

# сообщения <br> объединенного <br> ииститута <br> ядерных <br> исследований <br> дубна 

E2-90-396
M.E.Biagini ${ }^{1}$. A.Castro ${ }^{2}$, A.Z.Dubničkova, S. Dubnička

THE PREDICTION OF THE BEHAVIOUR
QF $\Lambda$-HYPERON ELECTROMAGNETIC
FORM FACTORS

[^0]The strange 1 -hyperon is a charge-neutral particle which according to the SU(3) symmetry classification belongs to the same octuplet like nucleons, $\Sigma$ - and $\Xi$ - hyperons. Its anomalous magnetic moment is nonzero and takes the value

$$
\begin{equation*}
\mu_{\Lambda}=-0.613 \pm 0.004\left[\mu_{\mathrm{N}}\right] \tag{1}
\end{equation*}
$$

in the nuclear magneton units $\mu_{\mathrm{N}}$. Consequently the $\Lambda$ - particle can in principle interact with electrons in elastic scattering experiments. However, oving to the fact that it decays in $2.631 \times 10^{-10} \mathrm{sec}$. (mainly into a nucleon and a pion) one cannot practically provide the $\Lambda$ - hyperon target for experiments of that type and there are no data on the cross section of the process $e^{-} \Lambda \rightarrow e^{-} \Lambda$ till now, from which one could extract the information on the electromagnetic (e.m.) structure of $\Lambda$ in the space-like region. Similarly to nucleons, the latter is completely described by two independent scalar functions of the photon momentum transfer squared $t<0$, which can be chosen in the form of the Dirac $F_{1}^{\Lambda}(t)$ and Pauli $F_{2}^{\Lambda}(t)$ form factors (ff's) or of the more practical electric ${ }_{G}{ }_{E} \Lambda_{( }(t)$ and magnetic $\mathrm{G}_{\mathrm{M}}^{\Lambda_{( }}(\mathrm{t})$ ones. The only difference from nucleons is that owing to the zero value of 1 sospin of the $\Lambda$-hyperon there is no splitting of the Dirac and Pauli ff's into the combinations of isoscalar and isovector parts, but $F_{1}^{\Lambda}(t)$ and $\mathrm{F}_{2}^{\Lambda}(\mathrm{t})$ are Just identified with the corresponding isoscalar ff's.

The e.m. structure of $\Lambda$ in the time-like region can be measured for $t>4 m_{\Lambda}^{2}$ in the annihilation process $e^{+} e^{-} \rightarrow \Lambda \Lambda$. The differential and total cross sections of the latter, expressed through the electric and magnetic if's, are
$\frac{d \sigma^{c \cdot m}}{d \Omega}\left(e^{+} e^{-} \rightarrow \Lambda \Lambda\right)=\frac{\alpha^{2}}{4 t} \beta\left[\frac{4 m_{\Lambda}^{2}}{t}\left|G_{E}^{\Lambda}\right|^{2} \sin ^{2} \vartheta+\left|G_{M}^{\Lambda}\right|^{2}\left(1+\cos ^{2} \vartheta\right)\right]$
and

$$
\begin{equation*}
\sigma\left(e^{+} e^{-} \rightarrow \Lambda \bar{\Lambda}\right)=\frac{4 \pi \alpha^{2}}{3 t} \beta\left[\frac{2 m^{2}}{t}\left|G_{E}^{\Lambda}\right|^{2}+\left|G_{M}^{\Lambda}\right|^{2}\right] \tag{3}
\end{equation*}
$$

respectively, where $t$ is the total c.m. energy squared, $\beta$ and $m_{\Lambda}$ are the velocity and mass of a preduced lambda or antllambda hyperon and $\vartheta$ is the angle of $\Lambda(\bar{\Lambda})$ relative to $e^{+}\left(e^{-}\right)$beam.

It is not easy task to 1dentify the produced neutral lambda-antilambda pairs among other particles created in the final state of the electron-positron anninilation process. Nevertheless, all technical problems connected with the latter. have been overcome recently and just one experimental point on the $e^{+} e^{-} \rightarrow \Lambda \bar{\Lambda}$ cross section

$$
\sigma\left(e^{+} e^{-} \rightarrow \Lambda \pi\right)=0.100 \pm \begin{gather*}
0.060  \tag{4}\\
0.035
\end{gather*} \quad[\mathrm{nb}] \quad \text { at } t=5.693 \mathrm{Gev}^{2}
$$

has been obtained [1] by the DM2 group at ORSAY for the first time. It exceeds a theoretically predicted [2] value of the cross section, assuming the t-dependence of the constraint-free ff's to arise from the coupling of many vector mesons in the form of a product of poles.

In this paper we do not try to predict the value (4) theoretically. We, however, employ it as a scale to determine a true behaviour of the predicted $e^{+} e^{-} \rightarrow \Lambda \bar{\Lambda}$ cross section and then from the latter to reproduce the electric and magnetic if's both in the time-like and in the space-like region.

In the next section we construct the modified VMD model of the e.m. structure of the $\Lambda$-hyperon with correct analytic properties and the asymptotic behaviour predicted by the quark model for baryon ff's. Section 3 is devoted, first, to the evolution of the resonance couplings to $\Lambda$ from known couplings to nucleons by utilizing the $S U(3)$ symmetry and the normalization conditions for Dirac and Pauli if's, and then to the prediction of the $\Lambda$-hyperon electric and magnetic if's behaviour. Here also the electric and magnetic mean square radil of the $\Lambda$-particle are evaluated numerically. Conclusions and summary are given in section 4.
$\therefore$ A modified VMD model for the electromagnetic structure of n-hyperon.

The electric and magnetic if's of the $\Lambda$-hyperon in (2) and (3) are defined through Dirac $F_{1}(t)$ and Pauli $F_{2}^{\Lambda}(t)$ if's, obtained [3] by the most general decomposition of the matrix element of the $\Lambda$-hyperon e.m. current into a maximal number of linearly independent covariants constructed from momenta and spin parameters. by means of the ralations

$$
\begin{align*}
& G_{E}^{\Lambda}(t)=F_{1}^{\Lambda}(t)+\frac{t}{4 m_{\Lambda}^{2}} F_{2}^{\Lambda}(t)  \tag{5}\\
& G_{M}^{\Lambda}(t)=F_{1}^{\Lambda}(t)+F_{2}^{\Lambda}(t)
\end{align*}
$$

Both sets of ff's are normalized as follows

$$
\begin{align*}
& G_{E}^{\Lambda}(0)=F_{1}^{\Lambda}(0)=0 \\
& G_{M}^{\Lambda}(0)=F_{2}^{\Lambda}(0)=\mu_{\Lambda} . \tag{6}
\end{align*}
$$

Since, as a consequence of the zero value of the $A$-hyperon isospin, contributing is only the isoscalar part $F_{1} \Lambda_{(t)}$ and $\mathrm{F}_{2}^{\Lambda_{( }}(t)$, according to the 1dea of the standard VMD model [4], each $F_{1,2}^{\Lambda}(t)$ in (5) is expressed in the form

$$
\begin{gather*}
\mathrm{F}_{1}^{\Lambda}(t)_{0}=\sum_{S=\omega, \phi, \phi^{\prime}} \frac{\mathrm{m}_{S}^{2}\left(\mathrm{f}_{S N K}^{(1)} / \mathrm{f}_{S}\right)}{\mathrm{m}_{S}^{2}-t} \\
\mathrm{~F}_{2}^{\Lambda}(t)_{0}=\sum_{s=\omega, \phi, \phi^{\prime}} \frac{\mathrm{m}_{S}^{2}\left(\mathrm{f}_{S \Lambda \Lambda}^{(2)} / \mathrm{I}_{S}\right)}{m_{S}^{2}-t} \tag{7}
\end{gather*}
$$

where the summation is carried out just through the well established isoscalar vector mesons. The J/U and $\mathbb{U}^{\prime}(3685)$ have
not been included as they are rather narrow resonances and they are found to be far above the $\Lambda \bar{\Omega}$ threshold. The subindex zero in the if's $F_{1,2}^{\Lambda}(t)_{0}$ means the zero width of the considered vector meson resonances. The ratios of the coupling constants in (7) are constrained by the equations

$$
\begin{align*}
& \sum_{S=\omega, \phi, \phi^{\prime}}\left(f_{S \Lambda \Lambda}^{(1)} / f_{S}\right)=0 \\
& \sum_{S=\omega, \phi, \phi^{+}}\left(f_{S \Lambda K}^{(2)} / f_{S}\right)=\mu_{\Lambda} \tag{8}
\end{align*}
$$

following from the normalization conditions (6).
The unitarization of the VMD model (7) is carried out by incorporation of the two-cut-approximation of the analytic structure of Dirac and Pauli $\Lambda$-hyperon if's. Th1s structure is generated by two branch points, $t_{0}$ and $t_{\text {in }}$, which correspond to the lowest normal or anomalous [5] threshold and to an effective threshold (to be left as a free parameter of the constructed model) simulating the contributions of other virtual processes, respectively.

For the pure 1soscalar ff's $\mathrm{F}_{1}^{\Lambda}(\mathrm{t})$ and $\mathrm{F}_{2}^{\Lambda}(t)$ the lowest normal threshold is at $t=9 m_{\mathbb{\pi}}^{2}$, where the ${\underset{m}{\pi}}$ is the pion mass. However, there is a triangle Feynman diagram of the e.m. vertex of the $\Lambda$-hyperon presented in Pig.1a, which could in principle generate the anomalous threshold below $9 m_{\pi}^{2}$ and therefore one has to determine its position explicitly.


Fig.1a


- Fig. 1 b

From the dual diagram [6] given in Fig. 1 b one finds

$$
\begin{equation*}
t_{a}=4 m_{K}^{2}-\left[1-\left[\frac{m_{\Lambda}^{2}-\left[m_{P}^{2}+m_{K}^{2}-\right]}{2 m_{P} m_{K}^{-}}\right]^{2}\right] \tag{9}
\end{equation*}
$$

or numerically

$$
\begin{equation*}
\mathrm{t}_{\mathrm{a}}=48.889 \mathrm{~m}_{\pi}^{2} \tag{10}
\end{equation*}
$$

The latter value reveals that the lowest anomalous threshold of the $\Lambda$-hyperon if is above the lowest normal threshold at $9 m_{\pi}^{2}$. As a result, the first branch point $t_{0}$ of $F_{1}^{\Lambda}(t)$, and $F_{2}^{\Lambda}(t)$ is then incorporated into Eqs.(7) by the transformation

$$
\begin{equation*}
t=t_{0}-\frac{4\left[t_{1 n}-t_{0}\right]}{\left[\frac{1}{w}-w\right]^{2}} \tag{11}
\end{equation*}
$$

that is transparent from the inverse transformation

which maps the whole $t$-plane into the left-half of the unit disc. For more details of this transformation see ref.[7].

If we, further, denote the zero-width VMD poles, corresponding to the considered resonances $\omega, \phi, \phi^{\prime}$, in the $W$-plane by $W_{s o}$ and the normalization point $t=0$ by $W_{N}$, one can write for the mass squared of resonances the relation

$$
\begin{equation*}
m_{S}^{2}=t_{0}-\frac{4\left(t_{1 n}-t_{0}\right)}{\left(1 / w_{S O}-w_{S O}\right)^{2}} \tag{13}
\end{equation*}
$$

and also the identity

$$
\begin{equation*}
0=t_{0}-\frac{4\left(t_{1 n}-t_{0}\right)}{\left(1 / W_{N}-W_{N}\right)^{2}} \tag{14}
\end{equation*}
$$

Now, substituting the transiormation (11), together with (1今) and (14) 1nto (7) one gets

 (15b)

To exhibit the reality of ( $15 \mathrm{a}, \mathrm{b}$ ) on the real axis for $t<t_{0}$ explicitly, we arrange them by using properties of VMD pole positions ${ }^{\prime} w_{s 0}$ in the $w$ plane, following directly from (12). Let us suppose, e.g., that

$$
\begin{array}{ll}
\text { then } \quad t_{0}<m_{\omega}^{2}<t_{\text {in }} \quad \text { and } \quad m_{\phi}^{2}, m_{\phi^{*}}^{2}>t_{\text {in }} \\
w_{\omega 0}=-w_{\omega 0}^{*} & \text { and } \quad w_{s o}=\left(w_{s o}^{*}\right), s=\phi, \phi^{\prime} \tag{17}
\end{array}
$$

respectively. As a result, eqs. ( $15 \mathrm{a}, \mathrm{b}$ ) take the form
$F_{1}^{\Lambda}(t)_{0}=\left[\frac{1-W^{2}}{1-W_{N}^{2}}\right]^{2}\left[\frac{\left(W_{N}-W_{\omega O}\right)\left(W_{N}-w_{\omega O}^{*}\right)\left(W_{N}-1 / W_{\omega O}\right)\left(W_{N}-1 / W_{\omega O}^{*}\right)}{\left(\bar{W}-W_{\omega O}\right)\left(W-W_{\omega O}^{*}\right)\left(W-1 / W_{\omega O}\right)\left(W-1 / W_{\omega O}^{*}\right)}-\left[f_{\omega \Lambda \Lambda^{(1)}}^{\left(f_{\omega}\right.}\right]+\right.$

$$
\begin{equation*}
\left.+\sum_{s=\phi, \phi} \frac{\left(W_{N}-W_{S O}\right)\left(W_{N}-W_{S O}^{*}\right)\left(W_{N}+W_{s O}\right)\left(W_{N}+W_{s O}^{*}\right)}{\left(W_{S O}\right)\left(W-W_{S O}^{*}\right)\left(W+W_{s O}\right)\left(W+W_{S O}^{*}\right)}\left[f_{s \Lambda \Lambda^{\prime}}^{(1)} f_{s}\right]\right] \tag{18a}
\end{equation*}
$$

$F_{2}^{\Lambda}(t)_{0}=\left[\frac{1-W^{2}}{1-W_{N}^{2}}\right]^{2}\left[\frac{\left(W_{N}-W_{\omega O}\right)\left(W_{N}-W_{\omega O}^{*}\right)\left(W_{N}-1 / W_{\omega O}\right)\left(W_{N}-1 / W_{\omega O}^{*}\right)}{\left(W-W_{\omega O}\right)\left(W-W_{\omega O}^{*}\right)\left(W-1 / W_{\omega O}\right)}\left(W-1 / W_{\omega O}^{*}\right)-\left[f_{\omega \Lambda K^{\prime}}^{(2)} f_{\omega}^{\prime}\right]+\right.$
from which the reality for $t$ c $t_{0}$, where $w(t)$ is real, is already transparent. We note, however, that the inequalities (16) should not be true ones and their precise form will be only known upon determining the value of $t_{\text {in }}$ numericaly in the next. section.

In expressions ( $15 \mathrm{~b}, \mathrm{~b}$ ), as well in $(18 \mathrm{a}, \mathrm{b})$, the vector mesons are still considered to be (see subindex zero in the corresponding ff's) stable particles. However, by introducing nonsero values of widths $\Gamma_{s} \neq 0\left(s=\omega, \phi, \phi^{\prime}\right)$ they can be defined as complex poles $t_{s}=\left(m_{s}-1 \Gamma_{s} / 2\right)$ on unphysical sheets of the four-sheeted Riemann surface in conformity with all required qualities of the constructed unitarized VMD model. The latter is ensured in (18a, b) by a simple change

$$
\begin{equation*}
\mathrm{W}_{\mathrm{so}} \rightarrow \mathrm{~W}_{\mathrm{s}} \tag{19}
\end{equation*}
$$

Which, leads to the shift of poles from the real axis to the complex region of the corresponding unphysical sheets.

Now we would like to draw attention to the factorization property of the used transformation (11). Which makes possible to accomodate the asymptotic behaviour of the $\Lambda$-hyperon e.m.
fi's in conformity with the quark model prediction for baryons. Really, the transformation (11) splits the resultant expressions (18a, b) (see also (15a, b)) Into a product of two factors. The first one

$$
\begin{equation*}
\left[\frac{1-w^{2}}{1-w_{N}^{2}}\right]^{2} \tag{20}
\end{equation*}
$$

determing the asymptotic behaviour of the VMD model (7), does not depend on the type of a contributing vector meson to the $\Lambda$-hyperon e.m. if's and soit is the same for all considered vector mesons. The second factor consists of a sum of pole terms and describes the nontrivial behaviour of if's in the resonance region. Since for $t \rightarrow \pm \infty$ it approaches a real constant, it does not contribute to the asyptotic behaviour of if's.

The particular form $\simeq t^{-1} \mid t \rightarrow \pm \infty$ of the asymptotic behaviour of the zero-w1dth VMD model is in the factor (20)
ensured by the power "2" above the round brackets. So, the change of the latter to any other even positive number leads to the change of the power asymptotic behaviour of considered e.m. If's, however, without a violation of any quality of the constructed unitarized VMD model.

In conformity with the quark model prediction [8] for baryon ff's the power "2" in (18a) will be changed to the power "4" and in (18b) to the power " 6 ". Taking into account these changes of the powers in (18a,b) and introducing nonzero values of vector meson widths by the replacement (19) one gets the modified VMD model


$$
\begin{equation*}
\left.+\sum_{s=\phi, \phi^{\prime}} \frac{\left(W_{N}-W_{s}\right)\left(W_{N}-W_{s}^{*}\right)\left(W_{N}+W_{s}\right)\left(W_{s}+W_{s}^{*}\right)}{\left(W-W_{s}\right)\left(W-W_{s}^{*}\right)\left(W+W_{s}\right)\left(W+W_{s}^{*}\right)}\left[f_{s \Lambda K}^{(1)} / f_{s}\right]\right] \tag{21a}
\end{equation*}
$$


for the $\Lambda$-hyperon e.m. if's with correct analytic properties the asymptotic behaviour predestined by the quark model for baryons. The latter is utilized for a prediction of the $G_{E}^{\Lambda}(t)$ and $G \Lambda_{M}(t)$ if's behaviour both in the time-like and in the space-like region. However, it is a subject of the next section.
3. The behaviour of the electric and magnetic form factors
of the $\Lambda$-hyperon.

The model for a calculation of $G_{E}^{\Lambda}(t)$ and $G_{M}^{\Lambda}(t)$ is obtained by substituting (21a,b) into the relations (5). It depends on the following 13 physical parameters $t_{\text {in }}, m_{s}, \Gamma_{s}, f_{S \Lambda \Lambda^{\prime} / 1}^{(2)} f_{S}$, $\mathrm{f}_{s \Lambda \Lambda^{\prime}}^{(2)} \mathrm{f}_{\mathrm{s}} \quad\left(\mathrm{s}=\omega, \phi, \phi^{\prime}\right)$, which are reduced to 11 linearly independent parameters by the constraints (8) on the coupling ratios following from the normalization conditions (6). As can be seen from subsequent, it is extremely useful to express Just $f_{\phi^{\prime}}^{(1)} \Lambda \Lambda^{\prime} f_{\phi}$, and $\mathcal{I}_{\phi^{\prime} \Lambda \Lambda^{\prime}}^{(2)} f_{\phi^{\prime}}$ through other coupling ratios by the relations (8) as follows

$$
\begin{align*}
& \mathrm{f}_{\phi^{\prime} \Lambda \Lambda^{\prime}}^{(1)} \mathrm{f}_{\phi^{*}}=-\mathrm{f}_{\omega \Lambda \Lambda}^{(1)} / \mathrm{I}_{\omega}-\mathbf{f}_{\phi \Lambda \Lambda^{(1)} / \mathbf{I}_{\phi}}  \tag{22}\\
& \mathrm{f}_{\phi \cdot \Lambda \Lambda}^{(2)} / \mathrm{f}_{\phi}=\mu_{\Lambda}-\mathrm{f}_{\omega \Lambda \Lambda}^{(2)} / \mathrm{f}_{\omega}-\mathrm{f}_{\phi \Lambda \Lambda}^{(2)} / \mathrm{f}_{\phi} .
\end{align*}
$$

Then one can predict the behaviour of $G_{E}^{\Lambda}(t)$ and $G_{N}^{\Lambda}(t)$ as soon as 11 remaining parameters are known numerically.

The masses and the widths of all three resonances are taken from the Review of Particle Properties [9].

The resonance couplings $\mathrm{I}_{\mathrm{S} \Lambda \bar{\Lambda}}(\mathrm{S}=\omega, \phi)$ to the $\Lambda$-hyperon are estimated from the resonance couplings to nucleons, determined In a new nucleon if analysis [10] by an improved model in comparison with the model presented in [7], utilizing the SU(3) symmetry.

Really, from the $S U(3)$ invariant Lagrangian for the vector-meson-baryon-antibaryon vertex

$$
\begin{align*}
L_{V B E}= & \frac{1}{\sqrt{2}} \mathrm{I}_{F}\left[B_{\beta}^{\alpha} \gamma_{\mu} B_{\gamma}^{\beta}-B_{\gamma}^{\beta} \gamma_{\mu} B_{\beta}^{\alpha}\right]\left[v_{\mu}\right]_{\alpha}^{\gamma}+ \\
& \frac{1}{\sqrt{2}} f_{D}\left[B_{\gamma}^{\beta} \gamma_{\mu} B_{\beta}^{\alpha}+{ }_{B}^{\alpha} \gamma_{\mu}^{\alpha} B_{\gamma}^{\beta}\right] \cdot\left[v_{\mu}\right]_{\alpha}^{\gamma}+  \tag{23}\\
& \frac{1}{\sqrt{2}} \perp_{S} B_{\beta}^{\alpha} \gamma_{\mu} B_{\alpha}^{\beta} \omega_{\mu}^{o}
\end{align*}
$$

with a consideration of the ideal $\omega-\phi$ mixing

$$
\begin{aligned}
& \phi^{0}=\phi_{8} \cos \vartheta-\omega_{1} \sin \vartheta \\
& \omega^{0}=\phi_{8} \sin \vartheta+\omega_{1} \cos \vartheta,
\end{aligned}
$$

where $B, B$ and $V$ are [11] baryon, antibaryon and vector-meson octuplet matrices, $\omega_{\mu}^{0}$ is the omega-meson singlet. $f_{F}, f_{D}$ and $f_{S}$ are the corresponding coupling constants and $i \cong 35.3^{\circ}$ is the mixing angle, one can obtain (besides other relations) the following expressions for couplings of $\rho-, \omega-, \phi-$ mesons to nucleons

$$
\begin{align*}
& f_{\rho N \bar{N}}=\frac{1}{2}\left(f_{D}+f_{F}\right) \\
& f_{\omega N \bar{N}}=\frac{1}{2} \cos \theta \mathrm{I}_{S}-\frac{2}{2 \sqrt{3}} \sin \theta\left(3 f_{F}-f_{D}\right)  \tag{24}\\
& f_{\phi N \bar{N}}=\frac{1}{\sqrt{2}} \sin \theta f_{S}+\frac{1}{2 \sqrt{3}} \cos \vartheta\left(3 f_{F}-f_{D}\right)
\end{align*}
$$

and to the $\Lambda$-hyperon

$$
\begin{align*}
& \mathbf{f}_{\omega N A}=\frac{1}{2} \cos \vartheta \mathrm{I}_{\mathrm{S}}+\sqrt{\frac{2}{3}} \sin \vartheta \mathrm{f}_{\mathrm{D}} \\
& \mathrm{f}_{\Phi \Lambda \Lambda}=\frac{1}{\sqrt{2}} \sin \vartheta \mathrm{f}_{\mathrm{S}}-\sqrt{\frac{2}{3}} \cos \vartheta \mathrm{I}_{\mathrm{D}} . \tag{25}
\end{align*}
$$

Now, calculating the values of the universal vector-meson coupling constants

$$
\begin{aligned}
& I_{\rho}=4.987 \\
& f_{\omega}=16.268 \\
& I_{\phi}=13.178
\end{aligned}
$$

by the relation

$$
\begin{equation*}
\frac{f_{v}^{2}}{4 \pi}=\frac{\alpha^{2}}{3} \frac{m_{v}}{I^{\prime}\left(V \rightarrow e^{+} e^{-}\right)} \tag{27}
\end{equation*}
$$

where $\Gamma\left(V \rightarrow \mathrm{e}^{+} \mathrm{e}^{-}\right)$is taken from the Review of Particle Properties [9], one finds from the results on the nucleon if analysis [10] the couplings to nucleons as follows

$$
\begin{array}{ll}
f_{\rho N \bar{N}}^{(1)}=1.875 & f_{\rho N \bar{N}}^{(2)}=6.361 \\
f_{\omega N \bar{N}}^{(1)}=6.803 & f_{\omega N \bar{N}}^{(2)}=-7.155 \\
f_{\phi N \bar{N}}^{(1)}=2.254 & \tag{28}
\end{array}
$$

Substituting the latter values into (24), by means of the solution of the corresponding three linear algebraic equations according to $f_{S}, f_{D}$ and $f_{P}$, the numerical values

$$
\begin{array}{ll}
f_{S}^{(1)}=8.291 & f_{S}^{(2)}=7.657 \\
f_{D}^{(1)}=4.016 & f_{D}^{(2)}=-0.717 \\
f_{F}^{(1)}=-0.266 & f_{F}^{(2)}=13.439 \tag{29}
\end{array}
$$

are found, which (together with the value of the mixing angle $\vartheta \cong 35.3^{\circ}$ ) through the relations (25) give

$$
\begin{array}{ll}
\mathbf{f}_{\omega M K}^{(1)}=6.680 & \mathbf{f}_{\omega \Lambda \Lambda}^{(2)}=4.081 \\
\mathbf{f}_{\phi \Lambda \Lambda}^{(1)}=0.712 & \mathbf{f}_{\phi \Lambda \Lambda}^{(2)}=3.607
\end{array}
$$

Dividing the values in (30) by the corresponding, universal vector-meson coupling constants (26), finally the coupling ratios of the modified VMD model for $\Lambda$-hyperon e.m. ff's are obtained

$$
\begin{align*}
& \mathrm{I}_{\omega \Lambda \Lambda^{\prime}}^{(1)}=0.411 \quad \mathrm{I}_{\omega \Lambda}^{(2)} \quad \bar{\Lambda}_{\omega}^{\prime I_{\omega}}=0.251 \\
& \mathrm{f}_{\phi \Lambda \Lambda^{\prime} / \mathrm{I}_{\phi}}^{(1)}=0.054 \quad \mathrm{f}_{\phi \Lambda \Lambda^{\prime} \mathrm{I}_{\phi}}^{(2)}=0.274 \tag{31}
\end{align*}
$$

The values of $\mathrm{f}_{\phi^{\prime}}^{(1)} \Lambda \Lambda^{/ f_{\phi^{\prime}}}$ and $f_{\phi^{\prime}}^{(2)} \Lambda K^{/ f_{\phi^{\prime}}}$, calculated by the relations (22) and the values (31) are as follows
$\mathrm{f}_{\phi^{\prime}}^{(1)} \Lambda \Lambda^{\prime \mathbf{f}_{\phi^{\prime}}}=-0.465 . \quad \mathrm{f}_{\phi^{\prime}}^{(2)} \Lambda \Lambda^{\prime / \mathbf{f}_{\phi^{\prime}}}=-1.13 \dot{8}$.
In this manner, finally, we are left only with one unknown parameter, $t_{i n}$, of the modified VMD model for a description of the e.m.structure of the $\Lambda$-hyperon. This effective threshold, common for both, Dirac and Pauli fi's, is determined from the f1t of the existing [1] ORSAY DM2 experimental point on the $\mathrm{e}^{+} \mathrm{e}^{-} \rightarrow \Lambda K$ cross section at $t=5.693 \mathrm{GeV}^{2}$. As a result, the following optimal value

$$
\begin{equation*}
t_{\text {in }}=1.462 \pm 0.020 \mathrm{GeV}^{2} \tag{33}
\end{equation*}
$$

is found. The corresponding behaviour of the total cross section (3) with (33) and the values of other parameters of the model given by (31) and (32) is presented in Fig. 2.

The predicted behaviour of the electric $G_{E}^{\Lambda}(t)$ and the magnetic $G_{M}^{\Lambda}(t) \Lambda$-hyperon ff's both in the space-like and in the time-like region are presented in Figs.3a,b and Figs.4a, b, respectively.

The electric and magnetic mean square radil defined by the relations

$$
\begin{align*}
& \left\langle r_{E}^{2}\right\rangle \Lambda=\left.6 \frac{d G_{E}^{\Lambda}(t)}{d t}\right|_{t=0} \\
& \left.<r_{M}^{2}\right\rangle \Lambda=\left.6 \frac{d G_{M}^{\Lambda}(t)}{d t}\right|_{t=0} \tag{34}
\end{align*}
$$

are found to be

$$
\left\langle r_{E}^{2}\right\rangle_{\Lambda}=0.021 \mathrm{~F}^{2} \text { and }\left\langle\mathrm{r}_{\mathrm{M}}^{2}\right\rangle_{\Lambda}=-0.135 \mathrm{~F}^{2} \cdot(35)
$$

So, unlike the neutron, the charge neutral $\Lambda$-hyperon has the positive electric mean square radius, though in the magnetic mean square radius both particles are coinciding in sign.
4. Conclusions and summary

The unitarized analytic VMD model with correct analytic properties and the asymptotic behaviour compatible with the quark model predictions, which has been shown to be successful In a description of the pion [12], kaon [13] and nucieon [7,10] e.m. fi's, is in this paper applied to the prediction of the behaviour of electric and magnetic $\Lambda$-hyperon if's.

The crucial point of the procedure is a determination of the numerical values of the free parameters $t_{i_{n}}$, $m_{s}, \Gamma_{s}$,
 meaning. There are three independent sources used to determine these parameters. First, the masses and widths of all considered resonances are fixed at the world averaged values, given by [9]. Second, the resonance couplings to the $\Lambda$-hyperon are evaluated from the known [10] couplings to nucleons by utilizing the $\operatorname{SU}(3)$ symmetry and normalization conditions for Dirac and Pauli ff's. Third, the optimal value of the effective inelastic threshold is determined from the fit of the only existing ORSAY DM2 experimental point on the $e^{+} e^{-} \rightarrow \Lambda \bar{\Lambda}$ cross section. So, the existing ORSAY DM2 point was used as a scale to reach a true behaviour of $\sigma\left(e^{+} e^{-} \rightarrow \Lambda \pi\right)$ up to $t=10 \mathrm{GeV}^{2}$ (see Fig.2), which seems to be interesting in connection with. the FENICE experiment at Frascati, where simultaneously with neutron ff's the experimental measurement of $n$-hyperon e.m. If's is expected to be realized too.

In conclusion we would like to note that a simplified
assumption was introduced that the effective inelastic threshold of the Dirac if is required in our model to be equal to the value of the effective inelastic threshold of the Pauli If. Though it was shown in .the nucleon if's case to be almost true [10], there is no deeper reason for them to be equal. However, without this simplification of the model and also W1thout the neglect of the $J / \psi$ and $\psi^{\prime}(3685)$ particle contributions at present any realistic prediction could not be possible for the $\Lambda$-hyperon e.m. ff's behaviour.

The autors are very much indebted to Prof. R. BaldiniFerroli for calling their attention to the problem of $n$-hyperon e.m.ff's in connection with the FENICE experiment.

References:

1. D. Bisello et.al., Preprint LAL-81-58, ORSAY (1988).
2. J.G.Koerner and M.Kuroda, Phys. Rev. D16 No7 (1977) 2165.
3. N.Cabibbo and R.Gatto, Phys.Rev. 124 No5 (1961) 1577
4. J.J.Sakurai: Currents and mesons, Univ. of Chicago press (1967).
5. R.Karplus, C. M. Sommerfield, E.H. Wichmann, Phys. Rev. 111 (1958), 1187
6. S. Dubnička, o. Dumbraje, Phys. Reports 190 (1975), 141.
7. S. Dubnička, Nuovo Cimento A100 No1 (1988) 1.
8. G.P.Lepage, S.J.Brodsky, Phys, Rev. D22 (1980) 2157.
9. Review of Partcle Properties, Phys, Lett. 204B, (1988).
10.S. Dubnička, Frascati report LNF-89, Frascat1 (1989) and Nuovo Cimento A (1990) ( to be published).
10. M.Gourdin: Unitary Symmetries, North-Holland Publ.Company Amsterdam (1967).
11. S. Dubnička, I. Furdik, V. A. Mescheryakov, Preprint JINR, E2-88-521, Dubna (1988).
12. S. Dubnička, Preprint JINR, E2-88-840, Dubna, (1988).

Received by Publishing Department
on June 6, 1990.


[^0]:    ${ }^{1}$ INFN - Laboratori Nazionali di Frascati, Frascati(Roma), Italy
    ${ }^{2}$ INFN - Sezione di Padova, Padova, Italy

