^2 + m_2^2) + (m_1^2 - m_2^2)^2 . \tag{2.21}$$

The more interesting thing is to observe the <u>equality</u> of the RHS of Eqs. (2.14) and (2.15) which provides the following <u>inter - connection</u> between the 4D wave function $\Phi(q)$ and its 3D counterpart $\phi(\hat{q})$:

$$\Gamma(\hat{q}) \equiv \Delta_1 \Delta_2\, \Phi(P,q) = D(\hat{q})\, \phi(\hat{q}) / 2\pi i . \tag{2.22}$$

Thus Eq.(2.22) directly defines the hadron-quark vertex function $\Gamma(\hat{q})$ in terms of the product $D \otimes \phi$ where the appearance of $\hat{q}_\mu$ (instead of q_μ) is a consequence of the ansatz (2.8).

The incorporation of the spin degree of freedom is a fairly straightforward matter. Indeed, under the on-shell conditions ($q\cdot P = 0$), see Sec.3 below, the NPA results of ref. 6 (§ 5,7) can be taken over to the present formalism, so that Eq.(2.22) may be simply replaced by

$$\Psi(P,q) = S_F(p_1)\, \Gamma_H(\hat{q})\, S_F(-p_2) \tag{2.23}$$

$$\Gamma_H(\hat{q}) = N_H\, \Gamma_i\, D(\hat{q})\, \phi(\hat{q}) / 2\pi i , \tag{2.24}$$

where Γ_i is a constant Dirac matrix (γ_5 for π, $i\gamma\cdot V$ for a V-meson, etc.) and N_H is the BS normalizer. (This is again subject to $q\cdot P \neq 0$ corrections, in principle).

Eq.(2.15) and Eqs.(2.22-24) respectively form a zero-order basis for making contact with the mass spectra of hadronic states on the one hand, and providing access to various types of transition amplitudes on the other (via appropriate 4D quark loop diagrams). This structure is fairly general and <u>independent</u> of the detailed assumptions on the (input) kernel K. In particular the denominator function $D(\hat{q})$, Eq.(2.18), has a <u>universal</u> and well defined meaning <u>off the mass shell</u> of either quark, and constitutes an important multiplicative ingredient of the hadron-quark vertex function (2.22) or (2.24). The function $\phi(\hat{q})$ is admittedly model-dependent, but together with $D(\hat{q})$ it controls the 3D equation (2.15) for the determination of mass spectra, so that its properties are directly traceable to the latter. Both quantities $D(\hat{q})$ and $\phi(\hat{q})$ have a common dependence on the quantity $\hat{q}^2$ whose most important property is its <u>positive-definiteness</u> on the hadron mass shell ($P^2 = -M^2$) throughout the 4D space:

$$\hat{q}^2 = q^2 - \frac{(q\cdot P)^2}{P^2} = q_\perp^2 + \frac{1}{4M_\perp^2}\left(q_+ P_- - q_- P_+\right)^2 ; \quad M_\perp^2 = M^2 + P_\perp^2 . \tag{2.25}$$

We now turn briefly to the connection of this formalism with other approaches, including our earlier efforts.

3. Past History: New Aspects

A detailed account of this subject may be found in a recent review[6]. We shall merely summarize some salient features which have a direct bearing on the present formalism. Many years ago, Feynman et al.[10] (FKR) had advocated a serious dynamical programme based on $q\cdot P=0$ for a unified understanding of spectroscopy and transition amplitudes. The spectroscopy was successful because the basical O(3) character of the data was consistent with Feynman's covariant 3D formulation. However their transition amplitude calculations were more sensitive to the 3D projections and had to be tempered with additional ad-hoc form factors to maintain unitarity, thus pointing to the nontrivial role of the fourth (time-like) direction for a proper dynamical understanding of these amplitudes. The next serious attempt [19] which did address the issues of spectra and transition amplitudes unfortunately relied on a Wick-rotated description of the 4D BSE which yielded an O(4)-like spectra picture, again in disagreement with data. Some years later Leutwyler and Stern[20] made a serious attempt to resurrect the FKR model through their covariant null-plane techniques, and indeed encountered the quantity $\hat{q}^2$, but they did not get-off the $P\cdot q=0$ surface, nor did they have anything akin to the denominator function $D(\hat{q})$.

An important lesson [3,6] to learn from these results is that the O(3)-like spectra are probably not very sensitive to the time-like d.o.f., but the transition amplitudes certainly require all four d.o.f.'s. Therefore the unfolding of the fourth d.o.f. should be gradual enough so as to make its relative insensitivity to the spectra compatible with its more active role w.r.t. to the transition amplitudes. The formulation attempted in this paper is designed precisely to give a concrete shape to this philosophy through the emphasis of interconnection between the 3D and 4D forms of the BSE, where-in the role of the former, Eq.(2.15), emphasises the spectra, while the latter, Eq.(2.22-24), forms the main ingredient for the evaluation of transition amplitudes. The exact connection between the two forms has admittedly been achieved with the special assumption, Eq.(2.8), on the BS kernel K, but it is realistic enough to cover most cases of practical interest. In particular it coincides with the definition of most confining potentials (linear, harmonic, etc.) in the instantaneous approximation, that have been employed in the literature. Such an "instantaneous point of view",

which automatically justifies the ansatz (2.8), has also been emphasized recently by Pervushin and co-workers [21].

We have for some years been stressing the above interconnection between the 3D and 4D forms of the BSE so as to provide a simultaneous access to spectroscopy and transition amplitudes within a unified framework. This was originally sought to be achieved in the instantaneous approximation [11] but its lack of Lorentz-covariance led us subsequently to formulate the same idea under the Null-plane-ansatz (NPA) [12] which exhibits a wider domain of stability under Lorentz transformations than does the former. This is most easily checked on the hypersurface $q\cdot P=0$ (on-shell) where the NPA definition [12] of the 3-vector $\vec{q}$, viz $[\vec{q}_\perp, q_3(=Mq_+/P_+)]$ agrees with that of $\hat{q}_\mu$, Eq.(2.5), as may be seen by making the substitution $q_- = -q_+M^2/P_+^2$ in Eq.(2.5).

This coincidence of the definition of $\hat{q}_\mu$ with that of the NPA 3-momentum $\vec{q}$ on the surface $q\cdot P=0$ enables us to make contact with our earlier results obtained under the null-plane formalism, while retaining the generality of the present formalism for possible applications to other potentials or kernels. In particular the results on the mass spectra of both $q\bar{q}$ [13-14] and qqq hadrons [15] which had earlier been obtained under a common ansatz for the kernel should be automatically valid under the present formalism under the on-shell ($q\cdot P=0$) conditions. The possible off-shell corrections ($q\cdot P\neq 0$) to the $q\bar{q}$ mass spectra, which will arise only from the spin-dependent effects (deviations from the conditions of Gordon reduction due to off-shellness), are calculable perturbatively and are expected to be small.

On the other hand the formal short-comings of our earlier NPA programme [6] have been more manifest at the level of the vertex function $\Gamma(\hat{q})$ through the need for defining different forms of extension (termed on-shell, off-shell and half-off-shell) for different types of applications [16], the preference for a particular for being governed by semi-intuitive considerations based on the topology of the amplitude under study. None of these extensions was truly 4D but constrained to lie on specific hypersurfaces ($P\cdot q=0$, $p_2^2+m_2^2=0$, etc.). These have had some successes [16] which are likely to be increasingly unreliable for arbitrary off-shell extensions. This is of course due to a formal lack of Lorentz covariance in these vertex functions [22] which the null-plane definition of the 3-vector $\vec{q}$ is not able to overcome fully. The present formalism has hopefully overcome this aspect of empiricity through a single Lorentz-invariant off-shell extension, Eqs.(2.22-25), coverin the entire 4D space, while retaining ground contact with the surface $P.q=0$ in respect of the 3D form, Eq. (2.15), of the BSE.

4. Simple Applications: $P \to \ell\bar{\ell}$ and $\pi^0 \to 2\gamma$

In this section we shall indicate the applications of the vertex function (2.24) to some simple cases, viz $P \to \ell\bar{\ell}$ and $\pi^0 \to 2\gamma$, partly to illustrate the general structure of these amplitudes within this formalism, and partly to bring out the nature of the difficulties encountered when extended to more complicated quark-loop diagrams. The calculational techniques which follow closely those of the NPA formalism /16/ will be omitted, except for drawing attention to the new features introduced by the off-shell structure, Eq.(2.25), of the quantity $\hat{q}^2$.

The f_P-amplitude:

The quantity f_P is defined by /16/

$$f_P P_\mu \equiv i\sqrt{3} \int d^4 q \; Tr \left[\Psi_P \, i\gamma_\mu \gamma_5 \right], \tag{4.1}$$

where Ψ_P is given by Eqs.(2.23-24). It reduces after some routine steps to the following expression, in the notation of Sec.2:

$$f_P = \frac{\sqrt{3}\, M N_P}{(2\pi)^{3/2}} \int d^3\hat{q}\; D(\hat{q})\, \phi(\hat{q})\, I_P(\hat{q}) \tag{4.2}$$

$$I_P(\hat{q}) = \frac{1}{2\pi i} \int d\sigma \left[2m_{12}\left(1-\frac{\delta m^2}{M^2}\right) - 4\sigma\,\delta m \right] \Big/ (\Delta_1 \Delta_2) \tag{4.3}$$

$$m_{12} = m_1 + m_2 \,; \qquad \delta m = m_1 - m_2 \tag{4.4}$$

where the numerator in (4.3) comes after trace evaluation and use of the full projections

$$p^{\mu}_{1,2} = P_\mu(\hat{m}_{1,2} \pm \sigma) \pm \hat{q}_\mu \tag{4.5}$$

to extract the 4-vector P_μ from the RHS. The σ-term is an off-shell effect arising directly from $P.q \neq 0$. The integral can be evaluated exactly as in Sec.2 to give

$$I_P(\hat{q}) = \frac{2m_{12}}{M}\left(1-\frac{\delta m^2}{M^2}\right) D^{-1}(\hat{q}) + \frac{\delta m}{M^3}\left(\frac{1}{\omega_2} - \frac{1}{\omega_1}\right). \tag{4.6}$$

The BS normalizer N_P, as defined in Eq.(4.2), with a factor taken out from the definition (2.24) for N_H, may be worked out from the current conservation condition /16/

$$2P_\mu i = (2\pi)^4 \int d^4 q \; Tr \left[\bar{\Psi} \frac{\partial}{\partial P_\mu} S_F^{-1}(p_1)\, \Psi\, S_F^{-1}(-p_2) \right] + (1 \rightleftarrows 2). \tag{4.7}$$

The P_μ-differentiation which must take account of the entire dependence of $p_{1\mu}$, $p_{2\mu}$ in accordance with (4.5), is expressed by

$$\frac{\partial}{\partial P_\mu}\, i\gamma.p_{1,2} = i\gamma_\mu(\hat{m}_{1,2} \pm \sigma). \tag{4.8}$$

The resulting structure for N_P, after evaluating the traces, etc, works out from Eq.(4.7) as /6,16/

$$N_P^{-2} = 2\int \frac{d^4 q}{2\pi i}\, \frac{D^2(\hat{q})\,\phi^2(\hat{q})}{\Delta_1^2 \Delta_2} \left[(M^2 - \delta m^2 + \Delta_2)(\hat{m}_1 + \sigma)^2 + \Delta_1(\hat{m}_1+\sigma) \right] + (1 \rightleftarrows 2). \tag{4.9}$$

where the σ-terms represent the off-shell effects due to $P.q \neq 0$ over and above those considered within the (earlier) null-plane formalism /16/. Before evaluating (4.9) further, a comment is in order regarding the appearance of the term Δ_2 in its numerator, /16/ since its effect had been zero in the null-plane pole formalism due to its cancellation with the corresponding denominator. Its present appearance must therefore be regarded as a specifically off-shell effect which characterizes the present formalism. It is particularly vexing in view of the quadratic multiplying factor $(\hat{m}_1 + \sigma)^2$ which makes its contribution <u>negative</u>, after σ-integration. This is one illustration of the kind of problems that are likely to arise in the present formalism as a price for demanding too much Lorentz covariance which involves strong off-shells effects. We have not yet been able to overcome this problem (perhaps some regularization is necessary) and in the meantime we are inclined to drop it, after proper identification, in strict analogy with the (on-shell) NPA formalism /16/. The final result for (4.9) after the σ-integration is:

$$N_P^{-2} = \int d^3\hat{q}\ \phi^2(\hat{q}) \left[\left(1 - \frac{\delta m^2}{M^2}\right) G + \frac{2\delta m^2}{M^2} D(\hat{q}) \right] \tag{4.10}$$

$$G = 4\left(\omega_1\omega_2 + M^2 \hat{m}_1 \hat{m}_2\right)\left(\frac{\hat{m}_1}{\omega_1} + \frac{\hat{m}_2}{\omega_2}\right)^{-1}. \tag{4.11}$$

So far the results are independent of any model for $\phi(\hat{q})$, but a few general statements can be made regarding the large M behaviour of f_P and N_P, especially for unequal mass kinematics ($m_1 \gg m_2$), if we make the reasonable assumption that the inverse length (β) associated with $\phi(\hat{q})$ is finite ($\beta \lesssim m_2 \ll m_1 \sim M$). In this limit, one easily finds the following asymptotic results:

$$N_P \sim \sqrt{M}\ , \quad f_P \sim \frac{1}{\sqrt{M}}\ . \tag{4.12}$$

This is in accordance with QCD predictions [23] for the $\sim M^{-\frac{1}{2}}$ behaviour of f_P which forms the basis for lattice extrapolations [24] from f_D to f_B. However, the full structure of f_P involves two distinct components for $m_1 \neq m_2$ and their relative strengths would determine its final magnitude. Some estimates [25] based on the null-plane parameters [14] for the wave function $\phi(\hat{q})$ in fact suggest that the second component ($\sim \delta m$) is particularly large in the (D,B) region, so that a single parameter extrapolation of the type (4.12) is likely to give misleading results. A detailed discussion of this problem, including the possible relevance of the second ($\sim \delta m$) term in (4.6) in the context of some current controverties on two groups of lattice results[26] is given separately[25].

$\pi^0 \to 2\gamma$ Amplitude :

This is another example where the present formalism gives a clean result without the problems of too much off-shellness. The effective coupling constant F_π may be defined in the standard notation through the invariant amplitude [27]

$$A_{eff} = F_\pi\, \varepsilon_{\mu\nu\rho\sigma}\, \varepsilon_\mu^{(1)} \varepsilon_\nu^{(2)}\, P_\rho\, Q_\sigma\ . \tag{4.13}$$

where the two outgoing photons have with 4-momenta $k^\mu_{1,2}$ and polarizations $\varepsilon^{(1,2)}_\mu$ with

$$k_1 + k_2 = P\ , \quad k_1 - k_2 = 2Q\ ; \quad P.Q = 0\ . \tag{4.14}$$

For brevity we omit the two standard triangle diagrams [6,27] which make equal contributions and give rise to the following net result:

$$F_\pi = \frac{8 m_1 e^2}{\sqrt{6}} \int d^3\hat{q}\ \frac{N_\pi D\phi}{(2\pi)^{3/2}} \int \frac{M\,d\sigma}{2\pi i}\ \frac{1}{\Delta_1 \Delta_2 \Delta_3}\ , \tag{4.15}$$

where all symbols are defined in sec.2 and $\Delta_3 = m_1^2 + (q - Q)^2$ taking $m_1 = m_2$. The final result on σ-integration is

$$F_\pi = \frac{\sqrt{6}\, N_\pi m_1 e^2}{(2\pi)^{3/2}} \int d^3\hat{q}\ \phi(\hat{q})\ \frac{1}{\omega^2 - \frac{1}{4}M^2}\ . \tag{4.16}$$

In the spirit of generality of this investigation we refrain from discussing numerical values, but for purposes of illustration with an NPA wave function for the pion, viz [6,14]

$$\phi_\pi = e^{-\hat{q}^2/2\beta^2}\ , \quad \beta^2 = 0.031\ GeV^2\ , \tag{4.17}$$

one gets

$$F_\pi = 0.029\ \left(\underline{Vs}.\ 0.027\ \underline{expt}\right). \tag{4.18}$$

Unitary problem:

We conclude this section with some remarks concerning the difficulties of extending this method to processes with quark loops involving more than one hadron-quark vertices:

$$\text{e.g.,}\quad h \to h' + \gamma\ , \quad h \to h' + h''\ , \quad \text{etc.} \tag{4.19}$$

In general such processes would be afflicted by Unitary cut-effects in the internal quark propagators, giving rise to complex amplitudes. It is only in simple processes involving only one hadron (e.g., $P \to l\bar{l}$, $\pi^0 \to 2\gamma$) or loops with two hadron vertices without external field lines (e.g., e.m. mass difference diagrams) that such complex amplitudes do not occur. This is because the appearance of all the four components of the common loop-integration variable q_μ in the product of the 3D wave functions $\phi\phi'\phi''$ generally prevents the choice of a single component say q_- from appearing only in the loop propagators and not in the (gaussian) functions representing the individual quantities ϕ, ϕ', ϕ'', etc. In the corresponding NPA formalism [16] this problem was sought to be circumvented by contraining these individual wave functions ϕ, ϕ', to lie on specific hypersurfaces (on-shell, half-off-shell, etc.) from where one would expect the main contributions to arise, thus keeping the variable q_- free to appear only in the propagators. This could at best be described as an intuitive physical procedure and cannot be regarded as a formal solution of the problem.

A more satisfactory procedure from the theoretical point of view has been developed by Efimov and co-workers [28] in which the confinement of the internal quark propagators (and hence the prevention of unitarity cuts) is ensured by a mathematical "smearing device" weighted by a suitably chosen, highly covergent, distribution function. They get good fits to processes like (4.19), similar to our NPA results [16], but their point-like hadron-quark vertices, motivated by non-local field considerations [28], do not yet incorporate the information on spectra as do the NPA form factors [16] or the ones represented by Eq. (2.24). Our present approach, which may be termed as a covariant instantaneous description has some obvious overlap with that of ref. [21] which would presumably have to face a similar problem.

A formal solution to the problem lies in the recognition of its basically infrared nature, namely, the need to modify the simple propagators S_F with fixed masses m_1, m_2 due to the gluonic self--energy corrections in the low momentum regime. These gluonic corrections in turn are non-perturbative and are in principle already implied in the structure assumed for the effective BS kernel K. Indeed for a kernel $K(\hat{q}-\hat{q}')$ the self-energy correction $\Sigma(p_1)$ to the propagator $S_F(p_1)$ may be written down from the identifications

$$p_1 = p_2', \quad p_2 = p_1' = p_1 - k ;$$
$$q = -q' = -k, \quad P = 2p_1 - k$$
$$2\hat{k}_\mu = 2k_\mu - \frac{2k\cdot P}{P^2} P_\mu .$$

Thus in the ladder approximation

$$\Sigma(p_1) = \frac{-i}{(2\pi)^4} \int S_F(p_1) \otimes K(2\hat{k})\, d^4k . \qquad (4.20)$$

Note the appearance of the 4-vector $\hat{k}_\mu$ instead of the more usual symbol k_μ in Eq.(4.20). This is in accordance with the ansatz (2.8) on K. The effect of this correction for confining kernels of the general type

$$K_n(\hat{q}_\mu) = \lim_{\lambda\to 0} \left(-\frac{\partial}{\partial\lambda}\right)^n \frac{g^2}{\lambda^2 + q_\mu^2} \qquad (4.21)$$

is currently under investigation [29]. This representation was first employed in ref. [12] with n=3 corresponding to harmonic confinement.

5. Summary and Conclusions

In this paper we have tried to put in perspective the interrelation between the 3D and 4D forms of the BSE when its kernel K is a function $K(\hat{q},\hat{q}')$ of the internal 4-momenta $\hat{q}_\mu$ which are orthogonal to the hadron 4-momentum P_μ ($\hat{q}\cdot P = 0$). Under these conditions the hadron quark vertex function is expressible as a product $D \otimes \phi$ of a universal denominator function $D(\hat{q}^2)$ and a 3D wave function $\phi(\hat{q})$ both of which are functions of the invariant $\hat{q}^2$, and are defined over all spaoe, on and off the energy shells of the respective constituents. The quantity $\phi(\hat{q})$ satisfies a 3D form of the BSE which is appropriate for making contact with the O(3)-like mass spectra. The vertex function $D\otimes\phi$ in turn represents the main ingredient for a fully Lorentz-invariant evaluation of hadronic transition amplitudes. Though the role of quantities like $\hat{q}^2$ has been long known [10] the new element lies in the use of this quantit in the context of the interconnection between the 3D and 4D forms of the BSE which is articulated through the universal denominator function $D(\hat{q})$ as the main connecting link between them. This last is what has been termed as a two-tier basis in our earlier efforts [11-16] in this direction in terms of null-plane dynamics [6]. The present formalism shows its Lorentz-covariance over all 4D space and is properly calibrated with the older form [12] on the mass shell $(P\cdot q = 0)$ in respect of the 3D structure of the BSE, Eq. (2.15), so that the results on the mass spectra [13-15] will remain essentially unaltered if the same parametrization [6] is employed for the general BS kernel K described here.

As simple illustrations of this new formalism we take offered two examples, viz. the general structures of the two amplitudes $f_P(P\to l\bar{l})$ and $F_\pi(\pi^0\to 2\gamma)$ with any arbitrary 3D wave function $\phi(\hat{q})$ without reference to a specific model. These quantities f_P have been shown to fall off-like $M^{-1/2}$ for markedly heavy-light mesons, in accordance with QCD perceptions [23], but their two-component structure additionally shows a strong off-shell sensitivity for such mesons.

The formal difficulties of extension of such calculations to bigger quark loops with more vertices and propagators have been traced to unitary cut effects on these propagators, a basically infrared problem of confinement is also recognized by other authors /28/. A possible line of attack has been suggested via a self-energy correction, Eq.(4.20), to the internal propagators arising out of the non-perturbative imput kernel $\underline{K}$ within this very formalism. A detailed investigation of this effect is in progress /29/.

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References

1. E.Eichten, F.Feinberg, Phys.Rev. D23, 2724 (1981).
2. M.Shifman et al. Nucl.Phys. B147, 385, 448 (1979).
3. A.N.Mitra, Prod.Ind.Nat.Sci.Acad. 54A, 179 (1988).
4. F.J.Dyson, Phys.Rev. 75, 1736 (1949);
   E.E.Salpeter, H.A.Bethe, Phys.Rev. 84, 1232 (1951).
5. A.Logunov, A.N.Tavkhelidze, Nuovo Cim. 29, 380 (1963).
   R.Blankenbecler, R.Sugar, Phys.Rev. 142, 105 (1966).
   V.Kadyshevsky, Nucl.Phys. B6, 125 (1968).
6. S.Chakrabarty et al. Prog.Part.Nucl.Phys. 22, 43 (1989).
7. Y.Nambu, G.Jona-Lasino, Phys.Rev. 122, 345 (1961).
8. See, e.g., ref.6, § 5.4.
9. V.G.Gusnin et al. Phys.Rev. D39, 2355 (1989).
10. R.P.Feynman et al., Phys.Rev. D3, 2706 (1971).
11. A.N.Mitra, I.Santhanam, Zeits.f.Phys. C8, 25, 33 (1981).
    D.S.Kulshreshtha, A.N.Mitra, Phys.Rev. D28, 588 (1983).
12. N.N.Singh et al. Few-Body Syst. 1, 47 (1986).
13. K.K.Gupta et al. Phys.Rev. D42, Sept.1 (1990).
14. A.Mittal, A.N.Mitra, Phys.Rev.Lett. 57, 290 (1986).
15. D.S.Kulshreshtha, A.N.Mitra, Phys.Rev.D37, 1268 (1988).
16. N.N.Singh et al. Phys.Rev. D38, 1454 (1988).
17. E.g., A.J.McFarlane, Rev.Mod.Phys. 34, 41 (1962).
18. H.D.Politzer, Nucl.Phys. B117, 397 (1976).
19. M.Bohm, et al. Acta Phys.Austr. Suppl. XI, 3 (1973).
20. H.Leutwyler, J.Stern, Ann.Phys. (N.Y.), 112, 94 (1978).

21. V.N.Pervushin et al. Fortschr.d.Phys. 38, N 4 (1990).
22. C.R.Ji et al. Phys.Rev.Lett. 62, April 9 (1990).
23. M.Shifman, M.Voloshin, Sov.J.Nucl.Phys. 45, 292 (1987).
    E.Eichten, Nucl.Phys. B4, 140 (1988); proc.suppl.
24. C.Bernard et al. Phys.Rev. D38, 3540 (1988).
25. S.Bhatnagar et al. Delhi Univ.Preprint, Aug.(1990).
26. A.Soni, talk at PANIC-XII, MIT, June 1990;
    G.Martinelli, ibid. (1990).
27. A.N.Mitra et al. Phys.Rev.Lett. 59, 2408 (1987).
28. G.Efimov, M.Ivanov, Intl. J.Mod.Phys. A3, 2911 (1988);
    Sov.J.Part.Nucl. 20, 1129 (1989).
29. In collaboration with A.G.Rusetsky (Tbilisi-Dubna)

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