объединенный
институт
ядерных
исследований
дубна

E2-90-361

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INTERFERENCE CORRELATIONS OF IDENTICAL BOSONS WITH SLIGHTLY DIFFERENT MOMENTA IN THE MODEL INCLUDING BOTH ONE-PARTICLE AND MULTIPARTICLE SOURCES

Submitted to International Workshop
on Correlations and Multiparticle Production,
Marburg, FRG, 14-16 May, 1990

1990

1. In the model of independent one-particle sources emitting unpolarized identical particles with spin j, the probability for the creation of two particles with 4-momenta p_1 and p_2 at the space-time points $x_1 = \{\vec{r}_1, t_1\}$ and $x_2 = \{\vec{r}_2, t_2\}$ is described by the expression

$$W(p_1, p_2) \sim 1 + \frac{(-1)^{2j}}{2j+1} \cos(qx) \quad , \tag{1}$$

where
$$q = p_1 - p_2 \ , \ x = x_1 - x_2 \ , \ qx = q_0 t - \vec{q}\vec{x} \ .$$

Formula (1), which characterizes the effect of Bose or Fermi statistics on pair correlations of identical particles, forms the basis of the method for determining the space-time parameters of the region of multiple elementary particle production /1-6/. In the spirit of statistical concepts the function $\cos(qx)$ in formula (1) should be averaged over the space-time distribution of chaotically located sources. As a result, the correlation function of two non-interacting identical particles in the region of small relative momenta takes a simple form

$$R(q) = 1 + \frac{(-1)^{2j}}{2j+1} |\mathcal{F}(q)|^2 , \tag{2}$$

where
$$\mathcal{F}(q) = \int W(\tilde{x}) e^{iq\tilde{x}} d^4\tilde{x} . \tag{3}$$

In formula (3) the function $W(\tilde{x})$ describes the probability distribution of 4-coordinates of one-particle sources and satisfies the normalization condition

$$\int W(\tilde{x}) d^4 x = 1 . \tag{4}$$

2. In the case of identical pions

$$R(q) = 1 + |\mathcal{F}(q)|^2 . \tag{5}$$

When analyzing the experimental data, eq.(5) is frequently replaced by a more complicated expression

$$R(q) = 1 + \Lambda |\mathcal{F}(q)|^2 , \tag{6}$$

where the coefficient $\Lambda \neq 1$. As is shown in papers /7/, there is a number of simple reasons for the appearance of the factor Λ, for example, the presence of two or several space-time parameters of the process. On the other hand, in many papers the introduction of Λ is related to the assumption of pion emission in coherent states and to the existence of a mixture of coherent and chaotic states (see, for example,[8-15]). The role of coherent states for the description of narrow pair correlations of bosons consists in the fact that there are no correlations due to symmetrization of wave functions between pions in the same coherent state.

But the absence of interference correlations is characteristic of all the situations when several bosons are created in the same quantum state, in particular, for any fixed number of particles as well. In connection with this, it is interesting to investigate the structure of pair correlations of pions with slightly different momenta in the presence of both one-particle and multi-particle sources. The corresponding model has been suggested earlier in our paper[16]. In the framework of such a model it is easy to explain not only the difference of Λ from unity but also the appearance of several characteristic dimensions (see also ref.[17]).

3. Following paper[16], let us consider a situation when a set of n identical pions is emitted simultaneously from the same point (or from different points located at distances which are very small in comparison with the wave length $\lambda \sim 1/p$). Then we can speak of an n-particle point-like source. It is essential that in this situation, in contrast to the case of one-particle sources, the symmetrization effect is absent, and there are no interference correlations. If the generation amplitudes are assumed to be constants, then the probability of pion registration with momenta p_1 and p_2 does not depend on momentum difference.

We are interested in pair correlations in the inclusive approach when the momenta of some two identical particles are fixed and averaging over the remaining momenta is performed. Then each pair of pions can be considered independently of all the other pairs. Besides, we suggest that the momenta of all the particles are so large that the interference maximum occupies only a very small part of the phase volume and the rare events with

three or more particles having nearly equal momenta can be neglected. Under these conditions it is sufficient to restrict ourselves to a simple summation of the two-particle probabilities corresponding to different pairs of pions, taking into account that the production probability is independent of momentum difference for two identical pions from the same multiparticle source, and this probability is proportional to $(1 + \langle \cos [q(x_1 - x_2)] \rangle)$ for pions from different sources.

Let us assume that in each interaction one n-particle source and m one-particle sources are excited. We denote the probability of a process, when the momenta of all the pions differ greatly, by W_0. How does this probability change when the momenta of any two pions become close to each other? The answer to this question depends on that which pair is chosen out of the total number of possible pairs $\frac{1}{2}(n + m)(n + m - 1)$. If the momenta of two pions from the n-particle source approach each other, then the probability does not change, but for other combinations the probability change follows the law $(1 + \langle \cos [q(x_1 - x_2)] \rangle)$. The number of pairs from the n-particle source is equal to $\frac{1}{2}n(n-1)$, the number of pairs from one-particle sources $\frac{1}{2}m(m-1)$ and the number of "mixed" pairs nm. After averaging over space-time distributions of sources, we obtain for the process probability at small relative momenta the following expression:

$$W = \frac{2W_0}{(n+m)(n+m-1)} \left\{ \frac{n(n-1)}{2} + \frac{m(m-1)}{2}\left(1 + |\mathcal{F}_1(q)|^2\right) + nm\left[1 + Re\left(\mathcal{F}_1(q)\mathcal{F}_2^*(q)\right)\right]\right\}. \quad (7)$$

Here $\mathcal{F}_1(q)$ is the Fourier transform of the function $W_1(\tilde{x})$ describing the probability distribution of 4-coordinates of one-particle sources, and $\mathcal{F}_2(q)$ is the Fourier transform of the analogous function $W_2(\tilde{x})$ for the n-particle source (see formula (4)). Thus, narrow pair correlations of identical pions are expressed by the formula

$$R(q) = \frac{W}{W_0} = 1 + \Lambda_1 |\mathcal{F}_1(q)|^2 + \Lambda_2 Re\left(\mathcal{F}_1(q)\mathcal{F}_2^*(q)\right), \quad (8)$$

where

$$\Lambda_1 = \frac{m(m-1)}{(n+m)(n+m-1)}, \qquad \Lambda_2 = \frac{2nm}{(n+m)(n+m-1)} . \quad (9)$$

In particular, if the numbers n and m fluctuate in accordance with the Poisson law when passing from one act of interaction to another, then we have

$$\Lambda_1 = \frac{1}{(1+\gamma)^2}, \quad \Lambda_2 = \frac{2\gamma}{(1+\gamma)^2}; \quad \gamma = \frac{\bar{n}}{\bar{m}} . \quad (10)$$

It should be emphasized that the result (10) takes place for arbitrary multiplicity distributions when the conditions $\bar{n} \gg 1$, $\bar{m} \gg 1$ are valid (in particular, for fixed numbers $n \gg 1$ and $m \gg 1$ as well).

It is interesting that formulae (10) are in full accordance with the relations obtained in paper[12] on the basis of the coherent state approach (see also ref.[13]). This coincidence is not mere chance: really, for the coherent state the probability distribution over the states with a given number n is the Poisson distribution[18].

4. Taking into account relations (10), we can rewrite formula (8) in the form

$$R(q) = 1 + p^2 \left|\mathcal{F}_1(q)\right|^2 + 2p(1-p)\, Re\left(\mathcal{F}_1(q)\mathcal{F}_2^*(q)\right) , \quad (11)$$

where

$$p = \frac{\bar{m}}{\bar{n}+\bar{m}} \quad (12)$$

is the "chaoticity" parameter introduced by Weiner[15]. In our model the parameter p describes a relative contribution of chaotically located one-particle sources.

Let us assume that the symmetry centres of the distributions $W_1(\vec{x})$ and $W_2(\vec{x})$ coincide and the root-mean-square radii are equal to r_1 and r_2, respectively. In accordance with the definition (4), in this case the equalities $Im\,\mathcal{F}_1(q)=0$ and $Im\,\mathcal{F}_2(q)=0$ hold. It is easy to see that the root-mean-square radius determining the dependence upon momentum difference q for the second term in formula (11), which is proportional to p^2, is equal to $R_1 = \sqrt{2}\, r_1$. The root-mean-square radius determining the q-dependence of the third term in formula (11), which is proportional to $2p(1-p)$, equals $R_2 = \sqrt{r_1^2 + r_2^2}$.

If $r_2 \ll r_1$, we obtain the relation presented in paper[15]:

$$R_1 = \sqrt{2}\, R_2 . \quad (13)$$

Let us consider the case when the point-like multiparticle source is in the centre of the spherical-symmetrical Gaussian distribution of independent one-particle sources. Then we have:

$$W_1(\vec{r},t) = \frac{1}{(2\pi)^4 r_0^3 \tau_0^3} \exp\left(-\frac{\vec{r}^{\,2}}{2r_0^2} - \frac{t^2}{2\tau_0^2}\right), \quad W_2(\vec{r},t) = \delta^3(\vec{r})\,\delta(t); \quad (14)$$

$$\mathcal{F}_1(q) = \exp\left(-\tfrac{1}{2}\vec{q}^{\,2} r_0^2 - \tfrac{1}{2} q_0^2 \tau_0^2\right), \quad \mathcal{F}_2(q) = 1 . \quad (15)$$

As a result, if the time parameter $\tau_0 = 0$, the correlation function takes the same form as in paper[15]:

$$R(q) = 1 + p^2 \exp\left(-\vec{q}^{\,2} r_0^2 - q_0^2 \tau_0^2\right) + 2p(1-p)\exp\left(-\tfrac{1}{2}\vec{q}^{\,2} r_0^2 - \tfrac{1}{2} q_0^2 \tau_0^2\right). \quad (16)$$

However, in the framework of our approach the relations (13) and (16) are not of a general character, and they are violated at $r_2 \sim r_1$. These relations are also changed in the case when $r_2 = 0$ but the multiparticle source is located at a definite distance $b = \{\vec{b}, T\}$ from the symmetry centre of the space-time distribution of one-particle sources. Under these conditions we should write instead of (16) the following expression for the correlation function of identical pions:

$$R(q) = 1 + p^2 \exp\left(-\vec{q}^{\,2} r_0^2 - q_0^2 \tau_0^2\right) + 2p(1-p)\cos(bq)\exp\left(-\tfrac{1}{2}\vec{q}^{\,2} r_0^2 - \tfrac{1}{2} q_0^2 \tau_0^2\right). \quad (17)$$

5. Other situations with the participation of multiparticle sources are also possible in the framework of our approach. Some of them have been analyzed in paper[16].

Let us consider, in particular, N multiparticle sources located at chance space-time points, each of them instantaneously emitting n identical pions. Let us assume that one-particle sources are generally absent. The number of pairs of pions from the same source is equal to $\frac{1}{2} n(n-1) N$, and the number of pairs of pions from different sources equals $n^2 \frac{N(N-1)}{2}$. This leads at once to a formula of the type (6) with one parameter Λ:

$$R(q) = 1 + \Lambda |\mathcal{F}(q)|^2, \qquad (18)$$

where

$$\Lambda = \frac{n(N-1)}{nN-1}, \qquad (19)$$

and $\mathcal{F}(q)$ is determined by the equality (4) with the function $W(\tilde{x})$ describing the probability distribution of 4-coordinates of multiparticle sources. Taking into account the fluctuations of n and N, we obtain*)

$$\Lambda = \frac{(\bar{n})^2 \overline{N(N-1)}}{(\bar{n})^2 \overline{N(N-1)} + \overline{n(n-1)}\,\bar{N}}. \qquad (20)$$

If the fluctuations correspond to the Poisson law, then

$$\Lambda = \left(1 - \frac{\bar{N}}{\bar{N}^2}\right). \qquad (21)$$

This result is also valid for any distributions when the condition $\bar{n} \gg 1$ is satisfied. The expression (21) coincides with the relation obtained in paper/12/ by means of the coherent state approach. We see that the appearance of the factor $\Lambda < 1$ needs not be always connected with the manifestation of a joint action of multiparticle and one-particle sources. The interpretation in the sense of formula (10) is quite possible in the presence of only multiparticle sources.

In the general case, in the model including both one-particle and multiparticle sources, the pair correlations of identical pions with nearly equal momenta may depend on several dimensions and several parameters Λ. Let us assume that m one-particle sources with 4-coordinate distribution $W_1(\tilde{x})$ are added to N former n-particle sources with 4-coordinate distribution $W_2(\tilde{x})$. Then $\frac{1}{2}$ n(n - 1) N pairs make a contribution to the generation probability which does not change when the pion momenta approach each other, $\frac{1}{2}$ m (m-1) pairs make a contribution proportional to $(1 + |\mathcal{F}_1(q)|^2)$, $\frac{1}{2} n^2 N(N-1)$ pairs make a contribution proportional to $(1 + |\mathcal{F}_2(q)|^2)$ and the contribution of nmN "mixed" pairs is proportional to $[1 + Re(\mathcal{F}_1(q)\mathcal{F}_2(q)]$. In this situation the pair correlations of identical pions with slightly different momenta are described by the expression:

$$R(q) = 1 + \Lambda_1 |\mathcal{F}_1(q)|^2 + \Lambda_2 Re\left(\mathcal{F}_1(q)\mathcal{F}_2^*(q)\right) + \Lambda_3 |\mathcal{F}_2(q)|^2, \qquad (22)$$

where

$$\Lambda_1 = \frac{\overline{m(m-1)}}{B}, \quad \Lambda_2 = \frac{2\bar{n}\bar{m}\bar{N}}{B}, \quad \Lambda_3 = \frac{\overline{N(N-1)}(\bar{n})^2}{B}, \qquad (23)$$

$$B = \overline{n(n-1)}\,\bar{N} + \overline{m(m-1)} + \overline{N(N-1)}(\bar{n})^2 + 2\bar{n}\bar{m}\bar{N}. \qquad (24)$$

If N = 1, then $\Lambda_3 = 0$, $B = \overline{(n+m)(n+m-1)}$, and we come to the results (8)-(9). If m = 0, we have $\Lambda_1 = \Lambda_2 = 0$, and the relations (18) - (20) follow from eqs. (22)-(24). When the multiplicities n and m are distributed in accordance with the Poisson law (and also for any distributions with $\bar{n} \gg 1$, $m \gg 1$),

$$\Lambda_1 = \frac{1}{(1+\gamma\bar{N})^2}, \quad \Lambda_2 = \frac{2\gamma\bar{N}}{(1+\gamma\bar{N})^2}, \quad \Lambda_3 = \frac{\overline{N(N-1)}\,\gamma^2}{(1+\gamma\bar{N})^2}, \qquad (25)$$

where $\gamma = \bar{n}/\bar{m}$. An analogous result has been obtained in paper/14/ by means of the coherent state approach.

6. Thus, the properties of correlations of bosons with nearly equal momenta, which are usually explained by means of a special role of coherent states, can be interpreted in the framework of the simple model including both multiparticle and one-particle sources. It should be emphasized that our results are related to arbitrary multiplicity distributions, in particular, to reactions with a fixed number of particles. In this sense they are of a more general character.

The coincidence of our concrete formulae with the relations obtained in terms of coherent states corresponds to the particular case of the Poisson distribution of bosons from sources (or to the limit of very large multiplicities for any distributions).

It should be noted once again that, from our point of view,

*) We assume that the multiplicity distributions are the same for all the sources, but the fluctuations of number n corresponding to different sources are independent.

the essence of the problem is that there are no interference correlations of the pions which are in the same quantum state. The concrete interpretation of coherent states as superpositions over the number of bosons is of no importance since correlations contain no contribution from the interference of states with a different number of particles; the superpositions of states with a different number of particles behave like non-coherent mixtures of such states.

It should also be noted that for charged pions the coherent states are the superpositions of states with different electric charges. In the framework of modern conceptions the existence of such superpositions is prohibited by the well-known rule of superselection following from the exact conservation of electric charge[19]. This circumstance prevents a literal application of the coherent state technique for the description of pair correlations of π^+ or π^--mesons with slightly different momenta. However, there is not any necessity in this since the superposition properties of coherent states are not really used. At the same time multiparticle sources were in fact first introduced just due to the conception of coherent states, which has led to a new view on the structure of narrow pair correlations of identical bosons.

The author is grateful to R.Lednicky and M.I.Podgoretsky for valuable discussions.

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Received by Publishing Department
on May 28, 1990.