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RELATIVISTIC BOUND STATES IN QCD

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1.Introduction and statement of the problem.

There exist different QCD - inspired phenomenological approaches to describe hadron spectra. Among them the potential model for the heavy - quarkonia spectroscopy [1] plays an important role because it is the most theoretically justified one and it is directly related to the definition of the fundamental parameters of the quark - antiquark interaction potential. One can say that the heavy - quarkonia spectroscopy is of the same importance for QCD as the Coulomb experiments for QED.

In the last time, the quark - quark potential from [1] has been applied to the description of the spectroscopy of light mesons in the framework of relativistic equations generalizing the Schrödinger equation [2] - [7]. It turned out that such a potential model succesfully describes the spontaneous breakdown of chiral symmetry (which is a pure relativistic effect), constituent quark masses, and the pion as a Goldstone particle, as well as the great mass difference between ρ - and π - mesons [4].

The relativistic covariant potential model has been suggested in ref. [8,9] in the framework of a bilocal field approach to get an unified description of the spectra and the interactions of light as well as heavy quarkonia (mesons). The success of the relativistic two - body potential model justifies its generalization to many - quark (gluon) systems. The investigation of this problem is the purpose of this paper.

For the derivation of equations for the relativistic quark - gluon bound states we use two, complementing each other, methods : the Green function method and the quasiparticle operator approach [10]. The Green function method enables one to obtain a self - consistent relativistic picture of the spectra and interaction of hadron [8,9], whereas the operator approach gives an adequate physical interpretation of the nonperturbative vacuum for many - quark systems, and other quantum states, and considerably simplifies the derivation of the many - quark
(gluon) bound state equations.

The paper is organized as follows. In section 2 we discuss the status of the potential model in gauge theories and apply the Green function method to the derivation of relativistic equation as quark - antiquark bound states. In section 3 we show the equivalence of the Green function method and Bethe - Salpeter equations applied to mesons. In section 4 we derive the equation for many - quark systems (namely, for baryons). In section 5 we consider the application of the relativistic potential model for the pure gluon sector (glueballs).

2.The status of the relativistic potential model for QCD and effective chiral Lagrangians.

It is widely believed that the potential models are related only to the nonrelativistic approximation. This opinion, however, is based on the experience of solving scattering and dissociation problems in QED, where the Coulomb propagator corresponding to transversal photon exchange, converts into the relativistic invariant Feynman propagator up to the longitudinal part vanishing on the mass shell. Such a "relativistic method" of the potential model is incorrect for the description of bound states where elementary particles are off their mass shells and the longitudinal part of the propagator differs from zero.

From the experience of the description of atoms in QED one knows that the bound state (atom) is formed by an instantaneous interaction (Coulomb) potential (with a singularity on the time axis) whereas the transversal photon exchange plays the role of a correction which does not destroy the instantaneous property of the bound state wave function [11]. However, in this case, unlike scattering and dissociation processes, the sum of the Coulomb and transversal propagators is not relativistic - invariant due to nonzero longitudinal photon contributions. It should be stressed that the propagators in manifestly covariant gauges (with a singularity on the light cone) themselves can not describe the bound states conserving their instantaneous property.

So to treat the bound states it is important to choose such transversal variables for which the instantaneous Coulomb potential separates.

In practice the relativistic description of the spectrum of a moving atom is done by means of the Coulomb potential moving together with the atom. This correspondens to the transformation to new transversal variables reffering to a new time axis (η_μ) which is parallel to atom's 4-momentum. To such a relativization there corresponds the following effective action [8,9] :

$$W_{eff}[\psi,\bar{\psi}] = \int d^4x[\bar{\psi}(x)(i\partial\!\!\!/ - m^0)\psi(x) + \\ + \frac{1}{2}\int d^4y(\psi(y)\bar{\psi}(x))\mathcal{K}^{(\eta)}(z^\perp \mid X)(\psi(x)\bar{\psi}(y))]. \quad (1)$$

Here $\partial\!\!\!/ = \partial^\mu\gamma_\mu$, $\mathcal{K}^{(\eta)}$ is the kernel

$$\mathcal{K}^{(\eta)}(z^\perp \mid X) = \eta\!\!\!/ V(z^\perp)\delta(z\cdot\eta)\eta\!\!\!/,\ (\eta\!\!\!/ = \eta^\mu\gamma_\mu), \quad (2)$$

where z and X are the relative and total coordinates defined, respectively, as

$$z = x - y,\ X = \frac{x+y}{2}, \quad (3)$$

and $V(z^\perp)$ is the potential depending on the transversal (with respect to the time axis η) component of the relative coordinate, $z^\perp_\mu = z_\mu - \eta_\mu(z\cdot\eta)$.

Notice that just such a relativistic transformation law of the fields with a simultaneous rotation of the time - axis has been used in the historically first formulation of QED [12] (a consistent construction of the gauge theories with such properties has been proposed recently [13]).

The next question is how to choose the time - axis in (1) for describing interacting atoms. It has been suggested in ref. [9] to take in this case as a time - axis the unit

vector which is proportional to the eigenvalue of the bound state total momentum operator, i.e.

$$\eta_\mu \sim i\frac{\partial}{\partial X_\mu}. \quad (4)$$

When this requirement is satisfied the bound state wave functions automatically belong to the irreducible representation of the Poincare group [14].

It should be stressed that in the relativistic theory the decomposition (3) is independ both of the variation of the quark masses and of the bound state.

The expression (1) with the kernel (2) and with the time - axis defined by (4) represents a relativistic covariant action. It seems that a most straightforward way for constructing a theory of bound states is the redefinition of action (1) in terms of bilocal fields by means of the Legendre transformation [15] - [17]

$$\frac{1}{2}\int d^4xd^4y(\psi(y)\bar{\psi}(x))\mathcal{K}(x,y)(\psi(x)\bar{\psi}(y)) = \\ = -\frac{1}{2}\int d^4xd^4y\mathcal{M}(x,y)\mathcal{K}^{-1}(x,y)\mathcal{M}(x,y) + \\ + \int d^4xd^4y(\psi(x)\bar{\psi}(y))\mathcal{M}(x,y) \quad (5)$$

where $\mathcal{K}^{-1}$ is the inverse of the kernel (2). Following ref. [16], we introduce the short - hand notation

$$\int d^4x\bar{\psi}(x)(i\partial\!\!\!/ - m^0)\psi(x) \equiv \int d^4xd^4y\psi(y)\bar{\psi}(x)(i\partial\!\!\!/ - m^0)\delta^{(4)}(x-y) = (\psi\bar{\psi}, -G_0^{-1}),$$
$$\int d^4xd^4y(\psi(x)\bar{\psi}(y))\mathcal{M}(x,y) = (\psi\bar{\psi},\mathcal{M}).$$

Then, from action (1) we get

$$W_{eff}[\mathcal{M}] = (\psi\bar{\psi},(-G_0^{-1}+\mathcal{M})) - \frac{1}{2}(\mathcal{M},\mathcal{K}^{-1}\mathcal{M}). \quad (6)$$

After quantization over N_c fermion fields and normal ordering, this action takes the form

$$W_{eff}[\mathcal{M}] = -\frac{1}{2}N_c(\mathcal{M},\mathcal{K}^{-1}\mathcal{M}) + iN_c\sum_{n=1}^{\infty}\frac{1}{n}\Phi^n. \quad (7)$$

Here $\Phi \equiv G_0\mathcal{M}, \Phi^2, \Phi^3$ etc. mean the following expressions

$$\Phi(x,y) \equiv G_0\mathcal{M} = \int d^4zG_0(x,z)\mathcal{M}(z,y), \\ \Phi^2 = \int d^4xd^4y\Phi(x,y)\Phi(y,z), \\ \Phi^3 = \int d^4xd^4yd^4z\Phi(x,y)\Phi(y,z)\Phi(z,x)\ etc \quad (8)$$

As a result of such quantization, only the the contributions with inner fermionic lines (but no scattering and dissociation channel contribution) are included in the effective action since we are interested only in the bound states.

The requirement for the choice of the time axis (4) in bilocal dynamics is equivalent to Markov - Yukawa condition [14]

$$z_\mu \cdot i\frac{\partial \mathcal{M}(z,X)}{\partial X_\mu} = 0 \qquad (9)$$

where $z_\mu = (x-y)_\mu$ and $X_\mu = (1/2)(x+y)_\mu$ are relative and total coordinates.

The first step to the quantization of the action (7) is the determination of its minimum

$$N_c^{-1}\frac{\delta W_Q(\mathcal{M})}{\delta \mathcal{M}} == -\mathcal{K}^{-1}\mathcal{M} + i\sum_{n=1}^{\infty} G_0(\mathcal{M}G_0)^n \equiv -\mathcal{K}^{-1}\mathcal{M} + \frac{i}{G_0^{-1}-\mathcal{M}} = 0. \qquad (10)$$

We denote the corresponding classical solution for the bilocal field by $\Sigma(x-y)$. It depends only on the differrence $x-y$ because of translation invariance of vacuum solutions.

The next step is the expansion of the action (1) around the point of minimum $\mathcal{M} = \Sigma + \mathcal{M}'$,

$$\begin{aligned} W_Q(\Sigma+\mathcal{M}') &= W_Q(\Sigma) + N_c[-\frac{1}{2}\mathcal{M}'\mathcal{K}^{-1}\mathcal{M}' + \frac{i}{2}(G_\Sigma \mathcal{M}')^2] + \\ &+ iN_c\sum_{n=3}^{\infty}\frac{1}{n}(G_\Sigma\mathcal{M}')^n, \quad (G_\Sigma = (G_0^{-1}-\Sigma)^{-1}), \end{aligned} \qquad (11)$$

and the representation of the small fluctuations $\mathcal{M}'$ as a sum over the complete set of classical solutions Γ,

$$\frac{\delta^2 W_Q(\Sigma+\mathcal{M}')}{\delta \mathcal{M}^2}|_{\mathcal{M}'=0}\cdot\Gamma = 0. \qquad (12)$$

Using the definitions (8) and (11) it is easy to obtain the standard form of equations (10) and (12) :

$$\Sigma(x-y) = m^0\delta^{(4)}(x-y) + i\mathcal{K}(x,y)G_\Sigma(x-y), \qquad (13)$$

$$\Gamma = i\mathcal{K}(x,y)\int d^4z_1 d^4z_2 G_\Sigma(x-z_1)\Gamma(z_1,z_2)G_\Sigma(z_2-y) \qquad (14)$$

which are, respectively, the Schwinger - Dyson (SD) and Bethe - Salpeter (BS) equations. They describe the spectrum of Dirac particles in bound states and the spectrum of the bound states themselves, respectively.

In the momentum space we obtain with

$$\begin{aligned} \underline{\Sigma}(k) &= \int d^4x \Sigma(x)e^{ikx}, \\ \underline{\Gamma}(q|\mathcal{P}) &= \int d^4x d^4y e^{i\frac{x+y}{2}\mathcal{P}} e^{i(x-y)q}\Gamma(x,y) \end{aligned}$$

for the kernel (2) the following equation for the mass operator ($\underline{\Sigma}$) and the vertex function ($\underline{\Gamma}$)

$$\underline{\Sigma}(k) = m^0 + i\int\frac{d^4q}{(2\pi)^4}\underline{V}(k^\perp - q^\perp)\not\eta \underline{G_\Sigma}(q)\not\eta, \qquad (15)$$

$$\underline{\Gamma}(k,\mathcal{P}) = i\int\frac{d^4q}{(2\pi)^4}\underline{V}(k^\perp-q^\perp)\not\eta[\underline{G_\Sigma}(q+\frac{\mathcal{P}}{2})\Gamma(q|\mathcal{P})\underline{G_\Sigma}(q-\frac{\mathcal{P}}{2})]\not\eta \qquad (16)$$

where $G_\Sigma(q) = (\not q - \underline{\Sigma}(q))^{-1}, \underline{V}(k^\perp)$ means the Fourier transform of the potential, $k^\perp_\mu = k_\mu - \eta_\mu(k\cdot\eta)$ is the transversal with respect to η_μ relative momentum, $\mathcal{P}_\mu$ is the total momentum.

The quantities $\underline{\Sigma}$ and $\underline{\Gamma}$ depend only on the transversal momentum

$$\underline{\Sigma}(k) = \underline{\Sigma}(k^\perp), \underline{\Gamma}(k|\mathcal{P}) = \underline{\Gamma}(k^\perp|\mathcal{P}),$$

because of the instantaneous form of the potential $\underline{V}(k^\perp)$ in any frame.

Therefore, we may integrate in (15) and (16) over the longitudinal momentum $q_0 = (q\cdot\eta)$ using the representation

$$\underline{\Sigma}_a(q) = \not q^\perp + E_a(q^\perp)S_a^{-2}(q^\perp) \qquad (17)$$

for the self - energy with

$$S_a^{-2}(q^\perp) = exp\{-\hat{\not q}^\perp 2v_a(q^\perp)\}, \; \hat{q}^\perp_\mu = q^\perp_\mu/|q^\perp| \qquad (18)$$

where S_a is the Foldy - Wouthuysen type transformation matrix with the parameter v_a.

Then, one has

$$\begin{aligned} \underline{G_{\Sigma_a}} &= [q_0\not\eta - E_a(q^\perp)S_a^{-2}(q^\perp)]^{-1} = \\ &= [\frac{\Lambda^{(\eta)}_{(+)a}(q^\perp)}{q_0 - E_a(q^\perp)+i\epsilon} + \frac{\Lambda^{(\eta)}_{(-)a}(q^\perp)}{q_0 + E_a(q^\perp)+i\epsilon}]\not\eta \end{aligned} \qquad (19)$$

where

$$\Lambda^{(\eta)}_{(\pm)a}(q^\perp) = S_a(q^\perp)\Lambda^{(\eta)}_{(\pm)}(0)S_a^{-1}(q^\perp), \; \Lambda^{(\eta)}_{(\pm)}(0) = (1\pm\not\eta)/2 \qquad (20)$$

are the operators separating the states with positive ($+E_a$) and negative ($-E_a$) energies.

As a result, we obtain the following equations for the one - particles energy E and the angle v :

$$E_a(k^\perp)\cos 2v(k^\perp) = m_a^0 + \frac{1}{2}\int\frac{d^3q^\perp}{(2\pi)^3}\underline{V}(k^\perp-q^\perp)\cos 2v(q^\perp) \qquad (21)$$

$$E_a(k^\perp)\sin 2v(k^\perp) = |k^\perp| + \frac{1}{2}\int \frac{d^3q^\perp}{(2\pi)^3}\underline{V}(k^\perp - q^\perp)|k^\perp \cdot q^\perp|\sin 2v(q^\perp) \quad (22)$$

Let us consider the Bethe - Salpeter equation (16) under such an integration. The vertex function is given by

$$\Gamma_{ab}(k^\perp|\mathcal{P}) = \int \frac{d^3q^\perp}{(2\pi)^3}\underline{V}(k^\perp - q^\perp)\not{\eta}\psi_{ab}(q^\perp)\not{\eta}, \quad (23)$$

where ψ_{ab} denotes the expression

$$\psi_{ab}(q^\perp) = \not{\eta}[\frac{\bar{\Lambda}_{(+)a}(q^\perp\Gamma_{ab}(q^\perp|\mathcal{P})\Lambda_{(-)b}(q^\perp)}{E_T - \sqrt{\mathcal{P}^2} + i\epsilon} + \frac{\bar{\Lambda}_{(-)a}(q^\perp\Gamma_{ab}(q^\perp|\mathcal{P})\Lambda_{(+)b}(q^\perp)}{E_T + \sqrt{\mathcal{P}^2} - i\epsilon}]\not{\eta} \quad (24)$$

and is the bound state wave function. Here $E_T = E_a + E_b$ means the sum of one - particles energies of the two particles (a) and (b) defined by (21,22) and the notation

$$\bar{\Lambda}_{(\pm)}(q^\perp) = S^{-1}(q^\perp)\Lambda_{(\pm)}(0)S(q^\perp) \quad (25)$$

has been introduced.

Acting with the operators (20) and (25) on equation (23) one gets the equations for the wavefunction ψ in an arbitrary moving reference frame

$$\begin{aligned}(\quad & E_T(k^\perp) \mp \sqrt{\mathcal{P}^2})\Lambda^{(\eta)}_{(\pm)a}(k^\perp)\psi_{ab}(k^\perp)\bar{\Lambda}^{(\eta)}_{(\mp)b}(k^\perp) = \\ = \quad & \Lambda^{(\eta)}_{(\pm)a}(k^\perp)[\int \frac{d^3q^\perp}{(2\pi)^3}\underline{V}(k^\perp - q^\perp)\psi_{ab}(q^\perp)]\bar{\Lambda}^{(\eta)}_{(\mp)b}(k^\perp). \quad (26)\end{aligned}$$

All these equations (21,22) and (26) have been derived without any assumption about the smallness of the relative momentum $|k^\perp|$ and for an arbitrary total momentum $\mathcal{P}_\mu = (\sqrt{M_A^2 + \vec{\mathcal{P}}^2}, \vec{\mathcal{P}} \neq 0)$.

If the atom is at rest ($\mathcal{P}_\mu = (M_A, 0, 0, 0)$) equation (26) coincides with the Salpeter equation [18]. If one assumes that the current mass m^0 is much larger than the relative momentum $|q^\perp|$ then the coupled equations (21,22) and (26) turn into the Schrödinger equation. In the rest frame ($\mathcal{P}_0 = M_A$) equations (21,22) for a large mass $(m^0/|q^\perp| \to \infty)$ describe a nonrelativistic particle

$$E_a(\mathbf{k}) = \sqrt{(m_a^0)^2 + \mathbf{k}^2} \simeq m_a^0 + \frac{1}{2}\frac{\mathbf{k}^2}{m_a^0},$$
$$\tan 2v = \frac{k}{m^0} \to 0;\ S(\mathbf{k}) \simeq 1;\ \Lambda_{(\pm)} \simeq \frac{1 \pm \gamma_0}{2}.$$

Then, in equation (26) only the state with positive energy remains

$$\Lambda_{(+)}\psi\Lambda_{(-)} \simeq \psi_{Sch},\ \ \Lambda_{(-)}\psi\Lambda_{(+)} \simeq 0,$$

and finally the Schrödinger equation results in

$$[\frac{1}{2\mu}\mathbf{k}^{-2} + (m_a^0 + m_b^0 - M_A)]\psi_{Sch}(\mathbf{k}) = \int \frac{d\mathbf{q}}{(2\pi)^3}\chi(\mathbf{k} - \mathbf{q})\psi_{Sch}(\mathbf{q}), \quad (27)$$

where $\mu = m_a \cdot m_b/(m_a + m_b)$. For an arbitrary total momentum $\mathcal{P}_\mu$ equation (26) takes the form

$$[-\frac{1}{2\mu}(k_\nu^\perp)^{-2} + (m_a^0 + m_b^0 - \sqrt{\mathcal{P}^2})]\psi_{Sch}(k^\perp) = \int \frac{d^3q^\perp}{(2\pi)^3}\underline{V}(k^\perp - q^\perp)\psi_{Sch}(q^\perp), \quad (28)$$

and describes a relativistic atom with nonrelativistic relative momentum $|k^\perp| << m^0_{a,b}$. In the framework of such a derivation of the Schrödinger equation it is sufficient to define the total coordinate according to (3), $X = (x + y)/2$, independently of the magnitude of the masses of the two particles forming an atom.

Now we consider the opposite case of massless particles, $m_a^0 = m_b^0 \to 0$. Suppose that in this case equations (21,22)

$$2E_a(k^\perp)\cos 2v(k^\perp) = \int \frac{d^3q^\perp}{(2\pi)^3}\underline{V}(k^\perp - q^\perp)\cos 2v(q^\perp) \quad (29)$$

$$2E_a(k^\perp)\sin 2v(k^\perp) = |k^\perp| + \int \frac{d^3q^\perp}{(2\pi)^3}\underline{V}(k^\perp - q^\perp)|\hat{k}^\perp \cdot \hat{q}^\perp|\sin 2v(q^\perp) \quad (30)$$

have a nontrivial solution $v(k^\perp) \neq 0$. This solution describes the spontaneous breakdown of chiral symmetry [2]-[9].

It can easily be seen that equations (29,30) are identical with (26) for the bound state wavefunction with zero eigenvalue, $\mathcal{P}_\mu^2 = 0$ and

$$\begin{aligned}\Lambda_{(+)}\psi\bar{\Lambda}_{(-)} &= \Lambda_{(-)}\psi\bar{\Lambda}_{(+)} \equiv \psi \\ 2E_a(k^\perp)\psi(k^\perp) &= \int \frac{d^3q^\perp}{(2\pi)^3}\underline{V}(k^\perp - q^\perp)\psi(q^\perp). \quad (31)\end{aligned}$$

Therefore,

$$\psi = \cos 2v(k^\perp)/F \quad (32)$$

where F is a proportionality constant.

In this way, the coupled equations (21,22) and (26) describe the pure relativistic effect of the appearance of the Goldstone mode due to spontaneous breakdown of chiral symmetry. Thus in the framework of instantaneous action (1) we get the proof of the Goldstone theorem in the bilocal variant.

Just this example represents a model for the construction of a low - energy theory of light mesons, in which the pion is considered in two different ways, as a quark - antiquark bound state and as a Goldstone particle. So it turns out that our relativistic instantaneous model for bound states can, in the lowest order of radiative corrections, also describes mesons.

Indeed, there is a number of paper (cf. [2]-[7] and references therein) where equations (21,22) and (26) are used for the calculation of the mass spectrum of light mesons, the constituent quark masses and the meson decay constants. In the papers the potentials are determined from the spectroscopy of heavy quarkonia as sums of rising and Coulomb potentials, for instance [4]

$$V(r) = \frac{\alpha_S}{r} - V_0 r^2, V_0^{1/3} \simeq 250MeV, \alpha_S \simeq 0.3 \,. \qquad (33)$$

Thereby, the heavy quarkonia ($m^0 >> 250MeV$) themselves are described by Schrödinger equation (28) which, as has been shown, can be derived from eq. (26) in the limit of large masses. In this limit, the effect of spontaneous breakdown of chiral symmetry also disappeares, and the constituent quark masses are identical to the current ones [19].

The advantage of such a potential approach compared with all other ones consists in the first constructive connection between the fundamental parameters for physics at short distances (the parameters of rising and Coulomb potentials and the current quark masses) with those of hadron physics for long distances (the pion mass and its weak decay constant F_π).

The shortcomings of this approach were the following: the nonrelativistic formulation (in the rest frame) of the bound state, the absence of an relativistic meson interaction Hamiltonian, and the open problem of the status of radiative QCD corrections. The first two disadvantages are absent in the new relativistic potential model [8,9] considered here. This model represents a logical interpretation of relativistic atomic physics, i.e. an interpretation of the "atomization" of QED.

From this point of view the "hadronization" of QCD qualitatively differs only by the short - range property of the quark - antiquark interaction potential for light quarkonia. Furthermore, the effective action for light mesons must be an action for a chiral Lagrangian. The proof of the fact that (11) leads to a chiral Lagrangian has been performed in [20] with the help of the separable approximation which can be used just for short - range potentials. For low orbital momenta such potentials can be represented with good accuracy as a product of two factors

$$< l = 0|V(\mathbf{p} - \mathbf{q})|l = 0 >= f(p^\perp) f(q^\perp).$$

The underlying model (11) becomes equivalent to one of versions of the Nambu - Jona - Lasinio model [21,22] with explicit indication of the formfactor $f^2(p)$ for the ultraviolet regularization. It is well known [21,22], that this model leads to chiral Lagrangians.

The validity of the separable approximation for short - range potentials explains the fact of the weak dependence of the low- energy physics for light mesons on the form of the potential. Therefore, there exists a number of models satisfactory describing the experimental data.

Here, one should mention also papers dealing with the derivation of nonlinear chiral Lagrangians from QCD (cf.ref. in [22]). The essence of those proofs consists in a formal derivation of the determinant (7), (11) by means of chiral transformation which are parametrized by the meson field. Thereby, in many cases no derivation of the equation for the meson spectrum is given to say nothing of its solution. The main aim of these papers is to find the coefficients in higher order terms in the expansion of chiral

Lagrangians in meson momenta and to establish the description of baryons in the form of "skyrmions" [23]. All these papers concerning the justification of chiral Lagrangian from QCD are not devoted to the determination of essential parameters of the low - energy physics (F_π, F_K, m_π, ...) from QCD.

The relativistic model for atoms and hadrons (1) - (26) as compared with the above - mentioned popular non - relativistic [4] and nonlinear [22] approaches unifies aspects of both approaches and gives a constructive generalization of chiral Lagrangians to heavy quarkonium physics, i.e. it allows to describe decays of heavy quarkonia into light ones in the framework of one unified action of the type (11) with a minimal number of parameters, defined in the short - range region where the perturbation theory begins to work.

The construction of such a quantum relativistic hadron theory on the basis of the action (11) has been given in paper [8,9].

3. The spontaneous breakdown of chiral symmetry and the physical vacuum.

In this section, for the potential model (1) we clear up to which physical vacuum there corresponds the spontaneous breakdown of chiral symmetry accompanied by the appearence of the Goldstone mode. We work in the operator approach [2]-[7] with the Hamiltonian given by

$$\mathcal{H} = \int d\mathbf{x} \bar{q}(i\partial_i \gamma_i + m^0) q + \frac{1}{2} \int d\mathbf{x} d\mathbf{y} (q_i^+(\mathbf{x}) \frac{\lambda_{ij}^a}{2} q_j(\mathbf{x})) V(\mathbf{x} - \mathbf{y}) (q_k^+(\mathbf{y}) \frac{\lambda_{kl}^a}{2} q_l(\mathbf{y})). \qquad (34)$$

The first step for constructing the physical states consists in the definition of the one - quasi - particle creation (a^+, b^+) and annihilation (a, b) operators with the help of the Bogolubov fermion expansion [10]

$$q_\alpha(\mathbf{x}) = \sum_s \int \frac{d\mathbf{q}}{(2\pi)^{3/2}} e^{i\mathbf{q}\mathbf{x}} [a_s(\mathbf{q}) \mu_\alpha(\mathbf{q}, s) + b_s^+(-\mathbf{q}) \nu_\alpha(-\mathbf{q}, s)]. \qquad (35)$$

Here $\mu_\alpha(\mathbf{q}, s)$ and $\nu_\alpha(-\mathbf{q}, s)$ are the coefficients determined from the Schrödinger equation for the one - particles energy

$$< a_s(\mathbf{q}) | \hat{H} | a_s^+(\mathbf{q}') >= E(\mathbf{q}) < 0 | a_s(\mathbf{q}) a_s^+(\mathbf{q}') | 0 > \,. \qquad (36)$$

They can be represented via the Foldy - Wouthuysen matrix (18) as

$$\mu_\alpha(\mathbf{q}, s) = S(\mathbf{q})_{\alpha\beta} \mu_\beta(0, s); \;\; \nu_\alpha(-\mathbf{q}, s) = S(\mathbf{q})_{\alpha\beta} \nu_\beta(0, s)$$

with

$$S_{\alpha\alpha'}(\mathbf{q}) [\sum_s \mu_{\alpha'}(0, s) \mu_{\beta'}^+(0, s)] S_{\beta'\beta}^{-1}(\mathbf{q}) = (S \frac{1 + \gamma_0}{2} S^{-1})_{\alpha\beta} \equiv (\Lambda_+^0(\mathbf{q}))_{\alpha\beta},$$

$$S_{\alpha\alpha'}(\mathbf{q}) [\sum_s \nu_{\alpha'}(0, s) \nu_{\beta'}^+(0, s)] S_{\beta'\beta}^{-1}(\mathbf{q}) = (S \frac{1 - \gamma_0}{2} S^{-1})_{\alpha\beta} \equiv (\Lambda_-^0(\mathbf{q}))_{\alpha\beta}.$$

Λ^0_+ and Λ^0_- are projection operators on states with positive, resp., negative energy. Then, equation (36) takes the form of the Schwinger - Dyson equation (29,30) which can compactly be written as

$$E(p)S^{-2}(\mathbf{p}) = m^0 + p_i\gamma_i + \frac{2}{3}\hat{I}_{\mathbf{pq}}S^{-2}(\mathbf{q}), \tag{37}$$

where $\hat{I}_{\mathbf{pq}}$ is a short - hand notation for the integral operator

$$\hat{I}_{\mathbf{pq}} = \int \frac{d\mathbf{q}}{(2\pi)^3}\underline{V}(\mathbf{p}-\mathbf{q})f(\mathbf{q}). \tag{38}$$

After inserting (35) into (34) the Hamiltonian can be given in the following manner :

$$\begin{aligned}
\mathcal{H} &= E_0 + H_1 + :H_4:, \\
E_0 &= <0|\mathcal{H}|0>, \\
H_1 &= \sum_{(1)} E(\mathbf{p}_1)(a_1^+a_1 + b_1^+b_1), \\
:H_4: &= \frac{2}{3}\sum_{1,2,3,4,} \delta^{(4)}(p_1 - p_2 + p_3 - p_4)\underline{V}(\mathbf{p}_1 - \mathbf{p}_3) \\
&\{\ a_1^+b_2^+a_3^+b_4^+\mu_1^*\nu_2^*\mu_3^*\nu_4^* + a_1^+b_2^+b_3a_1\mu_1^*\nu_2\nu_3^*\mu_4 + \\
&+\ b_1a_2a_3^+b_4^+\nu_1^*\mu_2\mu_3^*\nu_4 + b_1a_2b_3a_4\nu_1^*\mu_2\nu_3^*\mu_4 + \ \ldots\} + \ \ldots
\end{aligned} \tag{39}$$

In $:H_4:$ only terms forming colourless mesons as pair correlation [24,25,26] have been kept. The following abbreviations have been used in (39):

$$\sum_I = \sum_{s_I}\int \frac{d\mathbf{p}_I}{(2\pi)^{3/2}}, \{I\} = \{p_I, s_I\}, I = 1,2,3,4.$$

For diagonalizing the Hamiltonian (39) with respect to pair correlations $(a_1^+b_2^+)$, (b_3a_4) one defines a new vacuum as the coherent state

$$|0>>_\alpha = \exp\{\sum_{1,2,3,4}\alpha(1,2,3,4)[(a_1^{+i_1}b_2^{+i_1})(b_3^{+j_1}a_4^{+j_1})]\}|0> \tag{40}$$

and the creation operator for the bound state (of pair correlation)

$$B^+(n) = \sum_{1,2}\delta(\mathbf{p}_1 - \mathbf{p}_2)[X_+(1,2)a^{+i}(1)b^{+i}(2) - X_-(1,2)b^j(1)a^j(1)]. \tag{41}$$

The coefficient X_+ and X_- are determined from the Schrödinger equation for the two - particle energy M_B ,

$$_\alpha << 0|B(n)(H_1 + H_4)B^+(n)|0>>_\alpha = M_B\ _\alpha << 0|B(n)B^+(n)|0>>_\alpha, \tag{42}$$

and the parameter α in (40)is given with the help of the definition of the annihilation operator for the pair correlation

$$B^{(-)}(n)|0>>_\alpha = 0. \tag{43}$$

Equation (42) coincides with equation (10) in the rest frame (the Salpeter equation) for the meson spectrum

$$(E_1(\mathbf{p}) + E_2(\mathbf{p}) \mp M_B)\psi_{\pm\pm}(\mathbf{p}) = \frac{4}{3}\Lambda_\pm(\mathbf{p})[\hat{I}_{\mathbf{pq}}(\psi_{++}(\mathbf{q}) + \psi_{--}(\mathbf{q}))]\Lambda_\pm(\mathbf{p}) \tag{44}$$

up to the notation

$$\begin{aligned}
&\psi = \psi_{++} + \psi_{--};\psi_{\pm\pm} = \Lambda_\pm\psi\Lambda_\mp; \\
&\psi_{++}(\mathbf{p})_{\alpha\beta} = \sum_{s_1,s_2} X_+(\mathbf{p}_1,\mathbf{p}_1,s_1,s_2)\mu^+_\alpha(\mathbf{p}_1,s_1)\nu_\beta(\mathbf{p}_1,s_2), \\
&\psi_{--}(\mathbf{p})_{\alpha\beta} = \sum_{s_1,s_2} X_-(\mathbf{p}_1,\mathbf{p}_1,s_1,s_2)\nu^+_\alpha(\mathbf{p}_1,s_1)\mu_\beta(\mathbf{p}_1,s_2).
\end{aligned} \tag{45}$$

The one - particle energies $E_1(\mathbf{p})$, $E_2(\mathbf{p})$ in (44) are defined via the Schwinger - Dyson equation (37).

Notice that equations of the type (37), (44) are well - known from the nonrelativistic many - body theory (Landau's theory of fermi liquids [24], Randon Phase Approximation [25]) and play an essential role in the description of elementary excitation in atomic nuclei [26]. Their relativistic analogues describing the Goldstone pion and the constituent masses of the light quarks are equations (21,22) and (26).

The Green function method discussed in sect.2, and the operator approach lead to one and the same equations and complement each other. The first allows one to make easily the relativistic generalization and to construct the effective meson interaction Lagrangian, whereas the second yields an adequate interpretation of quantum states and enables one to describe more complicated system, e.g. baryons and other many - quark states [27].

4. The relativistic equation for many - quark systems.

Let us construct by means of the quasiparticle operator method the relativistis equation for the baryon as a three - quark system. In the meson "coherent" vacuum (40) the baryon creation operator consists not only of creation operators for quarks (a^+) but also of annihilation operatora for antiquarks (b) with the same quantum numbers

$$\begin{aligned}
\underline{B}^+ &= \sum_{1,2,3}\delta(\mathbf{p}_1 + \mathbf{p}_2 + \mathbf{p}_3)[X_{+++}(1,2,3)a^{i(+)}(1)a^{j(+)}(2)a^{l(+)}(3) + \\
&+\ X_{--+}(1,2,3)b^{i(+)}(1)b^{i(+)}(2)b^{i(+)}(3) + interchange\ of\ (1,2,3)]\epsilon^{ijk}.
\end{aligned} \tag{46}$$

The baryon functions are as follows:

$$\begin{aligned}
\psi_{+++}(1,2,3)_{\alpha\beta\gamma} &= \sum_{s_1s_2s_3}\mu^+_\alpha(1)\mu^+_\beta(2)\mu^+_\gamma(3)X_{+++}(1,2,3), \\
\psi_{--+}(1,2,3)_{\alpha\beta\gamma} &= \sum_{s_1s_2s_3}\nu^+_\alpha(1)\nu^+_\beta(2)\nu^+_\gamma(3)X_{--+}(1,2,3),
\end{aligned}$$

etc.
Then, the eigenvalue equation for the Hamiltonian operator,

$$ {}_{\alpha}<<0|\underline{B}H\underline{B}^{+}|0>>_{\alpha}, \tag{47}$$

is equivalent to the following system for the baryons wave functions $\psi_{+++}, \psi_{--+}, \psi_{-+-}, \psi_{+--}$.

$$\begin{aligned}
&\left[\begin{pmatrix}+\\+\\+\\-\end{pmatrix}E(1)\begin{pmatrix}+\\+\\-\\+\end{pmatrix}E(2)\begin{pmatrix}+\\-\\+\\+\end{pmatrix}E(3)\begin{pmatrix}-\\+\\+\\+\end{pmatrix}M_B\right]\psi\begin{pmatrix}+++\\--+\\-+-\\+--\end{pmatrix}^{(1,2,3)} = \\
&= \frac{2}{3}\Lambda\begin{pmatrix}+\\-\\-\\+\end{pmatrix}^{(1)}\Lambda\begin{pmatrix}+\\-\\+\\-\end{pmatrix}^{(2)}\Lambda\begin{pmatrix}+\\+\\-\\-\end{pmatrix}^{(3)} \\
&\left\{\ \hat{I}_{\underline{1}\underline{2}}\left[\psi\begin{pmatrix}+++\\--+\\-+-\\+--\end{pmatrix}^{(\underline{1},\underline{2},3)}+\psi\begin{pmatrix}--+\\+++\\+--\\-+-\end{pmatrix}^{(\underline{1},\underline{2},3)}\right]+\right. \\
&+\ \hat{I}_{\underline{2}\underline{3}}\left[\psi\begin{pmatrix}+++\\--+\\-+-\\+--\end{pmatrix}^{(1,\underline{2},\underline{3})}+\psi\begin{pmatrix}+--\\-+-\\--+\\+++\end{pmatrix}^{(1,\underline{2},\underline{3})}\right]+ \\
&\left.+\ \hat{I}_{\underline{1}\underline{3}}\left[\psi\begin{pmatrix}+++\\--+\\-+-\\+--\end{pmatrix}^{(\underline{1},2,\underline{3})}+\psi\begin{pmatrix}-+-\\+--\\+++\\--+\end{pmatrix}^{(\underline{1},2,\underline{3})}\right]\right\}
\end{aligned}\tag{48}$$

where

$$\begin{aligned}
I_{\underline{1}\underline{2}}\psi(\underline{1},\underline{2},3) &= \int\frac{d\mathbf{q}}{(2\pi)^3}\underline{V}(\mathbf{q})\psi(\mathbf{p}_1-\mathbf{q},\mathbf{p}_2+\mathbf{q},\mathbf{p}_3) \\
\mathbf{p}_1 &+ \mathbf{p}_2+\mathbf{p}_3 = 0.
\end{aligned}\tag{49}$$

Equation (48) is the analogue of the Salpeter equation (44) for a bound state consisting of three particles. The nonrelativistic reduction [18] from (26) to the Schrödinger equation (27),

$$E_a(\mathbf{p}) \simeq \sqrt{m_a^2+\mathbf{p}^2}\simeq m_a+\frac{1}{2}\frac{\mathbf{p}^2}{m_a},$$

$$S_a(\mathbf{p}) \simeq 1,\ \psi_{+++}\equiv\psi>>\psi\begin{pmatrix}+--\\-+-\\--+\end{pmatrix},$$

leads in our case to the well - known nonrelativistic equation for the wave function of three particle bound states

$$\begin{aligned}
&\left[\frac{\mathbf{p}_1^2}{2m_1}+\frac{\mathbf{p}_2^2}{2m_2}+\frac{\mathbf{p}_3^2}{2m_3}-(M_{\underline{B}}-m_1-m_2-m_3)\right]\psi(\mathbf{p}_1,\mathbf{p}_2,\mathbf{p}_3) = \\
&= \frac{2}{3}[\hat{I}_{\underline{1}\underline{2}}\psi(\underline{\mathbf{p}}_1,\underline{\mathbf{p}}_2,\mathbf{p}_3)+\hat{I}_{\underline{2}\underline{3}}\psi(\mathbf{p}_1,\underline{\mathbf{p}}_2,\underline{\mathbf{p}}_3)+\hat{I}_{\underline{1}\underline{3}}\psi(\underline{\mathbf{p}}_1,\mathbf{p}_2,\underline{\mathbf{p}}_3)].
\end{aligned}\tag{50}$$

Here, the condition (49), which means the choice of the rest frame $\mathcal{P}_\mu=(M_{\underline{B}},0,0,0)$, has to be fulfilled.

Notice that the Jacobi coordinates, which allow to write the Hamiltonian in the term of two relative momenta, have sense only in the nonrelativistic limit.

For the description of baryons in an arbitrary relativistic reference frame, one needs to generalize the Markov - Yukawa condition (9) for the bilocal field to the N- local field $\Phi(x_1,x_2,...,x_N)$. By the example of a bilocal system we have seen that the definition of the total and relative coordinate $X=(x_1+x_2)/2$ and $z=x_1-x_2$ respectively is universal for quarks with arbitrary mass, including also constituent masses depending on momenta. By analogy, we introduce for the N- local field the total and relative coordinates

$$X_\mu=\frac{1}{N}\sum_{i=1}^{N}x_{i\mu}\ ,\ z_\mu^{(i)}=x_{i\mu}-X_\mu \tag{51}$$

which are connected by the identity

$$\sum_{i=1}^{N}z_\mu^{(i)}=0.$$

Then, the generalization of the Markov - Yukawa condition takes the form

$$z_\mu^{(i)}\frac{\partial}{\partial X_\mu}\Phi(z_\mu^{(1)},z_\mu^{(2)},\dots,z_\mu^{(N)})=0\ \ (i=1,2,\dots N). \tag{52}$$

Let $\mathcal{P}_\mu$ be the eigenvalue of the operator for the total 4-momentum, and η_μ be the unit vector in the direction $\mathcal{P}(\eta_\mu\sim\mathcal{P}_\mu)$. Owing to the condition (52) the N- local function $\underline{\Phi}(p_\mu^{\perp(1)},p_\mu^{\perp(2)}\ ...\ ,p_\mu^{\perp(N)}|\mathcal{P})$, being the Fourier transform of $\Phi(z_\mu^{(1)},X_\mu)$ with respect to all coordinates, depends only on the transversal relative momenta

$$p_\mu^{(i)\perp}=p_\mu^{(i)}-\eta_\mu(p^{(i)}\cdot\eta),\ \ \sum_{i=1}^{N}p_\mu^{(i)\perp}=0. \tag{53}$$

To describe the baryon in an arbitrary reference frame it is sufficient to substitute in (48) all relative momenta $\mathbf{p}_i$ by the transversal ones, $p_\mu^{\perp(i)}$, and the projection operators $\Lambda_\pm(\mathbf{p})$ by the operators

$$\Lambda_\pm(\mathbf{p}^\perp)=S(\mathbf{p}^\perp)\frac{M_{\underline{B}}\pm\mathcal{P}}{2M_{\underline{B}}}S(\mathbf{p}^\perp)^{-1}.$$

In the same way one can generalize the equation (48) and its relativization for an arbitrary N- quark state.

The method for constructing relativistic wavefunctions of many - quark system explained above unambiguously enables one to build from the nonrelativistic bound state wave function

$$\chi_{\alpha_1,\alpha_2,\dots\alpha_N} \cdot e^{iMX_0} \cdot \Phi_{\alpha_1,\alpha_2,\dots\alpha_N}(\vec{p}^1,\vec{p}^2,\dots,\vec{p}^N),\quad \sum_i \vec{p}(i) = 0$$

relativistic wave functions for the same bound states with total momentum $\mathcal{P}_\mu = (\omega = \sqrt{\vec{P}^2 + M^2}, \vec{P})$,

$$\begin{aligned}\chi_{\alpha_1,\alpha_2,\dots\alpha_N} \quad &\cdot \quad e^{i\mathcal{P}X} \cdot \Lambda_{+\alpha_1\alpha_1'}(p^{(1)\perp}\Lambda_{+\alpha_2\alpha_2'}(p^{(2)\perp}) \dots \Lambda_{+\alpha_N\alpha_N'}(p^{(N)\perp}) \cdot \\ &\cdot \quad \Phi_{\alpha_1',\alpha_2',\dots\alpha_N'}(p^{(1)\perp},p^{(2)\perp}\dots p^{(N)\perp}), \\ &(\quad \sum_i p_\mu^{(i)\perp} = 0).\end{aligned}$$

Here $\chi_{\alpha_1,\alpha_2,\dots\alpha_N}$ is the matrix selecting one or another representation of the Lorentz group with a definite spin. (A representation of the Poincare group that preserves the one - time dependence of wave functions see in ref. [29]).

5. Relativistic equations for gluonic systems.

The quantization method considered above for fermion systems can also used to calculate the parameters of gluon states described by the QCD Hamiltonian [13,30]

$$\begin{aligned}H_{YM} \quad &= \quad \int d\mathbf{x}\frac{1}{2}[(E_i^{Ta})^2 + (B_i^a)^2] + \\ &+ \quad \frac{1}{2}\int d\mathbf{x}\int d\mathbf{y} f^{b_1c_1d_1}E_i^{Tc_1}(\mathbf{x})A_i^{Td_1}(\mathbf{x})V^{b_1b_2}(A|\mathbf{x}-\mathbf{y})f^{b_2c_2d_2}E_j^{Tc_2}(\mathbf{y})A_j^{Td_2}(\mathbf{y}) + \\ &+ \quad Schwinger\ terms. \qquad (54)\end{aligned}$$

Here $V(A|\mathbf{x})$ denotes the potential satisfying the equation

$$(\nabla_i(A)\partial_i)\frac{1}{\partial^2}(\nabla_j(A)\partial_j)V(A|\mathbf{x}) = -g^2\delta(\mathbf{x}),\ \nabla_i = \partial_i + gA_i^a\frac{\lambda^a}{2},$$

and the Schwinger terms are defined from the Lorentz covariance condition. It is important to note that the field operators in the Hamiltonian (54) are Weyl - ordered [13,30] due to the condition of relativistic invariance.

Let us represent the gluon fields as Bogolubov expansion in creation and annihilation operators

$$E_i^{Tb}(\mathbf{x}) \quad = \quad i\int\frac{d\mathbf{p}}{(2\pi)^{3/2}}\sqrt{\frac{\phi(\mathbf{p})}{2}}[a_\alpha^{(+)b}(\mathbf{p})e_i^\alpha - a_\alpha^{(-)b}(\mathbf{p})e_i^\alpha]e^{i\mathbf{p}\mathbf{x}},$$

$$A_i^{Tb}(\mathbf{x}) \quad = \quad \int\frac{d\mathbf{p}}{(2\pi)^{3/2}}\frac{1}{\sqrt{2\phi(\mathbf{p})}}[a_\alpha^{(+)b}(\mathbf{p})e_i^\alpha + a_\alpha^{(-)b}(\mathbf{p})e_i^\alpha]e^{-i\mathbf{p}\mathbf{x}}, \qquad (55)$$

where the function $\phi(\mathbf{p})$ is to be calculated from the diagonalization condition of the Hamiltonian (54) with respect to the operators $a^{(+)}, a^{(-)}$.

In the formal form, the gluon Hamiltonian reads as

$$\begin{aligned}H = E_0 + \int d\mathbf{k}[\quad (\quad &a_{\alpha_1}^{(+)}(-\mathbf{k})a_{\alpha_2}^{(+)}(\mathbf{k}) + a_{\alpha_1}^{(-)}(\mathbf{k})a_{\alpha_2}^{(-)}(-\mathbf{k}))\frac{C^{\alpha_1\alpha_2}(\phi)}{2} + \\ + \quad &a_{\alpha_1}^{(+)}a_{\alpha_2}^{(-)}\omega^{\alpha_1\alpha_2}(\phi) + O(a^4)]. \qquad (56)\end{aligned}$$

Here $C(\phi)$ and $\omega(\phi)$ are some defined functions the dependence on ϕ of which is determined from the expression of the Hamiltonian (54).

The diagonalization condition for the Hamiltonian (54) means that the coefficients $C(\phi)$ vanish

$$C(\phi) = 0 \qquad (57)$$

For the solutions (57) the function $\omega(\phi)$ defines the gluon energy spectrum in the same way as the Schwinger-Dyson equation (37) defines the energy spectrum for the quarks.The Green function for the transversal gluon A_i^T corresponding to equations (55) and (56) is given by

$$\mathcal{D}_{ij}(q_0,\mathbf{q}) = \frac{\omega(\mathbf{q})}{\phi(\mathbf{q})}\frac{1}{q_0^2 - \omega^2(\mathbf{q}) - i\epsilon}(\delta_{ij} - q_i\frac{1}{\mathbf{q}^2}q_j). \qquad (58)$$

From its meaning the quantity $\frac{\omega}{\phi} = Z(\mathbf{q})$ can be called the infrared renormalization constant of the wave function.

The Green functions for the quarks, eq. (19) and gluons, (58), are elements of a new quasiparticle perturbation theory in terms of which all matrix elements are calculated including the "running" coupling constant.

The phenomenon of dimensional transmutation appearing in the "running" coupling constant should be investigated in accordance with the logic of quantum theory at the stage of defining the quark and gluon energy spectrum and their one-particle Green functions.

Let us illustrate this scheme by calculating the one-particle energy of the gluon and its bound states for the simplest example of the theory (54) where the operator for the potential is substituted by an effective potential. This means we consider the sum of the free Hamiltonian

$$H_0 = \frac{1}{2}\int d\mathbf{x}[(E_i^T)^2 + (\partial_i A_j^T)^2],$$

and the Hamiltonian of potential interaction between colour gluon currents

$$\begin{aligned}H_I \quad = \quad &-\frac{1}{8}\int d\mathbf{p}_1 d\mathbf{q}_1 d\mathbf{p}_2 d\mathbf{q}_2 \delta(\mathbf{p}_1 - \mathbf{p}_2 + \mathbf{q}_1 - \mathbf{q}_2)\frac{V(\mathbf{p}_1 - \mathbf{p}_2)}{(2\pi)^3}\sqrt{\frac{\phi(\mathbf{p}_1)\phi(\mathbf{q}_1)}{\phi(\mathbf{p}_2)\phi(\mathbf{q}_2)}} \cdot \\ &\cdot \quad f^{ab_1b_2}f^{ac_1c_2}[a^{(+)}(-1) - a^{(-)}(-1)][a^{(+)}(2) + a^{(-)}(-2)] \cdot \\ &\cdot \quad [a^{(+)}(-1') - a^{(-)}(1')][a^{(+)}(2') + a^{(-)}(-2')].\end{aligned}$$

Here, the short - hand notation $(\pm \mathbf{p}_1, b_1, i_1) = (\pm 1), (\pm \mathbf{q}_1, c_1, j_1) = (\pm 2)$, $(\pm \mathbf{p}_2, b_2, i_2) = (\pm 1'), (\pm \mathbf{q}_2, c_2, j_2) = (\pm 2')$ has been used. In our case, the coefficients C and ω are given by the expressions

$$
\begin{aligned}
C^{\alpha_1\alpha_2}(\phi) &= [\frac{\mathbf{k}^2\delta_{ij} + (\mathcal{M}^2(\mathbf{k}))^{ij}}{2\phi} - \frac{\phi}{2} z^{ij}] e_i^{\alpha_1}(\mathbf{k}) e_j^{\alpha_2}(-\mathbf{k}) = 0, \\
\omega^{\alpha_1\alpha_2}(\phi) &= [\frac{\mathbf{k}^2\delta_{ij} + (\mathcal{M}^2(\mathbf{k}))^{ij}}{2\phi} + \frac{\phi}{2} z^{ij}] e_i^{\alpha_1}(\mathbf{k}) e_j^{\alpha_2}(\mathbf{k}) \qquad (59)
\end{aligned}
$$

with

$$
\begin{aligned}
\mathcal{M}^2(\mathbf{k}))^{ij} &= \frac{N_c}{2} \int \frac{d\mathbf{q}}{(2\pi)^3} \underline{V}(\mathbf{k}-\mathbf{q}) \phi(\mathbf{q}) (\delta_{ij} - q_i \frac{1}{\mathbf{q}^2} q_j), \\
z^{ij}(\mathbf{p}) &= \delta_{ij} + \frac{N_c}{2} \int \frac{d\mathbf{q}}{(2\pi)^3} \underline{V}(\mathbf{k}-\mathbf{q}) \frac{1}{\phi(\mathbf{q})} (\delta_{ij} - q_i \frac{1}{\mathbf{q}^2} q_j). \qquad (60)
\end{aligned}
$$

Since two gluons perform the simplest bound state, by analogy with the mesons (cf. (41)), we introduce a glueball creation operator

$$
\begin{aligned}
G^+ = \sum_b \int d\mathbf{k} \quad [& X_{\gamma_1\gamma_2}^{(++)}(\mathbf{k}) e_i^{\gamma_1}(\mathbf{k}) e_j^{\gamma_2}(-\mathbf{k}) a_{\gamma_1}^{(+)b}(\mathbf{k}) a_{\gamma_2}^{(+)b}(-\mathbf{k}) + \\
+ & X_{\gamma_1\gamma_2}^{(--)}(\mathbf{k}) e_i^{\gamma_1}(\mathbf{k}) e_j^{\gamma_2}(-\mathbf{k}) a_{\gamma_1}^{(-)b}(\mathbf{k}) a_{\gamma_2}^{(-)b}(-\mathbf{k})]
\end{aligned}
$$

and a "coherent" vacuum

$$
\begin{aligned}
|0>>_\alpha = \exp\{\sum_c \int & d\mathbf{k}_1 d\mathbf{k}_2 \alpha(\mathbf{k}_1, \mathbf{k}_2) (a_{\gamma_1}^{(+)c}(\mathbf{k}_1) a_{\gamma_2}^{(+)c}(\mathbf{k}_2)) \cdot \\
\cdot & (a_{\gamma_1}^{(+)b}(\mathbf{k}_1) a_{\gamma_2}^{(+)b}(\mathbf{k}_2))\}|0> .
\end{aligned}
$$

Then, the Schrödinger equation for eigenvalues of the Hamiltonian operator

$$
{}_\alpha << 0|GHG^+|0>>_\alpha = M_G \; {}_\alpha << 0|GG^+|0>>_\alpha, \qquad (61)
$$

is equivalent to the equation for the glueball wave function

$$
\begin{aligned}
(2\omega(\mathbf{k}) &- M_G) X_{ij}^{(++)}(\mathbf{k}) \\
&= \frac{N_c}{4} \hat{I}_{\mathbf{kq}} \{(W^+(\mathbf{q}|\mathbf{k}))^2 X_{ij}^{(++)}(\mathbf{q}) - (W^-(\mathbf{q}|\mathbf{k}))^2 X_{ij}^{(++)}(\mathbf{q})\} \qquad (62) \\
(2\omega(\mathbf{k}) &+ M_G) X_{ij}^{(--)}(\mathbf{k}) \\
&= \frac{N_c}{4} \hat{I}_{\mathbf{kq}} \{(W^+(\mathbf{q}|\mathbf{k}))^2 X_{ij}^{(--)}(\mathbf{q}) - (W^-(\mathbf{q}|\mathbf{k}))^2 X_{ij}^{(--)}(\mathbf{q})\} \qquad (63)
\end{aligned}
$$

with

$$
\begin{aligned}
W^\pm(\mathbf{q}|\mathbf{k}) &= [\sqrt{\frac{\phi(\mathbf{q})}{\phi(\mathbf{k})}} \pm \sqrt{\frac{\phi(\mathbf{k})}{\phi(\mathbf{q})}}], \\
\hat{I}_{\mathbf{kq}} f(\mathbf{q}) &= \int \frac{d\mathbf{q}}{(2\pi)^3} \underline{V}(\mathbf{k}-\mathbf{q}) f(\mathbf{q}). \qquad (64)
\end{aligned}
$$

Furthermore, one can write by analogy with the baryon (48) also the equation for the wave function of a antisymmetric three - gluon state $\psi_{+++}(\psi_{+--}, \psi_{-++}, \psi_{-+-})$ with eigenvalues M_{BG}

$$
\begin{aligned}
[\omega(1) &+ \omega(2) + \omega(3) - M_{BG}]\psi_{+++}(1,2,3) = \\
&= \frac{N_c}{2}\{\hat{I}_{1,\underline{2}}(\omega_{\underline{1}1}^+ \omega_{\underline{2}2}^+ \psi_{+++}(\underline{1},\underline{2},3) + \omega_{\underline{1}1}^- \omega_{\underline{2}2}^- \psi_{--+}(\underline{1},\underline{2},3)) + \; ... \}. \qquad (65)
\end{aligned}
$$

For an estimate of the solution to equation (62,63) we make use of the separable approximation for the potential

$$
\int \frac{d\mathbf{q}}{(2\pi)^3} \underline{V}(\mathbf{k}-\mathbf{q}) \phi(\mathbf{q}) (\delta_{ij} - q_i \frac{1}{\mathbf{q}^2} q_j) \simeq \frac{2}{3} \frac{1}{\mu_{QCD}^2} \int_0^L \frac{d\mathbf{q}}{(2\pi)^3} \phi(\mathbf{q}) \delta_{ij}, \qquad (66)
$$

with the parameters describing the physics of light quarkonia ($L = 1.6GeV, \mu_{QCD} = 0.35GeV$). Equation (60) take the form

$$
\begin{aligned}
\sqrt{Z} m_g^2 &= \frac{1}{\mu_{QCD}^2} \int_0^L \frac{d\mathbf{p}}{(2\pi)^3} \sqrt{\mathbf{p}^2 + m_g^2}, \\
Z &= 1 + \frac{\sqrt{Z}}{\mu_{QCD}^2} \int_0^L \frac{d\mathbf{p}}{(2\pi)^3} \frac{1}{\sqrt{\mathbf{p}^2 + m_g^2}}, \qquad (67) \\
\omega(\mathbf{p}) &= \sqrt{Z} \sqrt{\mathbf{p}^2 - m_g^2}
\end{aligned}
$$

and have the solutions $m_g \simeq 0.8GeV, \sqrt{Z} \simeq 1.18$. For the scalar glueball mass as an eigenvalue of equation (62,63) in the separable approximation

$$
\begin{aligned}
[2\sqrt{Z}\sqrt{\mathbf{k}^2 + m_g^2} &- M_G]X = \\
&= \frac{N_c}{4\mu_{QCD}^2} \int^L \frac{d\mathbf{q}}{(2\pi)^3} [(X - Y)W(\mathbf{k}|\mathbf{q}) + 2(X + Y)], \\
[2\sqrt{Z}\sqrt{\mathbf{k}^2 + m_g^2} &+ M_G]X = \\
&= \frac{N_c}{4\mu_{QCD}^2} \int^L \frac{d\mathbf{q}}{(2\pi)^3} [(Y - X)W(\mathbf{k}|\mathbf{q}) + 2(X + Y)], \qquad (68) \\
W(\mathbf{k}|\mathbf{q}) &= \sqrt{\frac{\mathbf{k}^2 + m_g^2}{\mathbf{q}^2 + m_g^2}} + \sqrt{\frac{\mathbf{q}^2 + m_g^2}{\mathbf{k}^2 + m_g^2}},
\end{aligned}
$$

one obtains the value $M_G \simeq 1.6GeV$.

The appearance of consituent masses for quarks and gluons does of course influence the determination of the "running" coupling constant which in the new theory cannot have any singularities in the whole Euclidean region of the transferred momenta, among them also at $\mathbf{q}^2 = 0$ [9,31].

6.Conclusions.

By analogy with QED in QCD one can separate the gluon interaction into the instantaneous and the retardation parts.The relative contributions of these components depend on the physical task under consideration. For the problems of scattering and dissociation they are of the same importance,whereas the definition of the bound states suggets the dominance of the instantaneous part. (The dominance is provided by the singularity of the instantaneous term at the equal-time limit.)Just this suggestion was the starting point of our work.

We gave the relativistic generalization of the instantaneous bound states in accordance with the condition of irreducibility of Poincaré group representation for the nonlocal objects proposed by Markov and Yukawa for two-body systems and extended it this paper to many-body systems.

This generalization includes the separation of the total coordinates for any current particle masses, the definition of the coherent vacuum for pair correlations and the construction of relativistic wave functions in an arbitrary frame.

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Калиновский Ю.Л. и др. E2-90-354
Релятивистские связанные состояния в КХД

Статья посвящена построение нелокальной теории адронов как связанных состояний кварков и глюонов. Исходным пунктом является релятивистски ковариантное определение одновременных связанных состояний и вытекающее из него доминантность одновременного глюонного взаимодействия. Мы обобщили описание нелокального релятивистского пиона на случай многокварковых и многоглюонных конфигураций, что включает правила построения релятивистских волновых функций в произвольной системе отсчета.

Работа выполнена в Лаборатории теоретической физики ОИЯИ.

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Kalinovsky Yu.L. et al. E2-90-354
Relativistic Bound States in QCD

This paper is devoted to the construction of the nonlocal theory of hadrons as bound states of quarks and gluons. The starting point of our investigation is relativistic covariant definition of instantaneous bound states and the supposition of the dominance of an instantaneous gluon interaction. We have generalized the description of the nonlocal relativistic pion to the case of many-quark and many-gluon configurations using an operator approach. This includes the rules for the construction of relativistic wave functions in an arbitrary frame.

The investigation has been performed at the Laboratory of Theoretical Physics, JINR.

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