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INSTANTANEOUS APPROXIMATION FOR QCD AND THE PROPERTIES OF MESONS (π, π', K, K$'$)

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I. Introduction

In the last time, a current of papers /1-8/ has been published which are devoted to the generalization of the potential model of heavy quarkonia to the sector of light quarks and their bound states (mesons). In these works, instead of the Schrödinger equation, one considers the QCD effective hamiltonian with the four-component Lorentz-vector potential of the instantaneous interaction of quarks. It has been shown that in this model the solutions of both the Schwinger—Dyson (SD) and the Bethe—Salpeter (BS) equations successfully describe the spontaneous breakdown of chiral symmetry (that is a purely-relativistic effect), the constituent quark masses and the Goldstone mode in pseudoscalar meson spectrum, and a large mass difference of ρ and π mesons /3/. As the same time, because of noncovariance of the potential such an approach could not give the wave functions of mesons in any moving frame. Furthermore, the authors did not reproduce the physical values of pion mass, and the pion decay constant (F_π) as well as the light quark condensates. However, one can suppose that the instantaneous approach to QCD, which qualitative describes the features of heavy and light quarkonium spectra, in its covariant form gives the solutions of the above-mentioned problems.

The relativistic covariant generalization of the instantaneous potential model (for a moving meson) and the effective bilocal lagrangian for composite meson interaction have been proposed in the framework of the field-theoretical approach in ref. /9/. In this approach, the effective hamiltonian of refs. /1-8/ is specified by the dependence of the kernel on the time-axis η_μ $(\eta_\mu^2=1)$ which leads to the following effective action:

$$S_{\text{eff}}^{\text{QCDinst.}} = \int d^4x \Big\{ \bar{q}(x)(i\not\partial-\hat{m}^\circ)\, q(x) -$$

$$\frac{1}{2}\int d^4y \,(q_{\beta_2}(y)\,\bar{q}_{\alpha_1}(x))\left[K^{(\eta)}(x,y)\right]_{\alpha_1\beta_1;\alpha_2\beta_2}(\bar{q}_{\beta_1}(x)\, q_{\alpha_2}(y))\Big\}. \qquad (1)$$

Here q is the quark field, $\hat{m}^\circ=\mathrm{diag}(m_u^\circ, m_d^\circ, m_s^\circ, \ldots)$ is the current quark mass matrix, $\not\partial = \partial^{\mu}\gamma_{\mu}$, and $K^{(\eta)}$ is the kernel defined as

$$[K^{(\eta)}(x,y)]_{\alpha_1\beta_1;\alpha_2\beta_2} = \frac{4}{3}(\not\eta)_{\alpha_1\beta_1} V(z^{\perp})\,\delta(z\cdot\eta)\,(\not\eta)_{\alpha_2\beta_2} \tag{2}$$

that is a colour-singlet, where $\not\eta = \eta^{\mu}\gamma_{\mu}$, $z^{\perp}_{\mu} = z_{\mu} - \eta_{\mu}(z\cdot\eta)$, and $z_{\mu} = x_{\mu} - y_{\mu}$ is the relative coordinate of quark-antiquark system. The potential $V(z^{\perp})$ is get from heavy quarkonium spectroscopy. The external vector η_{μ} must be "parallel" to the eigenvalue ($\mathcal{P}_{\mu}$) of the total momentum operator $(-i\,\partial/\partial X_{\mu})$ /9/ i.e.

$$\eta_{\mu} \sim -i\,\partial/\partial X_{\mu} \qquad \left(\eta_{\mu} = \mathcal{P}_{\mu}/\sqrt{\mathcal{P}^2}\right), \tag{3}$$

where $X_{\mu} = (x_{\mu} + y_{\mu})/2$ is the "total" coordinate of a bound state which does not depend on the masses of a quark and an antiquark.

In this paper we demonstrate that the relativistic model (1)-(3) gives a universal description of the quark dynamics for arbitrary current quark masses, and the spectroscopy of light as well as heavy mesons. This is exemplified by three quark flavours u,d, and s for which the oscillator potential approximation /3/ can be used. In this approximation we obtain the constituent quark masses, spectra of π and K mesons and their radial excitations (π' and K'), and leptonic decay constants F_{π} , $F_{\pi'}$, F_{K} and $F_{K'}$, and discuss the concept of quark condensates at low energies.

The paper is organized as follows. In section 2 the SD and BS equations for the oscillator potential and arbitrary current quark masses (in the rest frame: $\mathcal{P}_{\mu} = (M, \mathbf{0})$, $\eta_{\mu} = (1, \mathbf{0})$) are given. Here, the normalization condition for the meson composite fields, the expression for the decay constants, the chiral approximation solution for a pion, and the current algebra relation for the quark condensates are given too. Sections 3 and 4 are devoted to the numerical results and their discussion.

2. The quarks and their bound states in the oscillator potential approximation

Let us consider the SD and BS equations for the oscillator potential that in the momentum space is defined through the differential operator $\Delta_{\mathbf{p}}$

$$\underset{\sim}{V}(p) = (2\pi)^3 V_0 \, \Delta_{\mathbf{p}} \, \delta(\mathbf{p}) , \tag{4}$$

where V_0 is the phenomenological parameter for which we use the fits given in ref. /3/ .

For this potential the SD equation takes the form of the differential equation of the sine-Gordon type /3,9/

$$(p^2 \varphi_f'(p))' = 2p^3 \sin \varphi_f(p) - 2p^2 m_f^\circ \cos \varphi_f(p) - \sin 2\varphi_f(p) , \tag{5}$$

$$E_f(p) \equiv m_f^\circ \sin \varphi_f(p) + p \cos \varphi_f(p) - \frac{1}{2}\left[\varphi_f'(p)\right]^2 - \frac{1}{p^2} \cos^2 \varphi_f(p) , \tag{6}$$

where $p = |\mathbf{p}|$, m_f° is the current quark mass, f is the flavour index, the prime means the derivation over p , and $E_f(p)$ is the "dressed" quark energy. The function $\varphi_f(p)$ describes the constituent quark mass appearance, and it is connected with the known quark-antiquark pair function /1-8/ ψ_f as follows:

$$\sin \frac{\varphi_f(p)}{2} = \frac{\psi_f(p)}{\sqrt{1+\psi_f^2(p)}} . \tag{7}$$

Equations (5) and (6) are written in the dimensionless form by means of the substitutions

$$p \to \left(\tfrac{4}{3} V_0\right)^{1/3} p , \quad E_f(p) \to \left(\tfrac{4}{3} V_0\right)^{1/3} E_f(p) .$$

The solutions to equation (5) in the case of zero current quark mass ($m_f^\circ = 0$) have been obtained in ref. /3/ where the spontaneous breakdown of chiral symmetry leading to the constituent mass for the quark is proved.

The corresponding BS equation for a pseudoscalar meson in the rest frame of the meson is reduced to the Salpeter equation that for potential (4) takes the form of the differential coupled equations /9/

$$M L_{2(1)}(p) = \left[\frac{d^2}{dp^2} - E_T(p) + F_{+(-)}(p) \right] L_{1(2)}(p) . \tag{8}$$

Here M is the meson mass, $E_T = E_{f_1} + E_{f_2}$ is the sum of energies of the constituent quarks with flavours f_1 and f_2 defined by (6), and

$$F_{+(-)}(p) = \left\{ \frac{d}{dp}\left[\theta_{f_1}(p) +(-) \theta_{f_2}(p)\right]\right\}^2 + \frac{2}{p^2}\left[s_{+(-)}(p)\right]^2 ,$$

where

$$s_{+(-)}(p) = \sin\left[\vartheta_{f_1}(p) +(-) \vartheta_{f_2}(p)\right] ,$$

$$\vartheta_f(p) = \frac{1}{2}\left[\frac{\pi}{2} - \varphi_f(p)\right] .$$

The eigenfunctions of equations (8) are the components of the meson vertex function /9/

$$\Psi_{\mathcal{P}}(p^\perp) = S^{-1}_{f_1}(p^\perp)\gamma_5\left[L_1(p^\perp) + \frac{\not{\mathcal{P}}}{\sqrt{\mathcal{P}^2}} L_2(p^\perp)\right] S^{-1}_{f_2}(p^\perp), \qquad (9)$$

where $\not{\mathcal{P}} = \mathcal{P}^\mu\gamma_\mu = M\gamma_0$, S_f is the Foldy-Wouthuysen matrix

$$S^{-1}_f(p) = \exp\left[-\mathbf{p}\cdot\boldsymbol{\gamma}\,\vartheta_f(p)/p\right] \quad \left(p^\perp = (0, \mathbf{p})\right).$$

In refs. /1-7/ one uses other definitions of the wave functions (g,h) which can be related to $L_{1(2)}$ according to the relations

$$g(p) = \frac{2}{M} L_2(p),$$

$$h(p) = L_1(p) - \frac{M}{E_T(p)} L_2(p).$$

The covariant normalization condition for the wave functions has the form /9/

$$\frac{4N_c}{M}\int \frac{d^3p^\perp}{(2\pi)^3} L_1(p^\perp) L_2(p^\perp) = 1 \qquad (N_c = 3) . \quad (10)$$

The leptonic coupling constant (F) is defined by the following equation:

$$F = \frac{4N_c}{M}\int \frac{d^3p^\perp}{(2\pi)^3} L_2(p^\perp)\sin\varphi_{(+)}(p^\perp) , \qquad (11)$$

where $\varphi_{(+)} = (\varphi_{f_1} + \varphi_{f_2})/2 .$

Let us now give the low-energy properties of the wave functions L_1 and L_2. In the chiral limit ($m_f^\circ = 0$)

$$L_2(p)\Big|_{m_f^\circ=0} = 0\,,$$

equation (8) has the solution /9/

$$L_1(p)\Big|_{m_f^\circ=0} = \frac{1}{F_\pi}\ \sin\varphi_f(p) \qquad (f=u=d) \tag{12}$$

that satisfies the SD equation and describes the Goldstone mode of the composite pseudoscalar meson spectrum. Notice that for (12) equation (8) becomes an identity.

It is easily seen that in the small mass $\left(m_f^\circ = m_u^\circ = m_d^\circ \ll (\frac{4}{3}V_0)^{1/3}\right)$ approximations /9/, eq. (12) and

$$L_2(p) = \frac{2\,m_f^\circ}{M_\pi F_\pi} + O\left((m_f^\circ)^2\right) \tag{13}$$

lead to the well-known low-energy relation

$$m_f^\circ \langle u\bar{u} + d\bar{d}\rangle \simeq -M_\pi^2 F_\pi^2\,, \tag{14}$$

where the quark condensates are defined as

$$\begin{aligned}\langle q_f\bar{q}_f\rangle &= i\,2N_c\,\mathrm{tr}\int\frac{d^4p}{(2\pi)^4}\left[G_{\Sigma_f}(p) - G_{m_f^\circ}(p)\right] \\ &\simeq -2N_c\int\frac{d^3p^\perp}{(2\pi)^3}\ \sin\varphi_f(p^\perp)\,,\end{aligned} \tag{15}$$

$$\left(G_{\Sigma_f}(p) = (\not{p} - \Sigma_f)^{-1}\,,\ \ G_{m_f^\circ}(p) = (\not{p} - m_f^\circ)^{-1}\right).$$

Relation (14) for the quark condensates will be discuss using the numerical solutions to which the next section is devoted.

3. Numerical results

We have obtained the numerical solution to the SD (5) and BS(8) equations for three quark-flavours (u,d and s) with nonzero current quark masses ($m_u^0 = m_d^0 \neq 0$ and $m_s^0 \neq 0$). These masses as well as the potential parameter (V_0) are the free parameters of the model. We have fixed the current quark masses by comparing the eigenvalues of eq. (6) with the pion and kaon masses (M_π = 140 MeV and M_K= 497 MeV). For the parameter V_0 we have used the values given in ref.[3]. So the input parameters are the following:

$$
\begin{aligned}
m_u^0 = m_d^0 &= 0.007\ \left(\tfrac{4}{3}V_0\right)^{1/3} \\
m_s^0 &= 0.21\ \left(\tfrac{4}{3}V_0\right)^{1/3}
\end{aligned}
\qquad (16)
$$

where $\left(\frac{4}{3}V_0\right)^{1/3}$ = 289 MeV or 247 MeV.

We have solved Eq. (5) by combining the Newton continuum analogy[10] and the approximation over the parameters [11] (which are the current quark masses). The solutions φ_f , the constituent quark energies E_f , and the quark pair functions ψ_f are shown in figs. 1-3. It has been tested that our calculational scheme in the massless case ($m_f^0 = 0$) reproduces the results of ref. [3].

We have solved eq. (8) using the above numerical scheme and the "shooting" method which led to the identical results. The wave functions $L_{1(2)}$ for π and K mesons and their radial excitations (π' and K') are shown in figs. 4 and 5, respectively. Fig.6 shows the ratios of the pion BS wave functions to the low-energy approximation solutions (12) and (13)

$$
\begin{aligned}
R_1^\pi &= \frac{L_1^\pi(p)}{F_\pi^{-1}\sin\varphi_u(p)}\ , \\
R_2^\pi &= \frac{L_2^\pi(p)}{2m_u^0/(M_\pi F_\pi)}\ .
\end{aligned}
\qquad (17)
$$

The predictions for the masses of π' and K' mesons, and for the decay constants corresponding to the input parameters (16) are presented in table I.

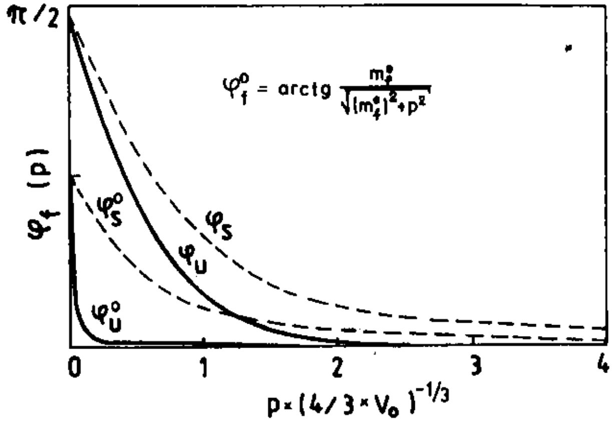


Fig.1. The solutions to SD equation (5) for quarks having the current masses (16).

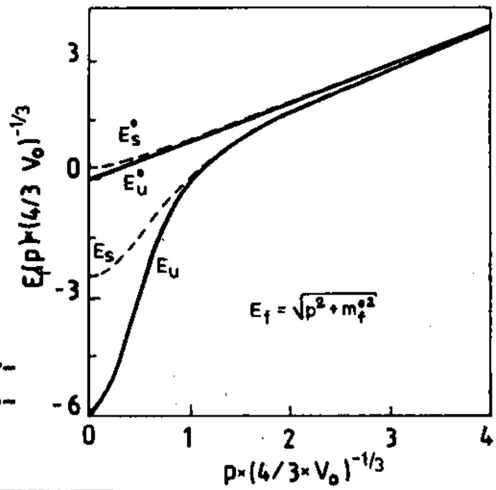


Fig.2. The constituent quark energies (6), corresponding to the solutions of eq.(5).

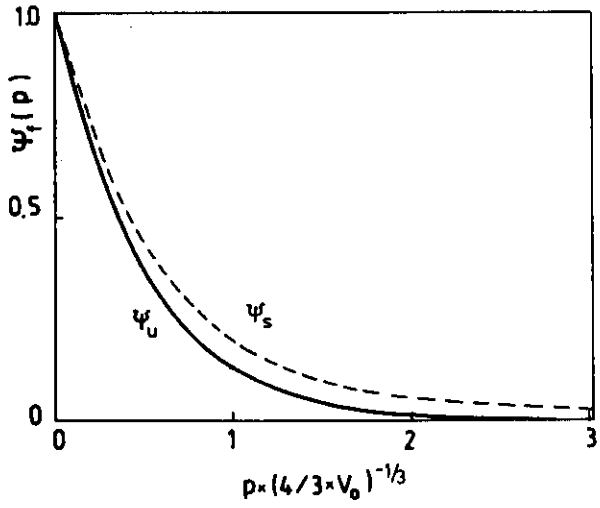


Fig.3. The quark pair functions (7) for the massive quarks.

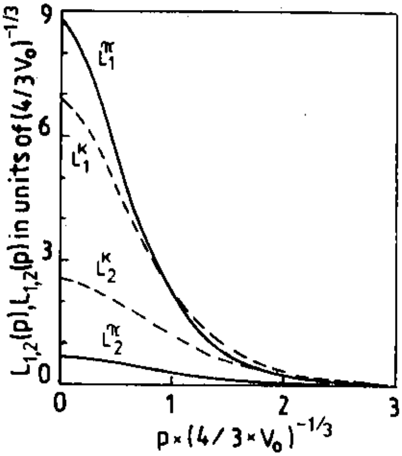
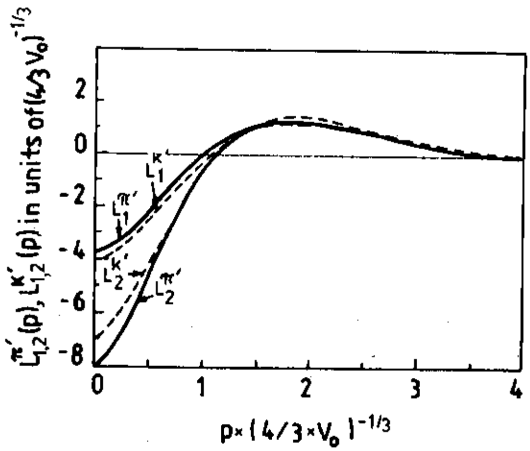


Fig.4. The solutions to BS equation (8) for π and K mesons.

Fig.5. The solution (with one node) to BS equation (8) for radial excitation states and K'.



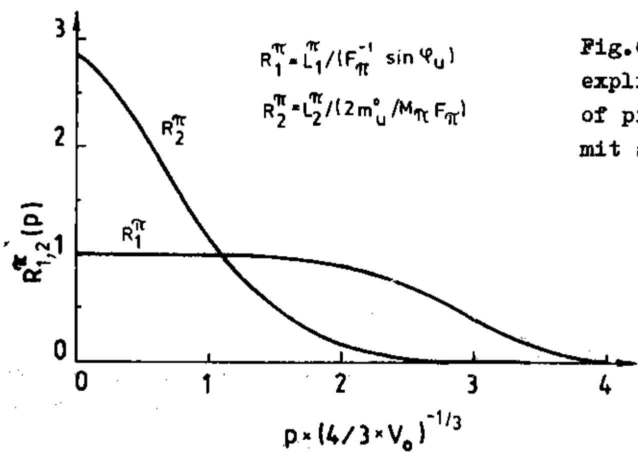


Fig.6. Ratios (17) of the explicit BS wave functions of pion to low energy limit solutions (12) and (13).

Table I. The masses and the coupling constants (11) of mesons and their radial excitations when the free parameters given by (16) (in the brakets the experimental data are indicated). All the quantities are given in MeV's

$\left(\frac{4}{3}V_0\right)^{1/3}$	F_π	$M_{\pi'}$	$F_{\pi'}$	F_K	$M_{K'}$	$F_{K'}$
289	90.4	1604	4	133	1653	61
247	77.3	1363.4	3.4	113	1405	52

4. Conclusion

The QCD-inspired instantaneous model (I)-(3) is applied to the description of the spectroscopy of quarks, having orbitrary masses in the case of three quarks u,d and s interacting through the oscillator potential (4), is considered.

It is shown that this model, unlike the nonvariant approaches /1-7/, qualitatively well reproduces the physical values of the pion mass and decay constant F_π. An analogous satisfactory description takes place for a kaon and the masses of π' and K' mesons too (see table I). For the decay constants of these mesons we have obtained $F_{\pi'}$ = 3.4 MeV and $F_{K'}$ = 52 MeV which in satisfactory agreement with the duality estimations /14/: $F_{\pi'}$ = (4-6) MeV and $F_{K'}$ = = 51 MeV. These predictions can be tested experimentally, for example, by means of the branching ratios for τ -lepton decays $\tau \to \pi' \nu_\tau$ and $\tau \to K' \nu_\tau$.

Figure 6 for ratios (17) allows us to conclude that approximation (13) is valid only in the momentum region near $p \sim \left(\frac{4}{3}V_0\right)^{1/3}$ and the quark condensates do not play the role of fundamental parameters.

Recently, the pion and kaon properties have been successfully described in the framework of the "separable version" of the Nambu--Jona-Lasinio model /13/ (the current quark masses, $m_u^0 = m_d^0$ = 2.1 MeV and m_s^0 = 56 MeV, used in this approach, are comparable with the ones in (16)). However, this task is solved in the model (I)-(3) without involving the cut-off parameters. Moreover, the advantage of the model (I)-(3) in comparison with the separable approach is the possibility of considering the radial and high-orbital states, and the systems of heavy quarks for which the Coulomb potential is significant.

To summarize, the reproducing of the physical properties of the mesons on the example of three- quark flavours indicates that the instantaneous model (I)-(3) is able to uniquely describe on the qualitative level the spectra of light and heavy mesons. A more detailed substantiation of this conclusions requires to include the Coulomb potential contribution, which is a subject of future works.

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