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ON CONSERVED NUMBERS IN SU(N) GAUGE THEORIES

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INTRODUCTION

This work is devoted to generalization of an additive number related to a global gauge transformation in U(1) and SU(N), N ≥ 2 vector theories without Higgs symmetry breaking or involving such Higgs symmetry breaking.

I. CHARGES IN U(1) THEORY

A global gauge transformation in U(1) theory [1] with the Lagrangian density

$$\mathcal{L}_1(x) = -\frac{1}{4} F_{\mu\nu} F^{\mu\nu} + i\bar{\psi}\mathfrak{D}_\mu \gamma^\mu \psi \tag{1}$$

and

$$F_{\mu\nu} = \partial_\mu A_\nu - \partial_\nu A_\mu, \quad \mathfrak{D}_\mu = \partial_\mu - ieA_\mu$$

is defined by the expressions

$$\psi'(x) = e^{ie\alpha}\psi(x)$$
$$\bar{\psi}'(x) = e^{-ie\alpha}\bar{\psi}(x), \quad U(1) \to e^{\pm ie\alpha}. \tag{2}$$

Since U(1) is topologically equivalent to the first homotopy group [2] $\Pi_1(U(1)) \equiv Z$, the U(1) transformation in the general case is of the form

$$U(1) \to e^{\pm ie_n\alpha}, \quad \text{then} \quad \psi \to \psi_n,$$

where $e_n = n \cdot e$, $n = 0,1,2,\ldots$ and

$$\psi_n'(x) = e^{ie_n\alpha}\psi_n(x)$$
$$\bar{\psi}_n'(x) = e^{-ie_n\alpha}\bar{\psi}_n(x) \tag{3}$$
$$\mathcal{L}_1(x) \to \mathcal{L}_{1n}(x) = -\frac{1}{4}F_{\mu\nu}F^{\mu\nu} + i\bar{\psi}_n\mathfrak{D}_\mu\gamma^\mu\psi_n.$$

The vector current [3,4]

$$J_n^{\mu}(x) = \bar{\psi}_n(x)\, \gamma^{\mu} \psi_n(x) \tag{4}$$

is conserved

$$\partial_{\mu} J_n^{\mu}(x) = 0. \tag{5}$$

In this theory one can define a number related to the charge

$$\rho_n = \pm \frac{e_n}{n} = \pm n \quad \text{and} \quad \mathcal{L}_1(x) \to \sum_n \mathcal{L}_{1n}(x). \tag{6}$$

It means that in the general case fermions $\bar{\psi}_n$, ψ_n are able to have multiple charges $e_n = n \cdot e$. For transition to the standard theory with Lagrangian density (1) it is necessary to assume that fermions $\bar{\psi}_n$ and ψ_n have a charge equal to n = 1. In a strict approach it is necessary to prove that spin 1/2 is related with charge n = 1.

In U(1) theory with a Higgs sector [1]

$$\mathcal{L}_2(x) = \mathcal{L}_1(x) - \frac{1}{2}\mu^2 \phi^+\phi + \lambda^2(\phi^+\phi)^2 - \frac{1}{2}(\mathcal{D}_{\mu}\phi)^+(\mathcal{D}^{\mu}\phi)$$
$$\mathcal{D}_{\mu} = \partial_{\mu} - i\delta A_{\mu}, \quad \mu^2 < 0, \ \lambda^2 > 0. \tag{7}$$

The gauge transformation of scalar fields

$$\phi'(x) = e^{i\delta a}\phi(x), \quad \phi'^+(x) = e^{-i\delta a}\phi^+(x) \tag{8}$$

cannot be interpreted as a gauge transformation with respect to the charge e because this leads to a contradiction. This is connected with a fact that before transferring the degree of freedom to the vector gauge field A_{μ} the field ϕ is a complex field and has a charge $\mp \phi(\phi^+)$, and after transferring the degree of freedom to the vector field, ϕ becomes a real field and the charge of this field has no sign. Making the transformation

$$\phi(x) = \frac{1}{\sqrt{2}}(\theta + \eta(x))\, e^{i\frac{\xi}{\theta}},$$

$$\phi^{\theta}(x) = e^{-i\frac{\xi}{\theta}}\phi(x) = \frac{1}{\sqrt{2}}(\theta + \eta(x)),$$

from the second part of (7) we have

$$\mathcal{L}_{\phi} = \frac{1}{2}\mu^2\eta^2 - \frac{1}{4}(\partial_\mu B_\nu - \partial_\nu B_\mu)^2 + \frac{1}{2}(e\theta)^2 B_\mu B^\mu + $$
$$+ \frac{1}{2}e^2 B_\mu B^\mu \eta^2 + \theta e^2 B_\mu B^\mu \eta + 4\lambda\theta^2\eta^3 + \lambda\eta^4 + \ldots \quad (9)$$
$$B_\mu = A_\mu - \frac{1}{e\theta}\partial_\mu \xi(x),$$

e.g. ϕ, $\phi^+ \to \eta$, ξ. At the same time the fermion field $\bar{\psi}$, ψ remains to be complex and accordingly fermions will possess a sign of the charge. For a reason to make this theory, in the presence of Higgs mechanism realisation with Lagrangian density (7), self-consistent, it is necessary that the fermion field must become a real field. To avoid these contradictions, it is enough to suppose that fermion fields and Higgs scalar fields have different charges. In this case the first part of formula (7) acquires form (3)

$$\mathcal{L}_2(x) = \sum_n \mathcal{L}_{in}(x) + \text{Higgs' part.} \quad (10)$$

II. CHARGES IN SU(N) THEORIES ($N \geq 2$)

The global U(1) charge transformation in SU(N) in gauge theories [5] with the Lagrangian density

$$\mathcal{L}_3(x) = \frac{i}{2}\bar{\psi}^i \gamma^\mu \mathfrak{D}_\mu \psi_i - \frac{1}{4} F^a_{\mu\nu} F^{a\mu\nu}$$

and

$$F^a_{\mu\nu} = \partial_\mu V^a_\nu - \partial_\nu V^a_\mu - g f^{abc} V^b_\mu V^c_\nu$$
$$\mathfrak{D}_\mu = \partial_\mu - igV_\mu, \quad i = 1 \div N, \quad a, b, c = 1 \div N^2 - 1 \quad (11)$$

is determined by the expressions

$$\psi_i'(x) = e^{ig\alpha}\psi_i(x), \quad \bar{\psi}'^i(x) = e^{-ig\alpha}\bar{\psi}^i(x). \quad (12)$$

Transformation (12) represents a U(1) transformation and is topologically equivalent to the first homotopy group $\Pi_1(U(1)) \equiv$ $\equiv Z$. Therefore, in the general case the following transformation

$$U(1) \to e^{\pm i g_n \alpha}, \quad \text{where} \quad g_n = n \cdot g, \; n = 0, 1, \ldots$$
$$\psi'_{in}(x) = e^{i g_n \alpha} \psi_{in}(x), \quad \bar{\psi}'^{i}_{n}(x) = e^{-i g_n \alpha} \bar{\psi}^{i}_{n}(x) \tag{13}$$

should exist, and the corresponding current

$$J^{\mu}_{n}(x) = \bar{\psi}^{i}_{n}(x) \gamma^{\mu} \psi_{in}(x) \tag{14}$$

is conserved,

$$\partial_{\mu} J^{\mu}_{n}(x) = 0. \tag{15}$$

Thus, in non-Abelian theories one can define a conserved number

$$B_n = \pm \frac{g_n}{g} = \pm n, \; n = 0, 1, 2, \ldots \tag{16}$$

From the duality

$$\bar{\psi}^{i}_{n} \gamma_{\mu} \psi_{i'n} \leftrightarrow A^{i'}_{\mu i}, \quad A'^{i'}_{\mu i} = e^{-ig\alpha} A^{i'}_{\mu i} e^{ig\alpha} = A^{i'}_{\mu i}$$

it follows that the number $B(A_{\mu}) = 0$.

Like in the first part of this work the Lagrangian density has the form

$$\mathcal{L}_3(x) = i \sum_n \bar{\psi}^{i}_{n} \gamma^{\mu} \mathcal{D}_{\mu} \psi_{in} - \frac{1}{4} F^{a}_{\mu\nu} F^{a\mu\nu}. \tag{17}$$

In the case of SU(N) gauge theories ($N \geq 2$) it should be supposed (or proved) why n takes only one value, n = 1.

Within the approach developed in [6] (when the SU(3) symmetry group space is inserted into V_4 space, e.g. there is constructed a stratified space E which is locally presented as a product $E \sim F \times V_4$, where the fundamental representation of the SU(3) group is realized in the F layer) where in V_4 space the fundamental representation of the SU(3) group is realized, one can receive an answer to the question, why n = 1.

To realize all the representations of the SU(3) group, the space of the dimension-8, is necessary, and in V_4 space there can be realized only the fundamental representation of the SU(3) group, and the fundamental representation of the SU(3) group has n = 1.

Now we consider the gauge SU(N) theory broken by the inclusion of the Higgs sector [1,7]

$$\mathcal{L}_4(x) = \mathcal{L}_3(x) - \frac{1}{2}[\mu^2\phi^+\phi + \lambda^2(\phi^+\phi)^2] - \frac{1}{2}(\mathcal{D}_\mu\phi)^+(\mathcal{D}^\mu\phi)$$
$$\mathcal{D}_\mu = \partial_\mu - igV_\mu, \quad V_\mu = T^a V_\mu^a, \quad \mu^2 < 0, \quad \lambda^2 > 0 \qquad (18)$$

up to the subgroup H so that K vector fields become massive, while L vector fields remain massless (T^a are generators of the SU(N) group, L is the number of generators of the subgroup H, $K + L = N^2 - 1$). From the duality $\bar{\psi}^{i'}\psi_i \leftrightarrow \phi_i^{i'}$ it follows that the number $B(\phi) = 0$. As a result of the SU(N) symmetry being broken by the Higgs sector up to the subgroup H, the number $B(\psi)$ determined by formula (16) does not change. In this case in the Lagrangian density $\mathcal{L}_4(x)$ term $\mathcal{L}_3(x)$ assumes the form (17).

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