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NONLOCAL STOCHASTIC QUANTIZATION ON GAUGE THEORY WITH FERMIONS

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1. INTRODUCTION

In previous papers[1,2] we have proposed a nonlocal stochastic quantization method for the quantum field theory by using the covariant-derivative regularization scheme due to Bern et al.[3-6] It turns out that the theory developed by Parisi and Wu[7] is useful for an adequate formulation of nonperturbative regularization of any field of interest, including theories of supersymmetry, general coordinate invariance and quantum gravity (for example, see[6-10]).

Here main attention is paid to the gauge theory with fermions. We generalize a useful regularization method[11] with meromorphical regulators within the framefork of the regularized Schwinger-Dyson equations. In the stochastic quantization shceme the two-noise equations developed by Sakita[12], Ishikawa[13] and Alfaro and Gravela[14] allow us to reformulate the Parisi-Wu program[7] for fermionic fields and provide an almost bosonic stochastic description of fermions and to apply, with stochastic regularization by fifth-time[15,16], to the study of anomalies in background gauge fields[14,17-19] and vacuum polarization in QED[19] (for other satisfactory stochastic formulations of fermions see [20]). It should be noted that in our shceme the noise term in these equations plays a double role in the theory; it controls the quantum behaviour of the theory and at the same time it carries nonlocality in stochastic equations. Further, we show that the scheme thus obtained is equivalent to the nonlocal theory with a regularized propagator of the type

$$D(x-y)=\frac{1}{(2\pi)^4 i}\int d^4P\ e^{-ip(x-y)}\frac{V(p^2L^2)}{m^2-p^2-i\varepsilon},$$

where $V(p^2L^2)$ is the Fourier transform of the nonlocal distribution $[K(x)]^2$,

$$K(x)=\sum_{n=0}^{\infty}\frac{c_n}{(2n)!}(\Box L^2)^n\,\delta^{(4)}(x) \qquad (1.1)$$

(for details, see Efimov [21] and Namsrai [22]).

Roughly speaking, formfactor (1.1) generalizes a minimal power-low regulator $R=(1-\Box/_{\Lambda}2)^{-2}$ used in the covariant-derivative regularization program for the continuum quantum field theory by Bern et al.[3-6].

The outline of the paper is as follows. **Section 2 has a** preliminary character and is devoted to the investigation of the Yang-Mills fields by means of the stochastic regularization with nonlocal formfactors. Here we obtain a useful representation of operator-valued formfactor which allows us to construct a gauge-invariant finite theory of elementary particle interactions . The method is demonstrated for the Yang-Mills fields . Section 3 deals with stochastic regularized Schwinger-Dyson equations for the gauge theory coupled to fermions. The weak coupling expansion of these systems is discussed. The fermionic contribution to the gluon mass is zero, thus verifying gauge-invariance of the regularized systems with any nonlocal white noise-type terms. Finally, in section 4 we study a simple gauge field model, QED, by means of the nonlocal covariant-derivative regularization method with fermions.

2. Yang-Mills Field and Nonlocal Regulator

In this section, we study stochastic quantization of the Yang-Mills fields within the framework of the nonlocal distribution (1.1) (for details see [1]). To obtain the covariant derivative regularization at the (d+1)-dimensional stochastic level for a d-dimensional SU(N) gauge theory, we consider the regularized Langevin equations for the Yang-Mills field

$$\frac{\partial A^a_\mu(x,t)}{\partial t} = -\frac{\delta S}{\delta A^a_\mu}(x,t)+D^{ab}\Lambda^b(x,t)+\int(dy)K^{ab}_{xy}(\Delta)\eta^b_\mu(y,). \qquad (2.1)$$

Here the action S given by

$$S=-\frac{1}{4}\int(dx)\ F^a_{\mu\nu}(x)\ F^a_{\mu\nu}(x)\ , \quad (dx)=d^dx$$

and

$$F^a_{\mu\nu}(x)=\partial_\mu A^a_\nu-\partial_\nu A^a_\mu+g\ f^{abc}A^b_\mu\ A^c_\nu\ .$$

The Zwanziger term is chosen as

$$\Lambda^a(x,t)=\frac{1}{\alpha}\partial_\mu A^a_\mu(x,t),$$

where α is the gauge-fixing parameter. The quantity $K^{ab}_{xy}(\Delta)$ is a nonlocal distribution for the white noise and is an entirely analytic function of its arguments. The covariant derivative is given in the standard form:

$$\Delta^{ab}_{xy}=\int(dz)D^{ac}_\mu(x)\delta(x-z)D^{cb}_\mu(z)\delta(z-y)=\Delta^{ab}(x)\delta(x-y).$$

The operator Δ^{ab} is defined by

$$\Delta^{ab}(x)=\delta^{ab}\ L^2\ \Box_x+g\Gamma^{ab}_1+g^2\ \Gamma^{ab}_2\ ,$$

where

$$\Gamma^{ab}_1(x)=L^2f^{abc}(\partial_\mu A^c_\mu+A^c_\mu\partial_\mu)$$

and

$$\Gamma^{ab}_2(x)=L^2f^{and}\ f^{nbc}\ A^d_\mu(x)\ A^c_\mu(x).$$

Further, we use the following notation:

$$\Gamma^{ab}(x)=g\ \Gamma^{ab}_1(x)+g^2\ \Gamma^{ab}_2(x).$$

From the Langevin equations it is easy to obtain the Schwinger-Dyson equation for the gauge-invariant functional F[A] at equilibrium:

$$<\int(dx)\{(\delta_{\mu\nu}\Box_x-\partial_\nu\partial_\mu+\frac{1}{\alpha}\partial_\nu\ \partial_\mu)\ A^a_\nu(x)+g\ f^{abc}[\partial_\nu(A^b_\nu\ A^c_\mu)-A^c_\nu\partial_\nu A^b_\mu+$$

$$A^c_\nu\ \partial_\mu\ A^b_\nu-\frac{1}{\alpha}A^c_\mu\ \partial_\nu\ A^b_\nu]-g^2\quad f^{adc}\ f^{dnm}A^c_\nu\ A^n_\nu\ A^m_\mu+\int(dy)[\frac{1}{2}\ \frac{\delta V^{ab}_{xy}(\Delta)}{\delta A^b_\mu(y)}$$

$$+V^{ab}_{xy}(\Delta)\ \frac{\delta}{\delta A^b_\mu(y)}\ ]\}\ \frac{\delta F[A]}{\delta A^a_\mu(x)}>=0\ . \qquad (2.3)$$

Here, by definition the formfactor:

$$V^{ab}_{xy}(\Delta)=\sum_{n=1}^{\infty}a_n\ (\Delta^n)^{ab}_{xy}\ , \qquad (2.4)$$

where $(\Delta^n)^{ab}_{xy}$ is defined by the contraction operation:

$$(\Delta^n)_{xy}=\int(dz_1)\dots(dz_{n-1})\,(\Delta_x^{\,ac_1}\delta(x-z_1))\,(\Delta_{z_1}^{c_1c_2}\,\delta(z_1-z_2))\dots$$

$$\dots(\Delta_{z_{n-1}}^{c_{n-1}b}\,\delta(z_{n-1}-y)). \qquad (2.5)$$

Taking into account (2.2) and an explicit form of the operator Δ we have from (2.4):

$$V^{ab}_{xy}(\Delta)=\sum_n a_n\, L^{2n}\,\delta^{ab}\Box^n_{xy}+\sum_n a_n L^{2(n-1)}\sum_{j=0}^{n-1}\int(dz_1)\dots(dz_{n-1})\,(\Box_{z_0}\delta(z_0-z_1))$$

$$(\Box_{z_1}\delta(z_1-z_2))\dots(\Box_{z_{j-1}}\delta(z_{j-1}-z_j))\,(\Gamma^{ab}(z_j)\delta(z_j-z_{j+1}))\dots(\Box_{z_{n-1}}\delta(z_{n-1}-z_n))$$

$$+\sum_n a_n\sum_{j=0}^{n-2}\sum_{s=j+1}^{n-1}L^{2(n-2)}\int(dz_1)\dots\int(dz_{n-1})\,(\Box_{z_0}\delta(z_0-z_1))\,(\Box_{z_1}\delta(z_1-z_2))$$

$$\cdot(\Gamma^{ac}(z_j)\delta(z_j-z_{j+1}))\dots(\Gamma^{cb}(z_{j+s})\delta(z_{j+s}-z_{j+s+1}))\dots(\Box_{z_{n-1}}\delta(z_{n-1}-z_n))$$

$$+\dots+\sum_n a_n\int(dz_1)\dots\int(dz_{n-1})\,(\Gamma^{ac_1}(z_0)\delta(z_0-z_1))\,(\Gamma^{c_1c_1}(z_1)\delta(z_1-z_2))$$

$$\dots(\Gamma^{c_{n-1}b}(z_{n-1})\delta(z_{n-1}-z_n)). \qquad (2.5)$$

Here we have used the notation $z_0=x$; $z_n=y$. From (2.5) it is seen that $V^{ab}_{xy}(\Delta)$ has the following decomposition in powers of the vertex $\Gamma^{ab}(x)$:

$$V^{ab}_{xy}(\Delta)=V^{ab}_{xy}(\Box)+V^{ab}_{xy}(\Box\Gamma)+V^{ab}_{xy}(\Box\Gamma\Gamma)+\dots+V^{ab}_{xy}(\Gamma).$$

Consider the first term of this expression.

$$V^{ab}_{xy}(\Box)=\sum_n a_n L^{2n}\delta^{ab}\Box^n_{xy}=\sum_n a_n\delta^{ab}L^{2n}\int(dz_1)\dots\int(dz_{n-1})*$$

$$(\Box_x\delta(x-z_1))\,(\Box_{z_1}\delta(z_1-z_2))\dots(\Box_{z_{n-1}}\delta(z_{n-1}-y)).$$

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$$\delta(x)=\int(dp)\,e^{-ipx}\,,\qquad (dp)=d^dp/(2\pi)$$

and

$$V^{ab}_{xy}(\Box)=\int(dp)\,e^{-ip(x-y)}V(p^2L^2).$$

Here $V(p^2L^2)$ is given by the following series:

$$V(p^2L^2)=\sum_n a_n\,(p^2L^2)^n$$

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$$V(p^2L^2)=\frac{1}{2i}\int_{-\beta+i\infty}^{-\beta-i\infty}d\xi\,\frac{\upsilon(\xi)}{\sin(\xi)}\,(p^2L^2)^\xi \qquad (0<\beta<1),$$

where the function $\upsilon(\xi)$ satisfies the conditions:

$$\upsilon(\xi)_{\xi=0}=1 \quad\text{and}\quad \upsilon(\xi)_{\xi=-1}=0.$$

Further, we restrict ourselves to the second order of the coupling constant g and pass to the momentum representation in (2.3). As a result we obtain the Schwinger-Dyson equation at equilibrium:

$$<\int(dp)\,[p^2\delta_{\mu\nu}-p_\nu p_\mu+\frac{1}{\alpha}p_\nu\,p_\mu]\,A^a_\nu(p)\,\frac{\delta F[A]}{\delta A^a_\mu(p)}>=<\{\int(dp_1)(dp_2)(dp_3)$$

$$V^{abc}_{\mu\nu\rho}(-p_1,p_2,p_3)\,A^b_\nu(p_2)\,A^c_\rho(p_3)\,\bar\delta(p_1-p_2-p_3)\frac{\delta F[A]}{\delta A^a_\mu(p)}-\int(dp_1)\dots(dp_4)$$

$$\bar\delta(p_1-p_2-p_3-p_4)\,V^{acmn}_{\mu\nu\rho\sigma}\,A^c_\nu(p_2)A^n_\sigma(p_3)\,A^m_\rho(p_4)\,\delta F[A]/\delta A^a_\mu(p_1)\,+$$

$$\int(dp\,V(p^2L^2)\,\frac{\delta F[A]}{\delta A^a_\mu(-p)\,\delta A^a_\mu(p)}+\sum_n a_n L^{2n}\sum_{j=1}^{n-1}\int(dp_1)(dp_2)(p_1^2)^j[\int(dq)$$

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$$*\bar\delta(q_1+q_2-p_1-p_2)\,]\,(p_2^2)^{n-j-1}\frac{\delta^2F[A]}{\delta A^b_\mu(p_2)\,\delta A^a_\mu(p_1)}+\sum_n a_n\sum_{j=0}^{n-2}\sum_{s=j+1}^{n-1}\int(dp_1)(dp_2)$$

$$*(dp_{j+1})(dq)(dk)\,(p_1^2)^j\,(p_{j+1}^2)^s\,(p_2^2)^{n-s-j-2}\,(q+p_{j+1})_\rho A^m_\rho(q)\;f^{amc}f^{cdb}$$

$$*\bar\delta(p_{j+1}-p_1+q)\,(k-2p_2)_\nu A^d_\nu(k)\,\bar\delta(k-p_2-p_{j+1})\,\frac{\delta^2F[A]}{\delta A^b_\mu(p_2)\,\delta A^a_\mu(p_1)}\}>=0 \qquad (2.6)$$

here $\bar\delta(p)=(2\pi)^d\delta(p)$.

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$$V^{abc}_{\mu\nu\rho}(-p_1,p_2,p_3)A^b_\nu(p_2)A^c_\rho(p_3)\bar\delta(p_1-p_2-p_3)\frac{\delta F[A]}{\delta A^a_\mu(p)}-\int(dp_1)\dots(dp_4)$$

$$\bar\delta(p_1-p_2-p_3-p_4)V^{acmn}_{\mu\nu\rho\sigma}A^c_\nu(p_2)A^n_\sigma(p_3)A^m_\rho(p_4)\,\delta F[A]/\delta A^a_\mu(p_1)+$$

$$\int(dp\,V(p^2L^2)\frac{\delta F[A]}{\delta A^a_\mu(-p)\delta A^a_\mu(p)}+\sum_n a_nL^{2n}\sum_{j=1}^{n-1}\int(dp_1)(dp_2)(p_1^2)^j[\int(dq)$$

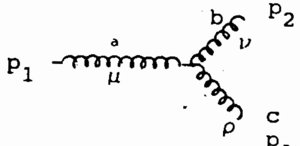
$$(-ig)f^{acb}\bar\delta(q-p_1-p_2)(q-2p_2)_\nu A^c_\nu(q)+\int(dq_1)(dq_2)g^2f^{aec}f^{cdb}A^e_\nu(q_1)A^d_\nu(q_2)$$

$$*\bar\delta(q_1+q_2-p_1-p_2)](p_2^2)^{n-j-1}\frac{\delta^2F[A]}{\delta A^b_\mu(p_2)\delta A^a_\mu(p_1)}+\sum_n a_n\sum_{j=0}^{n-2}\sum_{s=j+1}^{n-1}\int(dp_1)(dp_2)$$

$$*(dp_{j+1})(dq)(dk)(p_1^2)^j(p_{j+1}^2)^s(p_2^2)^{n-s-j-2}(q+p_{j+1})_\rho A^m_\rho(q)\,f^{amc}f^{cdb}$$

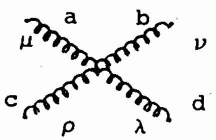
$$*\bar\delta(p_{j+1}-p_1+q)(k-2p_2)_\nu A^d_\nu(k)\bar\delta(k-p_2-p_{j+1})\frac{\delta^2F[A]}{\delta A^b_\mu(p_2)\delta A^a_\mu(p_1)}\}>=0 \qquad (2.6)$$

here $\bar\delta(p)=(2\pi)^d\delta(p)$.



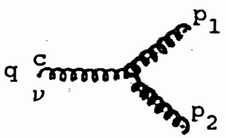
$$=V^{abc}_{\mu\nu\rho}(p_1,p_2,p_3)=-ig\; f^{abc}\{[p_1-p_2]_\rho\delta_{\mu\nu}+[p_2-p_3]_\mu\delta_{\nu\rho}+[p_3-p_1]_\nu\delta_{\mu\rho}+\frac{1}{\alpha}[p_{3\rho}\delta_{\mu\nu}-p_{2\nu}\delta_{\mu\rho}]\}.$$

a)



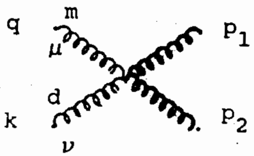
$$V^{abcd}_{\mu\nu\rho\lambda} = -\frac{1}{6}g^2\{f^{abn}f^{cdn}[\delta_{\nu\lambda}\delta_{\mu\rho}-\delta_{\nu\rho}\delta_{\mu\lambda}]+f^{acn}f^{dbn}[\delta_{\mu\lambda}\delta_{\nu\rho}-\delta_{\nu\mu}\delta_{\rho\lambda}]+f^{adn}f^{bcn} * [\delta_{\mu\nu}\delta_{\lambda\rho}-\delta_{\lambda\nu}\delta_{\mu\rho}]\}.$$

b)



$$=(-ig)f^{abc}A^c_\nu(q)\;\; [q-p_1-p_2]_\nu$$

c)



$$= g^2 f^{amn}\; f^{ndb}[2p_1-q]_\mu[k-2p_2]_\nu\bar{\delta}(k+q-p_1-p_2)$$

d)

Fig.1 The vertex diagram. Defined the gluon-gluon interactions.

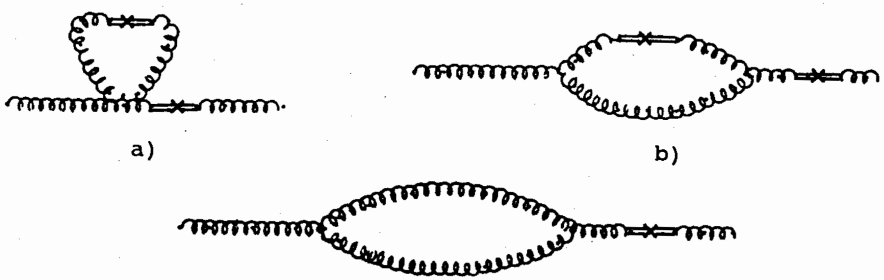


Fig.2. The diagrams giving nonvanishing contributions to the gluon mass in the nonlocal stochastic scheme

In this equation we have written only terms which contribute to vacuum polarization diagrams. Let us consider each term separately. The first and second terms in (2.6) define three-gluon and four-gluon vertices, respectively (the corresponding expressions are shown in figures 1a and 1b). The third term gives a modified propagator of the gluon. Let

$$F[A]=A^a_\nu(q_1)A^b_\mu(q_2)$$

then from (2.6) we get for g=0.

$$\langle A^a_\nu(q_1)A^b_\mu(q_2)\rangle = \frac{2\; d_{\nu\mu}(q_1)}{q_1^2+q_2^2}\; V(q_1^2L^2)\bar{\delta}(q_1+q_2), \qquad (2.7)$$

where $d_{\nu\mu}(q)=\delta_{\nu\mu}-q_\nu q_\mu/q^2 + \alpha\; q_\nu q_\mu/q^2$.

The fourth term in (2.6) defines a contribution to three and four-gluon vertices due to the decomposition of the nonlocal regulator $V^{ab}_{xy}(\Delta)$ and in the fifth term we have presented a subsequent pair of three-gluon vertices. The corresponding diagrams are shown in figures 1c and 1d.

Now we use the Feynman-like diagramatic rule[4] and calculate the vacuum polarization diagrams within the framework of the Schwinger-Dyson formalism for the Yang-Mills field. The corresponding diagrams giving contributions to the vacuum polarization are shown in Figure 2.

Thus the matrix element corresponding to the diagram shown in Fig.2a has the form:

$$\langle A^a_\rho(q_1)A^b_\lambda(q_2)\rangle^{(1)} = \frac{d_{\nu\rho}(q_1)}{q_1^2}\;\frac{d_{\mu\lambda}(q_2)}{q_2^2}\bar{\delta}(q_1+q_2)V(q_2^2L^2)\Pi^{(1)ab}_{\mu\nu}(q_1),$$

where

$$\Pi^{(1)ab}_{\mu\nu}(q) = -3Ng^2\;\delta^{ab}\delta_{\mu\nu}\int(dp)V(p^2L^2)/p^2=$$

$$= \frac{3Ng}{16\;\pi^2}\;\delta^{ab}\delta_{\mu\nu}\frac{\kappa}{L^2}\;\frac{3+\alpha}{4}\;. \qquad (2.8)$$

Here for SU(N) $f^{anm}f^{bnm}=\delta^{ab}N$ and $\kappa=\lim_{\xi\to -1} \upsilon(\xi)/(1+\xi)$.

Analogous calculation for the diagram (Fig.2b) gives:

$$<A^a_\rho(q_1)A^b_\lambda(q_2)> = \frac{d_{\rho\nu}(q_1)}{q_1^2}\ \frac{d_{\lambda\mu}(q_2)}{q_2^2}\ \bar\delta(q_1+q_2)\ V(q_2^2L^2)\Pi^{(2)ab}_{\mu\nu}(q_1)$$

and

$$\Pi^{(2)ab}_{\mu\nu}(q)=2\int(dp)\ V(p^2L^2)/_{p^2}\ V^{anm}_{\nu\lambda\rho}(-q,q-p,p)\ V^{nbm}_{\lambda\mu\rho}(p-q,q,-p)$$
$$*1/(p^2+(p-q)^2+q^2)= Ng^2\delta^{ab}\delta_{\mu\nu}/(16\pi^2)\ \kappa/L^2\ (-5-3\alpha)/4. \qquad (2.9)$$

The contribution to the two-point correlation function from the regularized vertices (Fig.2c) for the gluon field is gibven by:

$$<A^a_\rho(q_1)A^b_\lambda(q_2)> = -2ig\ \frac{d_{\rho\nu}(q_1)}{q_1^2}\int(dp)\ V^{anc}_{\nu\beta\sigma}(-q_1,p,q_1-p)\ f^{bnc}d_{\mu\lambda}(q_2)*$$
$$\bar\delta(q_1+q_2)\sum_{n=1}^{\infty}a_n\sum_{j=0}^{n-1}\{[(p-q_1)^2]^j[p^2]^{n-j-1}[q_2+2p]_\mu\delta_{\beta\sigma}V(q_2^2L^2)/q_2^2\ -[q_2^2]^j*$$
$$[p^2]^{n-j-1}[q_1+p]_\sigma\delta_{\beta\mu}V((p+q_2)^2L^2)/_{(p+q_2)^2}+[q_2^2]^j[(p-q_1)^2]^{n-j-1}\delta_{\sigma\mu}*$$
$$[p-2q_1]_\beta V(p^2L^2)/\ 2\}/(p^2+(p-q_1)^2+q_1^2).$$

After some simple calculations we get:

$$<A^a_\rho(q_1)A^b_\lambda(q_2)> = \frac{d_{\nu\rho}(q_1)}{q_1^2}\ \frac{d_{\mu\lambda}(q_2)}{q_2^2}\bar\delta(q_1+q_2)\ V(q_2^2L^2)\ \Pi^{(3)ab}_{\mu\nu}(q_1),$$

where

$$\Pi^{(3)ab}_{\mu\nu}(q) = -g^2N\delta^{ab}\sum_n a_n\sum_{j=0}^{n-1}\int(dp)2\ p_\mu\delta_{\beta\sigma}[-2p_\sigma\delta_{\beta\nu}+2p_\nu\delta_{\beta\sigma}-2p_\beta\delta_{\nu\sigma}]$$
$$1/(p^2+(p-q)^2+q^2)[(p-q)^2]^j[p^2]^{n-j-1}.$$

This formula is simplified by using the nonlocal formfactor $V(p^2L^2)$:

$$\Pi^{(3)ab}_{\nu\mu}(q)=-g^2N\ \delta^{ab}\delta_{\mu\nu}\lim_{\varepsilon\to 0}\frac{1}{2i}\int_{-\beta+i\infty}^{-\beta-i\infty}d\xi\ \frac{v(\xi)\xi L^{2\xi}}{\sin(\pi\xi)}\int(dp)[p^2+i\varepsilon]^{\xi-1}=$$

$$=-g^2N\ \delta^{ab}\ \delta_{\nu\mu}\ \frac{\kappa}{L^2}\ \frac{1}{16\ \pi^2}. \qquad (2.10)$$

Finally, one can easily verify that the sum of all contributions is zero, so the gluon remains massless to this order for the nonlocal stochastic quantization theory as in the previous case[1]. Thas, we have shown that the new representation (2.5) for the nonlocal formfactor $V^{ab}_{xy}(\Delta)$ constructed with the use of the covariant derivative operator Δ does not change our previous result and

allows us to prove gauge invariance of the theory.

3. Nonlocal Stochastic Regularization of a Gauge Theory With Fermions

To generalize our formalism to the fermions, we employ a covariant-derivative method[11]. Thus, we consider a d-dimensional SU(N) gauge theory coupled to Dirac fermions. the Euclidean action is:

$$S=\int(dx)\{\frac{1}{4}F^a_{\mu\nu}(x)F^a_{\mu\nu}(x)+\ \bar\Psi^A_i[\hat D^{AB}_{ij}+\delta^{AB}\delta_{ij}m]\Psi^B_j\}, \qquad (3.1)$$

where $F^a_{\mu\nu}(x)$ is the usual Yang-Mills field strength as a function of the gauge field A^a_μ. Our fermionic notation is the same as in the paper[11]. Like the Dirac field Ψ^A_i, $\bar\Psi^B_j$ carry spinor subscripts and capital letters which run over the representation, while the Dirac matrices $(\gamma_\mu)_{ij}$ and representation matrices $(T^a)^{AB}$ satisfy:

$$\{\gamma_\mu,\gamma_\nu\}=2\delta_{\mu\nu},\ \gamma^+_\mu=\gamma_\mu\ ;\ [-iT^a,-iT^b]=f^{abc}(-iT^c),\ (T^a)^+=T^a;$$

In (3.1) we have also used the following notation:

$$D^{ab}_\mu=\delta^{ab}\ \partial_\mu+\ g\ f^{abc}\ A^c_\mu\ ;\ \ \hat D^{AB}_{ij}=(\gamma_\mu)_{ij}D^{AB}_\mu\ ;\ D^{AB}_\mu=\ \delta^{AB}\partial_\mu+igA^a_\mu\ (T^a)^{AB}. \qquad (3.2)$$

These formulas are the relevant covariant derivatives. We carry out standard calculations as in the previous case of the gauge field and obtain the Schwinger-Dyson equation with fermions. Its form in the momentum representation is:

$$<\int(dp)d_{\mu\nu}(p)p^2A^a_\nu(p)\ \ \delta F[A]/_{\delta A^a_\mu(p)}> = <\{\int(dp)\ V(p^2L^2)\ \frac{\delta^2F[A]}{\delta A^a_\mu(-p)\delta A^a_\mu(p)}+$$
$$\int(dp_1)(dp_2)(dp_3)V^{abc}_{\mu\nu\rho}(-p_1,p_2,p_3)\bar\delta(p_1-p_2-p_3)A^b_\nu(p_2)A^c_\rho(p_3)\ \frac{\delta F[A]}{\delta A^a_\mu(p_1)}-$$
$$\int(dp_1)\dots(dp_4)\bar\delta(p_1-p_2-p_3-p_4)\ \ V^{acmn}_{\mu\nu\rho\sigma}\ A^c_\nu(p_2)A^n_\sigma(p_3)A^m_\rho(p_4)\delta F[A]/_{\delta A^a_\mu(p_1)}-$$
$$\int(dp_1)(dp_2)(dp_3)\bar\delta(p_2-p_3-p_1)ig(\gamma_\mu)_{ij}(T^a)^{AB}\bar\psi^A_i(p_3)\psi^B_j(p_2)\ \frac{\delta F[A]}{\delta A^a_\mu(p_1)}+$$

$$+\ \text{a contribution from the regularized vertex}\}>\ =\ 0. \qquad (3.3)$$

Fermionic parts in this expression are calculated in the same way as in the pure gauge case. We work in the weak coupling regime. In this approximation the regularized fermion vertex is given by:

$$[V_{xy}(\hat D^2)]^{AB}_{ij}=\sum_n a_n\{[\hat D^2]^n_{xy}\}^{AB}_{ij},$$

where

$$\{[\hat D^2]^n_{xy}\}^{AB}_{ij}=\int(dz_1)\dots(dz_{n-1})[\hat D^2_{xz_1}]^{AC_1}_{ik_1}\dots[\hat D^2_{z_{n-1}y}]^{C_{n-1}B}_{k_{n-1}j}$$

and

$$[\hat D^2_{xy}]^{AB}_{ij}=[\hat D^2_x]^{AB}_{ij}\cdot\delta(x-y)=\{\delta^{AB}\delta_{ij}\Box_x+\Gamma^{AB}_{ij}(x)\}\delta(x-y).$$

In turn, the function $\Gamma^{AB}_{ij}(x)$ is defined as:

$$\Gamma^{AB}_{ij}(x)=g\,\Gamma^{(1)AB}_{ij}(x)+g^2\,\Gamma^{(2)AB}_{ij}(x).$$

Here

$$\Gamma^{(1)AB}_{ij}=i(T^a)^{AB}\{(-i\sigma_{\mu\nu})_{ij}(\partial_\mu A^a_\nu)+\delta_{ij}[(\partial_\mu A^a_\mu)+2A^a_\mu\partial_\mu]\}$$

and

$$\Gamma^{(2)AB}_{ij}=-(T^aT^b)^{AB}[(-i\sigma_{\mu\nu})_{ij}A^a_\mu A^b_\nu+\delta_{ij}A^a_\mu A^b_\mu],\qquad \sigma_{\mu\nu}=[\gamma_\mu,\gamma_\nu].$$

After some simplification we have:

$$[V_{xy}(\hat D^2)]^{AB}_{ij}=\sum_n a_n\Box^n_{xy}\,\delta^{AB}\delta_{ij}+\sum_n a_n\sum_{m=0}^{n-1}\int(dz_1)\dots(dz_{n-1})\Box_{z_0z_1}\Box_{z_1z_2}$$

$$\dots\Box_{z_{m-1}z_m}(\Gamma^{AB}_{ij}(z_m)\delta(z_m-z_{m+1}))\dots\Box_{z_{n-1}z_n}+\sum_n a_n\sum_{m=0}^{n-2}\sum_{s=m+1}^{n-1}\int(dz_1)\dots$$

$$(dz_{n-1})\Box_{z_0z_1}\Box_{z_1z_2}\dots(\Gamma^{AC}_{ik}(z_m)\delta(z_m-z_{m+1}))\dots(\Gamma^{CB}_{kj}(z_{m+s})\delta(z_{m+s}-z_{m+s+1}))$$

$$\dots\Box_{z_{n-1}z_n}+\dots+\int(dz_1)\dots(dz_{n-1})(\Gamma^{AC_1}_{ik_1}(z_0)\delta(z_0-z_1))(\Gamma^{C_1C_2}_{k_1k_2}(z_1)\delta(z_1-$$

$$-z_2))\dots(\Gamma^{C_{n-1}B}_{k_{n-1}j}(z_{n-1})\delta(z_{n-1}-z_n)).\tag{3.4}$$

Here we have used the notation: $z_0=x$, and $z_n=y$. We will restrict ourselves to the lowest order of the coupling constant g. For the fermionic part we have finally:

$$\langle\{-\int(dp)\,\delta^{AC}\delta_{ik}(p^2+m^2)\psi^C_k(p)\frac{\delta F[\psi]}{\delta\psi^A_i(p)}\}\rangle=\langle\{-\int(dp)[\delta_{ij}m+ip_\rho(\gamma_\rho)_{ij}]$$

$$*V(p^2L^2)\,\delta^2/(\delta\bar\psi^A_j(p)\delta\psi^A_i(p))+ig(T^a)^{AB}(\gamma_\rho)_{ij}\int(dp)(dq)(dk)V(q^2L^2)$$

$$*A^a_\rho(k)\bar\delta(p-k-q)\frac{\delta^2}{\delta\bar\psi^B_j(q)\delta\psi^A_i(p)}-g\int(dp_1)(dp_2)(dq_1)\bar\delta(p_1-p_2-q_1)*$$

$$(T^a)^{AC}[-i(\sigma_{\mu\nu})_{ik}(q_{1\mu}A^a_\nu(q_1))+\delta_{ik}q_{1\mu}A^a_\mu(q_1)+2\delta_{ik}A^a_\mu(q_1)p_{2\mu}]\psi^C_k(p_2)\frac{\delta}{\delta\psi^A_i(p_1)}$$

$$+g^2\int(dp_1)(dp_2)(dq_1)(dq_2)\bar\delta(p_1-p_2-q_1-q_2)(T^aT^b)^{AC}[-i(\sigma_{\mu\nu})_{ik}A^a_\mu(q_1)*$$

$$*A^b_\nu(q_2)+\delta_{ik}A^a_\mu(q_1)A^b_\mu(q_2)]\psi^C_k(p_2)\frac{\delta}{\delta\psi^A_i(p_1)}-(g+O(g^2))(T^a)^{AC}*$$

$$\int(dp)(dq)\;[\delta_{ij}m+i(\gamma_\rho)_{ij}p_\rho]\sum_n a_n\sum_{m=0}^{n-1}\int(dq_1)[-i(\sigma_{\mu\nu})_{jk}(q_{1\mu}A^a_\mu(q_1))+$$

$$+\delta_{ik}q_{1\mu}A^a_\mu q_1)+2\delta_{jk}A^a_\mu(q_1)q_\mu](p^2)^m(q^2)^{n-1-m}\bar\delta(p-q_1-q)\frac{\delta^2}{\delta\bar\psi^C_k(q)\delta\psi^A_i(p)}\}*$$

$$*F[\psi\;\bar\psi]\rangle=0.\tag{3.5}$$

Now we use expressions (3.3) and (3.5) to calculate the fermionic contribution to the vacuum polarization of the gluon field. Diagrams giving a non-zero contribution to the vacuum polarization are shown in Figures 3. Consider each diagram separately.

The polarization operator is given by the two-point correlation function of the gluon:

$$\langle A^a_\rho(q_1)A^b_\lambda(q_2)\rangle=\frac{d_{\rho\nu}(q_1)}{q_1^2}\,\frac{d_{\lambda\mu}(q_2)}{q_2^2}\,\bar\delta(q_1+q_2)\Pi^{ab}_{\mu\nu}(q_1),$$

where $d_{\mu\nu}(q)=\delta_{\mu\nu}-(1+\alpha)q_\nu q_\mu/q^2$, q_j is the momentum of the gluon and $\Pi^{ab}_{\mu\nu}(q)$ is the usual polarization operator. The diagram shown Figure 3a. corresponds to the expression:

$$\Pi^{(1)ab}_{\mu\nu}(q)=-g^2\,Tr(T^aT^b)\int(dp)\;Tr\{\gamma_\mu[m+i(\hat p+\hat q)]\gamma_\nu(m+i\hat p)\}\,\frac{V(p^2L^2)}{m^2+(p+q)^2}$$

$$*V((p+q)^2L^2)/(m^2+(p+q)^2).$$

For the formfactor $V(p^2L^2)$ of the theory we use the following Mellin representation:

$$V(p^2L^2)= \frac{1}{2i}\int_{-\beta+i\infty}^{-\beta-i\infty} d\xi \; \frac{v(\xi)L^{2\xi}}{\sin(\pi\xi)}[m^2+p^2]^{\xi} \qquad (0<\beta<1).$$

Carrying out some simple calculations we get

$$\Pi^{(1)ab}_{\mu\nu}(q)= M^{ab}\delta_{\mu\nu}\frac{4\kappa}{L^2} +16\, M^{ab}(q_\mu q_\nu - q^2\delta_{\mu\nu})\int_0^1 dx\; x(1-x)[\, v'(0)+$$

$$+\ln(m^2L^2) - \ln(1-x)+\ln(1+\frac{q^2}{m^2}x\,(1-x))]+8\,\delta_{\mu\nu}\,M^{ab}(m^2+q^2/6)$$

where $M^{ab}=g^2\,\delta^{ab}\,N/(16\pi^2)$; for SU(3) N=3; and $\hat{p}=\gamma_\mu p_\mu$.

A subsequent contribution to the vacuum polarization of the gluon comes from the diagram shown in Figure 3b. Its explicit form is;

$$\Pi^{(2)ab}_{\mu\nu}(q)= g^2\, Tr(T^aT^b)\; Tr(\gamma_\nu\gamma_\mu)\int(dp)\,\frac{2\,V(p^2L^2)}{2m^2+p^2+(p-q)^2}.$$

Analogous calculations give:

$$\Pi^{(2)ab}_{\mu\nu}(q)=-\,M^{ab}\,\delta_{\mu\nu}\frac{4\kappa}{L^2} + M^{ab}\,\delta_{\mu\nu}4m^2[\,(1+\frac{1}{4}\frac{q^2}{m^2})(v'(0)-1+$$

$$+\ln(\,m^2L^2)) + \frac{3}{8}\,q^2/m^2]\,. \qquad (3.7)$$

Now we calculate the expression describing the diagram (Figure 3c) and it has the form:

$$\Pi^{(3)ab}_{\mu\nu}(q) = i\, g^2\, Tr(T^aT^b)\int(dp)\,\frac{2V(p^2L^2)}{m^2+p^2}\,\frac{1}{2m^2+(p+q)^2+p^2}\; Tr\{\gamma_\nu *$$

$$[\, i\sigma_{\beta\mu}q_\beta + q_\mu + 2p_\mu]\;(m+i\hat{p})\} = M^{ab}\delta_{\mu\nu}\,2\kappa/_{L^2} -4\,M^{ab}\delta_{\mu\nu}m^2[\,(1+$$

$$+q^2/_{m^2}\;1/_6)(v'(0)+\ln(m^2L^2)) - q^2/_{m^2}\,\frac{2}{9}\,]-4\,M^{ab}\int_0^1 dx\{1/_2\;x(1-x/2)\}*$$

$$*(q_\mu q_\nu-\delta_{\mu\nu}q^2)[\; v'(0)+\ln(m^2L^2)-\ln(1-x)+\ln(1+q^2/_{m^2}\;x/2\,(1-x/2))]\}-$$

$$-4\,M^{ab}\delta_{\mu\nu}q^2\;1/2\int_0^1 dx\; x(1-x)\;[\; v'(0)+\ln(m^2L^2)-\ln(1-x)]\;.$$

Finally, we consider the diagramm shown in Figure 3d. The corresponding matrix element is given by:

$$\Pi^{(4)ab}_{\mu\nu}(q)=i\,g^2 Tr(T^aT^b)\int(dp)\,2/_{[2m^2+(p+q)^2+p^2]}Tr[\,\gamma_\nu(m+i(\hat{p}+\hat{q}))*$$

$$(i\sigma_{\beta\mu}q_\beta+q_\mu+2p_\mu)]\{\,\sum_n a_n\sum_{j=0}^{n-1}[(p+q)^2]^j[p^2]^{n-j-1}\}.$$

Further, we restrict ourselves to the second order of the coupling constant and to terms proportional to q^2. In this approximation the expression for $\Pi^{(4)ab}_{\mu\nu}(q)$ takes the form:

$$\Pi^{(4)ab}_{\mu\nu}(q) = i g^2 Tr(T^aT^b)\;\int(dp)\;\;2\;/_{[2m^2+(p+q)^2+p^2]}\;Tr[\gamma_\nu(m+i(\hat{p}+\hat{q})*$$

$$(i\sigma_{\beta\mu}q_\beta+q_\mu+2p_\mu)]\,\frac{1}{2i}\int_{-\beta+i\infty}^{-\beta-i\infty}d\xi\;\frac{v(\xi)L^{2\xi}}{\sin(\pi\xi)}\;(m^2+p^2)^{\xi}[\frac{\xi}{m^2+p^2}+\frac{\xi(\xi-1)}{2}*$$

$$*\frac{(2pq+q^2)}{(m^2+p^2)^2}+\frac{1}{2}\frac{\xi(\xi-1)(\xi-2)}{3}\frac{(2pq+q^2)^2}{(m^2+p^2)^3}+\frac{1}{6}\frac{\xi(\xi-1)(\xi-2)(\xi-3)}{4}*$$

$$(2pq+q^2)/[m^2+p^2]^4]\;.$$

After some elementary calculations we have:

$$\Pi^{(4)ab}_{\mu\nu}(q)=-M^{ab}\;\delta_{\mu\nu}\;2\,\kappa/L^2 - M^{ab}\,\delta_{\mu\nu}4\,(m^2+q^2/6)-M^{ab}\delta_{\mu\nu}q^2 16/9\,. \quad (3.9)$$

Thus, it is evident that all sums of (3.6) -(3.9) give the following gauge-invariant expression for the gluon vacuum polarization:

$$\Pi^{ab}_{\mu\nu}(q)=\sum_{i=1}^{4}\Pi^{(i)ab}_{\mu\nu}(q)= (q_\mu q_\nu-\delta_{\mu\nu}q^2)\,\delta^{ab}\Pi(q^2),$$

where

$$\Pi(q^2)= g^2\,\frac{4N}{\pi^2}\int_0^1 dx\,x(1-x)\{v'(0)+\ln(m^2L^2)-\ln(1-x)+\ln(1+q^2/_{m^2}\;x(1-x)\}$$

$$+\,g^2\,\frac{N}{4\pi^2}\int_0^1 dx\,\frac{x}{2}(1-\frac{x}{2})[v'(0)+\ln(m^2L^2)-\ln(1-x)+\ln(1+\frac{q^2}{m^2}\frac{x}{2}(1-\frac{x}{2}))]. \qquad (3.10)$$

It should be noted that the usual fermionic contribution to the vacuum polazization of the gluon field does not violate gauge invariance of the theory.

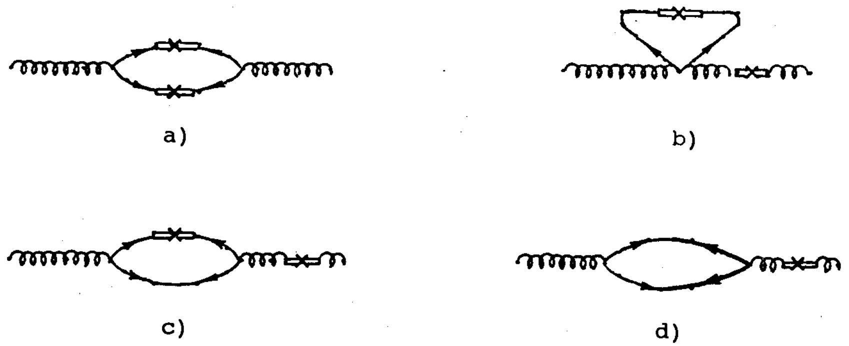


Fig.3. Diagrams giving a non-zero contribution to the vacuum polarization of the gluon field in the nonlocal stochastic scheme, with fermions.

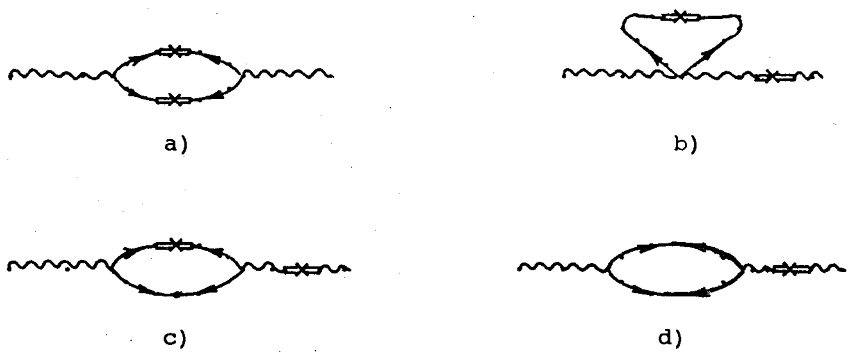


Fig.4. Diagrams of the vacuum polarization for the photon field in the nonlocal stochastic theory with Fermions.

4. Nonlocal Quantum Electrodynamics

In this section, as an appendix we apply the above-mentioned method to the simplest system of fields, i.e. QED. The Euclidean action has simple form:

$$S=\int (dx)\,[\tfrac{1}{4} F_{\mu\nu}(x)F_{\mu\nu}(x)+\bar{\psi}_i(\hat{D}_{ij}+m\,\delta_{ij})\psi_j]\,, \qquad (4.1)$$

where

$$\hat{D}_{ij}=(\gamma_\mu)_{ij}[\partial_\mu+ie A_\mu]; \quad [\hat{D}^2]_{ij}=\delta_{ij}\Box_x+e\,\Gamma^{(1)}_{ij}(x)+e^2\Gamma^{(2)}_{ij}(x)$$

and

$$\Gamma^{(1)}_{ij}(x)=i(-i(\sigma_{\mu\nu})_{ij}(\partial_\mu A_\nu)+\delta_{ij}[(\partial_\mu A_\mu)+2A_\mu\partial_\mu])$$

$$\Gamma^{(2)}_{ij}(x)=-\delta_{ij}A_\mu(x)A_\mu(x)\,.$$

In the quantum electrodynamics the Schwinger-Dyson equation have the following form (in the momentum representation):

$$<\{\int(dp)V(p^2L^2)\frac{\delta^2}{\delta A_\mu(-p)\delta A_\mu(p)}+ie(\gamma_\mu)_{ij}\int(dp_1)(dp_2)(dp_3)\;\bar{\delta}(p_1+p_2+$$
$$+p_3)\bar{\Psi}_i(p_3)\Psi_j(p_2)\delta/_{\delta A_\mu(p_1)}\}F[A]>$$

$$<[\int(dp)(p^2+m^2)\psi_j(p)\frac{\delta}{\delta\psi_j(p)}]F[\psi]>=<\{-\int(dp)(m+i\hat{p})_{ij}V(p^2L^2)\frac{\delta^2}{\delta\bar{\psi}_j(p)\delta\psi_i(-P)}$$

$$+ie\int(dp)(dq)(dk)(\gamma_\rho)_{ij}V(q^2L^2)A_\rho(k)\bar{\delta}(p-q-k)\frac{\delta^2}{\delta\bar{\psi}_j(q)\delta\psi_i(p)}-e\int(dp_1)*$$

$$*(dp_2)(dq_1)\bar{\delta}(p_1-p_2-q_1)[-i(\sigma_{\mu\nu})_{ik}(q_{1\mu}A_\nu(q_1))+\delta_{ik}q_{1\mu}A_\mu(q_1)+2\delta_{ik}*$$

$$*A_\mu(q_1)p_{2\mu}]\psi_k(p_2)\frac{\delta}{\delta\psi_i(p_1)}-e^2\int(dp_1)(dp_2)(dq_1)(dq_2)A_\mu(q_1)A_\mu(q_2)*$$

$$*\bar{\delta}(p_1-q_1-q_2-p_2)\psi_k(p_2)\frac{\delta}{\delta\psi_k(p_1)}-e\int(dp)(dq)[m+i\hat{p}]_{ij}\sum_n a_n\sum_{m=0}^{n-1}\int(dq_1)*$$

$$[-i(\sigma_{\mu\nu})_{kj}(q_{1\mu}A_\nu(q_1))+\delta_{jk}q_{1\mu}A_\mu(q_1)+2\delta_{kj}A_\mu(q_1)q_m](p^2)^m(q^2)^{n-m-1}*$$

$$\bar{\delta}(p-q-q_1)\frac{\delta^2}{\delta\bar{\psi}_k(q)\delta\psi_i(p)}\}\;F[\psi]> \qquad (4.2)$$

and so on. Let $F[A]=A_\nu(q)$, then $\delta F/_{\delta A_\mu(p)}=\bar{\delta}(p-q)\delta_{\mu\nu}$. The correlation function of the photon field the zero-order of the coupling constant e gives the photon propagator of the nonlocal theory:

$$\langle A_\rho(q_1)A_\lambda(q_2)\rangle = 2\,\frac{d_{\rho\lambda}(q_1)}{q_1^2+q_2^2}\,V(q_1^2L^2)\;\bar\delta(q_1+q_2).$$

Analogously, the correlationfunction of the Fermion defined the nonlocal propagator of the fermion field:

$$\langle\psi_j(q)\bar\psi_k(p)\rangle = -\,\frac{[m+i\hat q]_{jk}}{m^2+q^2}\;\bar\delta(p-q)\;V(q^2L^2).$$

To shown gauge invariance of the theory, we should study the simple diagrams shown in Figure 4.

The following simple expression corresponds to each diagram (Fig.4):

$$\Pi^{(1)}_{\mu\nu}(q) = e^2\int(dp)\;\frac{V((p+q)^2L^2)}{m^2+(p+q)^2}\;\frac{V(p^2L^2)}{m^2+p^2}\mathrm{Tr}[\gamma_\mu(m+i(\hat p+\hat q))\gamma_\nu(m+i\hat p)]=$$

$$=-\frac{\delta_{\mu\nu}e^2}{16\pi^2}\,\frac{4K}{L^2}+\frac{e^2}{\pi^2}(q^2\delta_{\mu\nu}-q_\mu q_\nu)\int_0^1 dx\;x(1-x)[\upsilon'(0)+\ln(m^2L^2)-\ln(1-x)+$$

$$+\ln(1+x(1-x)q^2/m^2)]-\delta_{\mu\nu}e^2q^2/(12\pi^2)+e^2\delta_{\mu\nu}m^2/(2\pi^2)\int dx\;[\upsilon'(0)+$$

$$+\ln(m^2L^2)-\ln(1-x)+\ln(1+\frac{q^2}{m^2}\,x\,(1-x))]-e^2m^2\delta_{\mu\nu}/(2\pi^2)$$

$$\Pi^{(2)}_{\mu\nu}(q) = e^2\,\mathrm{Tr}(\gamma_\mu\gamma_\nu)\;\int(dp)\;\frac{2V(p^2L^2)}{2m^2+p^2+(p+q)^2}=\frac{e^2\delta_{\mu\nu}}{16\pi^2}\,\frac{4K}{L^2}-\frac{e^2\delta_{\mu\nu}}{16\pi^2}4m^2*$$

$$*[(1+\frac{q^2}{4m^2})(\upsilon'(0)+\ln(m^2L^2))-1-\frac{q^2}{8m^2}]$$

$$\Pi^{(3)}_{\mu\nu}(q) = ie^2\int(dp)\;\frac{2V(p^2L^2)}{2m^2+p^2+(p+q)^2}\;\frac{1}{m^2+p^2}\;\mathrm{Tr}\{\gamma_\mu[\,i\sigma_{\rho\nu}q_\rho-q_\nu+2p_\nu](m+i\hat p)\}$$

$$=\frac{e^2\delta_{\mu\nu}}{16\pi^2}\,\frac{2K}{L^2}-\frac{e^2\delta_{\mu\nu}}{16\pi^2}\int_0^1 dx\,(4m^2+q^2x(1-x)[\upsilon'(0)+\ln(m^2L^2)-1-\ln(1-x)+$$

$$\ln(1+\frac{q^2}{m^2}\,\frac{x}{2}\,(1-\frac{x}{2}))]-\frac{e^2}{16\pi^2}\int_0^1 dx[2x(\delta_{\mu\nu}q^2-q_\nu q_\mu)+2q_\nu q_\mu x(1-x)]*$$

$$*[\upsilon'(0)+\ln(m^2L^2)-\ln(1-x)+\ln(1+\frac{q^2}{m^2}\,\frac{x}{2}\,(1-\frac{x}{2}))]$$

and

$$\Pi^{(4)}_{\mu\nu}(q) = ie^2\int(dp)\;\frac{2}{2m^2+p^2+(p-q)^2}\;\mathrm{Tr}\{\gamma_\mu[m+i(\hat p-\hat q)][i\sigma_{\rho\nu}q_\rho-q_\nu+2p_\nu]\}*$$

$$*\sum_n a_n\sum_{m=0}^{n-1}[(p-q)^2]^m[p^2]^{n-m-1}=-\frac{e^2\delta_{\mu\nu}}{16\pi^2}\,\frac{2K}{L^2}-\frac{e^2}{4\pi^2}\,\delta_{\mu\nu}(m^2+q^2/6)+$$

$$+\frac{e^2}{16\pi^2}\,\frac{5}{3}\,q_\mu q_\nu-\frac{10}{9}\,\frac{e^2}{16\pi^2}\,(\delta_{\mu\nu}q^2/2+q_\mu q_\nu).$$

The sum of these terms is a gauge-invariant quantity, the fermionic contribution to the vacuum polarization diagram for the photon field does not violate its gauge invariance.

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Нелокальное стохастическое квантование калибровочной теории с фермионами

Представляется стохастическое квантование калибровочной теории с фермионами. Найдено представление ковариантной производной нелокальных формфакторов, что позволяет формулировать конечную нелокальную калибровочную теорию, включая фермионы. Наша схема обобщает теорию мероморфных формфакторов, предложенную Гальпериным и др.

Работа выполнена в Лаборатории теоретической физики ОИЯИ.

Препринт Объединенного института ядерных исследований. Дубна 1990

Dineykhan M., Namsrai Kh. E2-90-373

Nonlocal Stochastic Quantization of Gauge Theory with Fermions

Nonlocal stochastic quantization of a gauge theory with fermions is presented. We have found the covariant-derivative representation of nonlocal formfactors which allows us to formulate a finite nonlocal gauge theory including fermion fields. Our scheme generalizes the theory with meromorphical formfactors proposed by Halperin with colleagues.

The investigation has been performed at the Laboratory of Theoretical Physics, JINR.

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