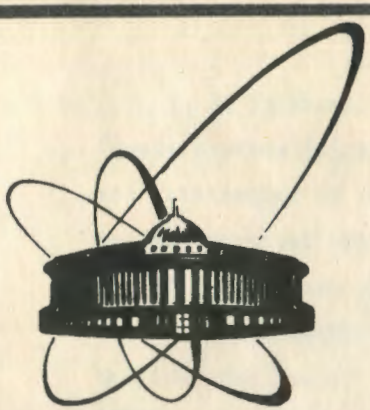

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PATH INTEGRAL FOR THE YANG-MILLS THEORY IN A NONPERTURBATIVE REGION

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1. It is known for systems with first-class constraints (*gauge systems*)[1] that, *generally speaking*, the quantization and elimination of unphysical variables do not commute [2-5] because physical variables (space of gauge orbits) are described by curvilinear coordinates. However, the introduction of curvilinear coordinates and quantization do not commute. Nevertheless, the usual way of deriving a Hamiltonian path integral (HPI) for constrained systems corresponds just to quantization of a system after eliminating all unphysical variables, and, *moreover, the phase space of physical degrees of freedom is* assumed a priori to be an even-dimensional Euclidean space. So, this HPI approach differs from the one in the operator formalism by Dirac[1]. It is shown for a simple model in [6] how one should modify the HPI approach so that it could correspond to the operator scheme by Dirac. It is necessary to take into account both the curvilinearity of physical variables and a possible reduction of a physical phase space [3,7] in order to find a correct HPI.

In the present letter the method of deriving HPI corresponding to the Dirac operator scheme is suggested for any way of a fixing physical variables (any gauge). It is also shown that the elimination of unphysical variables before quantization leads to a gauge-dependent quantum theory. In the framework of the found HPI approach the problem of non-existence of global gauge fixing in the Yang-Mills theory [8-10] is considered. It is argued in favour of that "copies" of intermediate field configurations (i.e. being between in- and out-field configurations in the transition amplitude) do not influence the HPI

description. However, unlike [11] "copies" of in- and out-field configurations should be taken into account in HPI. The found HPI modification does not change a perturbative Yang-Mills theory but it may turn out to be essential for a quasiclassical calculations [12].

2. Before the Yang-Mills theory, consider the simplest and well-known example [3,13] in order to illustrate the key point of the problem. The Lagrangian reads as

$$L = \frac{1}{2}\left(\dot{\underline{x}} + y T \underline{x}\right)^2 - V(\underline{x}^2) \qquad (1)$$

where a two-dimensional vector $\underline{x} = (x_1, x_2)$ and a scalar y are dynamical variables, $T = i\tau_2$ is a generator of rotations in a plane $\underline{x} \in \mathbb{R}^2$ (τ_2 is the Pauli matrix), V is a potential. Lagrangian (1) is invariant under gauge transformations

$$\underline{x} \to \exp(T\omega)\,\underline{x} \;, \quad y \to y - \dot{\omega} \;, \quad \omega = \omega(t). \qquad (2)$$

Canonical momenta are $\underline{p} = \partial L / \partial \dot{\underline{x}}$ and $\pi = \partial L / \partial \dot{y} = 0$, so the Hamiltonian has the form

$$H = \frac{1}{2}\,\underline{p}^2 + V(\underline{x}^2) - y\,\underline{p} T \underline{x} \,. \qquad (3)$$

There are two first-class constraints [1] in the theory: $\pi = 0$ and $\sigma = \{\pi, H\} = \underline{p} T \underline{x} = 0$, where σ is an angular momentum of a particle $\underline{x}$. So, the system has only one physical degree of freedom.

Apparently, gauge group orbits are circles with centers at $x_i = 0$ $(i=1,2)$. A line ℓ $x_i = f_i(u)$, u is a parameter, on a plane forms a gauge condition. The line ℓ should interseot every gauge orbit, at least, once so that u may describe the orbit space. The simplest case is $x_1 = u$, $x_2 = 0$ (unitary gauge). However, there remains a residual gauge group $\mathbb{Z}_2$.

This group acts in a physical configuration space: $x_i \to \pm x_i$ identifying points in it, i.e., the physical region of x_i is the semiaxis $x_i \geqslant 0$. This leads to a physical phase space reduction [3,7] and a modification of HPI [6,7,14] .

In the general case of arbitrary f_i , a highly intricate discrete group $S(u)$ acts on $u \in \mathbb{R}$: $u \to u_s(u)$. Obviously, the group $S(u)$ rearranges cyclically intersection points of the line ℓ with circles of the fixed radius $r(u) = (f_i^2(u))^{1/2}$. All functions $u_s(u)$ can be found from the equation

$$r^2(u_s) = r^2(u) . \tag{4}$$

(Here $r(0) = 0$ and $r(\pm\infty) = \infty$ are assumed). For describing the physical region of $u \in K$, we divide the axis $u \in \mathbb{R}$ into parts $\mathbb{R} = \bigcup_\alpha \mathbb{R}_\alpha$ so that Eq. (4) could have a fixed number of solutions at $u \in \mathbb{R}_\alpha$. Then $S(u) = \prod_\alpha \otimes S_\alpha$, i.e. $S(u) = S_\alpha$ when $u \in \mathbb{R}_\alpha$. In every $\mathbb{R}_\alpha$ we pick out a fundamental region K_α with respect to an action of S_α in $\mathbb{R}_\alpha$, i.e. $K_\alpha = \mathbb{R}_\alpha / S_\alpha$. Therefore, $K = \bigcup_\alpha K_\alpha$.

The quantum theory is given by equations [1]

$$\left[-\tfrac{1}{2} \Delta + V(\underline{x}^2) \right] \Phi_E(\underline{x}) = E \Phi_E , \tag{5}$$

$$\sigma \Phi_E(\underline{x}) = -i \underline{x}^T \frac{\partial}{\partial \underline{x}} \Phi_E(\underline{x}) = 0 , \tag{6}$$

where $\Delta = \left(\partial / \partial \underline{x} \right)^2$ (we do not consider the third trivial equation $\pi \Phi_E = -i \partial / \partial y \Phi_E = 0$). To get the correct quantum theory corresponding to a gauge condition $x_i = f_i(u)$, we introduce new curvilinear coordinates in (5),(6)

$$\begin{pmatrix} x_1 \\ x_2 \end{pmatrix} = \exp(T\theta) \begin{pmatrix} f_1(u) \\ f_2(u) \end{pmatrix} . \tag{7}$$

In the simplest case $f_1 = u \equiv r$, $f_2 = 0$ (7) gives polar coordinates. Since (7) should be the change of variables, one-to-one correspondence should exist between points $\underline{x} \in \mathbb{R}^2$ and $(\theta, u) \in \mathbb{R}^2$. So, $\theta \in (0, 2\pi)$ and $u \in \tilde{K} \subset \mathbb{R}$. To determine $\tilde{K}$, consider the symmetry group of the change of variables $\tilde{S}$: $\theta \to \theta + \theta_s(u)$, $u \to u_s(u)$ so that $\underline{x}$ in (7) does not change. It is easily seen that transformations from $\tilde{S}$ ($u \to u_s(u)$) can be found from (4). Indeed, $\tilde{S}$ is a composition of two transformations: 1) a point $x_i = x_i(\theta, u)$ being on a circle of a radius $r(u)$ passes at another point $x_i^s = x_i(\theta, u_s)$, 2) a point x_i^s returns to an initial point x_i by the rotation $\exp(T\theta_s(u))$. Thus, $\tilde{S} = S(u)$ and $\tilde{K} = K$ for $u \in \mathbb{R}$ *).

In Eqs. (5),(6) $\Phi_E(\theta, u) = \Phi_E(u)$ since $\sigma = -i\,\partial/\partial\theta$ in the new variables. So, in a physical subspace of states $\mathcal{H}_{ph}$ the scalar product reads as follows

$$\sum_{\alpha} \int_{K_\alpha} du\, \rho(u)\, \Phi_E^*(u)\, \Phi_{E'}(u) = \delta_{EE'}, \qquad (8)$$

where $d^2x = d\theta\, du\, \rho(u)$, $\rho(u) = f_i(u)\, \partial_u f_i(u)$ and the factor $\int d\theta = 2\pi$ is included into the norm of Φ_E.

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*) One should emphasize that the groups S and $\tilde{S}$ are different in nature in spite of the formal equality $S = \tilde{S}$. The group S is the residual discrete gauge group (group of "copies") acting in a configuration space of physical degree of freedom when unphysical variables are eliminated in a non-invariant way (i.e., by a gauge fixing). On the contrary, $\tilde{S}$ is the symmetry group of a change of variables $\underline{x} \to (\theta, u)$ where θ is an unphysical degree of freedom and u is a gauge-invariant one. So, saying below about "copies" we shall just imply the group $\tilde{S}$.

Eq. (5) in $\mathcal{H}_{ph}$ turns into

$$H_{ph}\Phi_E = \left(\tfrac{1}{2}P_u g(u) P_u + V_q(u) + V\right)\Phi_E = E\Phi_E \qquad (9)$$

where $V_q = \tfrac{1}{2}\mu^{-1/2}\partial_u\left(g(u)\partial_u\mu^{1/2}\right)$ is an effective quantum correction ($\sim\hbar^2$) to a potential, $g(u) = r'^2(u)/\mu^2(u)$ and $P_u = -i\mu^{-1/2}\partial_u\circ\mu^{1/2}$ is a Hermitian momentum operator. The first two terms in H_{ph} are, in fact, the Laplace - Beltrami operator Δ in coordinates (7) without terms containing $\partial/\partial\theta$. Amplitudes (scalar products in $\mathcal{H}_{ph}$) do not depend on the choice of f_i although the Hamiltonian H_{ph} depends on f_i. Indeed, making the substitution $\partial_u = \mu/r\,\partial_r$ in (9) we see that eq. (9) turns into the usual radial part of the Schroedinger equation (5) in polar coordinates, i.e. $\Phi_E(u) = \widetilde{\Phi}_E(r)$. Moreover, purely radial-excitations (S-states) should be even $\widetilde{\Phi}_E(r) = \widetilde{\Phi}_E(-r)$. So, all physical states are manifestly gauge-invariant

$$\Phi_E(u) = \widetilde{\Phi}_E(r^2) = \widetilde{\Phi}_E(\underline{x}^2). \qquad (10)$$

At last, by the definition of K the equality $\sum_{-\infty}\int_{K_\alpha} du\,\mu(u) = \int_0^\infty dr\,r$ should take place. As a result, amplitudes $\langle\Phi|\Phi'\rangle$ are independent of functions f_i.

If unphysical variables are eliminated in Hamiltonian (3) before quantization with the help of constraints and supplementary conditions $y = 0$, $x_i = f_i(u)$ (or $\chi(x_1, x_2) = 0$, where χ depends on f_i), the quantum theory (spectrum, amplitudes, etc.) depends on the choice of physical variables f_i (on a gauge), since eq. (10) is not valid and a scalar product does not coincide with (8).

3. Now consider the HPI approach. It follows from (10) and (4) that $\Phi_E(u_s) = \Phi_E(u)$. This property allows us to continue

analytically the unit operator kernel $\langle u|u'\rangle_{ph} = \sum_E \Phi_E(u)\Phi_E^*(u')$ into the unphysical region $u \in \mathbb{R}$. In accordance with (8) we may write

$$\langle u|u'\rangle_{ph} = \sum_S \left[\mu(u)\mu(u'_s)\right]^{-1/2} \delta(u-u'_s)\,. \qquad (11)$$

where $u'_s = u_s(u')$, $u' \in K, u \in \mathbb{R}$. The infinitesimal evolution operator kernel is defined as

$$U_\varepsilon^{ph}(u,u') \equiv \langle u|e^{-i\varepsilon H_{ph}}|u'\rangle_{ph} \approx \left(1-i\varepsilon H_{ph}(u)\right)\langle u|u'\rangle_{ph}\,. \qquad (12)$$

where $\varepsilon \to 0$. We transform the kernel (11) in (12) to the form

$$\langle u|u'\rangle_{ph} = \int_{-\infty}^{\infty} \frac{du''}{(\mu\mu'')^{1/2}} \left[\int_{-\infty}^{\infty} \frac{dp}{2\pi} \exp ip(u-u'')\right] Q(u'',u')\,, \qquad (13)$$

where $\mu = \mu(u)$, $\mu'' = \mu(u'')$ and

$$Q(u'',u') = \sum_S \delta(u''-u'_s)\,. \qquad (14)$$

Substituting (13) into (12) and taking $H_{ph}(u)$ from (9) we find

$$U_\varepsilon^{ph}(u,u') = \int_{-\infty}^{\infty} \frac{du''}{(\mu\mu'')^{1/2}} U_\varepsilon^{eff}(u,u'')Q(u'',u') \qquad (15)$$

accurate to $O(\varepsilon^2)$. Here

$$U_\varepsilon^{eff}(u,u'') = \int_{-\infty}^{\infty} \frac{dp}{2\pi} \exp\left[ip(u-u'') - i\varepsilon H^{eff}(u,p)\right], \qquad (16)$$

$$H^{eff} = \tfrac{1}{2} g(u)p^2 + \tfrac{i}{2} p\partial_u g(u) + V_q(u) + V \qquad (17)$$

is the effeotive Hamiltonian. Further, we should oalculate the convolution

$$U_{2\varepsilon}^{ph}(u,u') = \sum_\alpha \int_{K_\alpha} du''\mu(u'')U_\varepsilon^{ph}(u,u'')U_\varepsilon^{ph}(u'',u') \qquad (18)$$

to get the evolution operator kernel for a finite time interval. One may check that formula (15) is correct for the kernel (18) and what's more $U^{eff}_{2\varepsilon}(u,u') = \int_{-\infty}^{\infty} du''\, U^{eff}_{\varepsilon}(u,u'')\, U^{eff}_{\varepsilon}(u'',u')$. This statement follows directly from the equality

$$\sum_{\alpha} \int_{K_\alpha} du''\, \mu(u'')\, Q(u,u'')\, U^{ph}_{\varepsilon}(u'',u') = \mu(u)\, U^{ph}_{\varepsilon}(u,u') \quad (19)$$

which is a simple consequence of the equalities $H_{ph}(u_s) = H_{ph}(u)$ in (9) and $\langle u_s | u' \rangle_{ph} = \langle u | u' \rangle_{ph}$, $dr\, r = du\, \mu(u) = du_s\, \mu(u_s)$. On the whole, formula (15) is correct for a finite time interval t ($\varepsilon \to t$ in (15)) and the kernel U^{eff}_{t} is determined by the usual HPI

$$U^{eff}_{t}(u,u'') = \int \prod_{\tau=0}^{t} \left(\frac{dp(\tau)\, du(\tau)}{2\pi} \right) \exp i \int_{0}^{t} d\tau \left[p\dot{u} - H^{eff}(p,u) \right], \quad (20)$$

where $u = u(t)$, $u'' = u(0)$.

Thus, "copies" of intermediate points on a trajectory connecting u and u' in (15) ($\varepsilon \to t$) do not influence the transition amplitude. It is necessary to take into account only "copies" of initial or final points.

The problem of gauge fixing is usually connected with zeros of the Faddeev - Popov determinant [8] as if they prevent on pPI definition in a total configuration space. However, knowing only zeros of the determinant we cannot judge about the permissibility of a gauge. In this model, assuming $f_1 = u$, $f_2 = u - a$, i.e., $\chi = x_2 - x_1 + a = 0$ we find the determinant $M = \{\sigma, \chi\} = x_2 + x_1 = 2x_1 - a$. So, $M = 0$ at $x_1 = a/2$. Nevertheless, the gauge $\chi = 0$ is admissible only for $a = 0$ since the line $\chi = 0$ intersects all gauge orbits only at $a = 0$. Therefore, the only

criterion for a choice of physical variables is the possibility of making the change of variables in which we may solve constraints in a quantum theory.

4. Let us turn now to the Yang - Mills theory. Gauge transformations of vector potentials A_μ being elements of a Lie algebra of a simple compact group G read as follows

$$A_\mu \to \Omega A_\mu \Omega^{-1} + \frac{1}{g} \Omega \partial_\mu \Omega^{-1}, \qquad (21)$$

where $\Omega \in G$ and g is a coupling constant. The Hamiltonian formalism for this theory is well-known [15]. In fact, A_0 is the Lagrangian multiplier in the theory and we may ignore it in the quantum theory. So, physical states satisfy the following equation [15]

$$\sigma \Phi[\underline{A}] \equiv \left(\partial_\kappa \pi_\kappa + g[A_\kappa, \pi_\kappa]\right) \Phi[\underline{A}] = 0, \qquad (22)$$

where $\pi_\kappa(\underline{x}) = -i \delta/\delta A_\kappa(\underline{x})$ ($\kappa = 1,2,3$) are momentum operators canonically conjugated to $A_\kappa(\underline{x})$ and $[,]$ is a commutator in a Lie algebra. It follows from (22) that physical states $\Phi[\underline{A}]$ viewed as funotionals of dynamical variables $A_\kappa(\underline{x})$, must be invariant under transformations generated by σ, i.e.

$$\delta \Phi = \int d^3x \, \mathrm{Tr}\, \delta A_\kappa(\underline{x}) \frac{\delta \Phi}{\delta A_\kappa(\underline{x})} \equiv g^{-1} \int d^3x \, \mathrm{Tr}\left(\omega(\underline{x}) \sigma(\underline{x})\right) \Phi = 0, \qquad (23)$$

where $\delta A_\kappa = [\omega, A_\kappa] - g^{-1} \partial_\kappa \omega$ is an infinitesimal transformation (21) ($\Omega \approx 1 + \omega$).

The scalar product in a physical subspace $\mathcal{H}_{ph}$ has the form

$$\int D\underline{A} \; \Phi^*_E[\underline{A}] \, \Phi_{E'}[\underline{A}] = \delta_{E E'}. \qquad (24)$$

Here $\mathcal{D}\underline{A} = \prod_{\underline{x}} d\underline{A}(\underline{x})$ is a measure in a functional field space, the integration region $[\underline{A}]$ in (24) is defined so that every component of $\underline{A}(\underline{x})$ runs the real axis at fixed $\underline{x} \in \mathbb{R}^3$, and the states Φ_E satisfy the functional Schroedinger equation

$$\left[-\frac{1}{2} \left\langle \frac{\delta}{\delta A_\kappa}, \frac{\delta}{\delta A_\kappa} \right\rangle + V[\underline{A}] \right] \Phi_E = E \Phi_E . \qquad (25)$$

where $V[\underline{A}]$ is the Yang-Mills potential energy and $\langle , \rangle = \int d^3x \, \mathrm{Tr}$ is the scalar product in a space of dynamical variables.

Consider now a gauge condition $F[\underline{A}] = 0$ picking out physical degrees of freedom. It is necessary explicitly to solve Eq. (22) in order to guarantee the gauge invariance of the quantum theory (see (23)) in the physical configuration space $F[\underline{A}] = 0$. With this purpose we introduce new functional curvilinear coordinates w, α by analogy with (7)

$$A_\kappa = w \tilde{A}_\kappa w^{-1} + g^{-1} w \partial_\kappa w^{-1} . \qquad (26)$$

Here $w = w[\underline{A}] \in G$, $\tilde{A}_\kappa = \tilde{A}_\kappa[\alpha]$, and $\alpha = \alpha[\underline{A}]$ so that the equation $F[\tilde{\underline{A}}] = 0$ turns into the identity after the substitution $\tilde{A}_\kappa = \tilde{A}_\kappa[\alpha]$ into it, i.e., elements of a Lie algebra $\tilde{A}_\kappa$ and variables α are analogous to the vector-column f_i and the variable u in (7), respectively.

The condition $F = 0$ should not contradict the law of gauge transformations (21) and boundary conditions of $\underline{A}$ as $|\underline{x}| \to \infty$ [16]. In other words, we assume that any configuration $\underline{A}$ can be transformed to the form $F[\underline{A}] = 0$ by a non-singular gauge transformation (21) preserving its topology [17].

The HPI derivation is similar to (8)-(20). The operator of the constraint in (23) generates shifts of w in (26) and it

commutes with α being a gauge invariant, so Eq. (22) is equivalent to $\delta/\delta w\,\Phi = 0$. Thus, in $\mathcal{H}_{ph}$, the scalar product reads as follows

$$\int_K D\alpha\,\mu[\alpha]\;\Phi_E^*[\alpha]\,\Phi_{E'}[\alpha] = \delta_{EE'}\,, \qquad (27)$$

where $\mu[\alpha]$ is a measure in a physical configuration space: $D\underline{A} = \prod_x d\mu_G(w)\; D\alpha\,\mu[\alpha]$, $d\mu_G(w)$ is an invariant measure on G (the "volume" of an unphysical configuration space included in the norm of Φ_E), $K = [\alpha]/S$, $[\alpha]$ is a configuration space of α (all components of $\alpha(\underline{x})$ run a real axis at fixed $\underline{x} \in R^3$). The group S is determined from the symmetry group of the change of variables (26) $\tilde{S}: \alpha \to \alpha_s[\alpha]$, $w \to ww_s$ so that $\underline{A}$ is not varied, i.e., $F[\tilde{\underline{A}}_s] = F[\tilde{\underline{A}}]$, $\tilde{\underline{A}}_s = \tilde{\underline{A}}[\alpha_s]$. Therefore, S formally coincides with the residual gauge group determining "copies" of $\underline{A}$ in a gauge $F=0$ *).

Rewriting Hamiltonian in (25) in curvilinear coordinates (26) and rejecting terms with $\delta/\delta w$ we get, instead of (25),

$$\left[\frac{1}{2}\left(P_\alpha, g_{ph} P_\alpha\right) + V_q[\alpha] + V\right]\Phi_E = E\Phi_E\,, \qquad (28)$$

where $P_\alpha = -i\mu^{-1/2}\delta/\delta\alpha\cdot\mu^{1/2}$, $V_q[\alpha] = \frac{1}{2}\mu^{-1/2}\left(\delta/\delta\alpha, g_{ph}\delta/\delta\alpha\,\mu^{1/2}\right)$ is the effective quantum correction $\left(\sim(\hbar\delta^3(0))^2\right)$ to the potential

*) One has to have in mind that the transformations from $\tilde{S}$ should be non-singular to preserve the topology of fields $\tilde{\underline{A}}$, i.e., $\tilde{\underline{A}}_s$ and $\tilde{\underline{A}}$ belong to one and the same topology class [17] (compare with electrodynamics where gauge transformations with multi-valued functions are forbidden).

V , $(\,,\,)$ is a scalar product in a space of components of α (it is induced by $<\,,\,>$), $g_{ph} = g_{ph}[\alpha]$ is a metric tensor in a physical configuration space (is a Hermitian linear operator in a space of components of α). The metric tensor in coordinates (26) can be found from $<\delta A_\kappa, \delta A_\kappa> = (\delta q^a, g_{a\beta}\,\delta q^\beta)$, $a,b=1,2$. where $\delta q^1 = \delta\alpha$, $\delta q^2 = w^{-1}\delta w$ $\left(\mathcal{D}\underline{A} = (\det g_{a\beta})^{1/2}\, Dq^1 Dq^2\right)$. Then $g_{ph} = g^{11}$. where $g^{a\beta}\, g_{\beta c} = \delta^a_{\ c}$.

By analogy with (11) and (13), we write the unit operator kernel in $\mathcal{H}_{ph}$

$$<\alpha|\alpha'>_{ph} = \int \frac{\mathcal{D}\alpha''}{(\mu\mu'')^{1/2}} \left[\int \frac{\mathcal{D}\varepsilon}{2\pi} \exp i(\varepsilon, \alpha-\alpha'') \right] Q[\alpha'',\alpha'] \,.$$

where $\mu = \mu[\alpha]$, $\mu'' = \mu[\alpha'']$, $\alpha,\alpha'' \in [\alpha]$, $\alpha' \in K$ and

$$Q[\alpha,\alpha'] = \sum_S \delta[\alpha - \alpha'_s] \,, \quad \alpha'_s = \alpha_s[\alpha'] \,. \tag{29}$$

We assume $\int \mathcal{D}\alpha'\, \delta[\alpha-\alpha']\, \Phi[\alpha'] = \Phi[\alpha]$ by definition for a functional Φ . Repeating calculations (12),(15)–(20) we get the transition amplitude for the Yang – Mills theory

$$U^{ph}_{t-t'}[\alpha,\alpha'] = \int \frac{\mathcal{D}\alpha''}{(\mu\mu'')^{1/2}}\, U^{eff}_{t-t'}[\alpha,\alpha'']\, Q[\alpha'',\alpha'] \,. \tag{30}$$

where the kernel $U^{eff}_{t-t'}$ is determined by the standard HPI

$$U^{eff}_{t-t'}[\alpha,\alpha''] = \int \prod_{\tau=t'}^{t} \left(\frac{\mathcal{D}\varepsilon(\tau)\mathcal{D}\alpha(\tau)}{2\pi} \right) \exp i S^{eff} \,, \tag{31}$$

$$S^{eff} = \int_{t'}^{t} d\tau \left[(\varepsilon, \dot\alpha) - H^{eff}[\varepsilon,\alpha] \right], \tag{32}$$

$$H^{eff} = \tfrac{1}{2}\,(\varepsilon, g_{ph}\varepsilon) + V + \tilde V_q \,. \tag{33}$$

Here α , α'' are out- and in- field configurations, respectively $(\alpha(\underline{x},t) \equiv \alpha(\underline{x})$, $\alpha(\underline{x},t') = \alpha''(\underline{x}))$, $\tilde V_q = V_q + \tfrac{i}{2}\,(\delta/\delta\alpha, g_{ph}\varepsilon)$ (the second term $\sim \hbar\delta^3(0)$).

HPI (30) determines the evolution operator kernel, and as a consequence the S-matrix in the theory. We may see from (30) that the ambiguity in the choice of physical variables is essential only for in- and out-field configurations. "Copies" of intermediate configurations do not influence the HPI approach for the S-matrix because of the manifest invariance of the quantum Hamiltonian in (28) with respect to the group S (see (18),(19)). Note that the elimination of unphysical degrees of freedom before quantization breaks this property.

5. In conclusion we shall shortly discuss our result. The perturbative theory for the S-matrix corresponding to (30) does not change. Indeed, the following equality, obviously, should take place

$$\Phi_{out}[\alpha] = \lim_{\substack{t\to\infty \\ t'\to-\infty}} \hat{U}^{ph}_{t-t'}\,\Phi_{in,t'}[\alpha] = \tag{34}$$

$$= \lim \int \mathcal{D}\alpha' \left(\frac{\mu'}{\mu}\right)^{1/2} U^{eff}_{t-t'}[\alpha,\alpha']\,\Phi_{in,t'}[\alpha'] \equiv \lim \hat{U}^{eff}_{t-t'}\,\Phi_{in,t'}[\alpha]$$

according to (27) since $\hat{Q}\Phi[\alpha] = \int_K \mathcal{D}\alpha' \mu[\alpha'] Q[\alpha,\alpha']\,\Phi[\alpha'] = \mu\,\Phi[\alpha]$ for $\Phi[\alpha_s] = \Phi[\alpha] \in \mathcal{H}_{ph}$ (compare with (19) and (10)). Thus, the transition from $|in\rangle$ to $|out\rangle$ is determined by the S-matrix connected with HPI (31) which differs from the usual S-matrix in a gauge $F[\underline{A}] = 0$ (i.e. found by substituting δ-functions of both constraints and supplementary conditions into HPI over a total phase space) only by the terms of ordering in the effective action. However, these terms do not influence the perturbative theory [2]. Note that the factor $(\mu'/\mu)^{1/2}$ of $U^{eff}_{t-t'}$ in (34) can be eliminated by a redefinition of terms $\tilde{V}_q$ in S^{eff} (32).

Nevertheless, the modification of HPI (30) connected with

the operator $\hat{Q}$ may change nonperturbative calculations of the amplitude (30), for example, the quasiclassical one. For simpler models with a gauge symmetry this phenomenon really exists [12].

To include fermions into consideration one should make a change of variables for solving constraints in a superspace. The corresponding technique of an HPI derivation is suggested in [18]. In this case $\hat{Q}$ depends on fermi-degrees of freedom. Really, we must add to (26) the equality $\psi = W\xi$, where ξ is a gauge-invariant fermi-field, and $\tilde{S}$ in the fermi-sector acts as $\xi \to$ $\to W_S^{-1}\xi$ $(W \to WW_S,\ \alpha \to \alpha_s\ ;\ \psi \to \psi,\ A_\kappa \to A_\kappa)$.

Does the HPI modification influence complete Green's functions? The answer is positive. Put, for example, $F[\underline{A}] = \partial_\kappa A_\kappa = 0$ (Coulomb gauge), i.e. we may assume that physical variables α are two transversal components of $\tilde{A}_\kappa$: $\partial_\kappa \tilde{A}_\kappa \equiv \partial_\kappa \alpha_\kappa \equiv 0$. Consider then Green's functions $D^{ph}_{i_1 \dots i_n} = \langle T(\hat{\alpha}_{i_1}(t_1) \cdots \hat{\alpha}_{i_n}(t_n)) \rangle^{ph}_0$, $\hat{\alpha}_i(t) \equiv \hat{\alpha}_i(\underline{x}, t)$. The sign "ph" tells us that the scalar product is determined as (22), $\hat{\alpha}_i(t) = (\hat{U}^{ph}_t)^+ \hat{\alpha}_i \hat{U}^{ph}_t$. In the functional representation $\langle \underline{\alpha} | 0 \rangle = \Phi_0[\underline{\alpha}]$ is a vacuum state and $\hat{\alpha}_i \Phi_0 = \alpha_i \Phi_0$. Repeating calculations (34) for $\hat{U}^{ph}_t \hat{\alpha}_i \Phi_0 =$ $= \hat{U}^{ph}_t \alpha_i \Phi_0 = \hat{U}^{eff}_t \alpha^Q_i \Phi_0$ we conclude that $\alpha^Q_i = \alpha^Q_i[\underline{\alpha}] \neq \alpha_i$ (Φ_0 is gauge-invariant and, hence, is S-invariant) since the group S is always (i.e. for any F) nontrivial for fields $A_\kappa(\underline{x})$ tending to zero when $|\underline{x}| \to \infty$ [9,19]. Moreover $\alpha^Q_i[\underline{\alpha}_s] = \alpha^Q_i[\underline{\alpha}]$ because the action of $\hat{Q}$ defines a state $\hat{\alpha}_i \Phi[\alpha]$ outside the region K in an S-invariant way. For example, putting $f_1 = u$, $f_2 = 0$ in (7) we have $S = Z_2 : u \to \pm u$ $K = (0, \infty)$ and $u^Q(u) = |u|$. Hence $D^{ph}_{i_1 \dots i_n} = \langle T(\hat{\alpha}^Q_{i_1}(t_1) \cdots \hat{\alpha}^Q_{i_n}(t_n)) \rangle_0$,

where the scalar product is determined as (24) ($\underline{A} \to \underline{\alpha}$ in (24)), $\hat{\alpha}_i^Q(t) = \alpha_i^Q[\hat{\underline{\alpha}}(t)]$, $\hat{\alpha}_i(t) = (\hat{U}_t^{eff})^+ \hat{\alpha}_i \hat{U}_t^{eff}$, i.e. the function $D_{i_1 \dots i_n} = \langle T(\hat{\alpha}_{i_1}(t_1) \cdots \hat{\alpha}_{i_n}(t_n)) \rangle_0$ is the complete Green's function in the gauge $\partial_i \alpha_i \equiv 0$ which is determined without both $\hat{Q}$ in HPI and the scalar product modification. Then, $D^{ph} \neq D$ if all permissible configurations of $\underline{\alpha}$ are considered. However, for small $\underline{\alpha}$ (the perturbative theory) $S[\alpha] \approx 1, K \approx [\alpha]$, hence $\hat{Q} \approx 1$ and $\alpha_i^Q \approx \alpha_i$, i.e. $D^{ph} \approx D$.

Let $\Phi[\xi]$ is a fermion state in the holomorphic representation: $\hat{\xi}\Phi = \partial/\partial\xi\Phi$, $\hat{\xi}^+\Phi = \xi\Phi$. In this representation also $\hat{U}_t^{ph}\Phi = \hat{U}_t^{eff}\hat{Q}\Phi$ [18] and the kernel of $\hat{Q}$ coincides with the kernel of the operator of symmetrization in the group S : $\alpha \to \alpha_S[\alpha]$, $\xi \to w_S^{-1}\xi$, $w_S = w_S[\alpha]$ as in the above-considered case (29). However, in contrast with boson variables the integration region for Grassman ones in the scalar product is not changed for the change of variables $(A_\kappa, \psi) \to (\alpha, \xi, w)$ [18]. Because of this property, $\hat{Q}$ really symmetrizes in S states Φ converting them into gauge-invariant ones: $\hat{Q}\Phi[\xi] = \Phi^Q[\xi] = \Phi^Q[w^{-1}\psi] = \Phi^Q[\psi]$. So, $\hat{Q}\hat{\xi}\Phi_0[\xi] = 0$ since $\hat{\xi}\Phi_0$ is not S-invariant and then $\tilde{D}^{ph}_{(n)} = \langle T(\hat{\xi}^+(t_1) \cdots \hat{\xi}(t_n)) \rangle_0 = 0$ at $t_1 \neq \cdots \neq t_n$.

Thus, Green's functions of gauge-invariant (or S-invariant) objects such as $\hat{\xi}^+(\underline{x},t)\hat{\xi}(\underline{x},t) = \hat{\psi}^+(\underline{x},t)\hat{\psi}(\underline{x},t)$, $\hat{\xi}^+(\underline{x},t) \cdot P\exp\left(g\int_y^x \hat{\alpha}_i(\underline{z},t)dz_i\right)\hat{\xi}(\underline{y},t) = \hat{\psi}^+(\underline{x},t)P\exp\left(g\int_y^x \hat{A}_i(\underline{z},t)dz_i\right)\hat{\psi}(\underline{y},t)$ etc. are only non-trivial. In other words, after an elimination of unphysical degrees of freedom, the operator $\hat{Q}$ is the "memory" of the reduced system about its gauge origin:

gauge-invariant objects only have a physical sense, the way of their writing is only changed *) .

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*) For example, for the model from p.2, the only gauge-invariant object (others are functions of it) is written as $x^2 = f_i^2(u)$, where f_i determine a choice of a physical variable).

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